You have studied budgets, and you have studied preferences. Now is the time to put these two ideas together and do something with them. In this chapter you study the commodity bundle chosen by a utility-maximizing consumer from a given budget.

Given prices and income, you know how to graph a consumer's budget. If you also know the consumer's preferences, you can graph some of his indifference curves. The consumer will choose the "best" indifference curve that he can reach given his budget. But when you try to do this, you have to ask yourself, "How do I find the most desirable indifference curve that the consumer can reach?" The answer to this question is "look in the likely places." Where are the likely places? As your textbook tells you, there are three kinds of likely places. These are: (*i*) a tangency between an indifference curve and the budget line; (*ii*) a kink in an indifference curve; (*iii*) a "corner" where the consumer specializes in consuming just one good.

Here is how you find a point of tangency if we are told the consumer's utility function, the prices of both goods, and the consumer's income. The budget line and an indifference curve are tangent at a point (x_1, x_2) if they have the same slope at that point. Now the slope of an indifference curve at (x_1, x_2) is the ratio $-MU_1(x_1, x_2)/MU_2(x_1, x_2)$. (This slope is also known as the marginal rate of substitution.) The slope of the budget line is $-p_1/p_2$. Therefore an indifference curve is tangent to the budget line at the point (x_1, x_2) when $MU_1(x_1, x_2)/MU_2(x_1, x_2) = p_1/p_2$. This gives us one equation in the two unknowns, x_1 and x_2. If we hope to solve for the x's, we need another equation. That other equation is the budget equation $p_1x_1 + p_2x_2 = m$. With these two equations you can solve for (x_1, x_2).*

A consumer has the utility function $U(x_1, x_2) = x_1^2 x_2$. The price of good 1 is $p_1 = 1$, the price of good 2 is $p_2 = 3$, and his income is 180. Then, $MU_1(x_1, x_2) = 2x_1x_2$ and $MU_2(x_1, x_2) = x_1^2$. Therefore his marginal rate of substitution is $-MU_1(x_1, x_2)/MU_2(x_1, x_2) = -2x_1x_2/x_1^2 = -2x_2/x_1$. This implies that his indifference curve will be tangent to his budget line when $-2x_2/x_1 = -p_1/p_2 = -1/3$. Simplifying this expression, we have $6x_2 = x_1$. This is one of the two equations we need to solve for the two unknowns, x_1 and x_2. The other equation is the budget equation. In this case the budget equation is $x_1 + 3x_2 = 180$. Solving these two equations in two unknowns, we find $x_1 = 120$ and $x_2 = 20$. Therefore we know that

* Some people have trouble remembering whether the marginal rate of substitution is $-MU_1/MU_2$ or $-MU_2/MU_1$. It isn't really crucial to remember which way this goes as long as you remember that a tangency happens when the marginal utilities of any two goods are in the same proportion as their prices.

the consumer chooses the bundle $(x_1, x_2) = (120, 20)$.

For equilibrium at kinks or at corners, we don't need the slope of the indifference curves to equal the slope of the budget line. So we don't have the tangency equation to work with. But we still have the budget equation. The second equation that you can use is an equation that tells you that you are at one of the kinky points or at a corner. You will see exactly how this works when you work a few exercises.

A consumer has the utility function $U(x_1, x_2) = \min\{x_1, 3x_2\}$. The price of x_1 is 2, the price of x_2 is 1, and her income is 140. Her indifference curves are L-shaped. The corners of the L's all lie along the line $x_1 = 3x_2$. She will choose a combination at one of the corners, so this gives us one of the two equations we need for finding the unknowns x_1 and x_2. The second equation is her budget equation, which is $2x_1 + x_2 = 140$. Solve these two equations to find that $x_1 = 60$ and $x_2 = 20$. So we know that the consumer chooses the bundle $(x_1, x_2) = (60, 20)$.

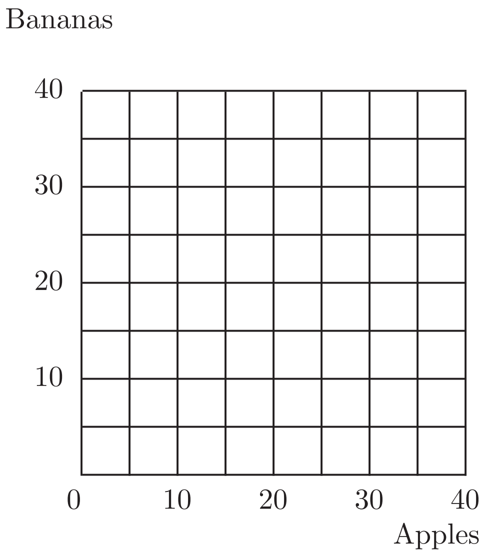
When you have finished these exercises, we hope that you will be able to do the following:

- Calculate the best bundle a consumer can afford at given prices and income in the case of simple utility functions where the best affordable bundle happens at a point of tangency.
- Find the best affordable bundle, given prices and income for a consumer with kinked indifference curves.
- Recognize standard examples where the best bundle a consumer can afford happens at a corner of the budget set.
- Draw a diagram illustrating each of the above types of equilibrium.
- Apply the methods you have learned to choices made with some kinds of nonlinear budgets that arise in real-world situations.

5.1 (0) We begin again with Charlie of the apples and bananas. Recall that Charlie's utility function is $U(x_A, x_B) = x_A x_B$. Suppose that the price of apples is 1, the price of bananas is 2, and Charlie's income is 40.

(a) On the graph below, use blue ink to draw Charlie's budget line. (Use a ruler and try to make this line accurate.) Plot a few points on the indifference curve that gives Charlie a utility of 150 and sketch this curve with red ink. Now plot a few points on the indifference curve that gives Charlie a utility of 300 and sketch this curve with black ink or pencil.

____



(b) Can Charlie afford any bundles that give him a utility of 150?______

____.

(c) Can Charlie afford any bundles that give him a utility of 300?______

(d) On your graph, mark a point that Charlie can afford and that gives him a higher utility than 150. Label that point A.

(e) Neither of the indifference curves that you drew is tangent to Charlie's budget line. Let's try to find one that is. At any point, (x_A, x_B), Charlie's marginal rate of substitution is a function of x_A and x_B. In fact, if you calculate the ratio of marginal utilities for Charlie's utility function, you will find that Charlie's marginal rate of substitution is $MRS(x_A, x_B) = -x_B/x_A$. This is the slope of his indifference curve at (x_A, x_B). The slope of Charlie's budget line is ________ (give a numerical answer).

(f) Write an equation that implies that the budget line is tangent to an indifference curve at (x_A, x_B). ______________ There are many solutions to this equation. Each of these solutions corresponds to a point on a different indifference curve. Use pencil to draw a line that passes through all of these points.

(g) The best bundle that Charlie can afford must lie somewhere on the line you just penciled in. It must also lie on his budget line. If the point is outside of his budget line, he can't afford it. If the point lies inside of his budget line, he can afford to do better by buying more of both goods. On your graph, label this best affordable bundle with an E. This happens where x_A=________ and x_B=_______ Verify your answer by solving the two simultaneous equations given by his budget equation and the tangency condition.

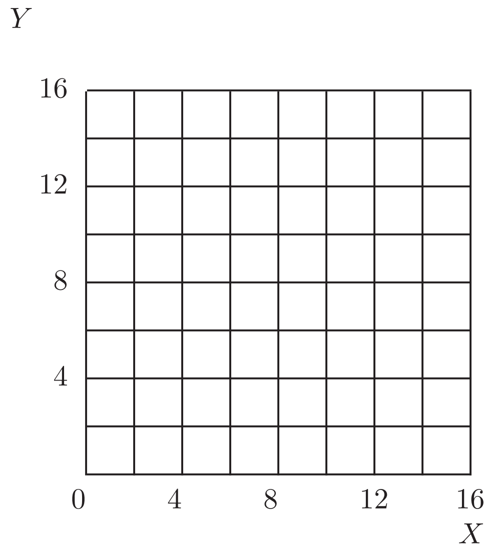
(h) What is Charlie's utility if he consumes the bundle $(20, 10)$?_______

____

(i) On the graph above, use red ink to draw his indifference curve through (20,10). Does this indifference curve cross Charlie's budget line, just touch it, or never touch it?____.

5.2 (0) Clara's utility function is $U(X, Y) = (X + 2)(Y + 1)$, where X is her consumption of good X and Y is her consumption of good Y.

(a) Write an equation for Clara's indifference curve that goes through the point $(X, Y) = (2, 8)$. $Y =$____________ On the axes below, sketch Clara's indifference curve for $U = 36$.



(b) Suppose that the price of each good is 1 and that Clara has an income of 11. Draw in her budget line. Can Clara achieve a utility of 36 with this budget?____.

____

(c) At the commodity bundle, (X, Y), Clara's marginal rate of substitution is________________.

(d) If we set the absolute value of the MRS equal to the price ratio, we have the equation________________.

(e) The budget equation is________________.

(f) Solving these two equations for the two unknowns, X and Y, we find $X =$________ and $Y =$________________.

5.3 (0) Ambrose, the nut and berry consumer, has a utility function $U(x_1, x_2) = 4\sqrt{x_1} + x_2$, where x_1 is his consumption of nuts and x_2 is his consumption of berries.

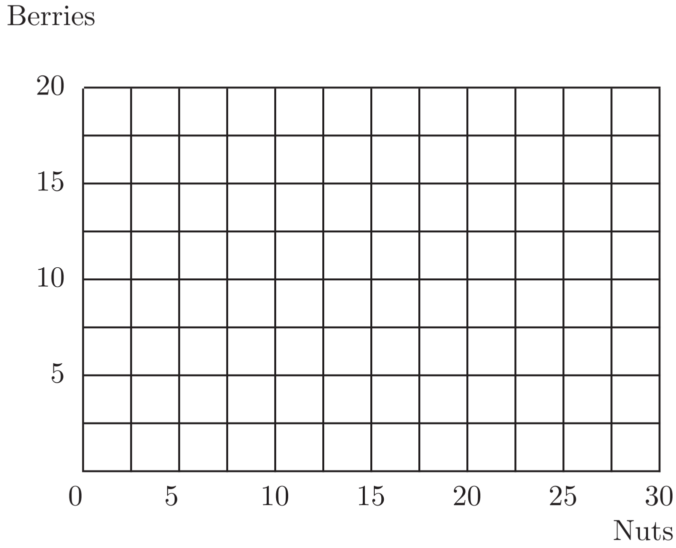
(a) The commodity bundle $(25, 0)$ gives Ambrose a utility of 20. Other points that give him the same utility are $(16, 4)$, (9,________), (4, ________), (1,________), and (0,________). Plot these points on the axes below and draw a red indifference curve through them.

(b) Suppose that the price of a unit of nuts is 1, the price of a unit of berries is 2, and Ambrose's income is 24. Draw Ambrose's budget line with blue ink. How many units of nuts does he choose to buy?________________

(c) How many units of berries?________________.

(d) Find some points on the indifference curve that gives him a utility of 25 and sketch this indifference curve (in red).

(e) Now suppose that the prices are as before, but Ambrose's income is 34. Draw his new budget line (with pencil). How many units of nuts will he choose? ____________ How many units of berries?____________.



(f) Now let us explore a case where there is a "boundary solution." Suppose that the price of nuts is still 1 and the price of berries is 2, but Ambrose's income is only 9. Draw his budget line (in blue). Sketch the indifference curve that passes through the point $(9, 0)$. What is the slope of his indifference curve at the point $(9, 0)$?________________________.

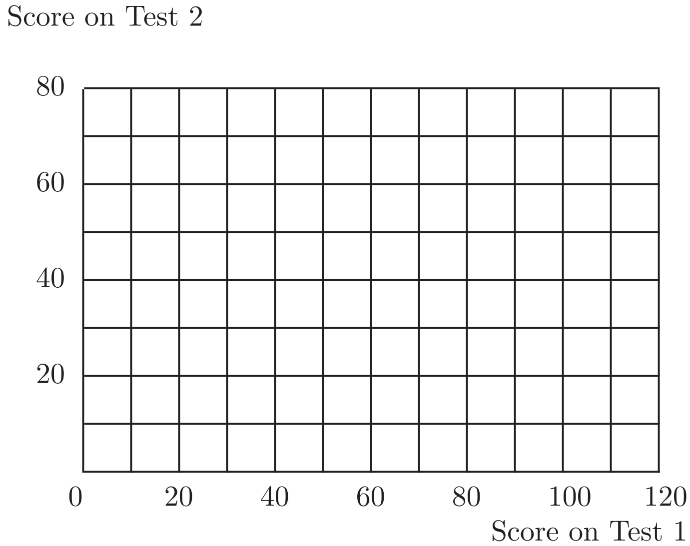
(g) What is the slope of his budget line at this point?________________.

(h) Which is steeper at this point, the budget line or the indifference curve?____.

(i) Can Ambrose afford any bundles that he likes better than the point $(9, 0)$?____.

5.4 (1) Nancy Lerner is trying to decide how to allocate her time in studying for her economics course. There are two examinations in this course. Her overall score for the course will be the *minimum* of her scores on the two examinations. She has decided to devote a total of 1,200 minutes to studying for these two exams, and she wants to get as high an overall score as possible. She knows that on the first examination if she doesn't study at all, she will get a score of zero on it. For every 10 minutes that she spends studying for the first examination, she will increase her score by one point. If she doesn't study at all for the second examination she will get a zero on it. For every 20 minutes she spends studying for the second examination, she will increase her score by one point.

(a) On the graph below, draw a "budget line" showing the various combinations of scores on the two exams that she can achieve with a total of 1,200 minutes of studying. On the same graph, draw two or three "indifference curves" for Nancy. On your graph, draw a straight line that goes through the kinks in Nancy's indifference curves. Label the point where this line hits Nancy's budget with the letter A. Draw Nancy's indifference curve through this point.



(b) Write an equation for the line passing through the kinks of Nancy's indifference curves.______________________________.

(c) Write an equation for Nancy's budget line.__________________.

(d) Solve these two equations in two unknowns to determine the intersection of these lines. This happens at the point $(x_1, x_2) =$__________.

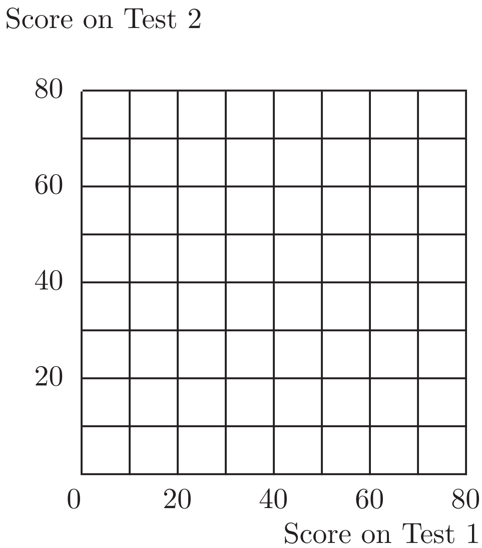
(e) Given that she spends a total of 1,200 minutes studying, Nancy will maximize her overall score by spending ______ minutes studying for the first examination and ______ minutes studying for the second examination.

5.5 (1) In her communications course, Nancy also takes two examinations. Her overall grade for the course will be the *maximum* of her scores on the two examinations. Nancy decides to spend a total of 400 minutes studying for these two examinations. If she spends m_1 minutes studying for the first examination, her score on this exam will be $x_1 = m_1/5$. If she spends m_2 minutes studying for the second examination, her score on this exam will be $x_2 = m_2/10$.

(a) On the graph below, draw a "budget line" showing the various combinations of scores on the two exams that she can achieve with a total of 400 minutes of studying. On the same graph, draw two or three "indifference curves" for Nancy. On your graph, find the point on Nancy's budget line that gives her the best overall score in the course.

(b) Given that she spends a total of 400 minutes studying, Nancy will maximize her overall score by achieving a score of ________ on the first examination and ________ on the second examination.

(c) Her overall score for the course will then be________________________.



5.6 (0) Elmer's utility function is $U(x, y) = \min\{x, y^2\}$.

(a) If Elmer consumes 4 units of x and 3 units of y, his utility is________.

(b) If Elmer consumes 4 units of x and 2 units of y, his utility is________.

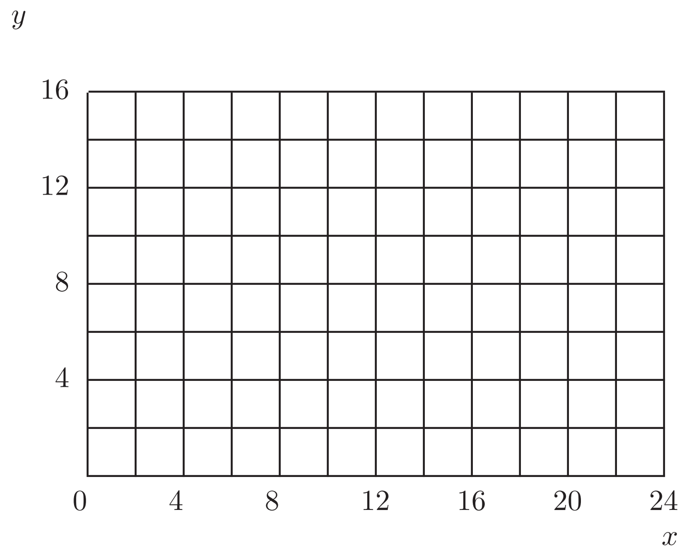
(c) If Elmer consumes 5 units of x and 2 units of y, his utility is________.

(d) On the graph below, use blue ink to draw the indifference curve for Elmer that contains the bundles that he likes exactly as well as the bundle $(4, 2)$.

(e) On the same graph, use blue ink to draw the indifference curve for Elmer that contains bundles that he likes exactly as well as the bundle $(1, 1)$ and the indifference curve that passes through the point $(16, 5)$.

(f) On your graph, use black ink to show the locus of points at which Elmer's indifference curves have kinks. What is the equation for this curve?____.

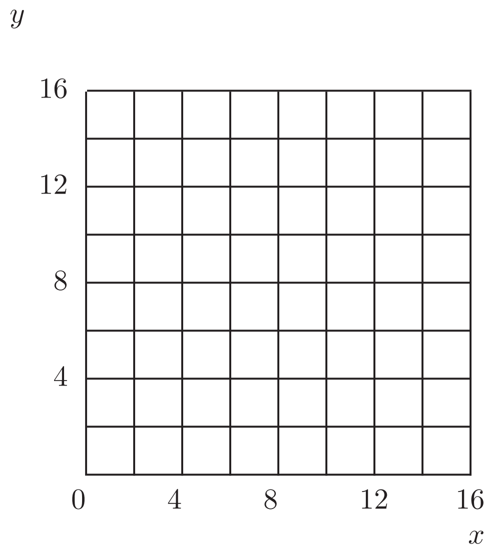
(g) On the same graph, use black ink to draw Elmer's budget line when the price of x is 1, the price of y is 2, and his income is 8. What bundle does Elmer choose in this situation?________________________.



(h) Suppose that the price of x is 10 and the price of y is 15 and Elmer buys 100 units of x. What is Elmer's income? __________ (Hint: At first you might think there is too little information to answer this question. But think about how much y he must be demanding if he chooses 100 units of x.)

5.7 (0) Linus has the utility function $U(x, y) = x + 3y$.

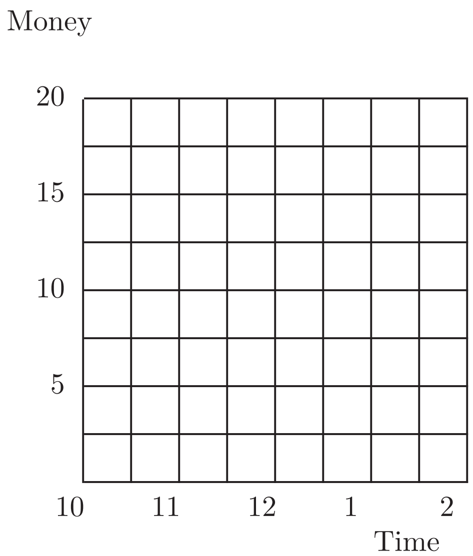
(a) On the graph below, use blue ink to draw the indifference curve passing through the point $(x, y) = (3, 3)$. Use black ink to sketch the indifference curve connecting bundles that give Linus a utility of 6.



(b) On the same graph, use red ink to draw Linus's budget line if the price of x is 1 and the price of y is 2 and his income is 8. What bundle does Linus choose in this situation?____________________.

(c) What bundle would Linus choose if the price of x is 1, the price of y is 4, and his income is 8?____________________.

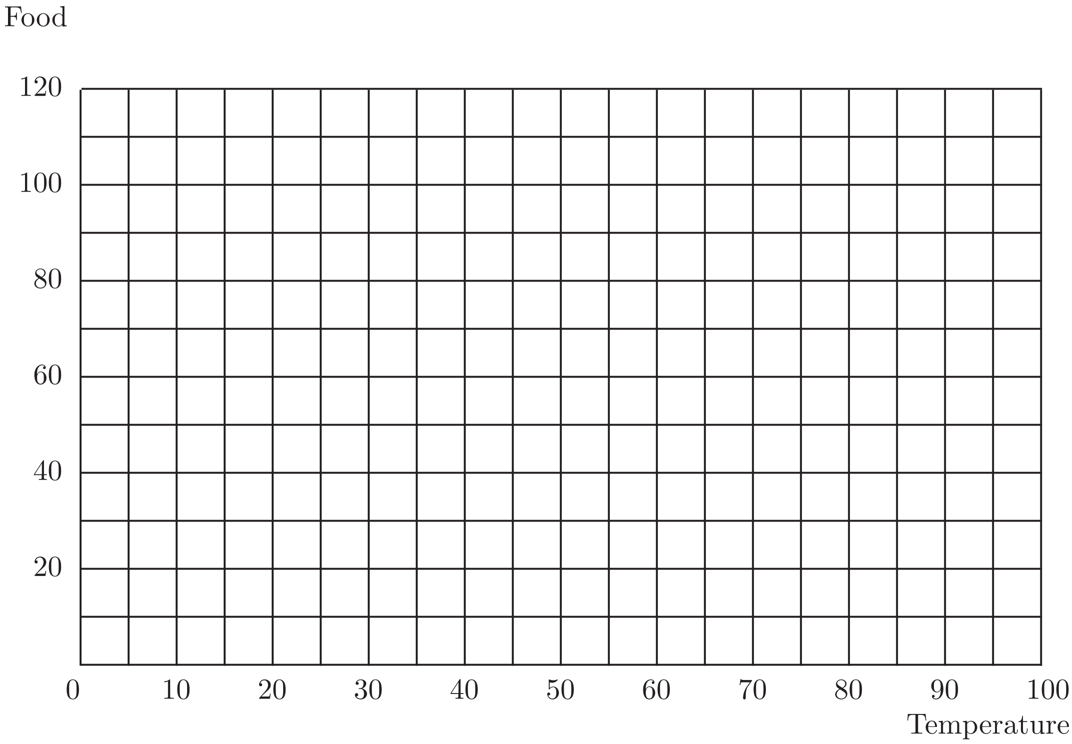
5.8 (2) Remember our friend Ralph Rigid from Chapter 3? His favorite diner, Food for Thought, has adopted the following policy to reduce the crowds at lunch time: if you show up for lunch t hours before or after 12 noon, you get to deduct t dollars from your bill. (This holds for any fraction of an hour as well.)



(a) Use blue ink to show Ralph's budget set. On this graph, the horizontal axis measures the time of day that he eats lunch, and the vertical axis measures the amount of money that he will have to spend on things other than lunch. Assume that he has \$20 total to spend and that lunch at noon costs \$10. (Hint: How much money would he have left if he ate at noon? at 1 P.M.? at 11 A.M.?)

(b) Recall that Ralph's preferred lunch time is 12 noon, but that he is willing to eat at another time if the food is sufficiently cheap. Draw some red indifference curves for Ralph that would be consistent with his choosing to eat at 11 A.M.

5.9 (0) Joe Grad has just arrived at the big U. He has a fellowship that covers his tuition and the rent on an apartment. In order to get by, Joe has become a grader in intermediate price theory, earning \$100 a month. Out of this \$100 he must pay for his food and utilities in his apartment. His utilities expenses consist of heating costs when he heats his apartment and air-conditioning costs when he cools it. To raise the temperature of his apartment by one degree, it costs \$2 per month (or \$20 per month to raise it ten degrees). To use air-conditioning to cool his apartment by a degree, it costs \$3 per month. Whatever is left over after paying the utilities, he uses to buy food at \$1 per unit.



(a) When Joe first arrives in September, the temperature of his apartment is 60 degrees. If he spends nothing on heating or cooling, the temperature in his room will be 60 degrees and he will have \$100 left to spend on food.

If he heated the room to 70 degrees, he would have ______ left to spend on food. If he cooled the room to 50 degrees, he would have ______ left to spend on food. On the graph below, show Joe's September budget constraint (with black ink). (Hint: You have just found three points that Joe can afford. Apparently, his budget set is not bounded by a single straight line.)

(b) In December, the outside temperature is 30 degrees and in August poor Joe is trying to understand macroeconomics while the temperature outside is 85 degrees. On the same graph you used above, draw Joe's budget constraints for the months of December (in blue ink) and August (in red ink).

(c) Draw a few smooth (unkinky) indifference curves for Joe in such a way that the following are true. (*i*) His favorite temperature for his apartment would be 65 degrees if it cost him nothing to heat it or cool it. (*ii*) Joe chooses to use the furnace in December, air-conditioning in August, and neither in September. (*iii*) Joe is better off in December than in August.

(d) In what months is the slope of Joe's budget constraint equal to the slope of his indifference curve?______________________________.

(e) In December Joe's marginal rate of substitution between food and degrees Fahrenheit is ______ In August, his MRS is______________.

(f) Since Joe neither heats nor cools his apartment in September, we cannot determine his marginal rate of substitution exactly, but we do know that it must be no smaller than ________ and no larger than ________ (Hint: Look carefully at your graph.)

5.10 (0) Central High School has \$60,000 to spend on computers and other stuff, so its budget equation is $C + X = 60,000$, where C is expenditure on computers and X is expenditures on other things. C.H.S. currently plans to spend \$20,000 on computers.

The State Education Commission wants to encourage "computer literacy" in the high schools under its jurisdiction. The following plans have been proposed.

Plan A: This plan would give a grant of \$10,000 to each high school in the state that the school could spend as it wished.

Plan B: This plan would give a \$10,000 grant to any high school, so long as the school spent at least \$10,000 *more* than it currently spends on computers. Any high school can choose not to participate, in which case it does not receive the grant, but it doesn't have to increase its expenditure on computers.

Plan C: Plan C is a "matching grant." For every dollar's worth of computers that a high school orders, the state will give the school 50 cents.

Plan D: This plan is like plan C, except that the maximum amount of matching funds that any high school could get from the state would be limited to $10,000.

(a) Write an equation for Central High School's budget if plan A is adopted.____________ Use black ink to draw the budget line for Central High School if plan A is adopted.

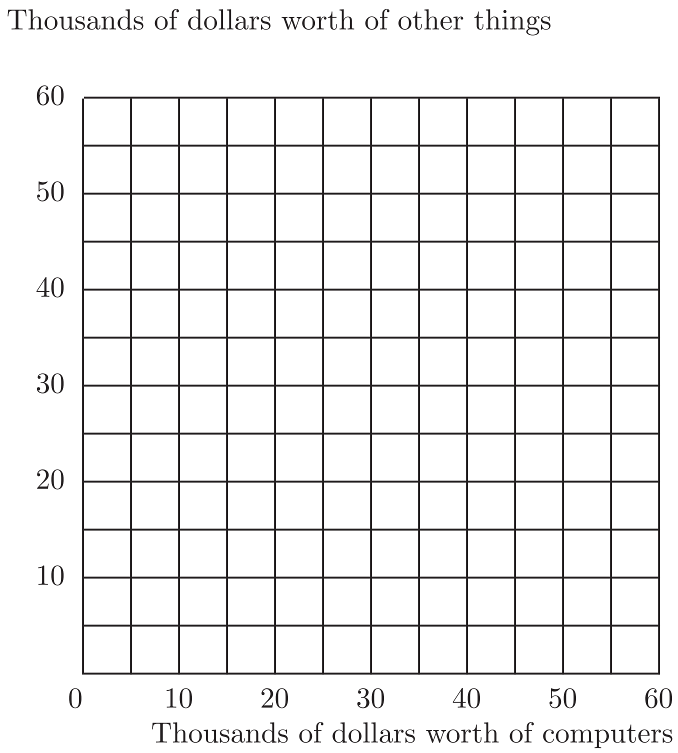
(b) If plan B is adopted, the boundary of Central High School's budget set has two separate downward-sloping line segments. One of these segments describes the cases where C.H.S. spends at least $30,000 on computers. This line segment runs from the point $(C, X) = (70,000, 0)$ to the point $(C, X) =$________________________________.

(c) Another line segment corresponds to the cases where C.H.S. spends less than $30,000 on computers. This line segment runs from $(C, X) =$ ____________ to the point $(C, X) = (0, 60,000)$. Use red ink to draw these two line segments.

(d) If plan C is adopted and Central High School spends C dollars on computers, then it will have $X = 60,000 - .5C$ dollars left to spend on other things. Therefore its budget line has the equation ____________ ______ Use blue ink to draw this budget line.

(e) If plan D is adopted, the school district's budget consists of two line segments that intersect at the point where expenditure on computers is ________ and expenditure on other instructional materials is______ ________________________________.

(f) The slope of the flatter line segment is ________ The slope of the steeper segment is ________ Use pencil to draw this budget line.



5.11 (0) Suppose that Central High School has preferences that can be represented by the utility function $U(C, X) = CX^2$. Let us try to determine how the various plans described in the last problem will affect the amount that C.H.S. spends on computers.

(a) If the state adopts none of the new plans, find the expenditure on computers that maximizes the district's utility subject to its budget constraint.________________________.

(b) If plan A is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint.____________.

(c) On your graph, sketch the indifference curve that passes through the point (30,000, 40,000) if plan B is adopted. At this point, which is steeper, the indifference curve or the budget line?____________________.

(d) If plan B is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint. (Hint: Look at your graph.)________________________.

____

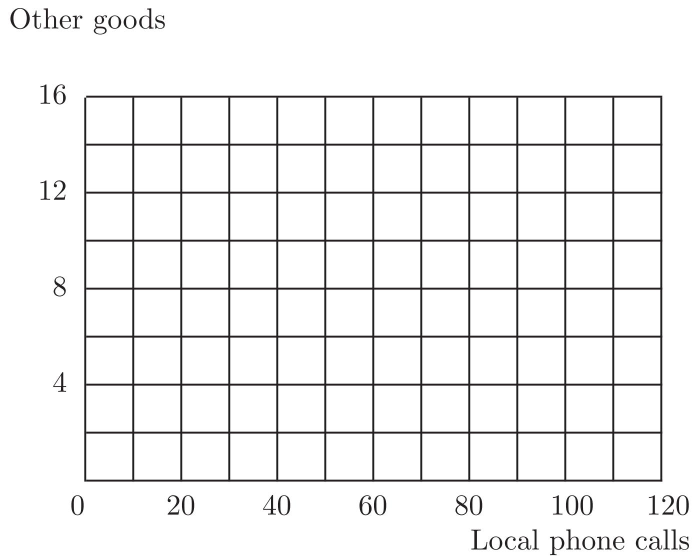
(e) If plan C is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint.____________.

(f) If plan D is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint.____________.

5.12 (0) The telephone company allows one to choose between two different pricing plans. For a fee of \$12 per month you can make as many local phone calls as you want, at no additional charge per call. Alternatively, you can pay \$8 per month and be charged 5 cents for each local phone call that you make. Suppose that you have a total of \$20 per month to spend.

(a) On the graph below, use black ink to sketch a budget line for someone who chooses the first plan. Use red ink to draw a budget line for someone who chooses the second plan. Where do the two budget lines cross?

____.



(b) On the graph above, use pencil to draw indifference curves for someone who prefers the second plan to the first. Use blue ink to draw an indifference curve for someone who prefers the first plan to the second.

5.13 (1) This is a puzzle—just for fun. Lewis Carroll (1832-1898), author of *Alice in Wonderland* and *Through the Looking Glass,* was a mathematician, logician, and political scientist. Carroll loved careful reasoning about puzzling things. Here Carroll's Alice presents a nice bit of economic analysis. At first glance, it may seem that Alice is talking nonsense, but, indeed, her reasoning is impeccable.

"I should like to buy an egg, please." she said timidly. "How do you sell them?"

"Fivepence farthing for one—twopence for two," the Sheep replied.

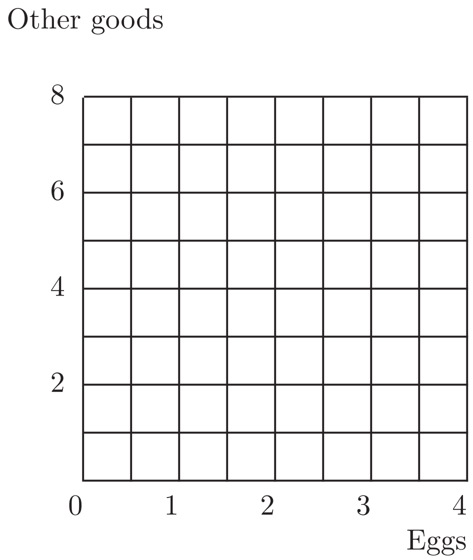
"Then two are cheaper than one?" Alice said, taking out her purse.

"Only you must eat them both if you buy two," said the Sheep.

"Then I'll have one please," said Alice, as she put the money down on the counter. For she thought to herself, "They mightn't be at all nice, you know."

(a) Let us try to draw a budget set and indifference curves that are consistent with this story. Suppose that Alice has a total of 8 pence to spend and that she can buy either 0, 1, or 2 eggs from the Sheep, but no fractional eggs. Then her budget set consists of just three points. The point where she buys no eggs is $(0, 8)$. Plot this point and label it A. On your graph, the point where she buys 1 egg is $(1, 2\frac{3}{4})$. (A farthing is 1/4 of a penny.) Plot this point and label it B.

(b) The point where she buys 2 eggs is __________ Plot this point and label it C. If Alice chooses to buy 1 egg, she must like the bundle B better than either the bundle A or the bundle C. Draw indifference curves for Alice that are consistent with this behavior.



In the last section, you were given a consumer's preferences and then you solved for his or her demand behavior. In this chapter we turn this process around: you are given information about a consumer's demand behavior and you must deduce something about the consumer's preferences. The main tool is the *weak axiom of revealed preference.* This axiom says the following. If a consumer chooses commodity bundle A when she can afford bundle B, then she will never choose bundle B from any budget in which she can also afford A. The idea behind this axiom is that if you choose A when you could have had B, you must like A better than B. But if you like A better than B, then you will never choose B when you can have A. If somebody chooses A when she can afford B, we say that for her, A is *directly revealed preferred* to B. The weak axiom says that if A is directly revealed preferred to B, then B is not directly revealed preferred to A.

Let us look at an example of how you check whether one bundle is revealed preferred to another. Suppose that a consumer buys the bundle $(x_1^A, x_2^A) = (2, 3)$ at prices $(p_1^A, p_2^A) = (1, 4)$. The cost of bundle (x_1^A, x_2^A) at these prices is $(2 \times 1) + (3 \times 4) = 14$. Bundle $(2, 3)$ is directly revealed preferred to all the other bundles that she can afford at prices $(1, 4)$, when she has an income of 14. For example, the bundle $(5, 2)$ costs only 13 at prices $(1, 4)$, so we can say that for this consumer $(2, 3)$ is directly revealed preferred to $(1, 4)$.

You will also have some problems about price and quantity indexes. A price index is a comparison of average price levels between two different times or two different places. If there is more than one commodity, it is not necessarily the case that all prices changed in the same proportion. Let us suppose that we want to compare the price level in the "current year" with the price level in some "base year." One way to make this comparison is to compare the costs in the two years of some "reference" commodity bundle. Two reasonable choices for the reference bundle come to mind. One possibility is to use the current year's consumption bundle for the reference bundle. The other possibility is to use the bundle consumed in the base year. Typically these will be different bundles. If the base-year bundle is the reference bundle, the resulting price index is called the *Laspeyres price index.* If the current year's consumption bundle is the reference bundle, then the index is called the *Paasche price index.*

Suppose that there are just two goods. In 1980, the prices were $(1, 3)$ and a consumer consumed the bundle $(4, 2)$. In 1990, the prices were $(2, 4)$ and the consumer consumed the bundle $(3, 3)$. The cost of the 1980 bundle at 1980 prices is $(1 \times 4) + (3 \times 2) = 10$. The cost of this same bundle at 1990 prices is $(2 \times 4) + (4 \times 2) = 16$. If 1980 is treated as the base year and 1990 as the current year, the Laspeyres price ratio is $16/10$. To calculate the Paasche price ratio, you find the ratio of the cost of the 1990 bundle at 1990 prices to the cost of the same bundle at 1980 prices. The 1990 bundle costs $(2 \times 3) + (4 \times 3) = 18$ at 1990 prices. The same bundle cost $(1 \times 3) + (3 \times 3) = 12$ at 1980 prices. Therefore the Paasche price index is 18/12. Notice that both price indexes indicate that prices rose, but because the price changes are weighted differently, the two approaches give different price ratios.

Making an index of the "quantity" of stuff consumed in the two periods presents a similar problem. How do you weight changes in the amount of good 1 relative to changes in the amount of good 2? This time we could compare the cost of the two periods' bundles evaluated at some reference prices. Again there are at least two reasonable possibilities, the *Laspeyres quantity index* and the *Paasche quantity index.* The Laspeyres quantity index uses the base-year prices as the reference prices, and the Paasche quantity index uses current prices as reference prices.

In the example above, the Laspeyres quantity index is the ratio of the cost of the 1990 bundle at 1980 prices to the cost of the 1980 bundle at 1980 prices. The cost of the 1990 bundle at 1980 prices is 12 and the cost of the 1980 bundle at 1980 prices is 10, so the Laspeyres quantity index is 12/10. The cost of the 1990 bundle at 1990 prices is 18 and the cost of the 1980 bundle at 1990 prices is 16. Therefore the Paasche quantity index is 18/16.

When you have completed this section, we hope that you will be able to do the following:

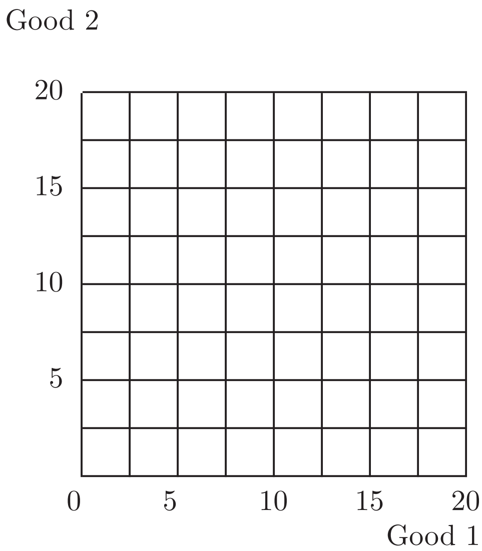
- Decide from given data about prices and consumption whether one commodity bundle is preferred to another.
- Given price and consumption data, calculate Paasche and Laspeyres price and quantity indexes.
- Use the weak axiom of revealed preferences to make logical deductions about behavior.
- Use the idea of revealed preference to make comparisons of well-being across time and across countries.

7.1 (0) When prices are $(4, 6)$, Goldie chooses the bundle $(6, 6)$, and when prices are $(6, 3)$, she chooses the bundle $(10, 0)$.

(a) On the graph below, show Goldie's first budget line in red ink and her second budget line in blue ink. Mark her choice from the first budget with the label A, and her choice from the second budget with the label B.

(b) Is Goldie's behavior consistent with the weak axiom of revealed preference?____________________________________.

____



7.2 (0) Freddy Frolic consumes only asparagus and tomatoes, which are highly seasonal crops in Freddy's part of the world. He sells umbrellas for a living, which provides a fluctuating income depending on the weather. But Freddy doesn't mind; he never thinks of tomorrow, so each week he spends as much as he earns. One week, when the prices of asparagus and tomatoes were each \$1 a pound, Freddy consumed 15 pounds of each. Use blue ink to show the budget line in the diagram below. Label Freddy's consumption bundle with the letter A.

(a) What is Freddy's income?____________________.

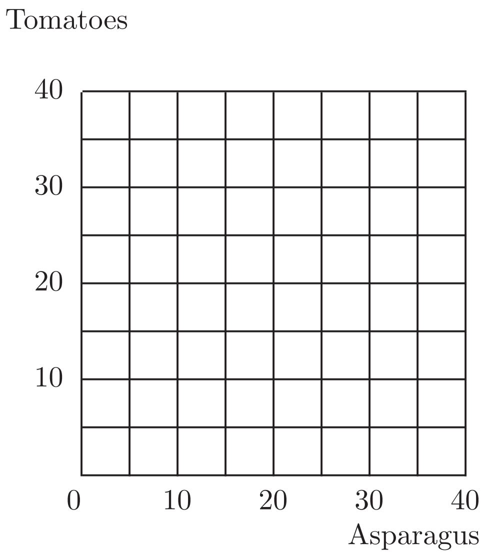
(b) The next week the price of tomatoes rose to \$2 a pound, but the price of asparagus remained at \$1 a pound. By chance, Freddy's income had changed so that his old consumption bundle of (15,15) was just affordable at the new prices. Use red ink to draw this new budget line on the graph below. Does your new budget line go through the point A? __________

What is the slope of this line?____________________.

(c) How much asparagus can he afford now if he spent all of his income on asparagus?____________________.

(d) What is Freddy's income now?____________________.

(e) Use pencil to shade the bundles of goods on Freddy's new red budget line that he definitely will *not* purchase with this budget. Is it possible that he would increase his consumption of tomatoes when his budget changes from the blue line to the red one?____________________________.



7.3 (0) Pierre consumes bread and wine. For Pierre, the price of bread is 4 francs per loaf, and the price of wine is 4 francs per glass. Pierre has an income of 40 francs per day. Pierre consumes 6 glasses of wine and 4 loaves of bread per day.

Bob also consumes bread and wine. For Bob, the price of bread is 1/2 dollar per loaf and the price of wine is 2 dollars per glass. Bob has an income of $15 per day.

(a) If Bob and Pierre have the same tastes, can you tell whether Bob is better off than Pierre or vice versa? Explain.________________________

____.

(b) Suppose prices and incomes for Pierre and Bob are as above and that Pierre's consumption is as before. Suppose that Bob spends all of his income. Give an example of a consumption bundle of wine and bread such that, if Bob bought this bundle, we would know that Bob's tastes are not the same as Pierre's tastes.____

____

____.

7.4 (0) Here is a table of prices and the demands of a consumer named Ronald whose behavior was observed in 5 different price-income situations.

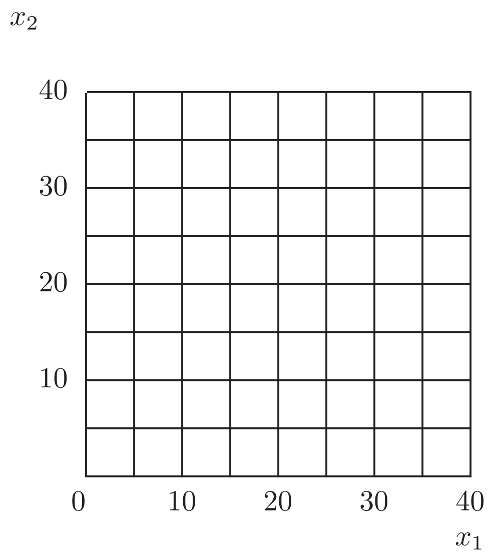
Situation	p_1	p_2	x_1	x_2
A	1	1	5	35
B	1	2	35	10
C	1	1	10	15
D	3	1	5	15
E	1	2	10	10

(a) Sketch each of his budget lines and label the point chosen in each case by the letters A, B, C, D, and E.

(b) Is Ronald's behavior consistent with the Weak Axiom of Revealed Preference?____.

(c) Shade lightly in red ink all of the points that you are certain are worse for Ronald than the bundle C.

(d) Suppose that you are told that Ronald has convex and monotonic preferences and that he obeys the strong axiom of revealed preference. Shade lightly in blue ink all of the points that you are certain are *at least as good as* the bundle C.



7.5 (0) Horst and Nigel live in different countries. Possibly they have different preferences, and certainly they face different prices. They each consume only two goods, x and y. Horst has to pay 14 marks per unit of x and 5 marks per unit of y. Horst spends his entire income of 167 marks on 8 units of x and 11 units of y. Good x costs Nigel 9 quid per unit and good y costs him 7 quid per unit. Nigel buys 10 units of x and 9 units of y.

(a) Which prices and income would Horst prefer, Nigel's income and prices or his own, or is there too little information to tell? Explain your answer.____

____.

(b) Would Nigel prefer to have Horst's income and prices or his own, or is there too little information to tell?________________________________

____.

7.6 (0) Here is a table that illustrates some observed prices and choices for three different goods at three different prices in three different situations.

Situation	p_1	p_2	p_3	x_1	x_2	x_3
A	1	2	8	2	1	3
B	4	1	8	3	4	2
C	3	1	2	2	6	2

(a) We will fill in the table below as follows. Where i and j stand for any of the letters A, B, and C in Row i and Column j of the matrix, write the value of the Situation-j bundle at the Situation-i prices. For example, in Row A and Column A, we put the value of the bundle purchased in Situation A at Situation A prices. From the table above, we see that in Situation A, the consumer bought bundle $(2,1,3)$ at prices $(1,2,8)$. The cost of this bundle A at prices A is therefore $(1\times 2)+(2\times 1)+(8\times 3) = 28$, so we put 28 in Row A, Column A. In Situation B the consumer bought bundle $(3,4,2)$. The value of the Situation-B bundle, evaluated at the situation-A prices is $(1 \times 3) + (2 \times 4) + (8 \times 2) = 27$, so put 27 in Row A, Column B. We have filled in some of the boxes, but we leave a few for you to do.

Prices/Quantities	A	B	C
A	28	27	
B		32	30
C	13	17	

(b) Fill in the entry in Row i and Column j of the table below with a D if the Situation-i bundle is directly revealed preferred to the Situation-j bundle. For example, in Situation A the consumer's expenditure is \$28. We see that at Situation-A prices, he could also afford the Situation-B bundle, which cost 27. Therefore the Situation-A bundle is directly revealed preferred to the Situation-B bundle, so we put a D in Row A, Column B. Now let us consider Row B, Column A. The cost of the Situation-B

bundle at Situation-B prices is 32. The cost of the Situation-A bundle at Situation-B prices is 33. So, in Situation B, the consumer could not afford the Situation-A bundle. Therefore Situation B is *not* directly revealed preferred to Situation A. So we leave the entry in Row B, Column A blank. Generally, there is a D in Row i Column j if the number in the ij entry of the table in part (a) is less than or equal to the entry in Row i, Column i. There will be a violation of WARP if for some i and j, there is a D in Row i Column j and also a D in Row j, Column i. Do these observations violate WARP?______________________________.

Situation	A	B	C
A	—		
B		—	D
C			—

(c) Now fill in Row i, Column j with an I if observation i is *indirectly* revealed preferred to j. Do these observations violate the Strong Axiom of Revealed Preference?______________________________.

7.7 (0) It is January, and Joe Grad, whom we met in Chapter 5, is shivering in his apartment when the phone rings. It is Mandy Manana, one of the students whose price theory problems he graded last term. Mandy asks if Joe would be interested in spending the month of February in her apartment. Mandy, who has switched majors from economics to political science, plans to go to Aspen for the month and so her apartment will be empty (alas). All Mandy asks is that Joe pay the monthly service charge of \$40 charged by her landlord and the heating bill for the month of February. Since her apartment is much better insulated than Joe's, it only costs \$1 per month to raise the temperature by 1 degree. Joe thanks her and says he will let her know tomorrow. Joe puts his earmuffs back on and muses. If he accepts Mandy's offer, he will still have to pay rent on his current apartment but he won't have to heat it. If he moved, heating would be cheaper, but he would have the \$40 service charge. The outdoor temperature averages 20 degrees Fahrenheit in February, and it costs him \$2 per month to raise his apartment temperature by 1 degree. Joe is still grading homework and has \$100 a month left to spend on food and utilities after he has paid the rent on his apartment. The price of food is still \$1 per unit.

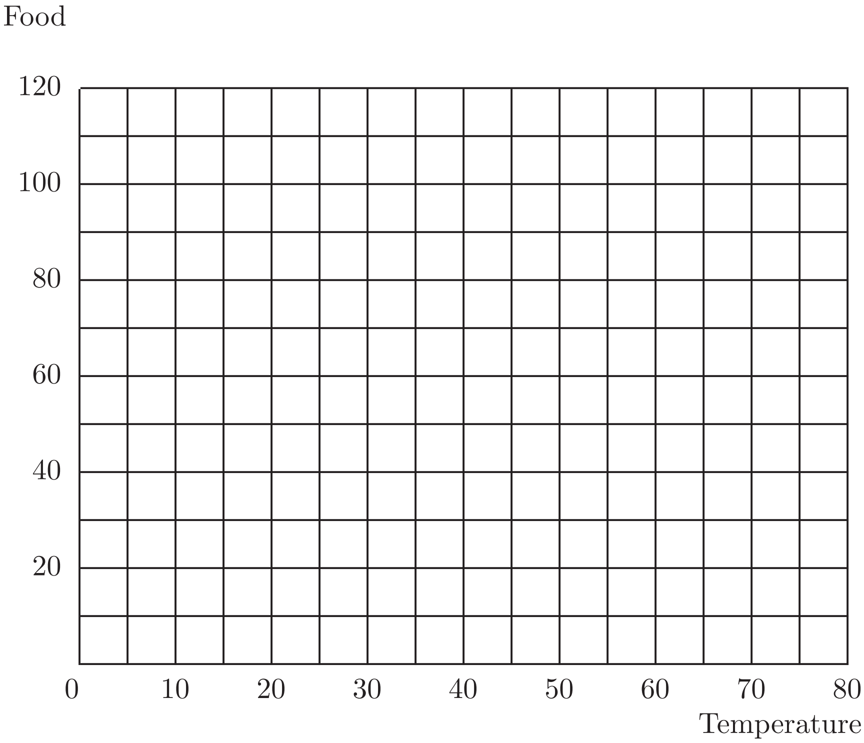
(a) Draw Joe's budget line for February if he moves to Mandy's apartment and on the same graph, draw his budget line if he doesn't move.

(b) After drawing these lines himself, Joe decides that he would be better off not moving. From this, we can tell, using the principle of revealed preference that Joe must plan to keep his apartment at a temperature of

less than____.

(c) Joe calls Mandy and tells her his decision. Mandy offers to pay half the service charge. Draw Joe's budget line if he accepts Mandy's new offer. Joe now accepts Mandy's offer. From the fact that Joe accepted this offer we can tell that he plans to keep the temperature in Mandy's

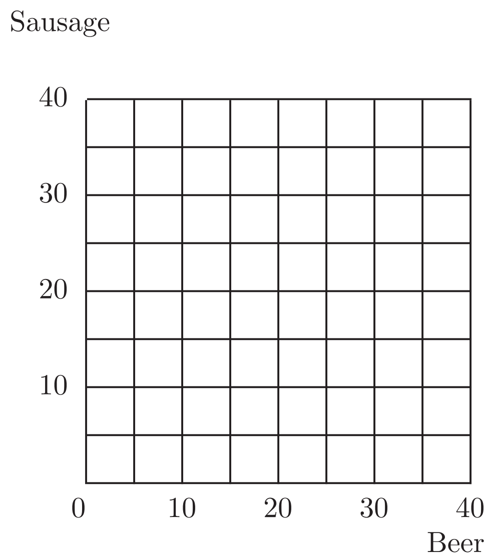
apartment above____.



7.8 (0) Lord Peter Pommy is a distinguished criminologist, schooled in the latest techniques of forensic revealed preference. Lord Peter is investigating the disappearance of Sir Cedric Pinchbottom who abandoned his aging mother on a street corner in Liverpool and has not been seen since. Lord Peter has learned that Sir Cedric left England and is living under an assumed name somewhere in the Empire. There are three suspects, R. Preston McAfee of Brass Monkey, Ontario, Canada, Richard Manning of North Shag, New Zealand, and Richard Stevenson of Gooey Shoes, Falkland Islands. Lord Peter has obtained Sir Cedric's diary, which recorded his consumption habits in minute detail. By careful observation, he has also discovered the consumption behavior of McAfee, Manning, and Stevenson. All three of these gentlemen, like Sir Cedric, spend their entire incomes on beer and sausage. Their dossiers reveal the following:

- **Sir Cedric Pinchbottom** — In the year before his departure, Sir Cedric consumed 10 kilograms of sausage and 20 liters of beer per week. At that time, beer cost 1 English pound per liter and sausage cost 1 English pound per kilogram.
- **R. Preston McAfee** — McAfee is known to consume 5 liters of beer and 20 kilograms of sausage. In Brass Monkey, Ontario beer costs 1 Canadian dollar per liter and sausage costs 2 Canadian dollars per kilogram.
- **Richard Manning** — Manning consumes 5 kilograms of sausage and 10 liters of beer per week. In North Shag, a liter of beer costs 2 New Zealand dollars and sausage costs 2 New Zealand dollars per kilogram.
- **Richard Stevenson** — Stevenson consumes 5 kilograms of sausage and 30 liters of beer per week. In Gooey Shoes, a liter of beer costs 10 Falkland Island pounds and sausage costs 20 Falkland Island pounds per kilogram.

(a) Draw the budget line for each of the three fugitives, using a different color of ink for each one. Label the consumption bundle that each chooses. On this graph, superimpose Sir Cedric's budget line and the bundle he chose.



(b) After pondering the dossiers for a few moments, Lord Peter announced. "Unless Sir Cedric has changed his tastes, I can eliminate one of the suspects. Revealed preference tells me that one of the suspects is innocent." Which one?________________________.

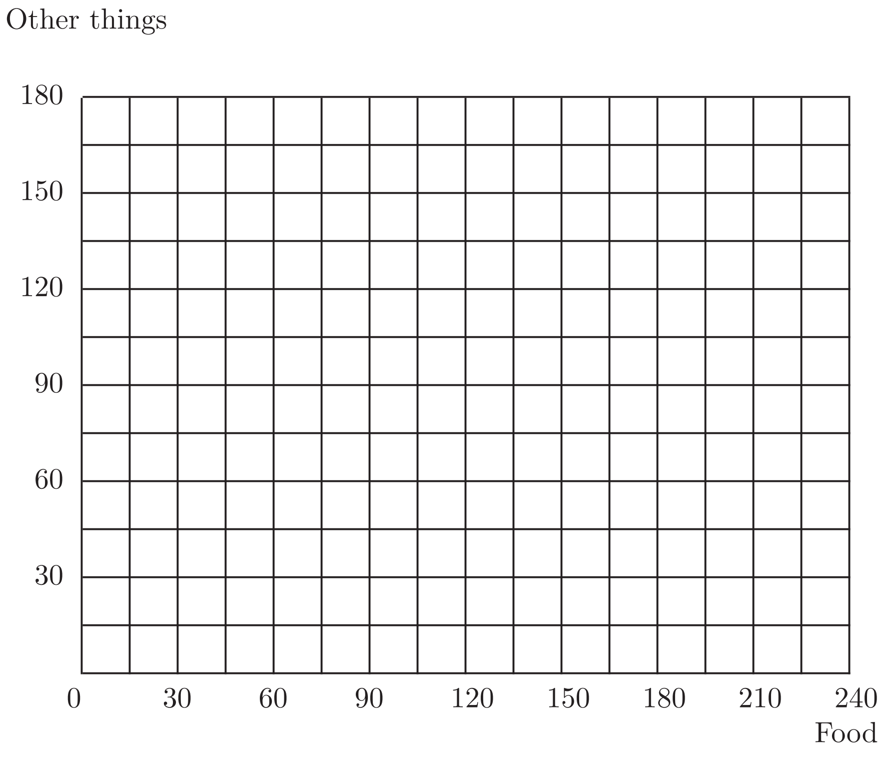
(c) After thinking a bit longer, Lord Peter announced. "If Sir Cedric left voluntarily, then he would have to be better off than he was before. Therefore if Sir Cedric left voluntarily and if he has not changed his tastes, he must be living in________________________.

7.9 (1) The McCawber family is having a tough time making ends meet. They spend \$100 a week on food and \$50 on other things. A new welfare program has been introduced that gives them a choice between receiving a grant of \$50 per week that they can spend any way they want, and buying any number of \$2 food coupons for \$1 apiece. (They naturally are not allowed to resell these coupons.) Food is a normal good for the McCawbers. As a family friend, you have been asked to help them decide on which option to choose. Drawing on your growing fund of economic knowledge, you proceed as follows.

(a) On the graph below, draw their old budget line in red ink and label their current choice C. Now use black ink to draw the budget line that they would have with the grant. If they chose the coupon option, how much food could they buy if they spent all their money on food coupons?

__________ How much could they spend on other things if they bought

no food? __________ Use blue ink to draw their budget line if they choose the coupon option.



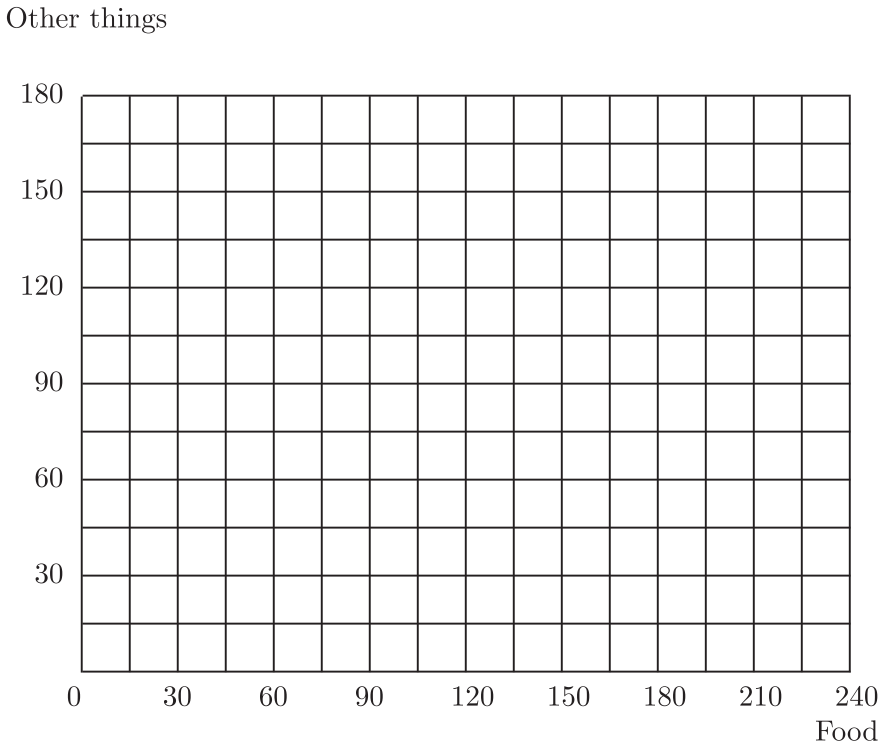
(b) Using the fact that food is a normal good for the McCawbers, and knowing what they purchased before, darken the portion of the black budget line where their consumption bundle could possibly be if they chose the lump-sum grant option. Label the ends of this line segment A and B.

____

(c) After studying the graph you have drawn, you report to the McCawbers. "I have enough information to be able to tell you which choice to make. You should choose the __________ because____________________

____.

(d) Mr. McCawber thanks you for your help and then asks, "Would you have been able to tell me what to do if you hadn't known whether food was a normal good for us?" On the axes below, draw the same budget lines you drew on the diagram above, but draw indifference curves for which food is not a normal good and for which the McCawbers would be better off with the program you advised them not to take.



7.10 (0) In 1933, the Swedish economist Gunnar Myrdal (who later won a Nobel prize in economics) and a group of his associates at Stockholm University collected a fantastically detailed historical series of prices and price indexes in Sweden from 1830 until 1930. This was published in a book called *The Cost of Living in Sweden*. In this book you can find 100 years of prices for goods such as oat groats, hard rye bread, salted codfish, beef, reindeer meat, birchwood, tallow candles, eggs, sugar, and coffee. There are also estimates of the quantities of each good consumed by an average working-class family in 1850 and again in 1890.

The table below gives prices in 1830, 1850, 1890, and 1913, for flour, meat, milk, and potatoes. In this time period, these four staple foods accounted for about 2/3 of the Swedish food budget.

Prices of Staple Foods in Sweden

Prices are in Swedish kronor per kilogram, except for milk, which is in Swedish kronor per liter.

	1830	1850	1890	1913
Grain Flour	.14	.14	.16	.19
Meat	.28	.34	.66	.85
Milk	.07	.08	.10	.13
Potatoes	.032	.044	.051	.064

Based on the tables published in Myrdal's book, typical consumption bundles for a working-class Swedish family in 1850 and 1890 are listed below. (The reader should be warned that we have made some approximations and simplifications to draw these simple tables from the much more detailed information in the original study.)

Quantities Consumed by a Typical Swedish Family

Quantities are measured in kilograms per year, except for milk, which is measured in liters per year.

	1850	1890
Grain Flour	165	220
Meat	22	42
Milk	120	180
Potatoes	200	200

(a) Complete the table below, which reports the annual cost of the 1850 and 1890 bundles of staple foods at various years' prices.

Cost of 1850 and 1890 Bundles at Various Years' Prices

Cost	1850 bundle	1890 bundle
Cost at 1830 Prices	44.1	61.6
Cost at 1850 Prices		
Cost at 1890 Prices		
Cost at 1913 Prices	78.5	113.7

(b) Is the 1890 bundle revealed preferred to the 1850 bundle?________.

(c) The Laspeyres quantity index for 1890 with base year 1850 is the ratio of the value of the 1890 bundle at 1850 prices to the value of the 1850 bundle at 1850 prices. Calculate the Laspeyres quantity index of staple food consumption for 1890 with base year 1850.________________.

____

(d) The Paasche quantity index for 1890 with base year 1850 is the ratio of the value of the 1890 bundle at 1890 prices to the value of the 1850 bundle at 1890 prices. Calculate the Paasche quantity index for 1890 with base year 1850.______________.

(e) The Laspeyres price index for 1890 with base year 1850 is calculated using 1850 quantities for weights. Calculate the Laspeyres price index for 1890 with base year 1850 for this group of four staple foods.______.

(f) If a Swede were rich enough in 1850 to afford the 1890 bundle of staple foods in 1850, he would have to spend ______ times as much on these foods as does the typical Swedish worker of 1850.

(g) If a Swede in 1890 decided to purchase the same bundle of food staples that was consumed by typical 1850 workers, he would spend the fraction ____ of the amount that the typical Swedish worker of 1890 spends on these goods.

7.11 (0) This question draws from the tables in the previous question. Let us try to get an idea of what it would cost an American family at today's prices to purchase the bundle consumed by an average Swedish family in 1850. In the United States today, the price of flour is about $.40 per kilogram, the price of meat is about $3.75 per kilogram, the price of milk is about $.50 per liter, and the price of potatoes is about $1 per kilogram. We can also compute a Laspeyres price index across time and across countries and use it to estimate the value of a current US dollar relative to the value of an 1850 Swedish kronor.

(a) How much would it cost an American at today's prices to buy the bundle of staple food commodities purchased by an average Swedish working-class family in 1850?______________.

(b) Myrdal estimates that in 1850, about 2/3 of the average family's budget was spent on food. In turn, the four staples discussed in the last question constitute about 2/3 of the average family's food budget. If the prices of other goods relative to the price of the food staples are similar in the United States today to what they were in Sweden in 1850, about how much would it cost an American at current prices to consume the same overall consumption bundle consumed by a Swedish working-class family in 1850?______________.

(c) Using the Swedish consumption bundle of staple foods in 1850 as weights, calculate a Laspeyres price index to compare prices in current American dollars relative to prices in 1850 Swedish kronor.__________ If we use this to estimate the value of current dollars relative to 1850 Swedish kronor, we would say that a U.S. dollar today is worth about ______ 1850 Swedish kronor.

7.12 (0) Suppose that between 1960 and 1985, the price of all goods exactly doubled while every consumer's income tripled.

(a) Would the Laspeyres price index for 1985, with base year 1960 be less than 2, greater than 2, or exactly equal to 2? ____________ What about the Paasche price index?__________________________________.

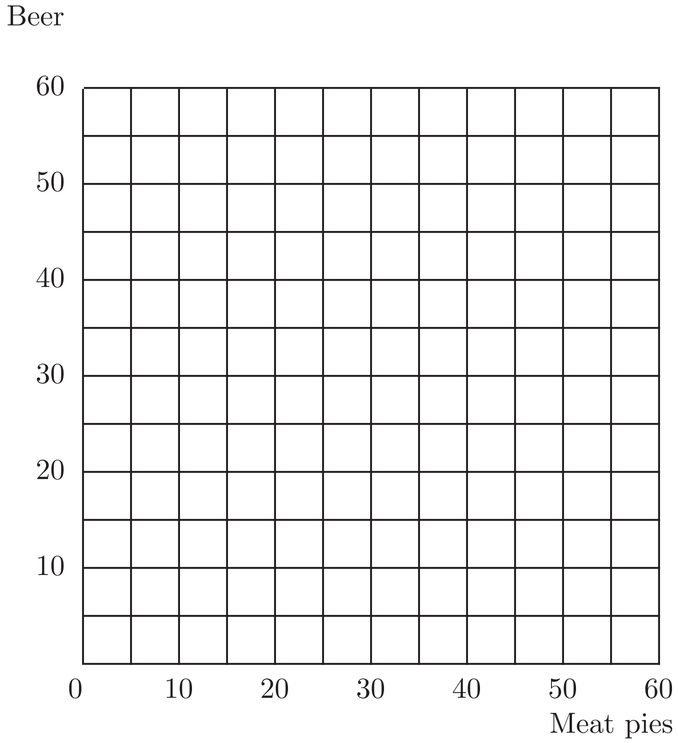
(b) If bananas are a normal good, will total banana consumption increase?__________ If everybody has homothetic preferences, can you determine by what percentage total banana consumption must have increased? Explain.____

____

____.

7.13 (1) Norm and Sheila consume only meat pies and beer. Meat pies used to cost \$2 each and beer was \$1 per can. Their gross income used to be \$60 per week, but they had to pay an income tax of \$10. Use red ink to sketch their old budget line for meat pies and beer.

____



(a) They used to buy 30 cans of beer per week and spent the rest of their income on meat pies. How many meat pies did they buy?____________.

(b) The government decided to eliminate the income tax and to put a sales tax of \$1 per can on beer, raising its price to \$2 per can. Assuming that Norm and Sheila's pre-tax income and the price of meat pies did not change, draw their new budget line in blue ink.

(c) The sales tax on beer induced Norm and Sheila to reduce their beer consumption to 20 cans per week. What happened to their consumption of meat pies? ____________________ How much revenue did this tax raise from Norm and Sheila?______________________________________.

(d) This part of the problem will require some careful thinking. Suppose that instead of just taxing beer, the government decided to tax *both* beer and meat pies at the *same* percentage rate, and suppose that the price of beer and the price of meat pies each went up by the full amount of the tax. The new tax rate for both goods was set high enough to raise exactly the same amount of money from Norm and Sheila as the tax on beer used to raise. This new tax collects \$_______ for every bottle of beer sold and \$________ for every meat pie sold. (Hint: If both goods are

taxed at the same rate, the effect is the same as an income tax.) How large an income tax would it take to raise the same revenue as the $1 tax on beer? __________ Now you can figure out how big a tax on each good is equivalent to an income tax of the amount you just found.

(e) Use black ink to draw the budget line for Norm and Sheila that corresponds to the tax in the last section. Are Norm and Sheila better off having just beer taxed or having both beer and meat pies taxed if both sets of taxes raise the same revenue? __________ (Hint: Try to use the principle of revealed preference.)