Choice and revealed preference

Varian, Intermediate Microeconomics, chapter 5 and sections 7.1–7.7

In this lecture, you will learn

- what the optimal choice is
- how to find it for different preferences
- whether Christmas is efficient
- how to find out from consumption choices whether the consumer is rational and what are his preferences

Optimal choice

The consumer chooses the most preferred bundle from her budget set.

If the consumer has monotonic and convex preferences and a smooth IC and the problem has an inner solution, it always holds in optimum:

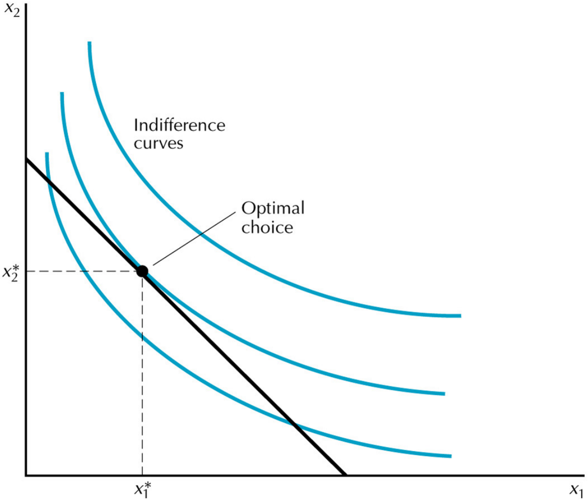
$$\text{slope IC} = \text{MRS} = -\frac{p_1}{p_2} = \text{slope BL}$$

Conversely, the slope of IC may not be equal to the slope of BL if:

- IC has a kink
- there is corner solution
- there are non-convex preferences
- there is a satiation point

Convex, monotonic and smooth IC and inner solution

The optimal choice: slope IC $=$ MRS $= -\frac{p_1}{p_2} =$ slope BL.



Cobb-Douglas preferences (general solution)

The consumer chooses the bundle from her budget set in order to maximize her utility:

$$\max_{x_1, x_2} u(x_1, x_2) = x_1^c x_2^d$$

$$\text{subject to } p_1 x_1 + p_2 x_2 \leq m$$

Cobb-Douglas preferences:

- monotonic preference $\implies p_1 x_1 + p_2 x_2 = m$
- monotonic, convex, and smooth IC, which do not touch the axes $\implies \text{MRS} = -p_1/p_2$

The optimal bundle (x_1^*, x_2^*) is the solution of the following equations:

$$MRS = -\frac{p_1}{p_2}$$

$$p_1 x_1^* + p_2 x_2^* = m$$

Cobb-Douglas preferences (general solution)

The optimal bundle:

$$(x_1^*, x_2^*) = \left(\frac{c}{c+d} \frac{m}{p_1}, \frac{d}{c+d} \frac{m}{p_2} \right)$$

Property of Cobb-Douglas preferences: In optimum the consumer spends a constant share of her income on each good:

$$\frac{p_1 x_1^*}{m} = \frac{p_1}{m} \frac{c}{c+d} \frac{m}{p_1} = \frac{c}{c+d}$$

$$\frac{p_2 x_2^*}{m} = \frac{p_2}{m} \frac{d}{c+d} \frac{m}{p_2} = \frac{d}{c+d}$$

Convenient to have C-D function with exponents adding up to 1, e.g.

$$u(x_1, x_2) = \sqrt{x_1 x_2}$$

Then the exponents are the shares of income spent on goods 1 and 2.

Example – Cobb-Douglas preferences

Romana buys only apples A and bananas B.
Utility function: $u(A, B) = A^2 B$
Prices and income: $p_A = 5$, $p_B = 10$, $m = 60$

What is the optimum bundle?

Cobb-Douglas preference $\implies$ in optimum it holds: $MRS = -p_A/p_B$

$$-2AB/A^2 = -5/10$$

$$A = 4B$$

In optimum Romana will buy 4 times as many A than B.

Substituting the ratio into the BL:

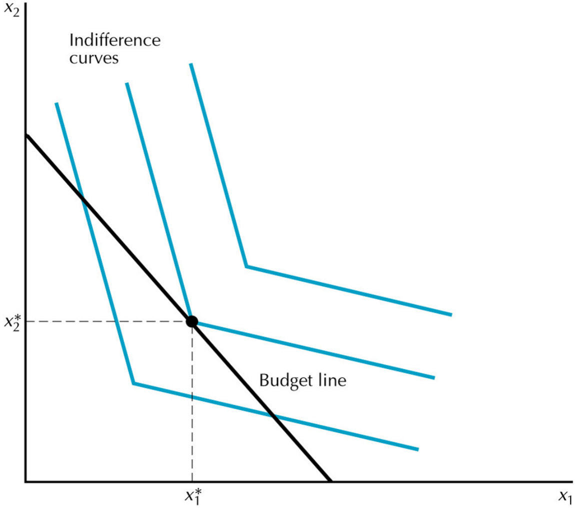
$$p_A 4B + p_B B = m$$

$$20 \times B + 10 \times B = 60$$

$$B = 2 \quad \text{a} \quad A = 4B = 8$$

Kink in the indifference curve

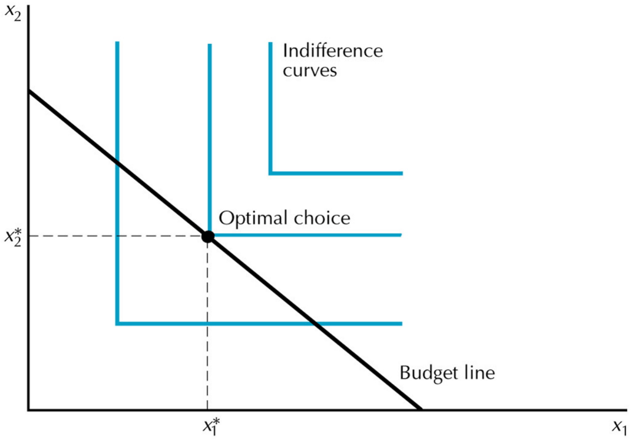
Graph: convex and monotonic IC – but not smooth
Optimal choice: the slope of IC is not defined.



Example – perfect complements

$u(x_1, x_2) = \min\{x_1, x_2\}$ – goods consumed at a constant ratio 1:1

If $p_1 > 0$ a $p_2 > 0$, it holds for the optimum that $x_1^* = x_2^*$.



Example – perfect complements

John consumes only tee T and cookies C at a ratio 1 : 2.
Price of tee: $p_T = 5$ CZK
Price of a cookie: $p_C = 2$ CZK
John's income: $m = 90$ CZK

Example of John's utility function + optimal consumption bundle?

John's utility function is e.g.

$$u(T, C) = \min\{2T, C\}$$

Positive prices $\implies$ the optimal combination of goods (= kinks of ICs): $C = 2T$. Substituting the ratio into the BL:

$$p_T T + p_C C = m$$

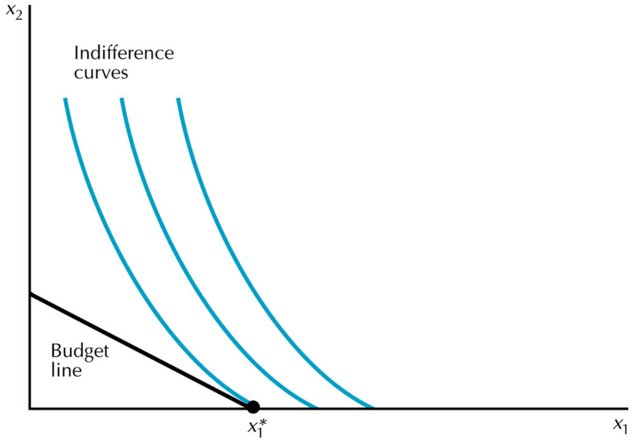
$$5 \times T + 2 \times 2T = 90$$

$$T = 10$$

$$C = 2T = 20$$

Corner solution

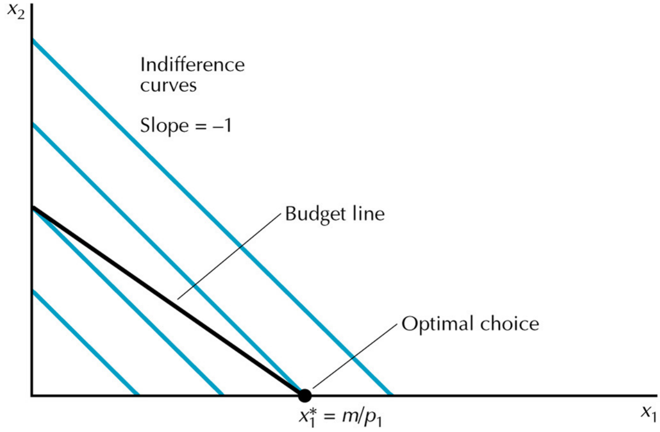
Graph: convex, smooth and monotonic IC – but touches the axes
Optimal choice: slope of IC $\neq$ slope of BL



Example – perfect substitutes

$u(x_1, x_2) = x_1 + x_2$ – willingness to exchange goods 1 a 2 at a ratio 1:1

If $p_1 < p_2$, the optimal choice is $(x_1^*, x_2^*) = (m/p_1, 0)$.



Example – perfect substitutes

Martha always willing to exchange 3 raspberries R for 1 blueberry B.
Price of raspberry: $p_R = 2$ CZK
Price of blueberry: $p_B = 5$ CZK
Martha's income: $m = 40$ CZK

Example of Martha's utility function + optimal consumption bundle?

Martha's utility function: $u(R, B) = R + 3B$

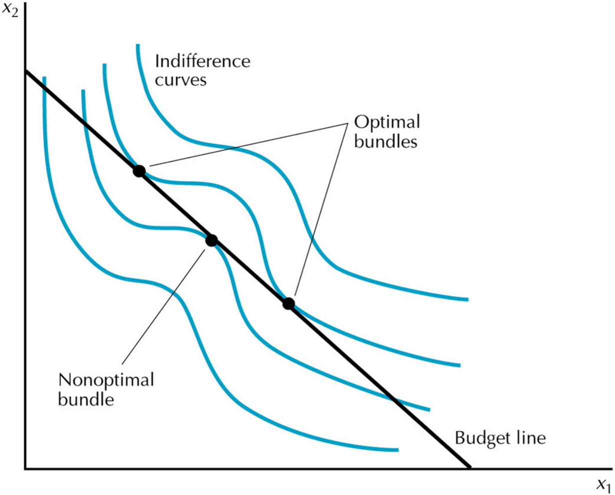
$$\text{MRS} = -1/3 \neq \text{slope BL} = -2/5$$

Which corner solution has a higher utility $u(R, B)$:

- $u(20, 0) = 20$
- $u(0, 8) = 24 \implies$ Optimal choice: $(R, B) = (0, 8)$.

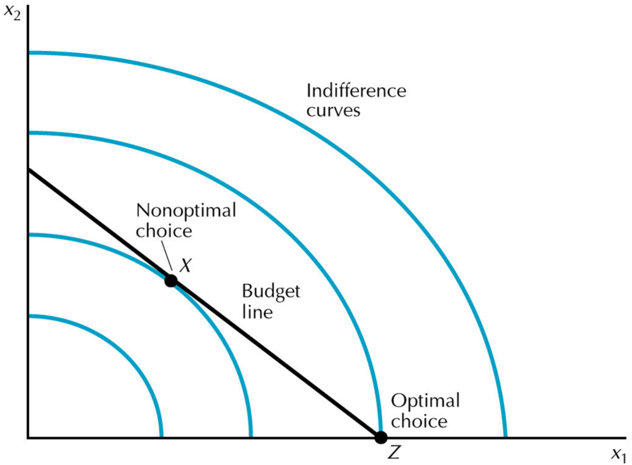
Nonconvex preferences

Graph: smooth and monotonic IC – but not convex
There may exist non-optimal bundles where slope of IC = slope of BL.



Example – concave preferences

Optimal choice (corner solution Z) – slope of IC $\neq$ slope of BL
Non-optimal choice (inner solution X) – slope of IC $=$ slope of BL



Example – concave preferences

Libor buys mushrooms M and cuckoos C
Libor's utility function: $u(M, C) = M^2 + C$
Price of mushrooms: $p_M = 20$ CZK
Price of cuckoos: $p_C = 10$ CZK
Libor's income: $m = 100$ CZK

What is Libor's optimal consumption bundle?

We derive the IC for the utility $u = 10$:

$$C = 10 - M^2$$

Second derivative IC: $C'' = -2 \implies$ Libor has concave preferences.

Finding out, which corner solution brings higher utility $u(M, C)$:

- $u(0, 10) = 10$
- $u(5, 0) = 25 \implies$ Optimal choice: $(M, C) = (5, 0)$

Example – concave preferences (non-optimal bundle)

Libor buys mushrooms M and cuckoos C
Libor's utility function: $u(M, C) = M^2 + C$
Price of mushrooms: $p_M = 20$ CZK
Price of cuckoos: $p_C = 10$ CZK
Libor's income: $m = 100$ CZK

What is Libor's optimal consumption bundle?

Assuming that in optimum the slope of IC = the slope of BL:

$$MRS = -p_1/p_2$$

$$-2M = -20/10$$

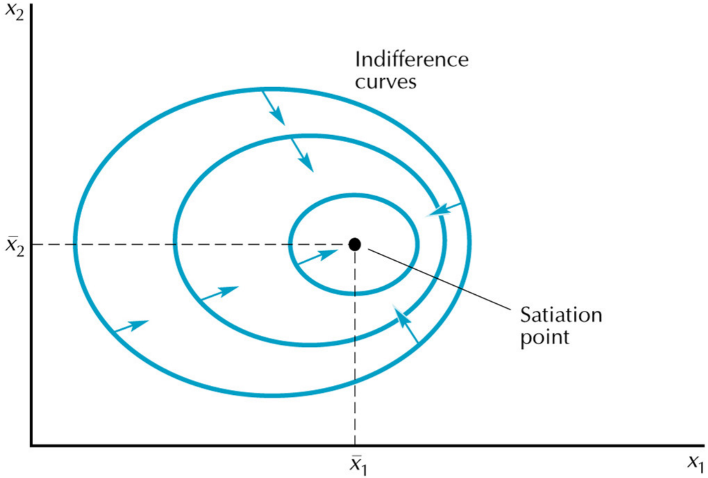
$$M = 1$$

$$C = m/p_C - (p_M/p_C)M = 8$$

Non-optimal choice: $X = (M, C) = (1, 8)$
X has the lowest utility of all bundles on BL: $u(1, 8) = 9$.

Satiation point

Graph: convex and smooth IC – but not monotonic
Optimal choice: the slope of IC in the satiation point not defined



Example – satiation point

Milena consumes only marmalade M and croissants C.
The bundle that maximizes her utility is $(M^*, C^*) = (10, 5)$.
Her ICs are concentric circles – the farther from (M^*, C^*), the worse.
Price of marmalade: $p_M = 10$ CZK
Price of tee: $p_C = 5$ CZK
Milena's income: $m = 140$ CZK

What is her optimal consumption bundle?

Her optimal bundle is the satiation point $(M^*, C^*) = (10, 5)$.
Can she afford the bundle?

$$p_M M^* + p_C C^* = 10 \times 10 + 5 \times 5 = 125.$$

The optimal bundle is available $(125 < 140)$.
Milena buys the bundle $(M^*, C^*) = (10, 5)$.

APPLICATION: Estimating a utility function

What utility function corresponds to this consumption data?

Rok	p_1	p_2	m	x_1	x_2	s_1	s_2	Užitek
1	1	1	100	25	75	.25	.75	57.0
2	1	2	100	24	38	.24	.76	33.9
3	2	1	100	13	74	.26	.74	47.9
4	1	2	200	48	76	.24	.76	67.8
5	2	1	200	25	150	.25	.75	95.8
6	1	4	400	100	75	.25	.75	80.6
7	4	1	400	24	304	.24	.76	161.1

Consumption shares (s_1, s_2) are roughly constant.

$\Longrightarrow$ Cobb-Douglas utility function $u(x_1, x_2) = x_1^{1/4} x_2^{3/4}$.

APPLICATION: Estimating a utility function (cont'd)

What is this estimation for?

For example, we can evaluate political choices.

Example:
New tax system leads to $(p_1, p_2) = (2, 3)$ and $m = 200$.
Is it a good or a bad result?

Demanded quantities of goods at these prices and income are:

$$x_1 = \frac{1}{4}\frac{200}{2} = 25$$
$$x_2 = \frac{3}{4}\frac{200}{3} = 50$$

Estimated utility of the bundle is $u(x_1, x_2) = 25^{1/4}50^{3/4} \approx 42$.

We can compare the results with the past – higher than in year 2 but lower than in year 3 (see the table in the previous slide).

APPLICATION: The cost of Christmas

Joel Waldfogel, "The Deadweight Loss of Christmas" (AER, 1993):

- „In the standard microeconomic framework of consumer choice, the best a gift-giver can do with, say, $10 is to duplicate the choice that the recipient would have made." (p. 1328) In most cases, the recipient is worse of.
- Gift-giving destroys 10 – 33 % of the value of gifts: loss ≈ $4 billion. (10 % of the estimated dead-weight loss of the income tax).

Revealed preference

In previous slides we derived choices from preferences.
In reality we do not observe preferences of people directly.

Conversely, revealed preferences derive preferences from choices.

We assume that consumer's preferences are *stable*
$=$ they do not change in the time we observe consumer's choices.

For simplicity we assume that the derived preferences are

- *strictly convex* $\implies$ exactly one bundle is demanded.
- *monotonic* $\implies$ consumer spends the entire income.

These two assumptions are not necessary for the theory of RP!

Concept of revealed preference

If the consumer chooses bundle X even though bundle Y is available, then she reveals that she prefers X to Y.

Revealed preference and preference

If X is revealed preferred to Y, does it mean that X is preferred to Y?

No. „X revealed preferred to Y“ only means that X is chosen when Y is available.

If the consumer maximizes utility (chooses best bundle available) then „revealed preference“ implies „preference“.

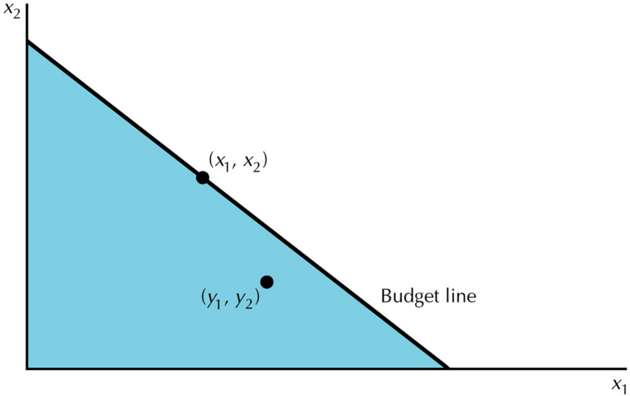
The following exposition in two steps:

1. Assuming utility maximization (and other assumptions) we use revealed preference to derive preferences from consumer choices.
2. We show how to test whether the consumer behaves in line with utility maximization.

Directly revealed preference

The chosen bundle (x_1, x_2) is **directly revealed preferred** to bundle (y_1, y_2) if the bundle (y_1, y_2) is available, that is if

$$p_1 x_1 + p_2 x_2 \geq p_1 y_1 + p_2 y_2.$$

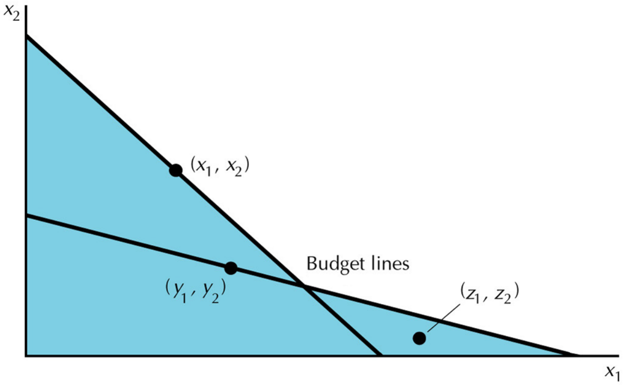


Indirectly revealed preference

It follows from transitivity that if

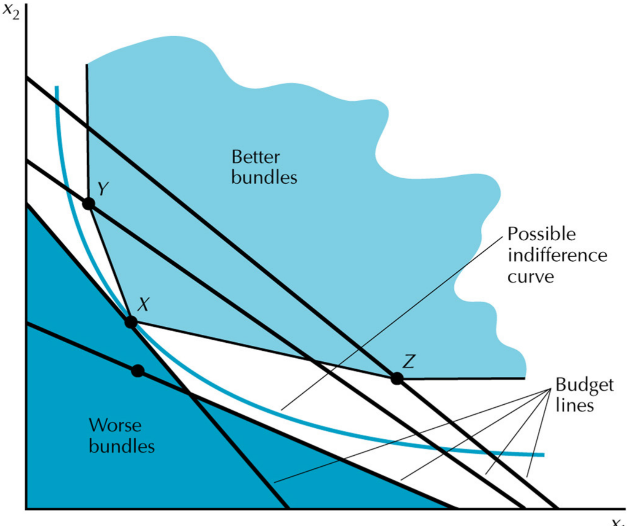
- bundle X is directly revealed preferred to bundle Y and
- bundle Y is directly revealed preferred to bundle Z

then bundle$X$ is **indirectly revealed preferred** to bundle Z.



Example – revealed preferrence

Derivation of an IC for strictly convex a monotonic preferences.



Weak axiom of revealed preference

Weak axion of revealed preference (WARP)
If bundle X is directly revealed preferred to bundle Y, then Y cannot be directly revealed preferred to X.

More formally: For each bundle (x_1, x_2) bought at prices (p_1, p_2) and a different bundle (y_1, y_2) bought at prices (q_1, q_2) holds that if

$$p_1 x_1 + p_2 x_2 \geq p_1 y_1 + p_2 y_2,$$

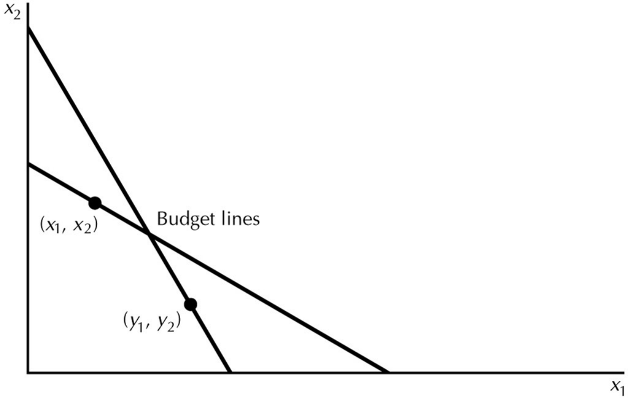
then it *must not* be true that

$$q_1 y_1 + q_2 y_2 \geq q_1 x_1 + q_2 x_2.$$

A necessary condition for consistency with utility maximization.

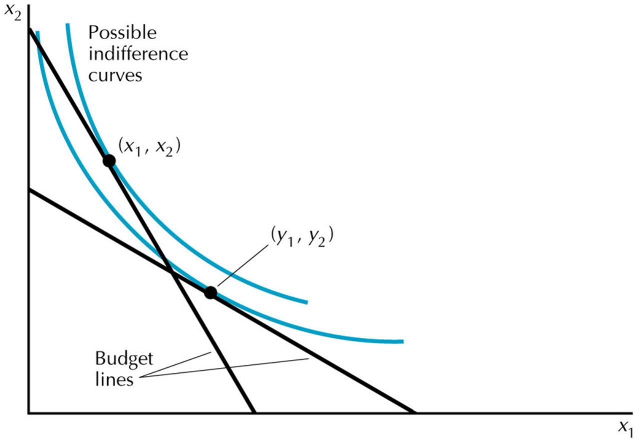
Weak axiom of revealed preference (cont'd)

Consumer choices that are not consistent with WARP:



Weak axiom of revealed preference (cont'd)

Consumer choices that are consistent with WARP:



How to test WARP?

Consider the following consumer data:

Observation	p_1	p_2	x_1	x_2
1	1	2	1	2
2	2	1	2	1
3	1	1	2	2

Costs of bundles 1, 2, and 3 at different prices:

		Bundles		
		1	2	3
Prices	1	5	4*	6
	2	4*	5	6
	3	3*	3*	4

The chosen bundles are directly revealed preferred to bundles with * in the same line (e.g. at prices 1 is bundle 1 preferred to bundle 2).

How to test WARP? (cont'd)

The WARP is violated if there is * in line t and column s and line s and column t (e.g. bundle 1 + price 2 and bundle 2 + price 1).

		Bundles 1	Bundles 2	Bundles 3
Prices	1	5	<u>4*</u>	6
	2	<u>4*</u>	5	6
	3	3*	3*	4

Data in the table violate WARP.

What does it mean that data violate WARP? Two options:

- The consumer does not choose the best available bundle.
- The consumer does not have stable or strictly convex preferences.

Strong axiom of revealed preference

WARP = necessary condition for consistency with utility maxim.
Does not test, though, whether the preferences are transitive.

Strong axiom of revealed preference (SARP)
If X is directly or indirectly revealed preferred to Y,
then Y cannot be directly or indirectly revealed preferred to X.

Necessary and sufficient condition for consistency with utility maxim.

If SARP holds we can find such preferences for which consumer behavior will be consistent with utility maximization.

How to test SARP?

The table below shows expenditures on bundles 1, 2, and 3 at different prices:

		Bundles 1	Bundles 2	Bundles 3
Prices	1	20	10*	22(*)
	2	21	20	15*
	3	12	15	10

At prices 1 (line 1) bundle 1 is indirectly revealed preferred to bundle 3 with $(*)$.

SARP is violated if each pair of diagonal fields with has $*$ or $(*)$.

SARP is not violated.

What should you know?

- The consumer chooses the most preferred bundle from her budget set.
- If we have monotonic, convex and smooth IC and an inner solution, MRS equals to the slope of BL in optimum.
- This equality does not hold for perfect substitutes (usually) and for prefect complements.
- It always holds for Cobb-Douglas preferences. The consumer also spends fixed shares of her income on each good.

What should you know? (cont'd)

- If a consumer buys bundle A when bundle B is available, she *reveals that she prefers* A to B.
- For a rational consumer, it also means that she *prefers* A to B.
- We can use WARP and SARP to test whether the consumer is rational.
- WARP: if the consumer prefers A to B, with a different BL she cannot prefer B to A.
- SARP $=$ WARP $+$ indirectly revealed preferences (transitivity).
- If SARP holds, we can use consumer choices to estimate preferences.