In this chapter you work with production functions, relating output of a firm to the inputs it uses. This theory will look familiar to you, because it closely parallels the theory of utility functions. In utility theory, an *indifference curve* is a locus of commodity bundles, all of which give a consumer the same utility. In production theory, an *isoquant* is a locus of input combinations, all of which give the same output. In consumer theory, you found that the slope of an indifference curve at the bundle (x_1, x_2) is the ratio of marginal utilities, $MU_1(x_1, x_2)/MU_2(x_1, x_2)$. In production theory, the slope of an isoquant at the input combination (x_1, x_2) is the ratio of the marginal products, $MP_1(x_1, x_2)/MP_2(x_1, x_2)$. Most of the functions that we gave as examples of utility functions can also be used as examples of production functions.

There is one important difference between production functions and utility functions. Remember that utility functions were only "unique up to monotonic transformations." In contrast, two different production functions that are monotonic transformations of each other describe different technologies.

If the utility function $U(x_1, x_2) = x_1 + x_2$ represents a person's preferences, then so would the utility function $U^*(x_1, x_2) = (x_1 + x_2)^2$. A person who had the utility function $U^*(x_1, x_2)$ would have the same indifference curves as a person with the utility function $U(x_1, x_2)$ and would make the same choices from every budget. But suppose that one firm has the production function $f(x_1, x_2) = x_1 + x_2$, and another has the production function $f^*(x_1, x_2) = (x_1 + x_2)^2$. It is true that the two firms will have the same isoquants, but they certainly do not have the same technology. If both firms have the input combination $(x_1, x_2) = (1, 1)$, then the first firm will have an output of 2 and the second firm will have an output of 4.

Now we investigate "returns to scale." Here we are concerned with the change in output if the amount of every input is multiplied by a number $t > 1$. If multiplying inputs by t multiplies output by more than t, then there are increasing returns to scale. If output is multiplied by exactly t, there are constant returns to scale. If output is multiplied by less than t, then there are decreasing returns to scale.

Consider the production function $f(x_1, x_2) = x_1^{1/2} x_2^{3/4}$. If we multiply the amount of each input by t, then output will be $f(tx_1, tx_2) = (tx_1)^{1/2}(tx_2)^{3/4}$. To compare $f(tx_1, tx_2)$ to $f(x_1, x_2)$, factor out the expressions involving t from the last equation. You get $f(tx_1, tx_2) = t^{5/4} x_1^{1/2} x_2^{3/4} = t^{5/4} f(x_1, x_2)$. Therefore when you multiply the amounts of all inputs by t, you multiply the amount of output by $t^{5/4}$. This means there are *increasing* returns to scale.

Let the production function be $f(x_1, x_2) = \min\{x_1, x_2\}$. Then

$$f(tx_1, tx_2) = \min\{tx_1, tx_2\} = \min t\{x_1, x_2\} = t\min\{x_1, x_2\} = tf(x_1, x_2).$$

Therefore when all inputs are multiplied by t, output is also multiplied by t. It follows that this production function has *constant* returns to scale.

You will also be asked to determine whether the marginal product of each single factor of production increases or decreases as you increase the amount of that factor without changing the amount of other factors. Those of you who know calculus will recognize that the marginal product of a factor is the first derivative of output with respect to the amount of that factor. Therefore the marginal product of a factor will decrease, increase, or stay constant as the amount of the factor increases depending on whether the *second* derivative of the production function with respect to the amount of that factor is negative, positive, or zero.

Consider the production function $f(x_1, x_2) = x_1^{1/2} x_2^{3/4}$. The marginal product of factor 1 is $\frac{1}{2} x_1^{-1/2} x_2^{3/4}$. This is a decreasing function of x_1, as you can verify by taking the derivative of the marginal product with respect to x_1. Similarly, you can show that the marginal product of x_2 decreases as x_2 increases.

18.0 Warm Up Exercise. The first part of this exercise is to calculate marginal products and technical rates of substitution for several frequently encountered production functions. As an example, consider the production function $f(x_1, x_2) = 2x_1 + \sqrt{x_2}$. The marginal product of x_1 is the derivative of $f(x_1, x_2)$ with respect to x_1, holding x_2 fixed. This is just 2. The marginal product of x_2 is the derivative of $f(x_1, x_2)$ with respect to x_2, holding x_1 fixed, which in this case is $\frac{1}{2\sqrt{x_2}}$. The TRS is $-MP_1/MP_2 = -4\sqrt{x_2}$. Those of you who do not know calculus should fill in this table from the answers in the back. The table will be a useful reference for later problems.

Marginal Products and Technical Rates of Substitution

$f(x_1, x_2)$	$MP_1(x_1, x_2)$	$MP_2(x_1, x_2)$	$TRS(x_1, x_2)$
$x_1 + 2x_2$			
$ax_1 + bx_2$			
$50x_1x_2$			
$x_1^{1/4}x_2^{3/4}$	$\frac{1}{4}x_1^{-3/4}x_2^{3/4}$		
$Cx_1^a x_2^b$	$Cax_1^{a-1}x_2^b$		
$(x_1 + 2)(x_2 + 1)$	$x_2 + 1$		
$(x_1 + a)(x_2 + b)$			
$ax_1 + b\sqrt{x_2}$			
$x_1^a + x_2^a$			
$(x_1^a + x_2^a)^b$	$bax_1^{a-1}\left(x_1^a + x_2^a\right)^{b-1}$	$bax_2^{a-1}\left(x_1^a + x_2^a\right)^{b-1}$	

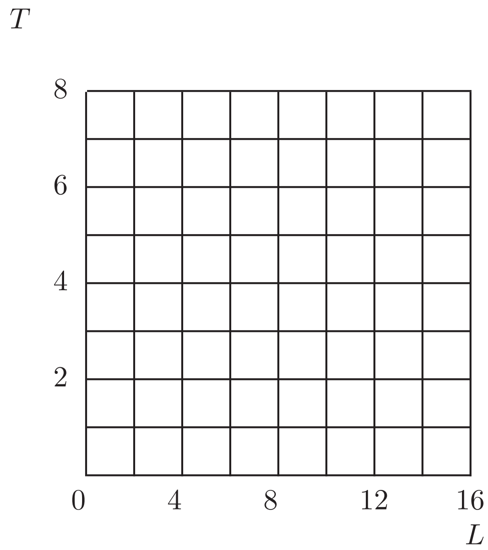
Returns to Scale and Changes in Marginal Products

For each production function in the table below, put an I, C, or D in the first column if the production function has increasing, constant, or decreasing returns to scale. Put an I, C, or D in the second (third) column, depending on whether the marginal product of factor 1 (factor 2) is increasing, constant, or decreasing, as the amount of that factor alone is varied.

$f(x_1, x_2)$	Scale	MP_1	MP_2
$x_1 + 2x_2$			
$\sqrt{x_1 + 2x_2}$			
$.2x_1x_2^2$			
$x_1^{1/4}x_2^{3/4}$			
$x_1 + \sqrt{x_2}$			
$(x_1 + 1)^{.5}(x_2)^{.5}$			
$\left(x_1^{1/3} + x_2^{1/3}\right)^3$			

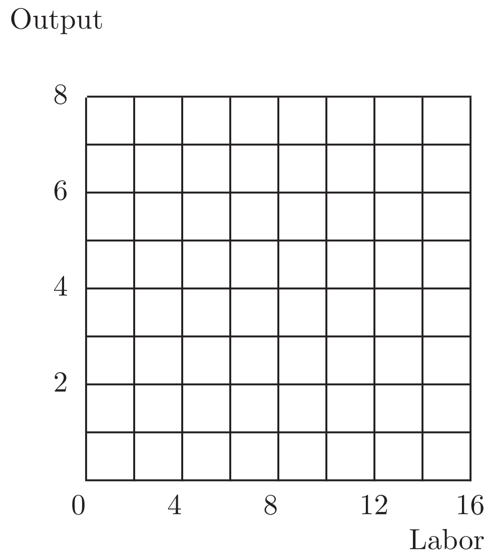
18.1 (0) Prunella raises peaches. Where L is the number of units of labor she uses and T is the number of units of land she uses, her output is $f(L, T) = L^{\frac{1}{2}}T^{\frac{1}{2}}$ bushels of peaches.

(a) On the graph below, plot some input combinations that give her an output of 4 bushels. Sketch a production isoquant that runs through these points. The points on the isoquant that gives her an output of 4 bushels all satisfy the equation $T =$____________________________________.



(b) This production function exhibits (constant, increasing, decreasing) returns to scale.______________________________.

(c) In the short run, Prunella cannot vary the amount of land she uses. On the graph below, use blue ink to draw a curve showing Prunella's output as a function of labor input if she has 1 unit of land. Locate the points on your graph at which the amount of labor is 0, 1, 4, 9, and 16 and label them. The slope of this curve is known as the marginal __________ of __________ Is this curve getting steeper or flatter as the amount of labor increase?______________________________.



(d) Assuming she has 1 unit of land, how much extra output does she get from adding an extra unit of labor when she previously used 1 unit of labor? ________________ 4 units of labor? ________________ If you know calculus, compute the marginal product of labor at the input combination $(1, 1)$ and compare it with the result from the unit increase in labor output found above.____

____.

(e) In the long run, Prunella can change her input of land as well as of labor. Suppose that she increases the size of her orchard to 4 units of land. Use red ink to draw a new curve on the graph above showing output as a function of labor input. Also use red ink to draw a curve showing marginal product of labor as a function of labor input when the amount of land is fixed at 4.

18.2 (0) Suppose x_1 and x_2 are used in fixed proportions and $f(x_1, x_2) = \min\{x_1, x_2\}$.

(a) Suppose that $x_1 < x_2$. The marginal product for x_1 is ________ and (increases, remains constant, decreases)____________________ for small increases in x_1. For x_2 the marginal product is ________ , and (increases, remains constant, decreases)____________________ for small increases in x_2. The technical rate of substitution between x_2 and x_1 is __________ ______ This technology demonstrates (increasing, constant, decreasing) _____________ returns to scale.

(b) Suppose that $f(x_1, x_2) = \min\{x_1, x_2\}$ and $x_1 = x_2 = 20$. What is the marginal product of a small increase in x_1? ________ What is the marginal product of a small increase in x_2? ________ The marginal product of x_1 will (increase, decrease, stay constant)______________ if the amount of x_2 is increased by a little bit.

18.3 (0) Suppose the production function is Cobb-Douglas and $f(x_1, x_2) = x_1^{1/2} x_2^{3/2}$.

(a) Write an expression for the marginal product of x_1 at the point (x_1, x_2).____.

____

(b) The marginal product of x_1 (increases, decreases, remains constant) ______________ for small increases in x_1, holding x_2 fixed.

(c) The marginal product of factor 2 is ______________ , and it (increases, remains constant, decreases)______________ for small increases in x_2.

(d) An increase in the amount of x_2 (increases, leaves unchanged, decreases) ______________ the marginal product of x_1.

(e) The technical rate of substitution between x_2 and x_1 is______________.

(f) Does this technology have diminishing technical rate of substitution?

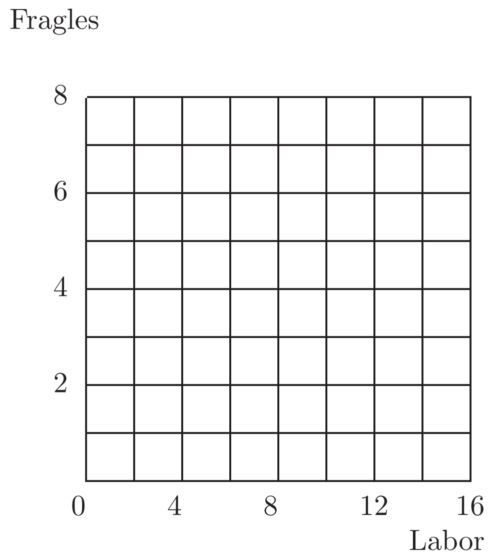
____.

(g) This technology demonstrates (increasing, constant, decreasing) ______________ returns to scale.

18.4 (0) The production function for fragles is $f(K, L) = L/2 + \sqrt{K}$, where L is the amount of labor used and K the amount of capital used.

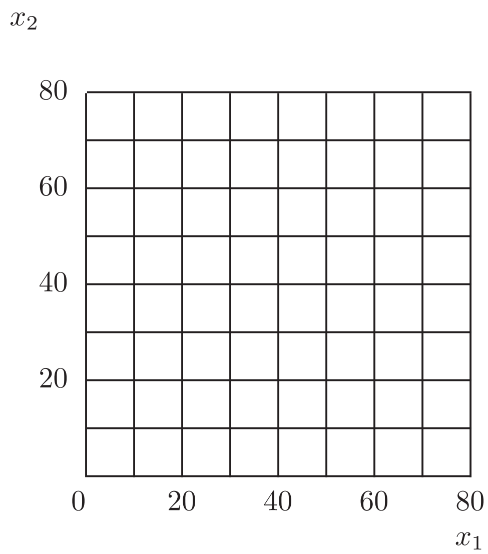
(a) There are (constant, increasing, decreasing)______________ returns to scale. The marginal product of labor is ______________ (constant, increasing, decreasing).

(b) In the short run, capital is fixed at 4 units. Labor is variable. On the graph below, use blue ink to draw output as a function of labor input in the short run. Use red ink to draw the marginal product of labor as a function of labor input in the short run. The average product of labor is defined as total output divided by the amount of labor input. Use black ink to draw the average product of labor as a function of labor input in the short run.



18.5 (0) General Monsters Corporation has two plants for producing juggernauts, one in Flint and one in Inkster. The Flint plant produces according to $f_F(x_1, x_2) = \min\{x_1, 2x_2\}$ and the Inkster plant produces according to $f_I(x_1, x_2) = \min\{2x_1, x_2\}$, where x_1 and x_2 are the inputs.

(a) On the graph below, use blue ink to draw the isoquant for 40 juggernauts at the Flint plant. Use red ink to draw the isoquant for producing 40 juggernauts at the Inkster plant.



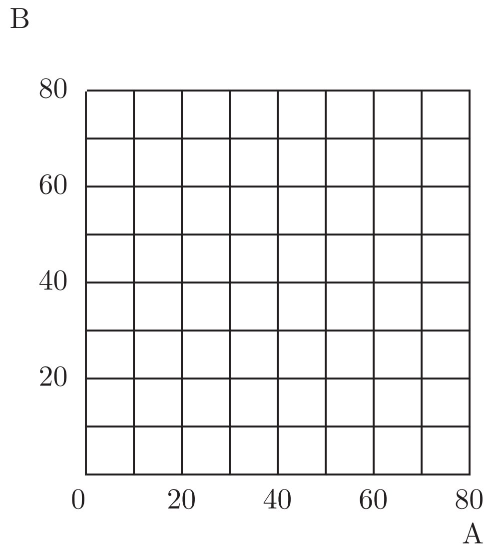
____

(b) Suppose that the firm wishes to produce 20 juggernauts at each plant. How much of each input will the firm need to produce 20 juggernauts at the Flint plant? ________________ How much of each input will the firm need to produce 20 juggernauts at the Inkster plant? ________________ Label with an a on the graph, the point representing the total amount of each of the two inputs that the firm needs to produce a total of 40 juggernauts, 20 at the Flint plant and 20 at the Inkster plant.

(c) Label with a b on your graph the point that shows how much of each of the two inputs is needed in toto if the firm is to produce 10 juggernauts in the Flint plant and 30 juggernauts in the Inkster plant. Label with a c the point that shows how much of each of the two inputs that the firm needs in toto if it is to produce 30 juggernauts in the Flint plant and 10 juggernauts in the Inkster plant. Use a black pen to draw the firm's isoquant for producing 40 units of output if it can split production in any manner between the two plants. Is the technology available to this firm convex?____.

18.6 (0) You manage a crew of 160 workers who could be assigned to make either of two products. Product A requires 2 workers per unit of output. Product B requires 4 workers per unit of output.

(a) Write an equation to express the combinations of products A and B that could be produced using exactly 160 workers. ________________ On the diagram below, use blue ink to shade in the area depicting the combinations of A and B that could be produced with 160 workers. (Assume that it is also possible for some workers to do nothing at all.)



(b) Suppose now that every unit of product A that is produced requires the use of 4 shovels as well as 2 workers and that every unit of product B produced requires 2 shovels and 4 workers. On the graph you have just drawn, use red ink to shade in the area depicting combinations of A and B that could be produced with 180 shovels if there were no worries about the labor supply. Write down an equation for the set of combinations of A and B that require exactly 180 shovels.____________________________.

(c) On the same diagram, use black ink to shade the area that represents possible output combinations when one takes into account both the limited supply of labor and the limited supply of shovels.

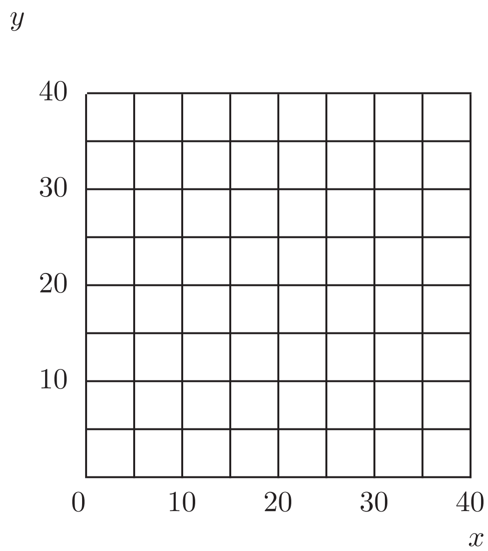
(d) On your diagram locate the feasible combination of inputs that use up all of the labor and all of the shovels. If you didn't have the graph, what equations would you solve to determine this point?______________

____.

(e) If you have 160 workers and 180 shovels, what is the largest amount of product A that you could produce? ______________ If you produce this amount, you will not use your entire supply of one of the inputs. Which one? ______________ How many will be left unused?____________________.

18.7 (0) A firm has the production function $f(x, y) = \min\{2x, x + y\}$. On the graph below, use red ink to sketch a couple of production isoquants for this firm. A second firm has the production function $f(x, y) = x + \min\{x, y\}$. Do either or both of these firms have constant returns to scale?

______________ On the same graph, use black ink to draw a couple of isoquants for the second firm.



18.8 (0) Suppose the production function has the form

$$f(x_1, x_2, x_3) = Ax_1^a x_2^b x_3^c,$$

where $a + b + c > 1$. Prove that there are increasing returns to scale.

____.

18.9 (0) Suppose that the production function is $f(x_1, x_2) = Cx_1^a x_2^b$, where a, b, and C are positive constants.

(a) For what positive values of a, b, and C are there decreasing returns to scale?________________ constant returns to scale? ________________ increasing returns to scale?________________.

(b) For what positive values of a, b, and C is there decreasing marginal product for factor 1?________________________________.

(c) For what positive values of a, b, and C is there diminishing technical rate of substitution?________________________________.

18.10 (0) Suppose that the production function is $f(x_1, x_2) = (x_1^a + x_2^a)^b$, where a and b are positive constants.

(a) For what positive values of a and b are there decreasing returns to scale?__________ Constant returns to scale? __________ Increasing returns to scale?________________________________.

18.11 (0) Suppose that a firm has the production function $f(x_1, x_2) = \sqrt{x_1} + x_2^2$.

(a) The marginal product of factor 1 (increases, decreases, stays constant) ______________ as the amount of factor 1 increases. The marginal product of factor 2 (increases, decreases, stays constant) ______________ as the amount of factor 2 increases.

(b) This production function does not satisfy the definition of increasing returns to scale, constant returns to scale, or decreasing returns to scale.

How can this be? ____

__________________________ Find a combination of inputs such that doubling the amount of both inputs will more than double the amount of output. __________________________ Find a combination of inputs such that doubling the amount of both inputs will less than double output.

____.

A firm in a competitive industry cannot charge more than the market price for its output. If it also must compete for its inputs, then it has to pay the market price for inputs as well. Suppose that a profit-maximizing competitive firm can vary the amount of only one factor and that the marginal product of this factor decreases as its quantity increases. Then the firm will maximize its profits by hiring enough of the variable factor so that the value of its marginal product is equal to the wage. Even if a firm uses several factors, only some of them may be variable in the short run.

A firm has the production function $f(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. Suppose that this firm is using 16 units of factor 2 and is unable to vary this quantity in the short run. In the short run, the only thing that is left for the firm to choose is the amount of factor 1. Let the price of the firm's output be p, and let the price it pays per unit of factor 1 be w_1. We want to find the amount of x_1 that the firm will use and the amount of output it will produce. Since the amount of factor 2 used in the short run must be 16, we have output equal to $f(x_1, 16) = 4x_1^{1/2}$. The marginal product of x_1 is calculated by taking the derivative of output with respect to x_1. This marginal product is equal to $2x_1^{-1/2}$. Setting the value of the marginal product of factor 1 equal to its wage, we have $p2x_1^{-1/2} = w_1$. Now we can solve this for x_1. We find $x_1 = (2p/w_1)^2$. Plugging this into the production function, we see that the firm will choose to produce $4x_1^{1/2} = 8p/w_1$ units of output.

In the long run, a firm is able to vary all of its inputs. Consider the case of a competitive firm that uses two inputs. Then if the firm is maximizing its profits, it must be that the value of the marginal product of each of the two factors is equal to its wage. This gives two equations in the two unknown factor quantities. If there are decreasing returns to scale, these two equations are enough to determine the two factor quantities. If there are constant returns to scale, it turns out that these two equations are only sufficient to determine the *ratio* in which the factors are used.

In the problems on the weak axiom of profit maximization, you are asked to determine whether the observed behavior of firms is consistent with profit-maximizing behavior. To do this you will need to plot some of the firm's isoprofit lines. An isoprofit line relates all of the input-output combinations that yield the same amount of profit for some given input and output prices. To get the equation for an isoprofit line, just write down an equation for the firm's profits at the given input and output prices. Then solve it for the amount of output produced as a function of the amount of the input chosen. Graphically, you know that a firm's behavior is consistent with profit maximization if its input-output choice in each period lies below the isoprofit lines of the other periods.

19.1 (0) The short-run production function of a competitive firm is given by $f(L) = 6L^{2/3}$, where L is the amount of labor it uses. (For those who do not know calculus—if total output is aL^b, where a and b are constants, and where L is the amount of some factor of production, then the marginal product of L is given by the formula abL^{b-1}.) The cost per unit of labor is $w = 6$ and the price per unit of output is $p = 3$.

(a) Plot a few points on the graph of this firm's production function and sketch the graph of the production function, using blue ink. Use black ink to draw the isoprofit line that passes through the point $(0, 12)$, the isoprofit line that passes through $(0, 8)$, and the isoprofit line that passes through the point $(0, 4)$. What is the slope of each of the isoprofit lines?

______________________ How many points on the isoprofit line through

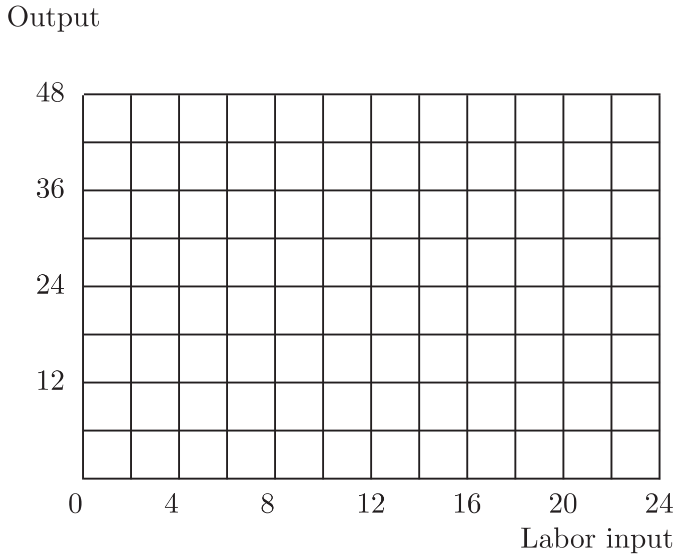
$(0, 12)$ consist of input-output points that are actually possible?______

______ Make a squiggly line over the part of the isoprofit line through $(0, 4)$ that consists of outputs that are actually possible.

(b) How many units of labor will the firm hire? ________ How much

output will it produce? ______ If the firm has no other costs, how much

will its total profits be?________________________________.



(c) Suppose that the wage of labor falls to 4, and the price of output remains at p. On the graph, use red ink to draw the new isoprofit line for the firm that passes through its old choice of input and output. Will

the firm increase its output at the new price?__________ Explain why,

____

referring to your diagram.____

____.

19.2 (0) A Los Angeles firm uses a single input to produce a recreational commodity according to a production function $f(x) = 4\sqrt{x}$, where x is the number of units of input. The commodity sells for \$100 per unit. The input costs \$50 per unit.

(a) Write down a function that states the firm's profit as a function of the amount of input.______________________________________.

(b) What is the profit-maximizing amount of input? ______ of output? ______ How much profits does it make when it maximizes profits? ____.

(c) Suppose that the firm is taxed \$20 per unit of its output and the price of its input is subsidized by \$10. What is its new input level? ______ What is its new output level? ______ How much profit does it make now?__________ (Hint: A good way to solve this is to write an expression for the firm's profit as a function of its input and solve for the profit-maximizing amount of input.)

(d) Suppose that instead of these taxes and subsidies, the firm is taxed at 50% of its profits. Write down its after-tax profits as a function of the amount of input.____________________ What is the profit-maximizing amount of output? ______ How much profit does it make after taxes?____.

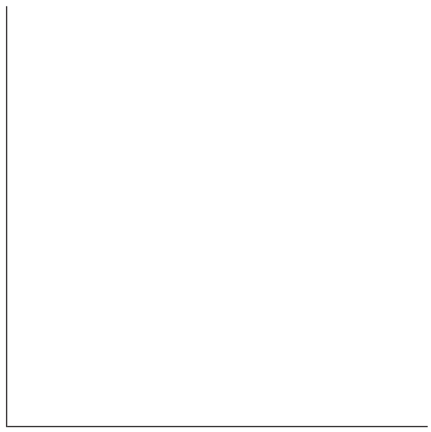
19.3 (0) Brother Jed takes heathens and reforms them into righteous individuals. There are two inputs needed in this process: heathens (who are widely available) and preaching. The production function has the following form: $r_p = \min\{h, p\}$, where r_p is the number of righteous persons produced, h is the number of heathens who attend Jed's sermons, and p is the number of hours of preaching. For every person converted, Jed receives a payment of s from the grateful convert. Sad to say, heathens do not flock to Jed's sermons of their own accord. Jed must offer heathens a payment of w to attract them to his sermons. Suppose the amount of preaching is fixed at $\bar{p}$ and that Jed is a profit-maximizing prophet.

(a) If $h < \bar{p}$, what is the marginal product of heathens? ________ What is the value of the marginal product of an additional heathen?________.

(b) If $h > \bar{p}$, what is the marginal product of heathens? ________ What is the value of the marginal product of an additional heathen in this case?

____.

(c) Sketch the shape of this production function in the graph below. Label the axes, and indicate the amount of the input where $h = \bar{p}$.



(d) If $w < s$, how many heathens will be converted? ________ If $w > s$, how many heathens will be converted?________________________.

19.4 (0) Allie's Apples, Inc. purchases apples in bulk and sells two products, boxes of apples and jugs of cider. Allie's has capacity limitations of three kinds: warehouse space, crating facilities, and pressing facilities. A box of apples requires 6 units of warehouse space, 2 units of crating facilities, and no pressing facilities. A jug of cider requires 3 units of warehouse space, 2 units of crating facilities, and 1 unit of pressing facilities. The total amounts available each day are: 1,200 units of warehouse space, 600 units of crating facilities, and 250 units of pressing facilities.

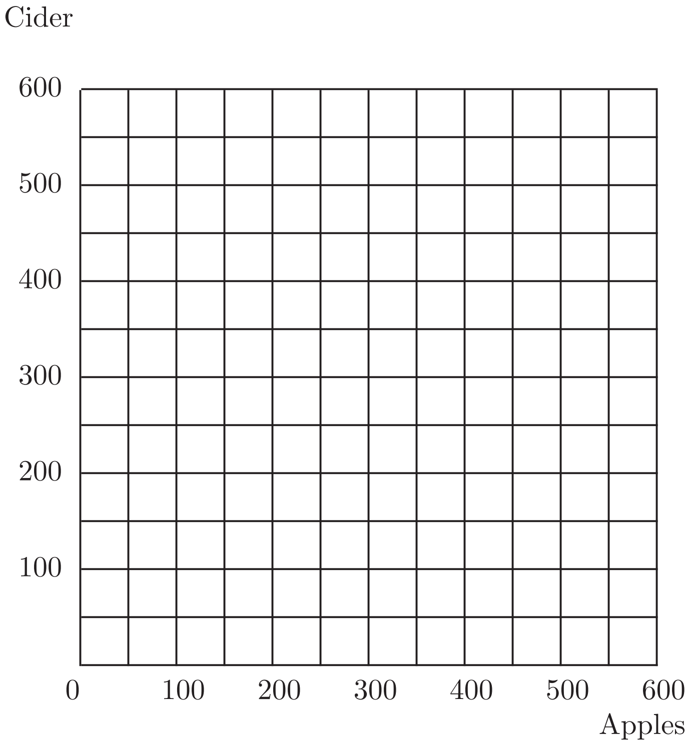
(a) If the only capacity limitations were on warehouse facilities, and if all warehouse space were used for the production of apples, how many boxes of apples could be produced in one day? ________ How many jugs of cider could be produced each day if, instead, all warehouse space were used in the production of cider and there were no other capacity constraints? ________ Draw a blue line in the following graph to represent the warehouse space constraint on production combinations.

(b) Following the same reasoning, draw a red line to represent the constraints on output to limitations on crating capacity. How many boxes of apples could Allie produce if he only had to worry about crating capacity?

__________ How many jugs of cider?________________________________.

(c) Finally draw a black line to represent constraints on output combinations due to limitations on pressing facilities. How many boxes of apples could Allie produce if he only had to worry about the pressing capacity and no other constraints? ____________________ How many jugs of cider?________________________________.

(d) Now shade the area that represents feasible combinations of daily production of apples and cider for Allie's Apples.



(e) Allie's can sell apples for \$5 per box of apples and cider for \$2 per jug. Draw a black line to show the combinations of sales of apples and cider that would generate a revenue of \$1,000 per day. At the profit-maximizing production plan, Allie's is producing ______ boxes of apples and ________ jugs of cider. Total revenues are________________________.

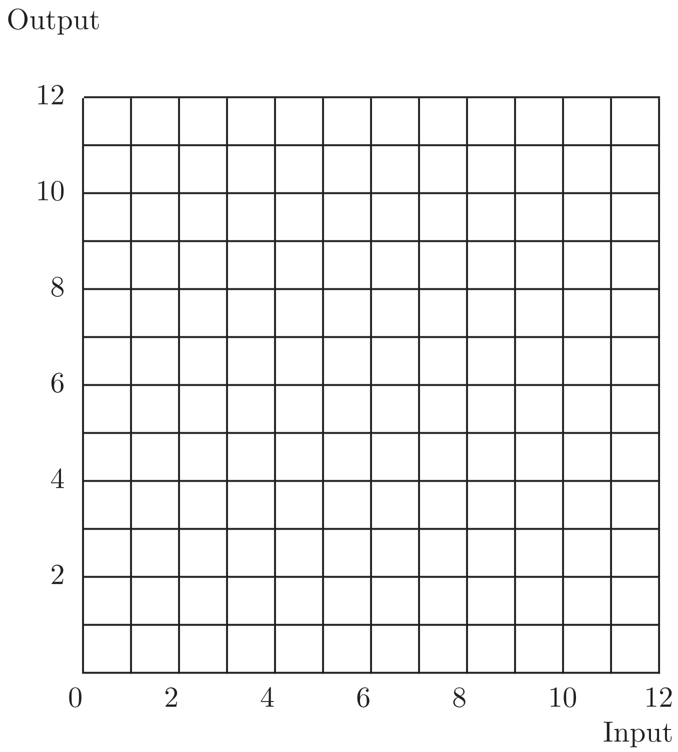
19.5 (0) A profit-maximizing firm produces one output, y, and uses one input, x, to produce it. The price per unit of the factor is denoted by

w and the price of the output is denoted by p. You observe the firm's behavior over three periods and find the following:

Period	y	x	w	p
1	1	1	1	1
2	2.5	3	.5	1
3	4	8	.25	1

(a) Write an equation that gives the firm's profits, π, as a function of the amount of input x it uses, the amount of output y it produces, the per-unit cost of the input w, and the price of output p.________________.

(b) In the diagram below, draw an isoprofit line for each of the three periods, showing combinations of input and output that would yield the same profits that period as the combination actually chosen. What are the equations for these three lines?______________________________ Using the theory of revealed profitability, shade in the region on the graph that represents input-output combinations that could be feasible as far as one can tell from the evidence that is available. How would you describe this region in words?____.

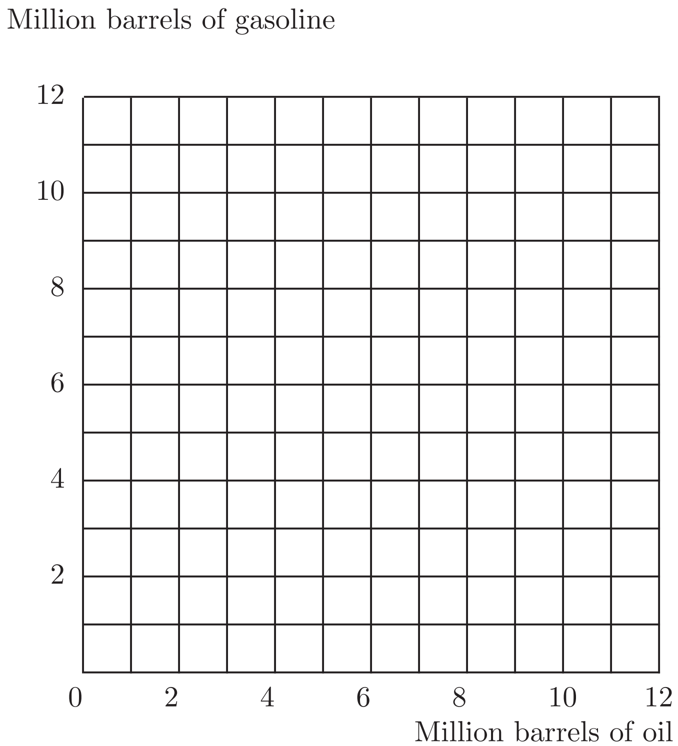


19.6 (0) T-bone Pickens is a corporate raider. This means that he looks for companies that are not maximizing profits, buys them, and then tries to operate them at higher profits. T-bone is examining the financial records of two refineries that he might buy, the Shill Oil Company and the Golf Oil Company. Each of these companies buys oil and produces gasoline. During the time period covered by these records, the price of gasoline fluctuated significantly, while the cost of oil remained constant at $10 a barrel. For simplicity, we assume that oil is the only input to gasoline production.

Shill Oil produced 1 million barrels of gasoline using 1 million barrels of oil when the price of gasoline was $10 a barrel. When the price of gasoline was $20 a barrel, Shill produced 3 million barrels of gasoline using 4 million barrels of oil. Finally, when the price of gasoline was $40 a barrel, Shill used 10 million barrels of oil to produce 5 million barrels of gasoline.

Golf Oil (which is managed by Martin E. Lunch III) did exactly the same when the price of gasoline was $10 and $20, but when the price of gasoline hit $40, Golf produced 3.5 million barrels of gasoline using 8 million barrels of oil.

(a) Using black ink, plot Shill Oil's isoprofit lines and choices for the three different periods. Label them 10, 20, and 40. Using red ink draw Golf Oil's isoprofit line and production choice. Label it with a 40 in red ink.



(b) How much profits could Golf Oil have made when the price of gasoline was \$40 a barrel if it had chosen to produce the same amount that it did when the price was \$20 a barrel? ________ What profits did Golf actually make when the price of gasoline was \$40?________.

(c) Is there any evidence that Shill Oil is not maximizing profits? Explain.

____.

(d) Is there any evidence that Golf Oil is not maximizing profits? Explain.

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19.7 (0) After carefully studying Shill Oil, T-bone Pickens decides that it has probably been maximizing its profits. But he still is very interested in buying Shill Oil. He wants to use the gasoline they produce to fuel his delivery fleet for his chicken farms, Capon Truckin'. In order to do this Shill Oil would have to be able to produce 5 million barrels of gasoline from 8 million barrels of oil. Mark this point on your graph. Assuming that Shill always maximizes profits, would it be technologically feasible for it to produce this input-output combination? Why or why not?

____

____.

19.8 (0) Suppose that firms operate in a competitive market, attempt to maximize profits, and only use one factor of production. Then we know that for any changes in the input and output price, the input choice and the output choice must obey the Weak Axiom of Profit Maximization, $\Delta p \Delta y - \Delta w \Delta x \geq 0$.

Which of the following propositions can be proven by the Weak Axiom of Profit Maximizing Behavior (WAPM)? Respond yes or no, and give a short argument.

(a) If the price of the input does not change, then a decrease in the price of the output will imply that the firm will produce the same amount or less output.________________________________

____.

(b) If the price of the output remains constant, then a decrease in the input price will imply that the firm will use the same amount or more of the input.________________________________

____.

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(c) If both the price of the output and the input increase and the firm produces less output, then the firm will use more of the input.__________

____.

19.9 (1) Farmer Hoglund has discovered that on his farm, he can get 30 bushels of corn per acre if he applies no fertilizer. When he applies N pounds of fertilizer to an acre of land, the *marginal product* of fertilizer is $1 - N/200$ bushels of corn per pound of fertilizer.

(a) If the price of corn is \$3 a bushel and the price of fertilizer is \$$p$ per pound (where $p < 3$), how many pounds of fertilizer should he use per acre in order to maximize profits?______________________.

(b) (Only for those who remember a bit of easy integral calculus.) Write down a function that states Farmer Hoglund's yield per acre as a function of the amount of fertilizer he uses.______________________.

(c) Hoglund's neighbor, Skoglund, has better land than Hoglund. In fact, for any amount of fertilizer that he applies, he gets exactly twice as much corn per acre as Hoglund would get with the same amount of fertilizer. How much fertilizer will Skoglund use per acre when the price of corn is \$3 a bushel and the price of fertilizer is \$$p$ a pound? __________
(Hint: Start by writing down Skoglund's marginal product of fertilizer as a function of N.)

(d) When Hoglund and Skoglund are both maximizing profits, will Skoglund's output be more than twice as much, less than twice as much or exactly twice as much as Hoglund's? Explain.______________

____

____.

(e) Explain how someone who looked at Hoglund's and Skoglund's corn yields and their fertilizer inputs but couldn't observe the quality of their land, would get a misleading idea of the productivity of fertilizer.______________________________

____.

19.10 (0) A firm has two variable factors and a production function, $f(x_1, x_2) = x_1^{1/2} x_2^{1/4}$. The price of its output is 4. Factor 1 receives a wage of w_1 and factor 2 receives a wage of w_2.

(a) Write an equation that says that the value of the marginal product of factor 1 is equal to the wage of factor 1 ________________ and an equation that says that the value of the marginal product of factor 2 is equal to the wage of factor 2.________________ Solve two equations in the two unknowns, x_1 and x_2, to give the amounts of factors 1 and 2 that maximize the firm's profits as a function of w_1 and w_2. This gives $x_1 =$______________ and $x_2 =$______________ (Hint: You could use the first equation to solve for x_1 as a function of x_2 and of the factor wages. Then substitute the answer into the second equation and solve for x_2 as a function of the two wage rates. Finally use your solution for x_2 to find the solution for x_1.)

(b) If the wage of factor 1 is 2, and the wage of factor 2 is 1, how many units of factor 1 will the firm demand?________ How many units of factor 2 will it demand?________ How much output will it produce? ________ How much profit will it make?__________________________.

19.11 (0) A firm has two variable factors and a production function $f(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. The price of its output is 4, the price of factor 1 is w_1, and the price of factor 2 is w_2.

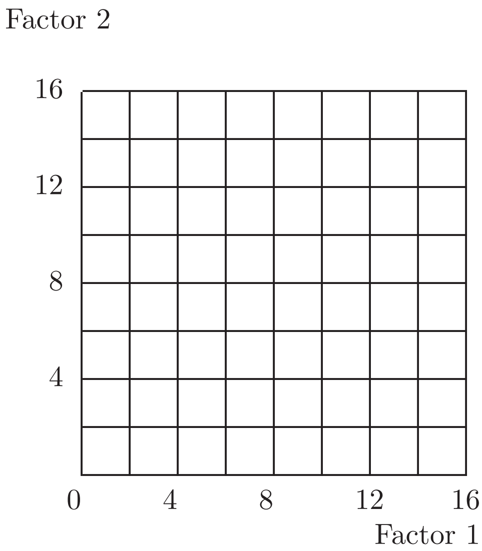
(a) Write the two equations that say that the value of the marginal product of each factor is equal to its wage. ______________________________ _______ If $w_1 = 2w_2$, these two equations imply that $x_1/x_2 =$__________.

(b) For this production function, is it possible to solve the two marginal productivity equations uniquely for x_1 and x_2?______________________.

19.12 (1) A firm has two variable factors and a production function $f(x_1, x_2) = \sqrt{2x_1 + 4x_2}$. On the graph below, draw production isoquants corresponding to an ouput of 3 and to an output of 4.

(a) If the price of the output good is 4, the price of factor 1 is 2, and the price of factor 2 is 3, find the profit-maximizing amount of factor 1 ________ , the profit-maximizing amount of factor 2__________ , and the profit-maximizing output____________________________________.

____

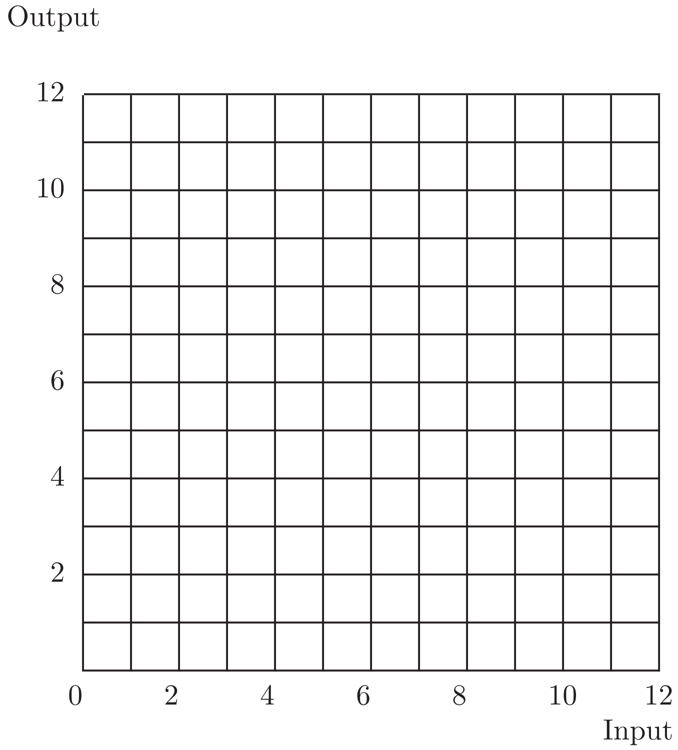


19.13 (0) A profit-maximizing firm produces one output, y, and uses one input, x, to produce it. The price per unit of the factor is denoted by w and the price of the output is denoted by p. You observe the firm's behavior over three periods and find the following:

Period	y	x	w	p
1	1	1	1	1
2	2.5	3	.5	1
3	4	8	.25	1

(a) Write an equation that gives the firm's profits, π, as a function of the amount of input x it uses, the amount of output y it produces, the per-unit cost of the input w, and the price of output p.______________.

(b) In the diagram below, draw an isoprofit line for each of the three periods, showing combinations of input and output that would yield the same profits that period as the combination actually chosen. What are the equations for these three lines?____________________________ Using the theory of revealed profitability, shade in the region on the graph that represents input-output combinations that could be feasible as far as one can tell from the evidence that is available. How would you describe this region in words?____.



19.14 (0) T-bone Pickens is a corporate raider. This means that he looks for companies that are not maximizing profits, buys them, and then tries to operate them at higher profits. T-bone is examining the financial records of two refineries that he might buy, the Shill Oil Company and the Golf Oil Company. Each of these companies buys oil and produces gasoline. During the time period covered by these records, the price of gasoline fluctuated significantly, while the cost of oil remained constant at \$10 a barrel. For simplicity, we assume that oil is the only input to gasoline production.

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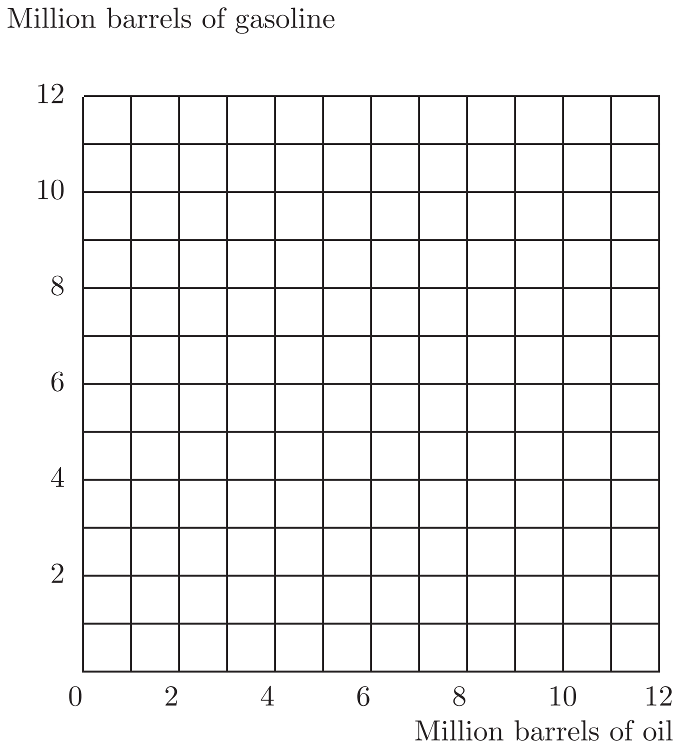

(b) How much profits could Golf Oil have made when the price of gasoline was \$40 a barrel if it had chosen to produce the same amount that it did when the price was \$20 a barrel? ____________ What profits did Golf actually make when the price of gasoline was \$40?____________.

(c) Is there any evidence that Shill Oil is not maximizing profits? Explain.

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(d) Is there any evidence that Golf Oil is not maximizing profits? Explain.

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19.15 (0) After carefully studying Shill Oil, T-bone Pickens decides that it has probably been maximizing its profits. But he still is very interested in buying Shill Oil. He wants to use the gasoline they produce to fuel his delivery fleet for his chicken farms, Capon Truckin'. In order to do this Shill Oil would have to be able to produce 5 million barrels of gasoline from 8 million barrels of oil. Mark this point on your graph. Assuming that Shill always maximizes profits, would it be technologically feasible for it to produce this input-output combination? Why or why not?

____

____.

19.16 (0) Suppose that firms operate in a competitive market, attempt to maximize profits, and only use one factor of production. Then we know that for any changes in the input and output price, the input choice and the output choice must obey the Weak Axiom of Profit Maximization, $\Delta p \Delta y - \Delta w \Delta x \geq 0$.

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____.

(c) If both the price of the output and the input increase and the firm produces less output, then the firm will use more of the input.__________

____.