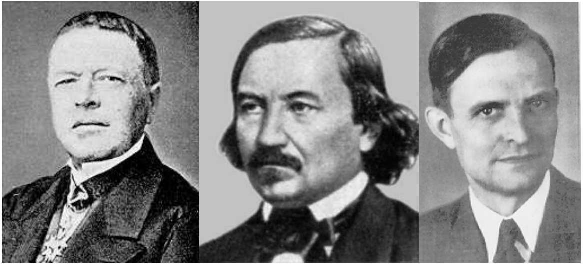
Game theory and oligopoly

Varian: Intermediate Microeconomics, sections 27.1-9, 28.1-3, 28.7-8

In this lecture you learn

- what oligopoly and game theory are
- how to find equilibrium in simultaneous and sequential games
- how the Cournot, Bertrand, Stackelberg, and price-leadership models work
- how to shoot penalty kicks
- how internet shops can make profit



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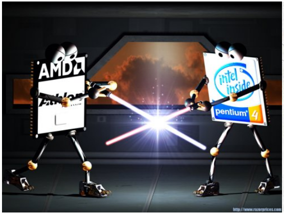
Examples of oligopolistic markets:

- microprocessors (Intel vs. AMD)
- soft drinks (Coke vs. Pepsi)
- cell phones
- cars
- ...



Oligopoly

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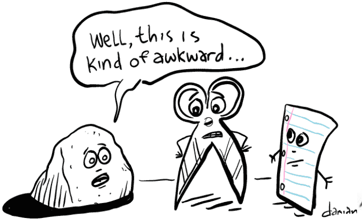
Game = a mathematical model of conflict/cooperation between rational decision-makers.

In this lecture we will discuss two types of games:

- simultaneous game – players decide simultaneously
- sequential game – players decide in a given order



Simultaneous game

Simultaneous game consists of

- a set of players
- a set of actions (strategies) for each player
- each player has preferences defined over the **action profiles** (action profile = one combination of actions of all payers)

Example of a simultaneous game

Let's have a simultaneous game consisting of

- players A and B
- a set of actions for
  - player A (top T, bottom B)
  - player B (left L, right R)
- preferences defined by payoffs for all action profiles, where
  - player A's preferences are $TL \succ BR \succ BL \sim TR$
  - player B's preferences are $BR \succ TL \sim BL \sim TR$

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  - player B's preferences are $BR \succ TL \sim BL \sim TR$

The game may have the following payoff matrix:

		Player B	
		Left	Right
Player A	Top	2, 1	0, 0
	Bottom	0, 0	1, 2

Nash equilibrium

Reaction function shows the optimal actions of a player for all combinations of actions of other players.

		Player B	
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Table: e.g. the reaction function of A is $f_A(L) = T$ and $f_A(R) = B$.

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Table: e.g. the reaction function of A is $f_A(L) = T$ and $f_A(R) = B$.

Nash equilibrium is the action profile at which each player chooses the optimal action given actions of all other players.

The game in the table has two Nash equilibria TL and BR:

- TL because $f_A(L) = T$ a $f_B(T) = L$
- BR because $f_A(R) = B$ a $f_B(B) = R$

		Player B	
		Left	Right
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Equilibrium in dominant strategies

Equilibrium in dominant strategies is an action profile at which each player chooses her dominant strategy.

Dominanant strategy is such an action of a player that is optimal given all possible actions of the other players.

		Player B	
		Left	Right
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In this table, the dominant strategy of player A is **bottom** and of player B is **left**. The equilibrium in dominant strategies is *BL*.

		Player B	
		Left	Right
Player A	Top	$1, 2^*$	$0, 1$
	Bottom	$2^*, 1^*$	$1^*, 0$

Pure and mixed strategies

Pure strategies = actions that each player chooses with certainty.
Some games do not have any Nash equilibrium in pure strategies.

Mixed strategies = probability distribution over actions.
E.g. player A chooses **top** with 75% and **bottom** with 25% probab.
Each game has a Nash equilibrium in mixed strategies.

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Does *rock-paper-scissors* have an equilibrium in pure strategies?

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What is the mixed equilibrium *rock-paper-scissors*?

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Each player chooses the actions randomly with a probability of $1/3$.

EXAMPLE: Penalty kicks

Chiappori, Groseclose and Levitt (AER, 2002) studied how football players shoot and goalkeepers catch penalties.

The players' actions divided in 3 variants: left, right, middle

Finding: The success rate of penalties the same in all variants. $\implies$ Players and keepers select actions randomly with a probability of $1/3$.

Cournot model

Cournot model is a simultaneous game in which

- players are firms,
- actions are quantities produced by the firms,
- preferences are given by profits of the firms.

Profit-maximizing firms choose quantities simultaneously.

Nash (Cournot) equilibrium is a combination of firms' quantities at which each firm reacts optimally to quantities of the other firms.

Cournot equilibrium with 2 firms

The basic version of the Cournot model:

- two firms – firm 1 and firm 2 produce quantities y_1 and y_2
- zero costs – $c_1(y_1) = 0$ and $c_2(y_2) = 0$
- identical product – the inverse market demand is $p = a - b(y_1 + y_2)$

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Firms maximize profits

$$\max_{y_1} \pi_1(y_1, y_2) \quad \text{and} \quad \max_{y_2} \pi_2(y_1, y_2)$$

where the profits of firms 1 and 2 are

$$\pi_1(y_1, y_2) = r(y_1, y_2) - c(y_1) = (a - b(y_1 + y_2))y_1 = ay_1 - by_1^2 - by_1 y_2$$

$$\pi_2(y_1, y_2) = r(y_1, y_2) - c(y_2) = (a - b(y_1 + y_2))y_2 = ay_2 - by_2^2 - by_1 y_2$$

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- identical product – the inverse market demand is $p = 10 - (y_1 + y_2)$

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Cournot equilibrium with 2 firms (cont'd)

Differentiating both profit functions, we get the first order conditions:

$$a - 2by_1 - by_2 = 0 \quad \text{and} \quad a - 2by_2 - by_1 = 0.$$

Deriving the reaction functions of firms 1 and 2 from the FOC:

$$f_1(y_2) = y_1 = \frac{a - by_2}{2b} \quad \text{and} \quad f_2(y_1) = y_2 = \frac{a - by_1}{2b} \tag{1}$$

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Cournot equilibium is a combination of outputs (y_1^*, y_2^*) such that

$$y_1^* = f_1(y_2^*) \quad \text{and} \quad y_2^* = f_2(y_1^*).$$

Solving the equations (1) we get the Cournot equilibrium

$$(y_1^*, y_2^*) = \left(\frac{a}{3b}, \frac{a}{3b}\right).$$

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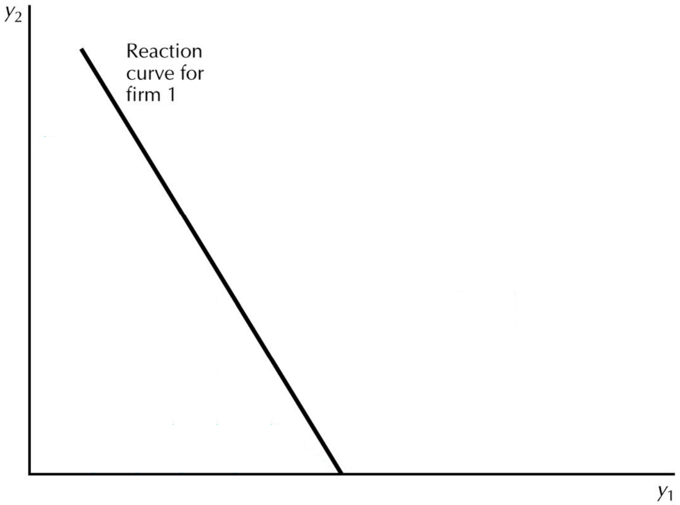
$$(y_1^*, y_2^*) = \left(\frac{10}{3}, \frac{10}{3}\right).$$

Cournot equilibrium with 2 firms (graph)



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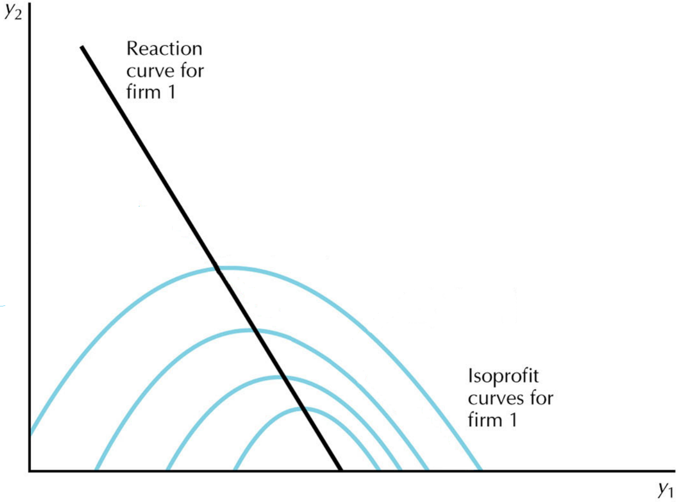

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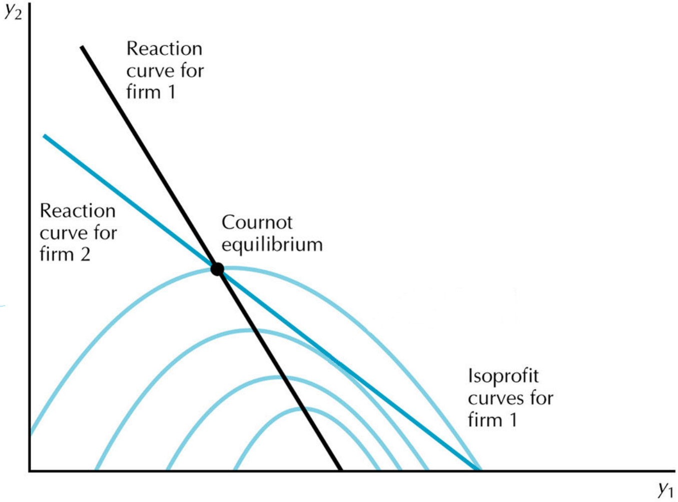

Cournot equilibrium with n firms

The total output of an industry is $Y = y_1 + \cdots + y_n$, where n is the number of industry firms. Firm i maximizes profit:

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Multiplying $\frac{dp(Y)}{dY} y_i$ on the LHS by $\frac{p(Y)}{p(Y)} \frac{Y}{Y}$, we get

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By rearranging the LHS, we get

$$p(Y)\left(1 + \frac{dp(Y)}{dY} \frac{Y}{p(Y)} \frac{y_i}{Y}\right) = MC(y_i).$$

Cournot equilibrium with n firms (cont'd)

$$p(Y)\left(1+\frac{dp(Y)}{dY}\frac{Y}{p(Y)}\frac{y_i}{Y}\right)=MC(y_i)$$

$$p(Y)\left(1-\frac{s_i}{|\epsilon(Y)|}\right)=MC(y_i)$$

where

- $s_i = y_i/Y$ is the share of firm i in the market,
- $\epsilon(Y) = \frac{dY}{dp(Y)}\frac{p(Y)}{Y}$ is the elasticity of demand.

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The market contains

- 1 firm ($s_i = 1$), it is the monopoly markup,
- many firms ($s_i \to 0$), $p_i \to MC$ as in perfect competition.

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Control question – decreasing demand, firms the same MC ($F = 0$):

What happens with p and Q, if another firm enters the market?

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Control question – decreasing demand, firms the same MC $(F = 0)$:

What happens with p and Q, if another firm enters the market?
The price p decreases and market quantity Q increases.

Bertrand model

Bertrand model is a simultaneous game in which

- players are firms,
- actions are prices,
- preferences are given by profits of the firms.

Profit-maximizing firms choose prices simultaneously.

Nash (Bertrand) equilibrium is such a combination of firms' prices at which each firm reacts optimally to prices of the other firms.

Bertrand equality with 2 firms

The basic version of Bertrand model:

- firms 1 and 2 with marginal costs $MC_1(y_1) = MC_2(y_2) = c$ $(FC = 0)$
- inverse market demand $p(y) = a - by$, where $y = y_1 + y_2$
- demand for firms' production depends on their prices:
    - $p_1 < p_2 \implies$ firm 1 sells the entire market quantity
    - $p_1 = p_2 \implies$ both firms sell half the market quantity
    - $p_1 > p_2 \implies$ firm 2 sells the entire market quantity

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Nash (Bertrand) equilibrium is $(p_1^*, p_2^*) = (c, c)$.

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$p_1 = c$ is an optimal reaction of firm 1 to $p_2 = c$ (and vice-versa)

- at price $p_1 > c$, firm 1 would not sell anything (is not better off)
- at price $p_1 < c$, firm 1 would be in loss (is worse off)

EXAMPLE: Internet sales

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Firms try to avoid zero profits using obfuscation.

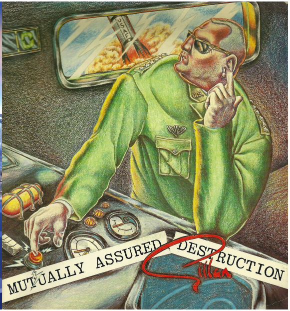
Strategy:

1. offer a cheap but inferior product for price search engine
2. customers click on the product – are redirected to the firm's website
3. on the website they find an attractive offer of
    - a more expensive product upgrade
    - product add-ons (additional screws or a snazzy mouse pad)

Sequential game

Sequential game consists of

- a set of players
- a finite set of histories
- a player function that determines whose turn it is
- preferences defined over the set of histories for each player.



Example of a sequential game

Let's have a sequential game that contains

- players A and B,
- a set of finite histories (TL, TR, BL, BR),
- a player function: first player A, than player B,
- preferences are given by the payoffs for all histories:
  - player A's preferences:
    $BR \succ TL \sim TR \succ BL$
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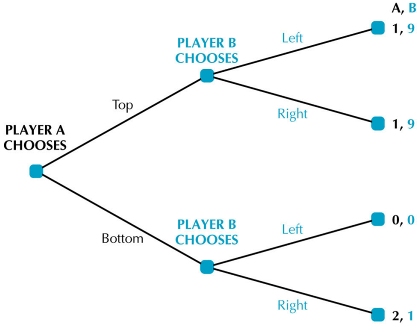
The game may have the following game tree (the game in **extensive form**):

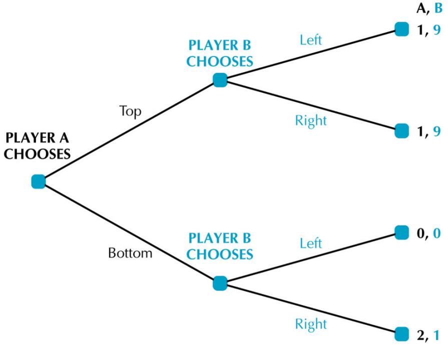


Subgame perfect equilibrium (SPE)

The game has two Nash equilibria: *TL* and *BR*. Only *BR* is the **subgame perfect equilibrium (SPE)**.

SPE is the action profile in which each player in each subgame (i.e. in each moment she chooses) plays the optimal action given the actions of all other players.

The game is solved using backward induction:



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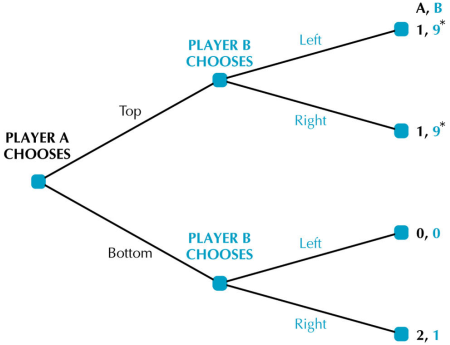
- player B chooses according to her reaction function:
  - $f_B(T) = L, R$



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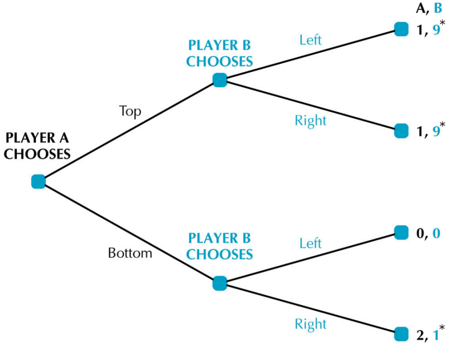

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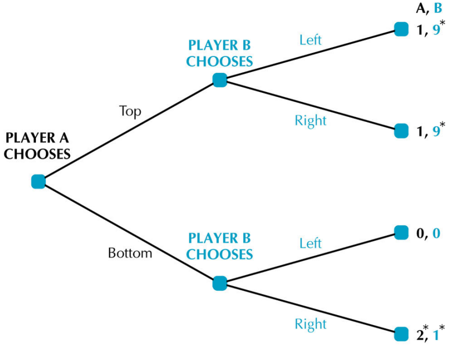
The game is solved using backward induction:

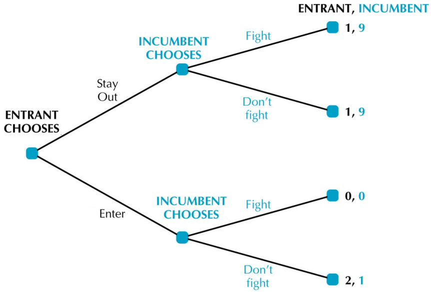
- player B chooses according to her reaction function:
  - $f_B(T) = L, R$
  - $f_B(B) = R$
- player A chooses B, because $BR \succ TL \sim TR$



Example – entry deterrence

Similar game as in the previous example – a different motivation.

If the payoffs are the same as in the previous game (the left figure), the entrant picks *enter* and the incumbent *does not fight*.



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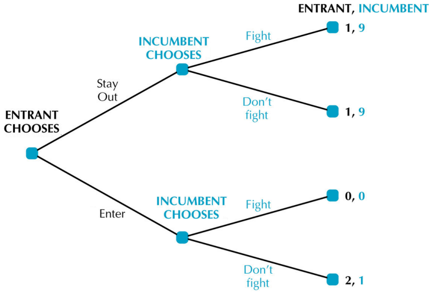

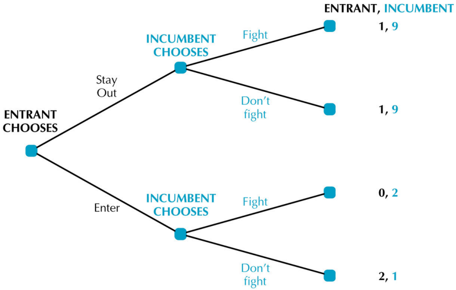

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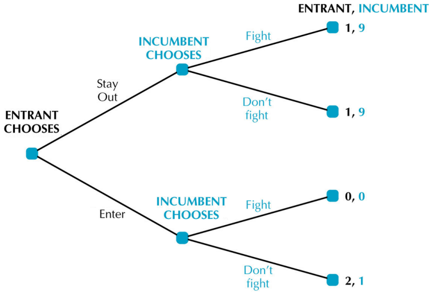

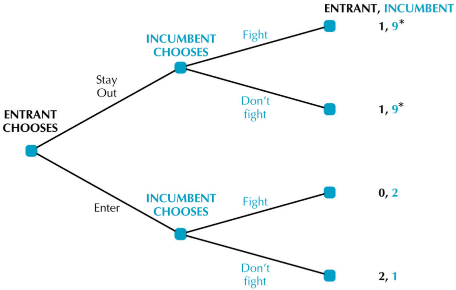

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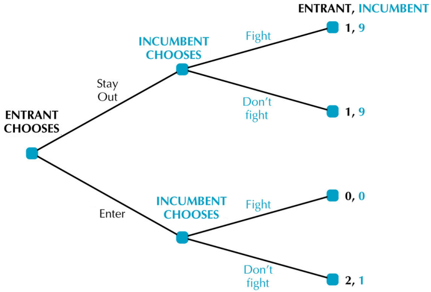

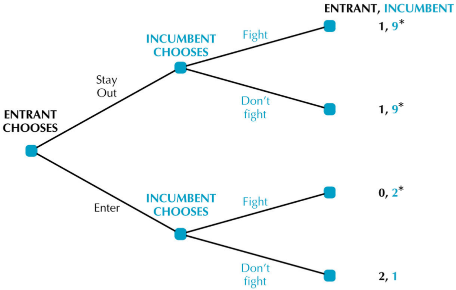

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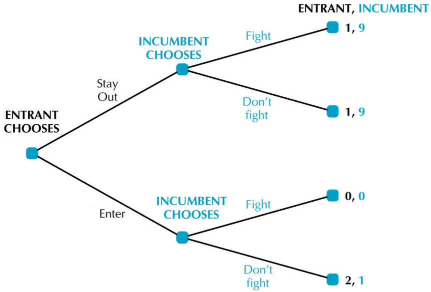

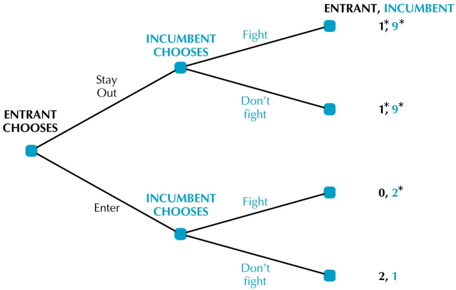

Stackelberg model

Stackelberg model (quantity leadership) is a sequential game where

- players are firms – leader and follower,
- the leader chooses quantity first and the follower second,
- preferences are given by profits of the firms.

Examples:

- the industry contains one dominant firm – effect of capacities
- models with entry and building capacities (ports)

Stackelberg equilibrium is the subgame prefect equilibrium (SPE) – the leader chooses the optimal quantity given the reaction function of the follower.

Stackelberg equilbrium

A basic version of Stackelberg model:

- firm 1 is the leader, firm 2 is the follower
- zero costs – $c_1(y_1) = 0$ and $c_2(y_2) = 0$
- identical product – the inverse market demand is $p = a - b(y_1 + y_2)$

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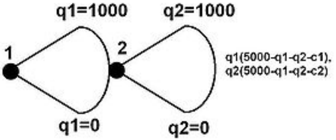
Backward induction:

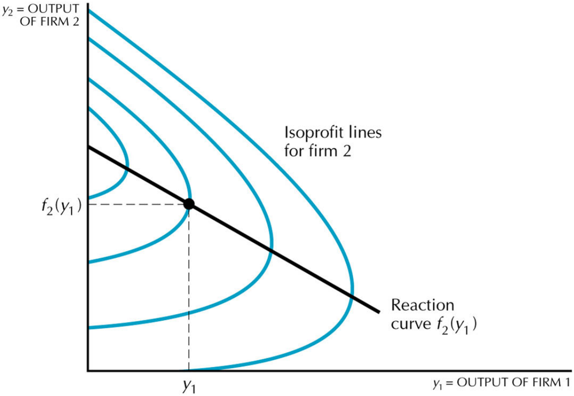
- For each possible output of the leader, the follower chooses the profit-maximizing quantity (the reaction function of firm 2).
- The leader chooses the profit-maximizing quantity given the reaction function of the follower.



Stackelberg equilbrium (cont'd)

The follower maximizes her profit $\pi_2(y_1, y_2)$. From the FOC $a - 2by_2 - by_1 = 0$, we calculate the reaction function of firm 2

$$f_2(y_1) = y_2 = \frac{a - by_1}{2b}.$$



Stackelberg equilbrium (cont'd)

The leader solves the profit-maximizing problem

$$\max_{y_1} \pi(y_1, y_2) = p(y_1 + f_2(y_1))y_1 - c_1(y_1).$$

Substituting the demand and the reaction function of firm 2:

$$\max_{y_1} \pi(y_1, y_2) = \left(a - b\left(y_1 + \frac{a - by_1}{2b}\right)\right) y_1 = \frac{a}{2}y_1 - \frac{b}{2}y_1^2.$$

From the first order condition we get the leader's optimal quantity:

$$y_1^* = \frac{a}{2b}$$

Stackelberg equilbrium (cont'd)

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$$y_2^* = \frac{a - b\frac{a}{2b}}{2b} = \frac{a}{4b}$$

Stackelberg equilibrium: $(y_1^*, y_2^*) = \{\frac{a}{2b}, \frac{a}{4b}\}$.

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Is the market quantity higher than in Cournot?

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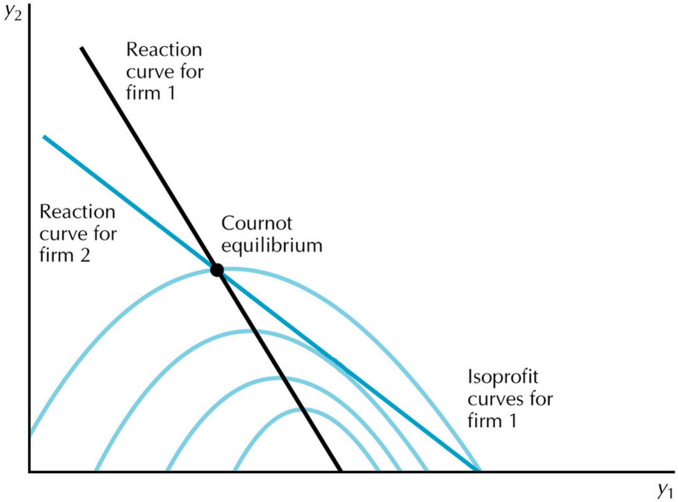
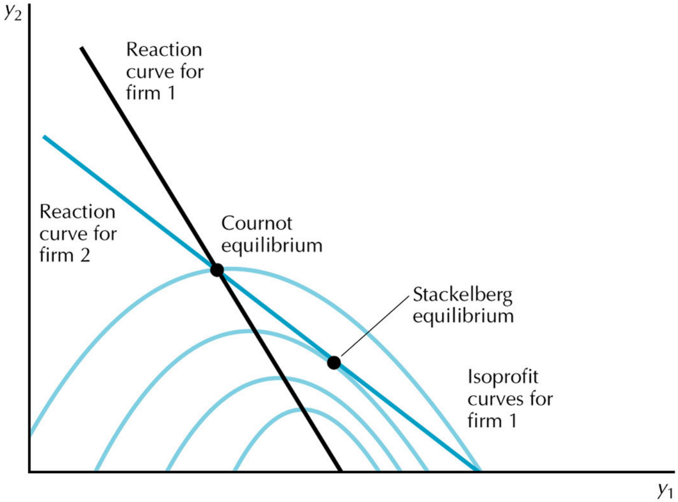
Stackelberg equilibrium: $(y_1^*, y_2^*) = \{\frac{a}{2b}, \frac{a}{4b}\}$. Market quantity: $\frac{3a}{4b}$
Is the market quantity higher than in Cournot? Yes – Cournot: $\frac{2a}{3b}$

Stackelberg equilbrium (graph)



Stackelberg equilbrium (graph)

The leader does not choose the quantity on her reaction function.
She must make a credible commitment to her quantity of output.



Price leadership

Price leadership is a sequential game in which

- players are firms,
- the leader chooses price first and follower(s) second,
- preferences are given by profits of the firms.

Example: One dominant firm sending catalogues with prices.

Solution: the subgame prefect equilibrium (SPE) – the leader chooses the optimal price given a known reaction function of the follower(s).

Price leadership with identical product

The basic version of price leadership:

- firm 1 is the leader, firm 2 is the follower
- the leader has constant $MC = c$, the follower has increasing $MC(y_2)$ (fixed costs $FC_1 = FC_2 = 0$)
- both firms produce identical product

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Backward induction:

- The follower will supply the profit-maximizing quantity at the leader's price ($p_2 = p_1$).
- The leader chooses the price of production that maximizes her profit given the known supply of the follower.

Price leadership with identical product (cont'd)

The follower accepts the leader's price $p_2 = p_1$ and chooses the quantity such that

$$\max_{y_2} p_1 y_2 - c_2(y_2).$$

From the FOC, we derive the profit-maximizing condition (competitive firms have the same condition)

$$p_1 = MC_2(y_2).$$

The reaction function of the follower is $y_2 = S_2(p_1)$, where $S_2(p_1)$ is the competitive supply of the follower.

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Why does the follower choose the price $p_2 = p_1$?

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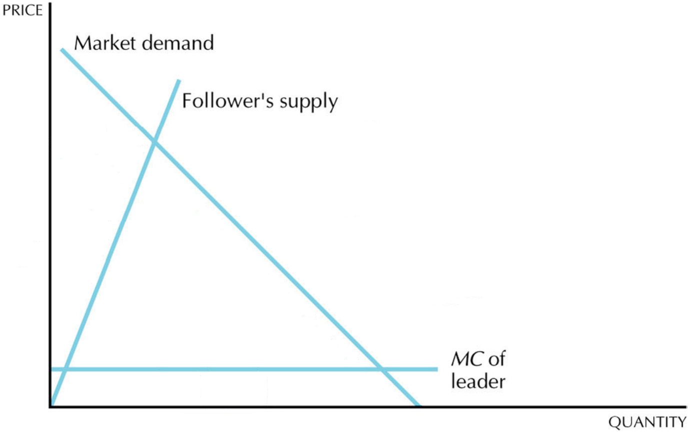
Why does the follower choose the price $p_2 = p_1$?

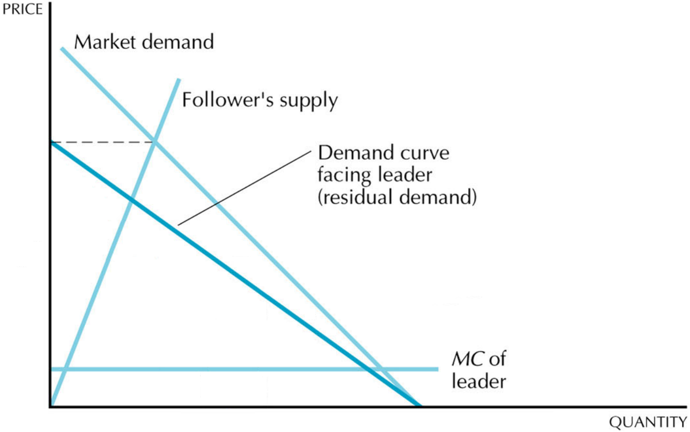
- If she chose $p_2 > p_1$, she would not sell anything (worse off).
- If she chose $p_2 < p_1$, she would have a lower profit (worse off).

Price leadership with identical product (cont'd)



Price leadership with identical product (cont'd)

The leader faces the **residual demand** $R(p_1) = D(p_1) - S_2(p_1)$.



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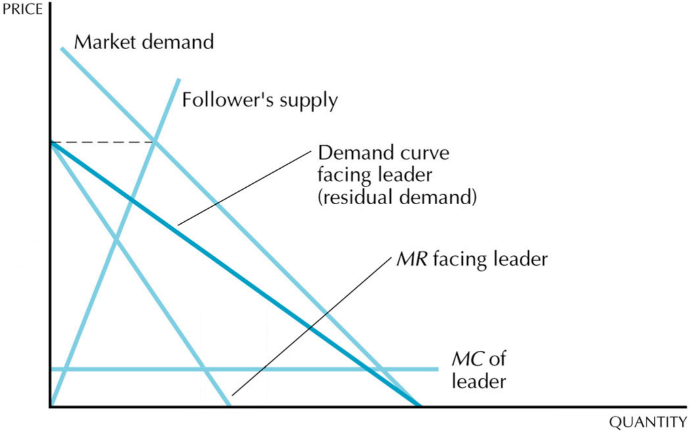
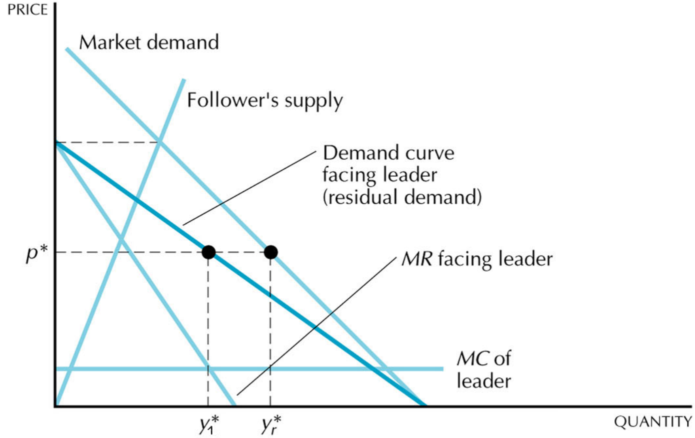


Price leadership with identical product (cont'd)

The leader faces the **residual demand** $R(p_1) = D(p_1) - S_2(p_1)$.

The leader chooses the price $p_1 = p^*$ that maximizes profit $\pi_1(p_1) = (p_1 - c)R(p_1)$. At this price $MC(y_1^*) = MR(y_1^*)$.



Example – price leadership with identical product

Demand curve: $D(p) = 10 - p$
Cost function of the leader: $c_1(y_1) = 2y_1$
Cost function of the follower: $c_2(y_2) = y_2^2/2$

What is the optimal price? What are the quantities of the firms?

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Residual demand for leader's production:

$$R(p_1) = D(p_1) - S_2(p_1) = 10 - p_1 - p_1 = 10 - 2p_1$$

Inverse demand for the leader's production:

$$p_1 = 5 - \frac{y_1}{2}$$

Example – price leadership with identical product (cont'd)

We proceed as in the case of a profit-maximizing monopoly:

$$\max_{y_1} \left(5 - \frac{y_1}{2}\right) y_1 - 2y_1 \quad \text{subject to } y_1 \geq 0$$

The first order condition $MR_1 = MC_1$ is

$$5 - y_1 = 2.$$

The optimal quantity of the leader:

$$y_1^* = 3$$

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The optimal quantity of the leader:

$$y_1^* = 3$$

The equilibrium price and the optimal quantity of the follower:

$$p^* = y_2^* = 5 - \frac{y_1^*}{2} = \frac{7}{2}$$

What should you know?

- In the Nash equilibrium, each player reacts optimally to actions of the other players.
- Cournot equilibrium is the Nash equilibrium of the game, in which firms choose quantities simultaneously.
- Bertrand equilibrium is the Nash equilibrium of the game, in which firms choose prices simultaneously.
- In sequential games we can use backward induction to find out which Nash equilibria are subgame prefect (SPE).
- Stackelberg equilibrium is the SPE of the game, in which firms choose quantities sequentially.
- The equilibrium in price leadership is the SPE of the game, in which firms choose prices sequentially.