

# Budget constraint, preferences, utility

Varian, Intermediate Microeconomics, 8e, chapters 2, 3, and 4

## In this lecture, you will learn

- what budget set and budget line are
- how their shape is influenced by taxes and food stamps
- what preferences are and how they are derived
- what the basic types of preferences are – why some indifference curves are straight and some curved, or circle-shaped
- what we need a utility function for
- how to find out whether to reconstruct a stadium



## Budget constraint

We assume that the consumer chooses a bundle  $(x_1, x_2)$ , where  $x_1$  and  $x_2$  are quantities of goods 1 and 2.

Budget constraint is  $p_1x_1 + p_2x_2 \leq m$ :

- $p_1$  and  $p_2$  are prices of goods 1 and 2
- $m$  is income

**Budget set** – bundles for which:  $p_1x_1 + p_2x_2 \leq m$ .

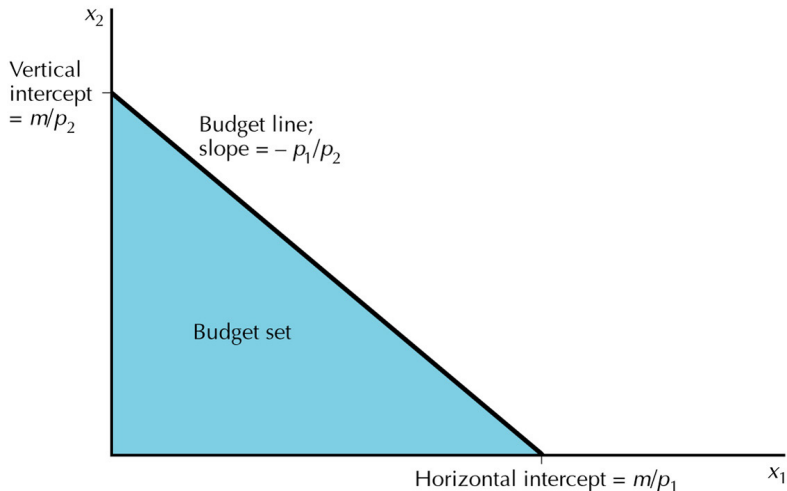
**Budget line (BL)** – bundles for which:  $p_1x_1 + p_2x_2 = m$ .

## Budget set and budget line (graph)

Budget line:  $p_1x_1 + p_2x_2 = m$

## Budget set and budget line (graph)

$$\text{Budget line: } p_1x_1 + p_2x_2 = m \iff x_2 = m/p_2 - (p_1/p_2)x_1$$



## Composite good

The theory works for more than two goods.  
How to plot it in a 2D graph?



## Composite good

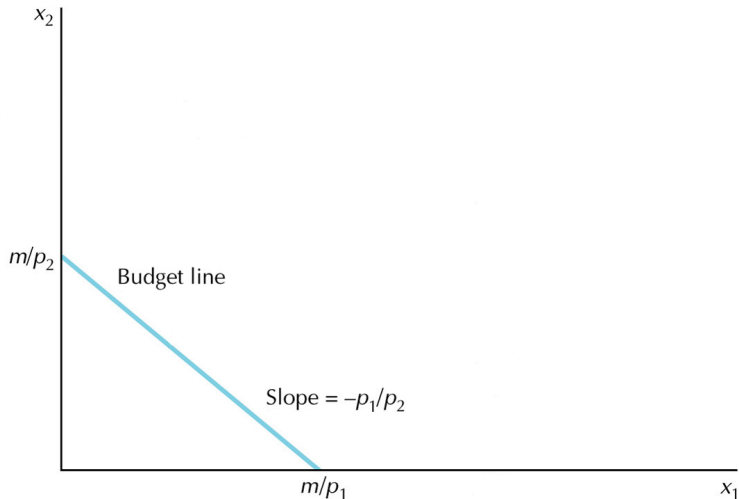
The theory works for more than two goods.  
How to plot it in a 2D graph?

On the  $y$  axis we can plot the **composite good**  
= money value of all other consumed goods.



## Change in income

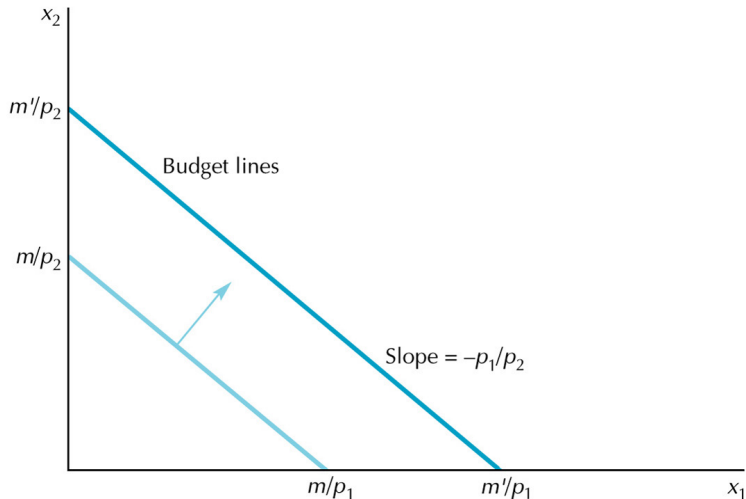
A rise in income from  $m$  to  $m'$





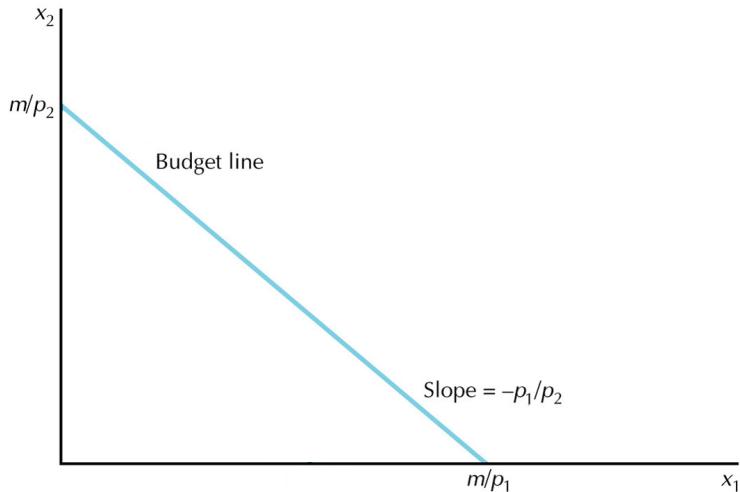
## Change in income

A rise in income from  $m$  to  $m'$   $\implies$  parallel shift out



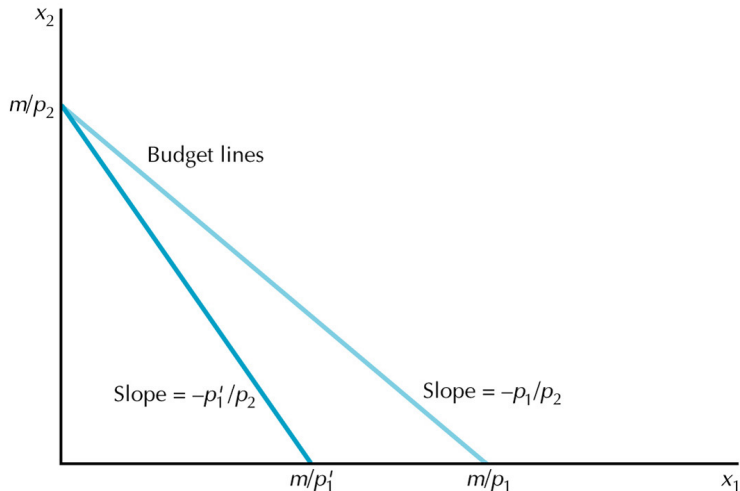
## Change in price

A rise in price from  $p_1$  to  $p'_1$



## Change in price

A rise in price from  $p_1$  to  $p'_1 \implies$  pivot around the vertical intercept



## Change in more variables

Multiplying all prices and income by  $t$ ...

$$tp_1x_1 + tp_2x_2 = tm$$

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Multiplying all prices and income by  $t$  does not change BL:

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Multiplying all prices by  $t$  has the same effect as dividing income by  $t$ :

$$tp_1x_1 + tp_2x_2 = m \iff p_1x_1 + p_2x_2 = \frac{m}{t}$$

# Numeraire

Any price or income can be normalized to 1 and adjust all variables so that the BL stays the same.

**Numeraire** = an item with its value normalized to 1



# Nnumeraire

Any price or income can be normalized to 1 and adjust all variables so that the BL stays the same.

**Nnumeraire** = an item with its value normalized to 1

Budget line  $p_1x_1 + p_2x_2 = m$ :

- Good 1 is numeraire – the same BL:

$$x_1 + \frac{p_2}{p_1}x_2 = \frac{m}{p_1}$$

- Good 2 is numeraire – the same BL:

$$\frac{p_1}{p_2}x_1 + x_2 = \frac{m}{p_2}$$

- The income is numeraire – the same BL:

$$\frac{p_1}{m}x_1 + \frac{p_2}{m}x_2 = 1$$

# Taxes and subsidies

Three types of taxes:

- **Quantity tax** – consumer pays the amount  $t$  for each unit.  
→ Price of good 1 increases to  $p_1 + t$ .



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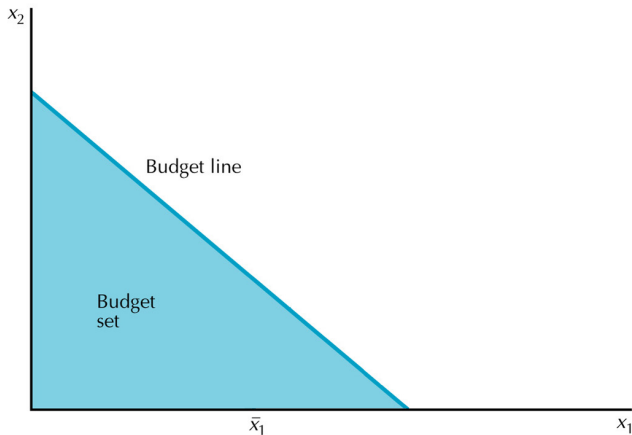
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Subsidy = a tax with a negative sign



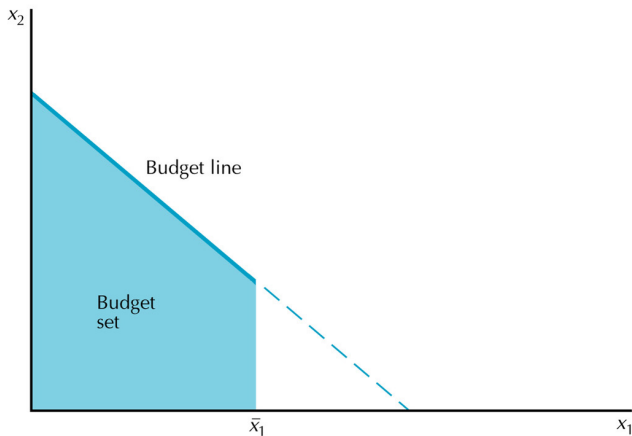
# Rationing

If there is rationing imposed on good 1, no consumer is allowed to buy a higher quantity of good 1 than  $\bar{x}_1$ .



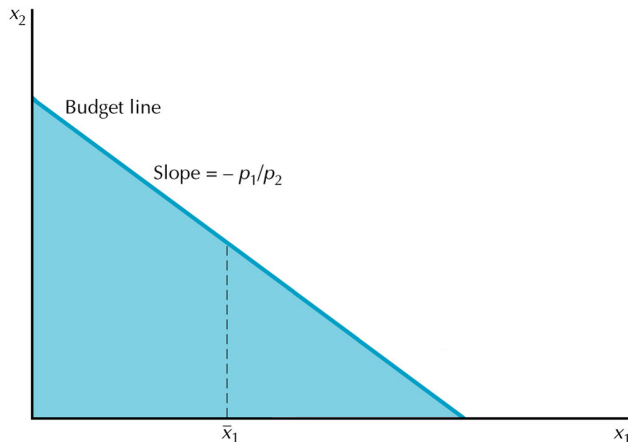
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## Taxing consumption greater than $\bar{x}_1$

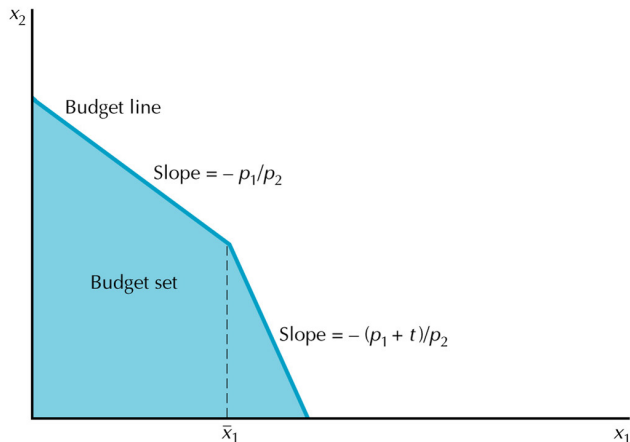
If consumer pays a tax only on the consumption of good 1 that is in excess of  $\bar{x}_1$ ...





## Taxing consumption greater than $\bar{x}_1$

If consumer pays a tax only on the consumption of good 1 that is in excess of  $\bar{x}_1$ , budget line is steeper to the right of  $\bar{x}_1$ .



# CASE: The food stamp program

Before 1979 (left graph):

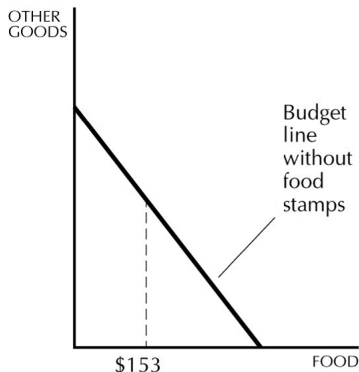
- value subsidy – people pay a part of the value of the food stamp
- rationing – maximum value of stamps (e.g. 153 \$)



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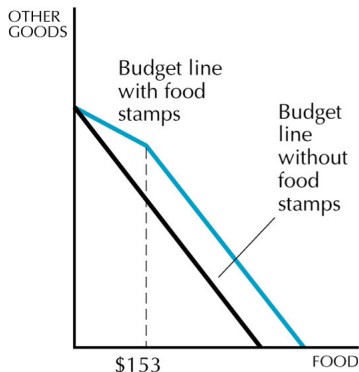
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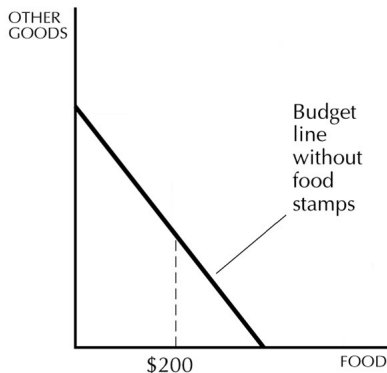
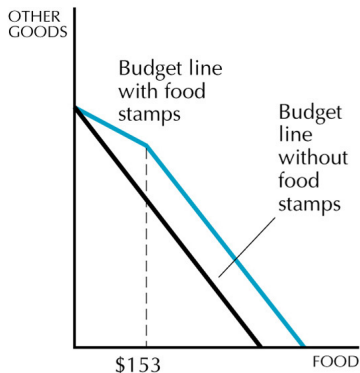


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After 1979 (right graph) – a specific number of food stamps for free

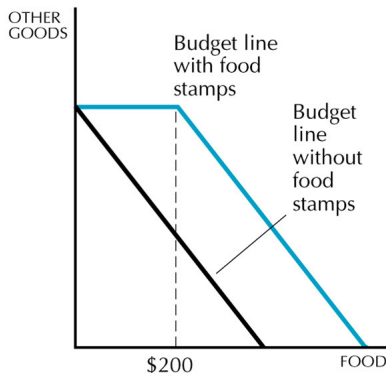
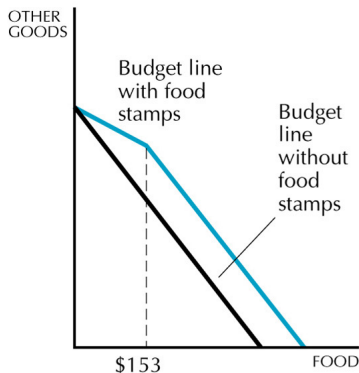


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# Preferences

Consumers compare bundles according to their preferences.

Preference relations – three symbols:

- bundle  $X$  is **strictly preferred** to bundle  $Y$ :

$$(x_1, x_2) \succ (y_1, y_2)$$

- bundle  $X$  is **weakly preferred** to bundle  $Y$   
(bundle  $X$  is at least as good as bundle  $Y$ ):

$$(x_1, x_2) \succeq (y_1, y_2)$$

- consumer is **indifferent** between bundles  $X$  and  $Y$ :

$$(x_1, x_2) \sim (y_1, y_2)$$

## Assumptions about preferences

Assumptions that allow ordering of bundles according to preferences:

- **Completeness** — any two bundles can be compared:  
 $(x_1, x_2) \succeq (y_1, y_2)$ , or  $(x_1, x_2) \preceq (y_1, y_2)$ , or both



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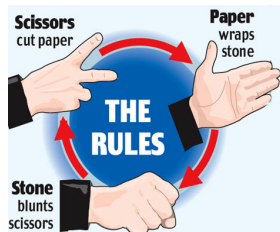
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- **Reflexivity** — each bundle is at least as good itself:  $(x_1, x_2) \succeq (x_1, x_2)$

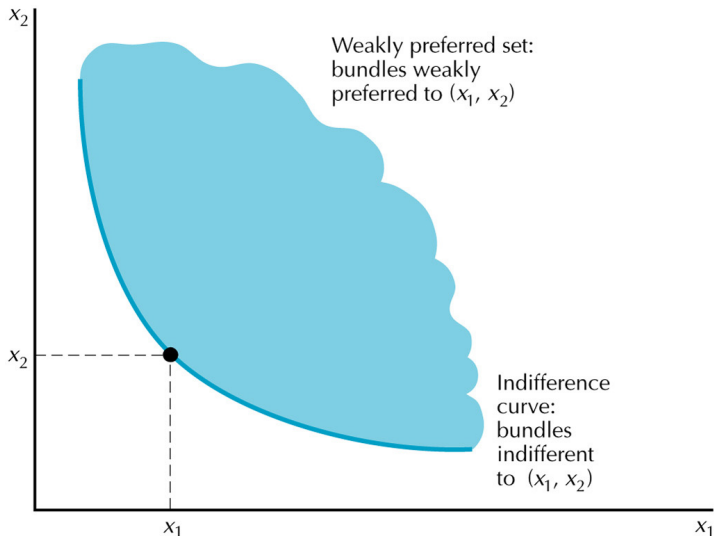
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- **Reflexivity** — each bundle is at least as good itself:  $(x_1, x_2) \succeq (x_1, x_2)$
- **Transitivity** — if  $(x_1, x_2) \succeq (y_1, y_2)$  and  $(y_1, y_2) \succeq (z_1, z_2)$ , then  $(x_1, x_2) \succeq (z_1, z_2)$

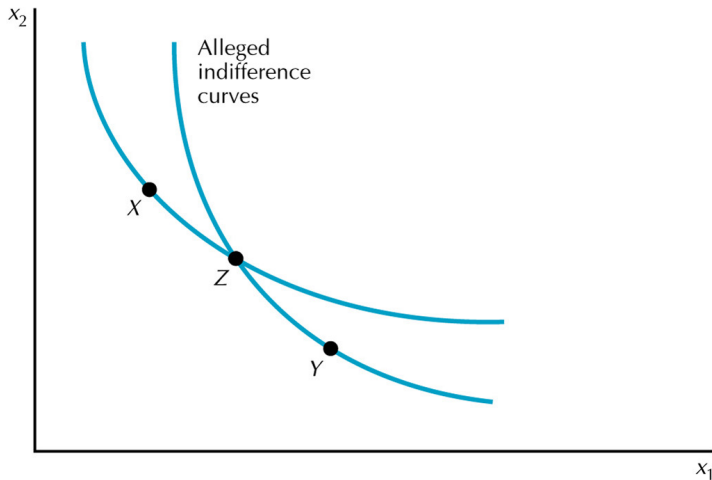


# Weakly preferred set and indifference curves



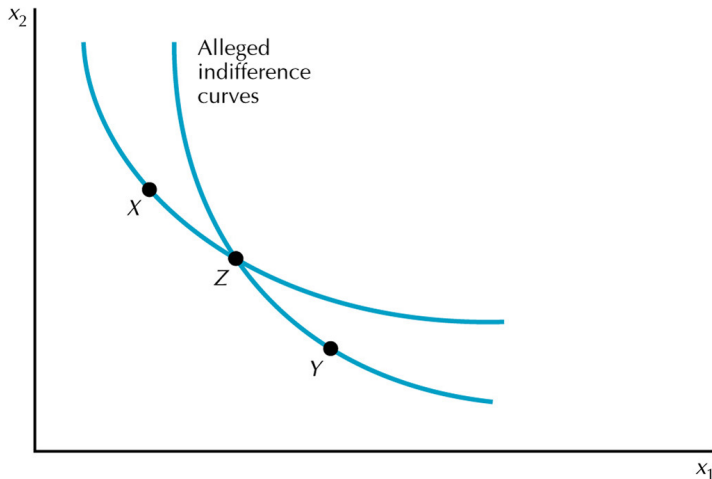
## Two indifference curves cannot cross

Two different IC such that  $X \succ Y$ . Why cannot they cross?



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Two different IC such that  $X \succ Y$ . Why cannot they cross? It follows from transitivity that if  $X \sim Z$  and  $Z \sim Y$  then  $X \sim Y$ .



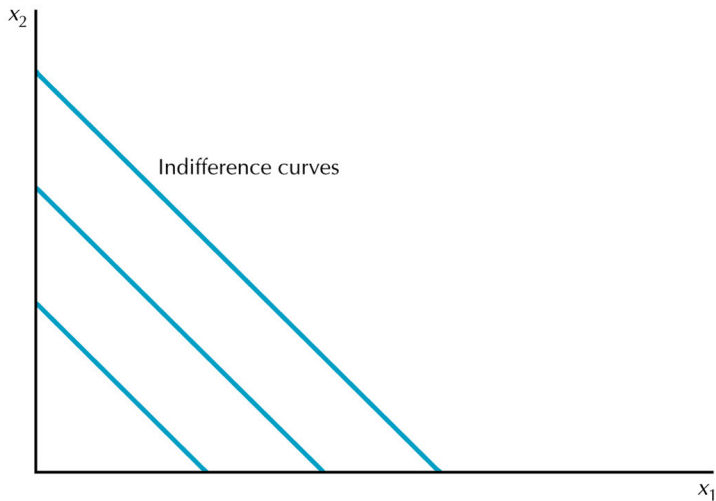
## Examples of preferences – perfect substitutes

Willingness to substitute one good for the other at a constant rate



## Examples of preferences – perfect substitutes

Willingness to substitute one good for the other at a constant rate  $\implies$  constant slope of the indifference curve (not necessarily  $-1$ ).



## Examples of preferences – perfect complements

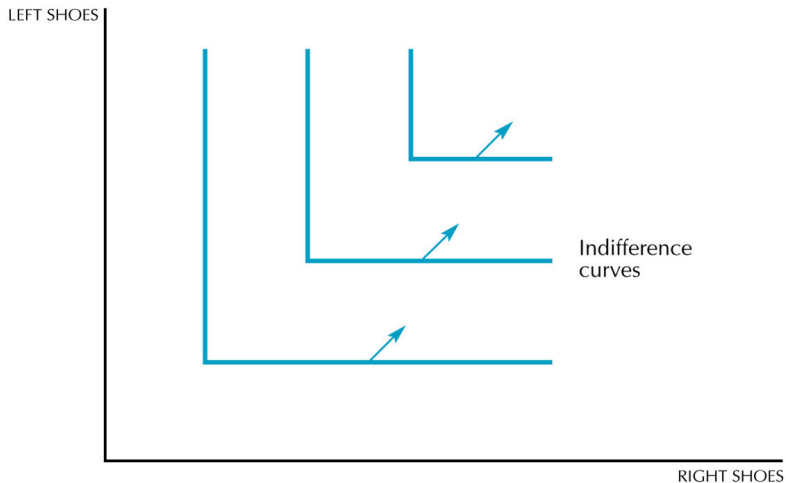
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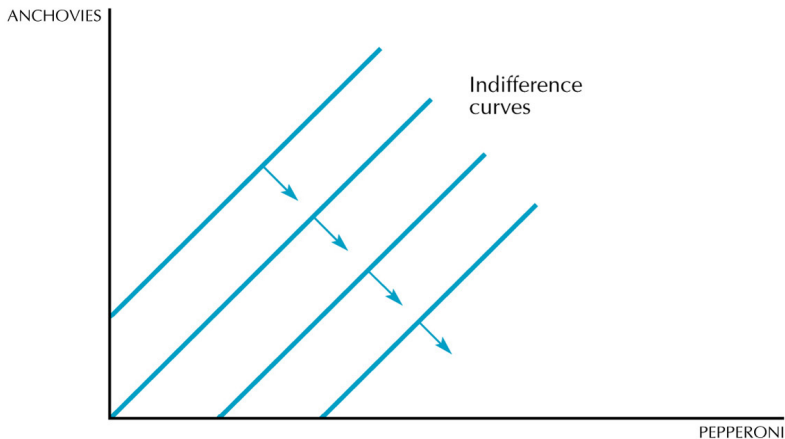
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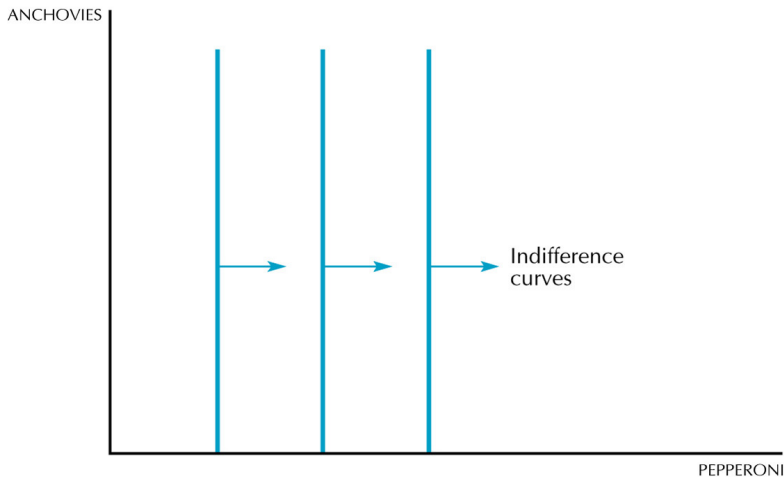
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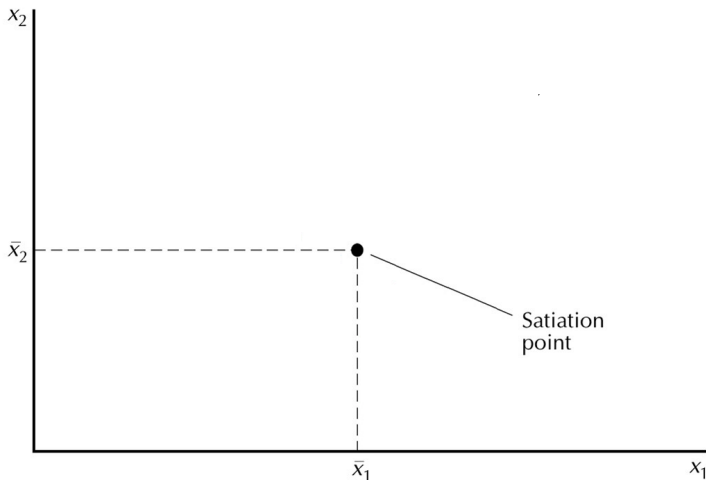
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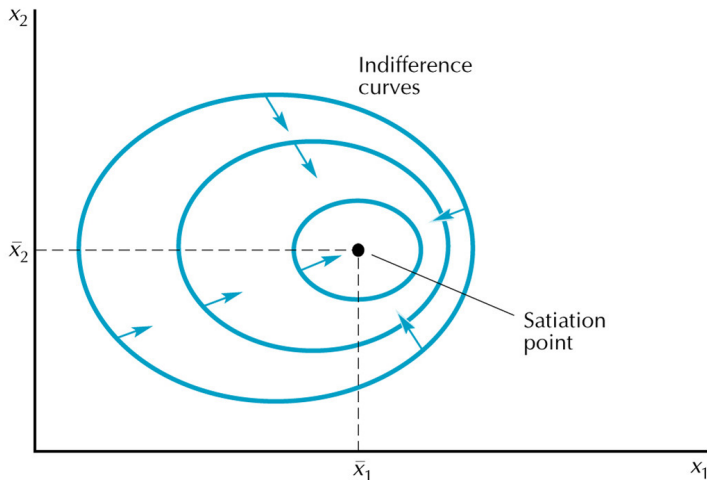
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**Satiation point** is the most preferred point  $(\bar{x}_1, \bar{x}_2)$ . When the consumer has too much of one of the goods, it becomes a bad.



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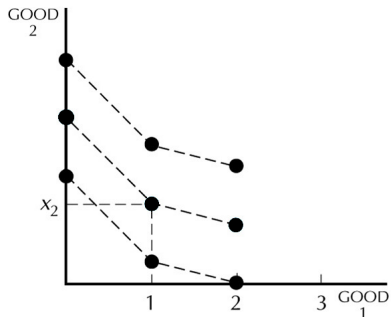
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## Examples of preferences – discrete goods

A **discrete good** is not divisible – consumption in integer amounts:

- indifference „curves“



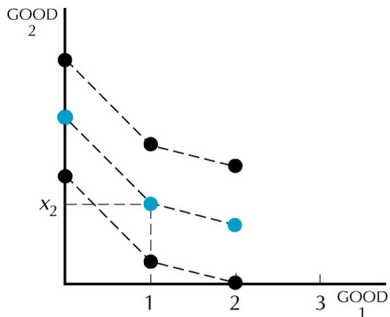
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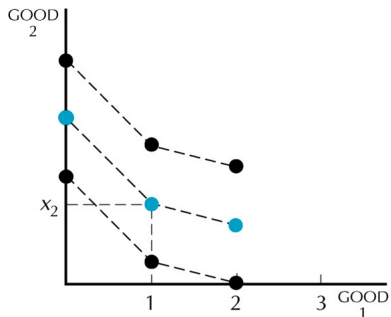


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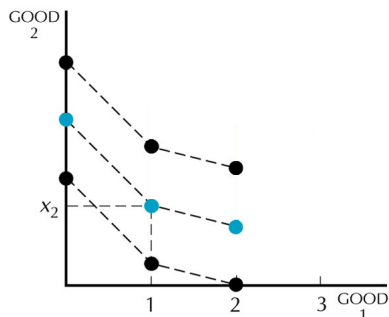
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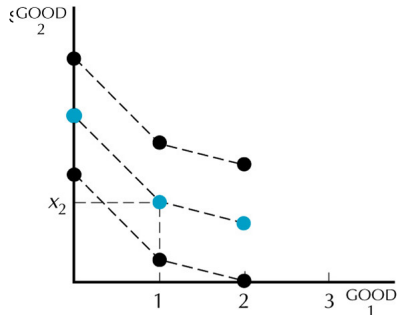


B Weakly preferred set

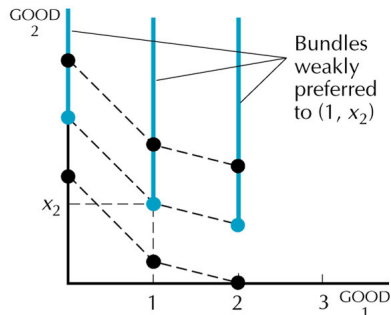
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- a weakly preferred set – a set of line segments



A Indifference “curves”



B Weakly preferred set

## Well-behaved preferences

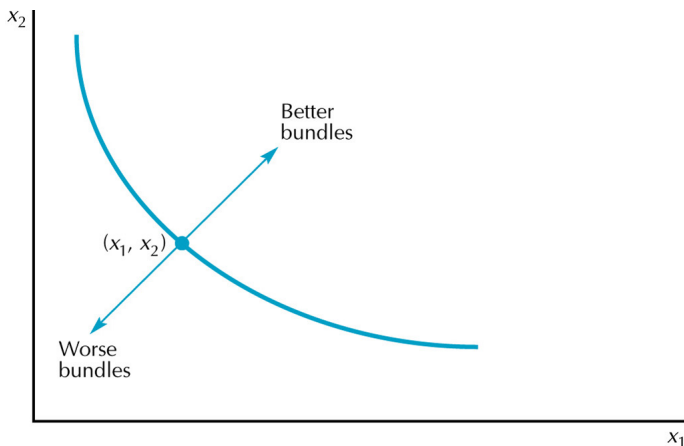
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# Well-behaved preferences

Assumptions of well-behaved preferences: monotonicity and convexity

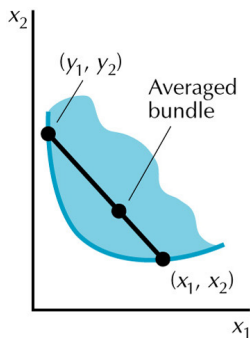
**Monotonicity** – more is better (it excludes bads)

⇒ indifference curves have negative slope.

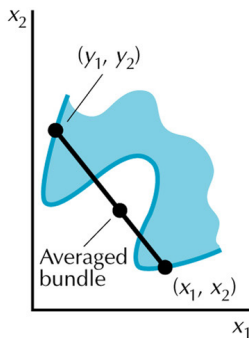


## Well-behaved preferences (cont'd)

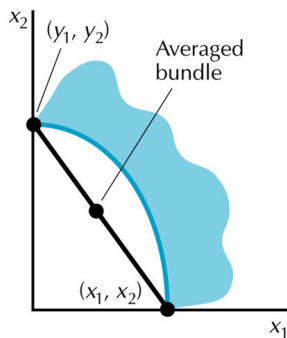
**Convexity** – if  $(x_1, x_2) \sim (y_1, y_2)$ , then it holds for all  $0 \leq t \leq 1$  that  $(tx_1 + (1 - t)y_1, tx_2 + (1 - t)y_2) \succeq (x_1, x_2)$ .



**A** Convex preferences



**B** Nonconvex preferences

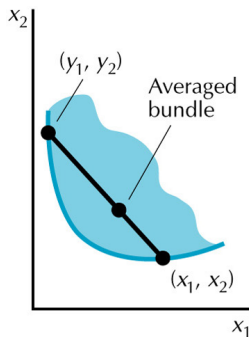


**C** Concave preferences

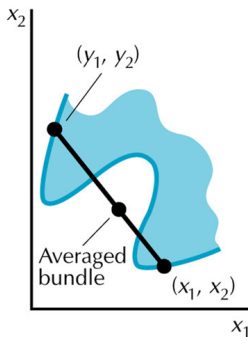
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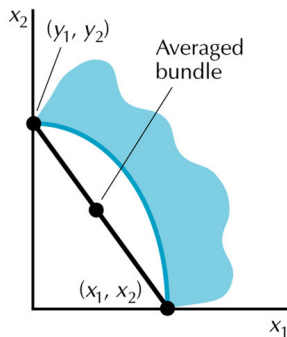
**Strict convexity** – if  $(x_1, x_2) \sim (y_1, y_2)$ , then it holds for all  $0 \leq t \leq 1$  that  $(tx_1 + (1 - t)y_1, tx_2 + (1 - t)y_2) \succ (x_1, x_2)$ .



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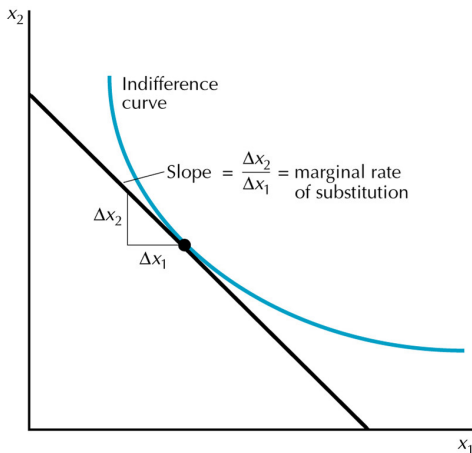
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# Marginal rate of substitution

**Marginal rate of substitution**  
(MRS) = slope of the indifference  
curve:

$$\text{MRS} = \frac{\Delta x_2}{\Delta x_1} = \frac{dx_2}{dx_1}$$

**Diminishing marginal rate of substitution** – absolute value of MRS decreases as we increase  $x_1$ .





# Interpretation of marginal rate of substitution

## Interpretation of MRS:

- The amount of good 2 one is willing to pay for one unit of good 1.
- If good 2 is measured in money:  $MRS =$  **marginal willingness to pay** = how many dollars you would just be willing to give up for an additional unit of good 1.

## APPLICATION: Build a stadium for Minnesota Vikings?

The club does not like the stadium – considers leaving Minnesota.

Fenn a Crooker (SEJ, 2009) measure how much households are willing to pay for Vikings staying in Minnesota = MRS between composite good and Vikings in Minnesota.

MRS of an average household: 531 \$

Value of the stadium: 531 \$  $\times$  1,323 million households = 702 mil. \$



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Estimated costs are 1 billion \$.

The new stadium opens in 2016  
– the state provided 500 million \$.



# Utility

Two concepts of utility:

Cardinal utility – attach a significance to the magnitude of utility:

- difficult to assign the magnitude
- not needed to describe choice behavior

Numbers	
1 one 	6 six 
2 two 	7 seven 
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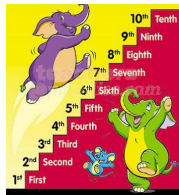
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**Ordinal utility** – important is only the order of preference:

- easy to set the utility – 1 rule: preferred bundle has a higher utility
- we can derive a complete theory of demand

We will use the ordinal utility.

Numbers	
1 one 	6 six 
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4 four 	9 nine 
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## Ordinal utility

**Utility function** is a way of assigning a number to every possible consumption bundle such that more-preferred bundles get assigned larger numbers than less-preferred bundles.

If  $(x_1, x_2) \succ (y_1, y_2)$ , then  $u(x_1, x_2) > u(y_1, y_2)$ .

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Different ways to assign utilities that describe the same preferences:

Bundle	$U_1$	$U_2$	$U_3$
A	3	17	-1
B	2	10	-2
C	1	.002	-3

## Monotonic transformation

**Positive monotonic transformation**  $f(u)$  = any increasing function of  $u$ .  
Describes the same preferences as the original utility function  $u$ .

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Two bundles  $X$  and  $Y$ , preferences:  $X \succ Y$

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Yes:

- $f_1(u) = 3u$ :  $f_1(u(X)) = 3 > -3 = f_1(u(Y))$
- $f_2(u) = u + 3$ :  $f_2(u(X)) = 4 > 2 = f_2(u(Y))$

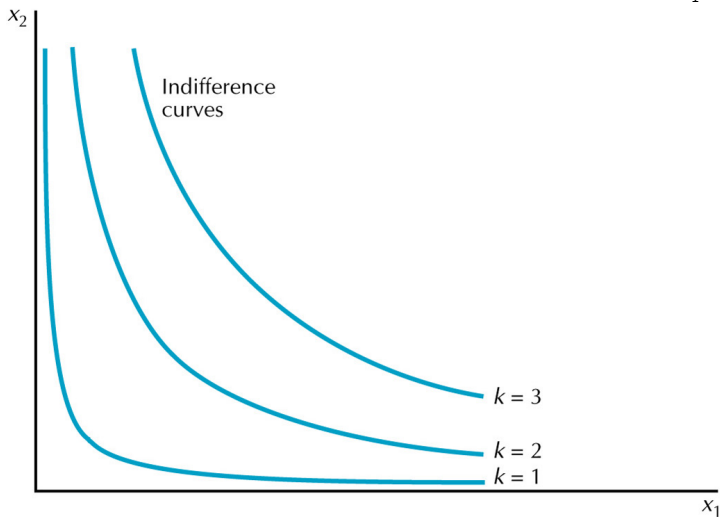
## Construction of indifference curves from utility function

Utility function  $u(x_1, x_2) = x_1 x_2$



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Utility function  $u(x_1, x_2) = x_1x_2 \implies$  indifference curves  $x_2 = \frac{k}{x_1}$



## PROBLEM: The slope of indifference curves

The slope of indifference curves for two utility functions:

1. What is the slope of IC  $x_2 = 4/x_1$  v point  $(x_1, x_2) = (2, 2)$ ?

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baguette has a double weight: e.g.  $u(R, H) = R + 2H$



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goal: same numbers in the bracket – we need only 1/5 of coke:

e.g.  $u(R, K) = \min\{5R, K\}$

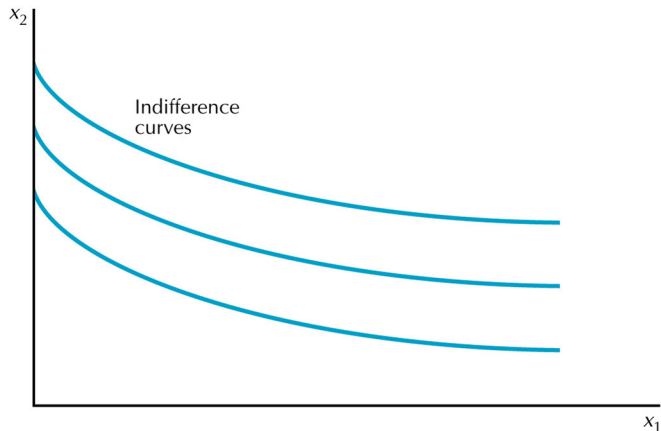




## Examples of utility functions – quasilinear preferences

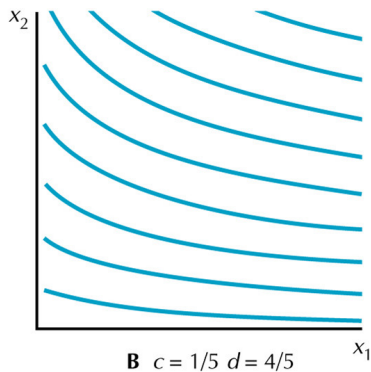
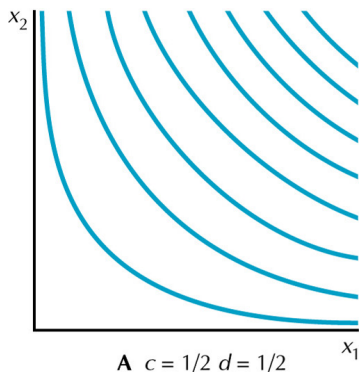
Indifference curves are vertically parallel (a practical property)

Utility function  $u(x_1, x_2) = v(x_1) + x_2$ , e.g.  $u(x_1, x_2) = \sqrt{x_1} + x_2$



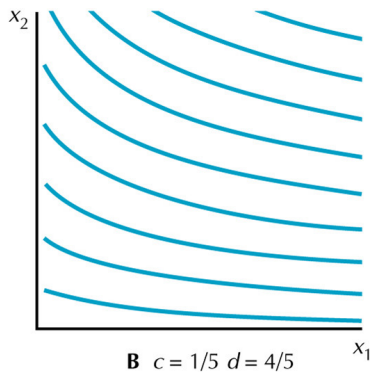
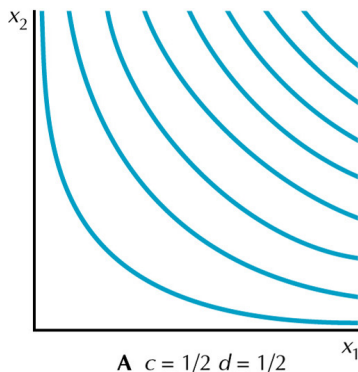
## Examples of utility functions – Cobb-Douglas preferences

- A simple utility function representing well-behaved preferences.
- Utility function of the form  $u(x_1, x_2) = x_1^c x_2^d$ .
- More convenient to use the transformation  $f(u) = u^{\frac{1}{c+d}}$  and write  $x_1^a x_2^{1-a}$ , where  $a = c/(c + d)$ .



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## Marginal utility

**Marginal utility** ( $MU$ ) is the change in utility from an increase in consumption of one good, while the quantities of other goods are constant.

Partial derivatives of  $u(x_1, x_2)$  with respect to  $x_1$  or  $x_2$ .

Příklady:

- $u(x_1, x_2) = x_1 + x_2 \rightarrow MU_1 = \partial u / \partial x_1 = 1$
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The value of  $MU$  changes with a monotonic transformation of the utility function. If we multiply utility times 2,  $MU$  increases times 2.

## Relationship between $MU$ and MRS

We want to measure MRS = slope of IC  $u(x_1, x_2) = k$ , where  $k$  is a constant.

We are interested in  $(\Delta x_1, \Delta x_2)$ , for which the utility is constant:

$$MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0$$

$$\text{MRS} = \frac{\Delta x_2}{\Delta x_1} = -\frac{MU_1}{MU_2}$$

We can calculate MRS from the utility function. E.g. for  $u = \sqrt{x_1 x_2}$ :

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The value of MRS does not change with monotonic transformation.

If we multiply utility function times 2,  $MRS = -\frac{2MU_1}{2MU_2} = -\frac{MU_1}{MU_2}$ .

## APPLICATION: Utility from commuting

People decide whether to take bus or car.

Each type of transport represents a bundle with different characteristics, e.g.:

- $x_1$  is walking time
- $x_2$  is time taking a bus or car
- $x_3$  is the total cost of commuting
- ...

Assume that the utility function has a linear form

$$U(x_1, \dots, x_n) = \beta_1 x_1 + \dots + \beta_n x_n.$$

Then we use statistical techniques to estimate the parameters  $\beta_i$  that best describe choices.





## APPLICATION: Utility from commuting (cont'd)

Domenich and McFadden (1975) estimated the following utility function:

$$U(TW, TT, C) = -0,147TW - 0,0411TT - 2,24C$$

- $TW$  = total walking time in minutes
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The parameters can be used for different purposes.

For instance, we can:

- calculate the marginal rate of substitution between two characteristics
- forecast consumer response to proposed changes
- estimate whether a change is worthwhile in a benefit-cost sense

## What should you know?

- Budget set = consumption bundles available at given prices and income
- Budget line are bundles for which the entire income is spent.
- If the preference relation is complete, reflexive and transitive, consumer can order bundles according to preferences.
- Monotonicity and convexity are reasonable assumptions – easier to find the optimum bundle.



## What should you know? (cont'd)

- Utility function assigns numbers to different bundles so that the bundles are ordered according to preferences.
- The numbers have no meaning in itself. Monotonic transformation of  $u$  represents the same preferences..
- MRS measures the slope of IC.
- The slope of IC measures the willingness to pay for good 1 (in units of good 2)
- The slope of BL measures the opportunity cost of good 1(in units of good 2)

