

Consumer's surplus and market

Varian: Intermediate Microeconomics, 8e, chapters 14, 15 and 16

In this lecture you will learn

- how to measure consumers' welfare
- how to evaluate the impact of a tax reform
- how to derive a market demand
- what the connection between MR and elasticity of demand is
- what properties has market equilibrium
- how president Obama solved a problem with paparazzi



Consumer's surplus

How do we measure consumer's welfare?

Until now you probably used consumer's surplus (the area between below the demand curve and above the price price).

Now we learn two more general methods for measuring welfare:

- compensating variation (CV)
- equivalent variation (EV)

Consumer's surplus is only a special case of CV and EV.



Compensating variation (CV)

How much extra money would you need *after* a price change to be as well off as you were before the price change?

An alternative definition: What is our net revenue if we compensate a consumer for a change in price that happened in the past.

Example:

A Czech firm sends an employee to a more expensive country (USA).

- CV: By how much the employee's salary has to be raised so that she has the same utility as at the original salary in the CR.
- CV alternatively: What is the firm's cost of the income compensation at which the employee has the same utility as with the original salary in the CR.

Equivalent variation (EV)

How much extra money would you need *before* a price change to be just as well off as you were before the price change?

An alternative definition: What change in welfare would be equivalent to a change in price from the point of view of the consumer.

Example:

A Czech firm sends an employee to a more expensive country (USA).

- EV: By how much money the employee's salary has to be reduced so that she has the same utility as with the original salary in the US?
- EV alternatively: What salary reduction in the CR is equivalent to the employee working with the Czech salary in the US.

Example – Cobb-Douglas preferences

Utility function: $u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$

Income: $m = 90$

Price of good 2: $p_2 = 1$

Price of good 1 increases from $p_1^* = 1$ to $\hat{p}_1 = 2$

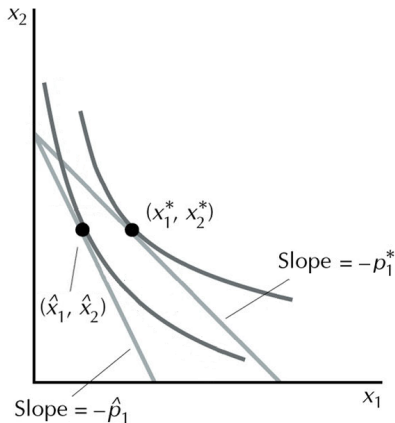
What is CV and EV?

The optimal bundle at $p_1^* = 1$:

$$(x_1^*, x_2^*) = \left(\frac{m}{3p_1^*}, \frac{2m}{3p_2} \right) = (30, 60)$$

The optimal bundle at $\hat{p}_1 = 2$:

$$(\hat{x}_1, \hat{x}_2) = \left(\frac{m}{3\hat{p}_1}, \frac{2m}{3p_2} \right) = (15, 60)$$



Example – Cobb-Douglas preferences (CV)

How much extra money would the consumer need at $(\hat{p}_1, p_2) = (2, 1)$ to be just as well off as with the bundle $(x_1^*, x_2^*) = (30, 60)$?

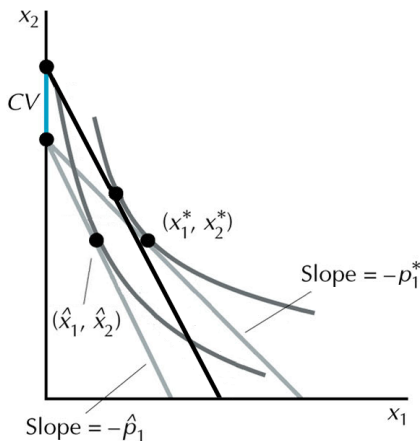
What income m^* would bring her the original utility $u(30, 60)$?

$$\left(\frac{m^*}{6}\right)^{\frac{1}{3}} \left(\frac{2m^*}{3}\right)^{\frac{2}{3}} = 30^{\frac{1}{3}} 60^{\frac{2}{3}}$$

$$m^* \approx 113$$

Compensating variation:

$$CV = m^* - m = 113 - 90 = 23$$



Example – Cobb-Douglas preferences (EV)

How much extra money would the consumer need at $(p_1^*, p_2) = (1, 1)$, to be just as well off as with the bundle $(\hat{x}_1, \hat{x}_2) = (15, 60)$?

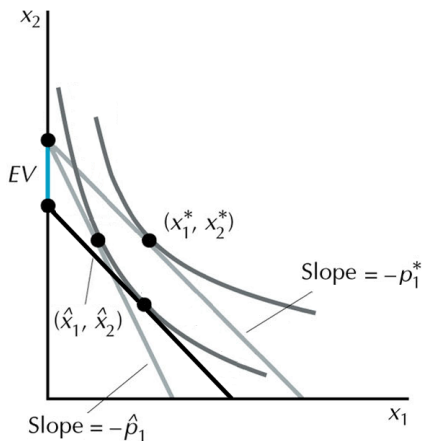
What income \hat{m} would bring her the original utility $u(15, 60)$?

$$\left(\frac{\hat{m}}{3}\right)^{\frac{1}{3}} \left(\frac{2\hat{m}}{3}\right)^{\frac{2}{3}} = 15^{\frac{1}{3}} 60^{\frac{2}{3}}$$

$$\hat{m} \approx 71$$

Equivalent variation:

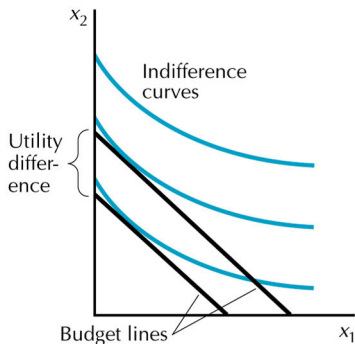
$$EV = m - \hat{m} = 90 - 71 = 19$$



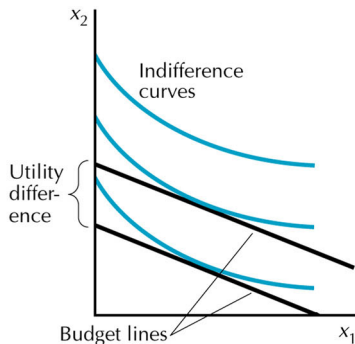
Compensating vs. equivalent variation

CV and EV = two ways of measuring the vertical distance between the ICs
 \implies usually $CV \neq EV$ (e.g. in the previous example).

Exception: quasilinear preferences – the vertical distance among ICs is the same for all relative prices $\implies CV = EV$ (see the graph below).



A

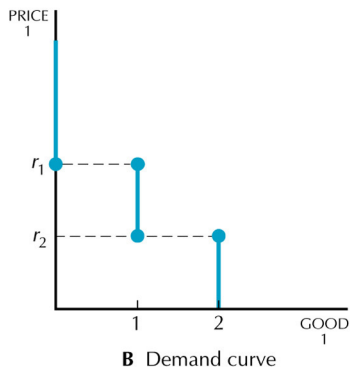
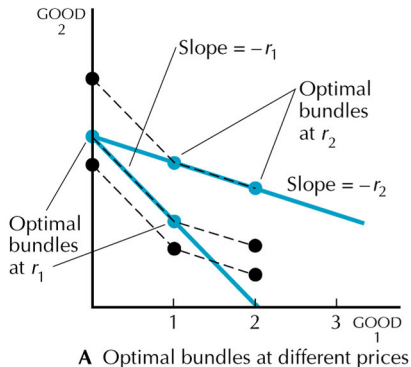


B

Consumer's surplus – quasilinear preferences

Consumer buys a discrete good 1 and a composite good 2 and has a quasilinear utility function $u(x_1, x_2) = v(x_1) + x_2$, where $v(x_1) = 0$.

We can use reservation prices to derive the demand:



Consumer's surplus – quasilinear preferences (cont'd)

For each x_1 holds: corresponding reservation price = $MU_1(x_1)$

Example:

At $p = r_1$ the consumer is indifferent between $u(x_1, x_2)$ for $x_1 = 0$ and 1:

$$u(0, m) = u(1, m - r_1)$$

$$v(0) + m = v(1) + m - r_1$$

$$r_1 = v(1)$$

At $p = r_2$ the consumer is indifferent between $u(x_1, x_2)$ for $x_1 = 1$ and 2:

$$u(1, m - r_2) = u(2, m - 2r_2)$$

$$v(1) + m - r_2 = v(2) + m - 2r_2$$

$$r_2 = v(2) - v(1)$$

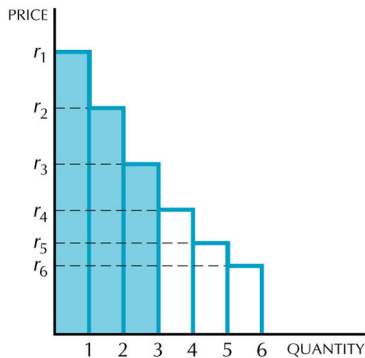
The utility from n units of good 1 = $r_1 + r_2 + \dots + r_n = v(n)$.

Example: Utility from $n = 2$ is $r_1 + r_2 = v(1) + v(2) - v(1) = v(2)$.

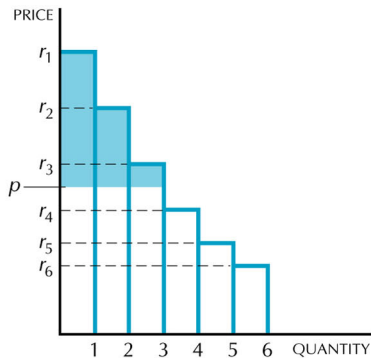
Consumer's surplus – quasilinear preferences (cont'd)

Gross surplus $GCS = v(n)$ is the utility from n units of good 1.

Net surplus $CS = v(n) - pn$ is the utility minus the expenditure on n units of good 1.



A Gross surplus



B Net surplus

Approximating a continuous demand – quasilinear pref.

Consumer's surplus associated with a continuous demand can be approximated by a discrete demand with small changes in quantity.



A Approximation to gross surplus

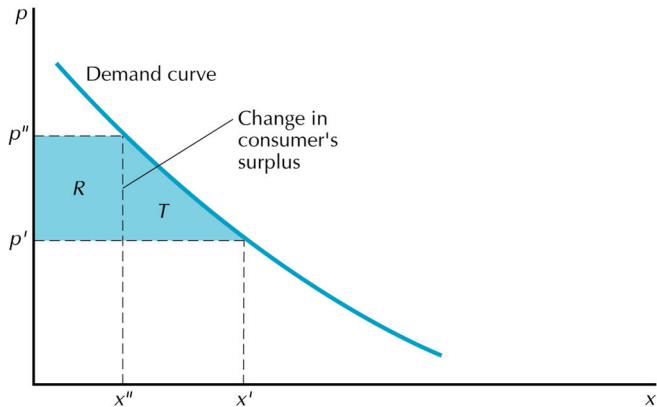


B Approximation to net surplus

A change in consumer's surplus – quasilinear preferences

Let us assume that the price of a good increases from p' to p'' .

The change in consumer's surplus ΔCS has a shape of a trapezoid.



Example – GCS for quasilinear preferences

Consumer buys good 1 and composite good 2

Utility function: $u = v(x_1) + x_2$, where $v(x_1) = 10x_1 - x_1^2/2$

Budget line: $m = p_1x_1 + x_2$

What is the gross consumer surplus at $p_1 = 8$?

I substitute the BL $x_2 = m - p_1x_1$ into the utility u : $v(x_1) + m - p_1x_1$

Looking for x_1 that maximizes utility:

$$MU_1(x_1) - p_1 = 0$$

$$p_1 = MU_1(x_1) = 10 - x_1$$

Price of good 1 measures marginal utilities for given quantities x_1 .

GCS – area under the inverse demand for $x_1 \in (0, 2)$:

- CS (area of the triangle) $+ p_1x_1 = \frac{2 \times 2}{2} + 8 \times 2 = 18$

- $\int_0^2 (10 - x_1) dx_1 = 10x_1 - x_1^2/2 = 18$

GCS equals to the utility from $x_1 = 2$: $v(x_1) = 10x_1 - x_1^2/2 = 18$.

Example – ΔCS pro quasilinear preferences

Inverse demand of a consumer: $p = 10 - x$

The price of good decreases from $p' = 8$ to $p'' = 7$

What is the change in the net consumer's surplus (ΔCS)?

The original consumer's surplus:

Quantity demanded: $x' = 10 - p' = 2$

Consumer's surplus: $CS' = \frac{(10-p')x'}{2} = \frac{(10-8) \times 2}{2} = 2$

New consumer's surplus:

Quantity demanded: $x'' = 10 - p'' = 3$

Consumer's surplus: $CS'' = \frac{(10-p'')x''}{2} = \frac{(10-7) \times 3}{2} = 4.5$

The change in consumer's surplus: $\Delta CS = CS'' - CS' = 2.5$

ΔCS , CV and EV for quasilinear preferences

Utility function: $u(x_1, x_2) = v(x_1) + x_2$, price increase from p_1' to p_1''

A change in consumer's surplus

$$\Delta CS = (v(x_1') - p_1'x_1') - (v(x_1'') - p_1''x_1'')$$

CV: How much to give the consumer at the price p_1'' to keep her at the same utility as with the price p_1' : $u(x_1'', x_2'') + CV = u(x_1', x_2')$

$$v(x_1'') + m - p_1''x_1'' + CV = v(x_1') + m - p_1'x_1'$$

$$CV = (v(x_1') - p_1'x_1') - (v(x_1'') - p_1''x_1'') = \Delta CS$$

EV: How much to take from the consumer at the price p_1' to keep her at the same utility as with the price p_1'' : $u(x_1', x_2') - EV = u(x_1'', x_2'')$

$$v(x_1') + m - p_1'x_1' - EV = v(x_1'') + m - p_1''x_1''$$

$$EV = (v(x_1') - p_1'x_1') - (v(x_1'') - p_1''x_1'') = \Delta CS$$

SUPPLEMENT: ΔCS , CV and EV for other preferences

Quasilinear preferences ($IE = 0$), where good 2 is composite:

- Prices = reservation prices = marginal utilities measured in money
- Utility from n units = area below the demand curve for $x_1 \in (0, n)$
- ΔCS measures the changes in utility in money units correctly

For quasilinear preferences holds:

$$\Delta CS = CV = EV$$

Other preferences with $IE \neq 0$:

- If price of good 1 decreases, I become richer.
- Normal good ($IE < 0$): demand additional unit for price $> MU$
(Inferior good ($IE > 0$): demand additional unit for price $< MU$)
- Utility from n units \neq area below the demand curve for $x_1 \in (0, n)$
- ΔCS **doesn't measure** the changes in utility correctly.

For other preferences with $IE \neq 0$ holds:

$$\Delta CS \neq CV \neq EV$$

APPLICATION: Evaluating social policies

If we know the market demand function we can calculate ΔCS .

This approach suffers from two defects:

- ① ΔCS = change in welfare only for quasilinear preferences.
- ② ΔCS we get an estimate of cost across the population. Important to know more about the distribution of costs.

King (J Publ Econ, 1983) uses a method similar to EV to evaluate a proposed reform of tax treatment of housing in Britain.

Results:

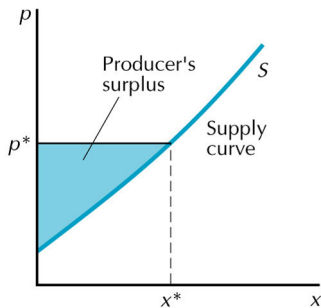
- 4 888 out of 5 895 studied households are better off.
- Worse off are especially for very poor households.

This information is crucial for a correct setup of the reform.

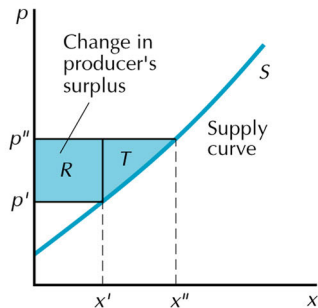
Producer's surplus PS

Producer's surplus (PS) = the difference between the revenue from x^* and the minimal amount at which the producer is willing to sell x^* .

A rise in price from p' to p'' reduces producer's surplus by ΔPS .



A



B

Market demand

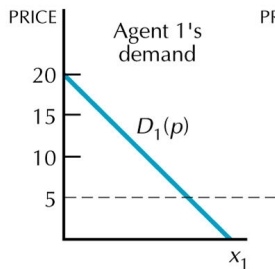
Market demand = sum of individual demand curves

Example:

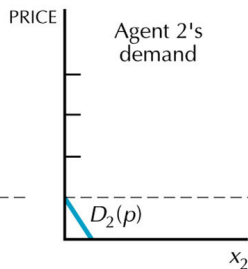
Individual demand curves:

- $D_1(p) = \max\{20 - p, 0\}$
- $D_2(p) = \max\{2.5 - p/2, 0\}$

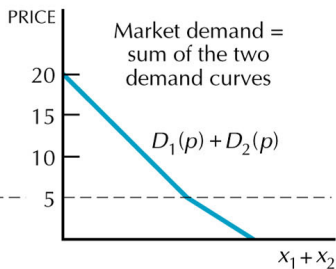
Market demand: $D(p) = D_1(p) + D_2(p) = \max\{22.5 - 1.5p, 20 - p, 0\}$



A



B



C

Price elasticity of demand

Price elasticity of demand measures sensitivity of demand to price.

Two ways of calculating:

1) Percentage change in quantity/percentage change in price:

$$\epsilon = \frac{\Delta q}{q} / \frac{\Delta p}{p} = \frac{\Delta q}{\Delta p} \frac{p}{q}$$

2) Point elasticity (small changes):

$$\epsilon = \frac{dq}{dp} \frac{p}{q}$$

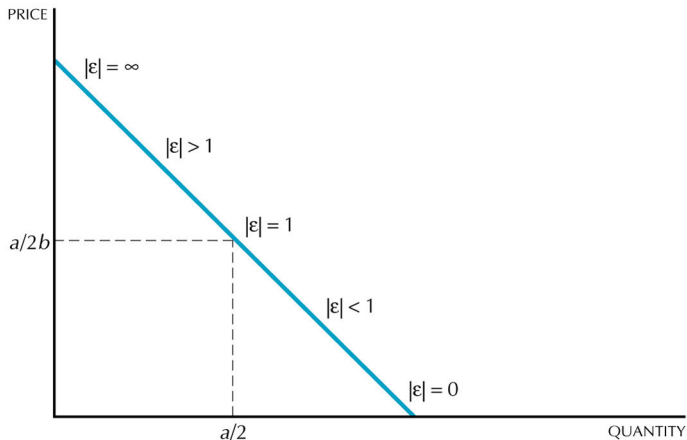
Elasticity often shown in absolute value:

- $|\epsilon| < 1$ – **inelastic demand**
- $|\epsilon| = 1$ – **unit elastic demand**
- $|\epsilon| > 1$ – **elastic demand**

Example – linear demand function

Linear demand $q = a - bp$:

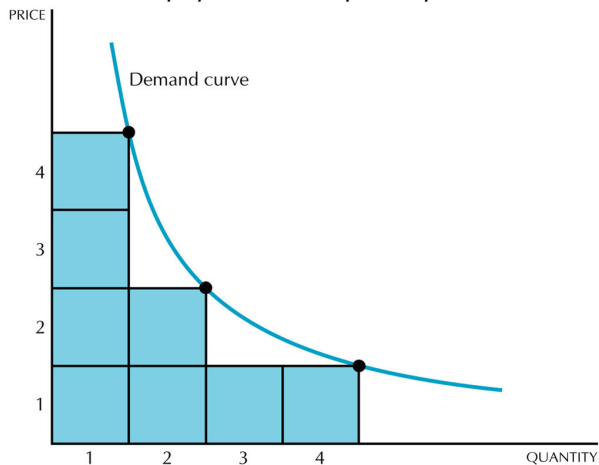
$$\epsilon = \frac{dq}{dp} \frac{p}{q} = -b \frac{p}{q}$$



Example – constant elasticity demand

Demand function $q = Ap^\epsilon$:

$$\epsilon = \frac{dq}{dp} \frac{p}{q} = \epsilon Ap^{\epsilon-1} \frac{p}{q} = \frac{\epsilon Ap^\epsilon}{Ap^\epsilon} = \epsilon$$



Elasticity and marginal revenue

Marginal revenue MR – what is the change in total revenue if quantity increases by 1 unit:

$$MR(q) = \frac{dR(q)}{dq} = \frac{d(pq)}{dq} = p + \frac{dp}{dq}q \quad (1)$$

By expanding (1) by p/p we get the relationship between MR and ϵ :

$$MR(q) = p + \frac{dp}{dq}q \frac{p}{p} = p \left(1 + \frac{dp}{dq} \frac{q}{p} \right) = p \left(1 + \frac{1}{\epsilon} \right) = p \left(1 - \frac{1}{|\epsilon|} \right)$$

Control questions:

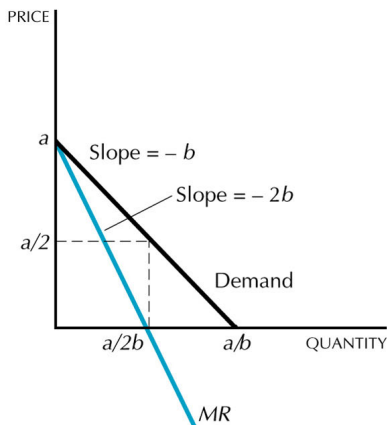
- A monopoly charges a price $p = 10$ at which $|\epsilon| = 4$.
What is monopoly's marginal revenue? $MR = 10 \times (1 - 1/4) = 7.5$
- Can a profit-maximizing monopoly choose a price at which $|\epsilon| < 1$?
No (It would be Yes only if $MC < 0$).

Example of MR – linear demand

Marginal revenue for a linear inverse demand curve $p(q) = a - bq$:

$$MR(q) = (R(q))' = ((a - bq) \times q)' = (aq - bq^2)' = a - 2bq$$

The marginal revenue curve has the same vertical intercept as the demand curve, but has twice the slope.



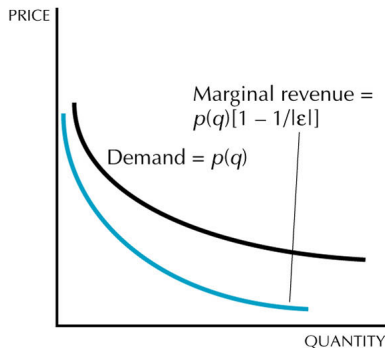
Example of MR – constant elasticity demand

Marginal revenue for a demand with constant elasticity $q(p) = Ap^\epsilon$:

$$MR(q) = p \left(1 + \frac{dp(q)}{dq} \frac{q}{p} \right) = p \left(1 + \frac{1}{\epsilon} \right) = p \left(1 - \frac{1}{|\epsilon|} \right)$$

Relationship between ϵ and MR :

- $|\epsilon| < 1 \implies MR < 0$
- $|\epsilon| = 1 \implies MR = 0$
- $|\epsilon| > 1 \implies MR > 0$



CASE: Strikes and Profits

In 1979 the United Farm Workers called for a strike against lettuce growers in California.

Result of the strike:

- production was cut in half
- price of lettuce rose 4x
- profits of farmers doubled

Producers eventually settled the strike. Why?

They were afraid of the long-run supply response. Most of the lettuce consumed in U.S. in winter is grown in California. If the strike had held for several seasons, lettuce could be planted in other regions.



Equilibrium – assumptions

Market demand $D(p)$ = sum of individual demands of consumers

Market supply $S(p)$ = sum of individual supplies of firms

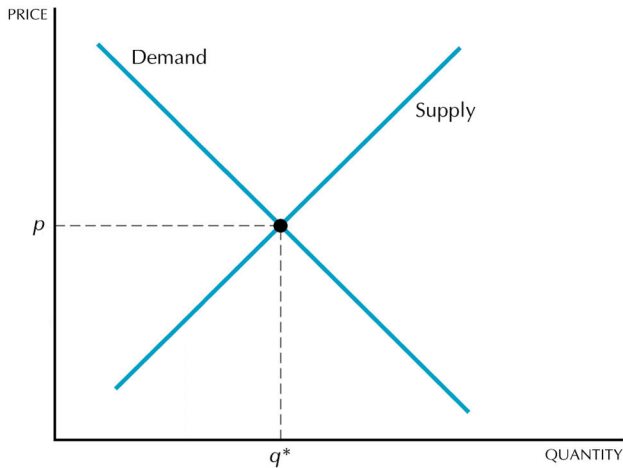
Competitive market – each agent takes prices as outside of her control. The market price is independent of any one agent's actions, but determined by the actions of all the agents together.



Equilibrium

At the equilibrium price quantities supplied and demanded are equal:

$$D(p^*) = S(p^*) = q^*$$



Example – equilibrium in the market with linear curves

Demand curve: $D(p) = a - bp$

Supply curve: $S(p) = c + dp$

What is the equilibrium price and quantity?

In equilibrium supply equals to demand:

$$D(p) = a - bp^* = c + dp^* = S(p)$$

Equilibrium price:

$$p^* = \frac{a - c}{b + d}$$

Substituting the price into $D(p)$ or $S(p)$ we get equilibrium quantity:

$$q^* = \frac{ad + bc}{b + d}$$

Taxes

When a tax is present in a market, there are two prices of interest:

- **Demand price** p_D – the price the demander pays.
- **Supply price** p_S – the price the supplier gets.

The prices differ by the amount of the tax

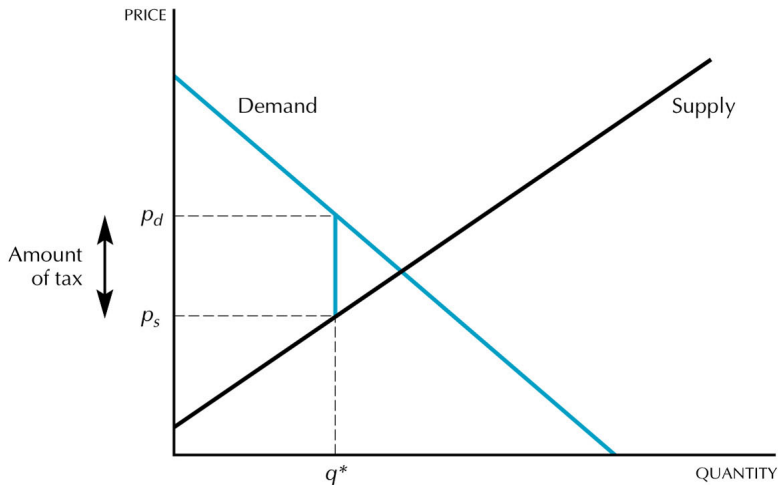
Quantity tax: $p_D = p_S + t$.

Value tax: $p_D = (1 + \tau)p_S$.



Example – quantity tax

Equilibrium with quantity tax: $q^* = D(p_D) = S(p_S)$ and $p_S = p_D - t$



Example – quantity tax with linear curves

Quantity tax: $t > 0$

Demand curve: $D(p) = a - bp$

Supply curve: $S(p) = c + dp$

What determines the division of the tax between demand and supply?

Equilibrium conditions with quantity tax:

$$a - bp_D^* = c + dp_S^* \quad \text{and} \quad p_D^* = p_S^* + t$$

Solution – supply a demand prices:

$$p_S^* = \frac{a - c - bt}{b + d} \quad \text{and} \quad p_D^* = \frac{a - c + dt}{b + d}$$

Difference between the equilibrium price $p^* = (a - c)/(b + d)$ and

- supply price p_S^* is $bt/(b + d)$,
- demand price p_D^* is $-dt/(b + d)$.

The division of the tax depends on slopes of supply and demand.

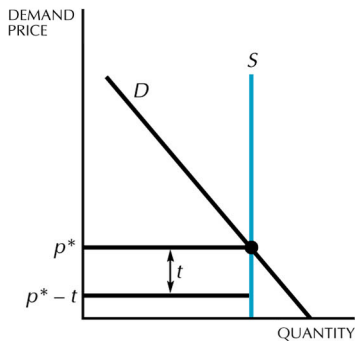
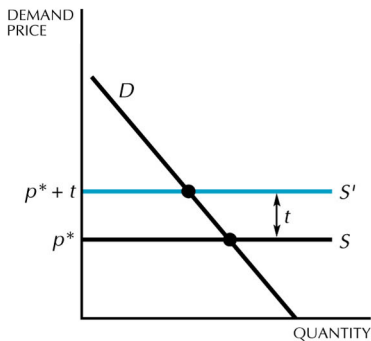
Passing along a tax

Horizontal supply ($d = \infty$) – demanders pay the entire tax:

$$p_S^* = p^* \text{ a } p_D^* = p^* + t.$$

Vertical supply ($d = 0$) – suppliers pay the entire tax:

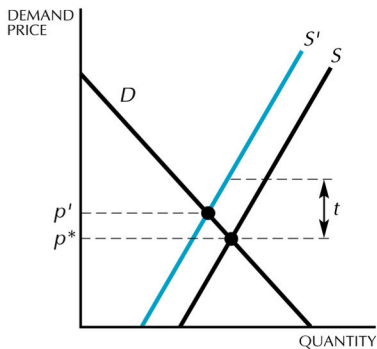
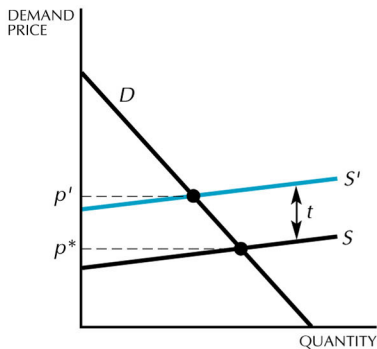
$$p_S^* = p^* - t \text{ a } p_D^* = p^*.$$



Passing along a tax

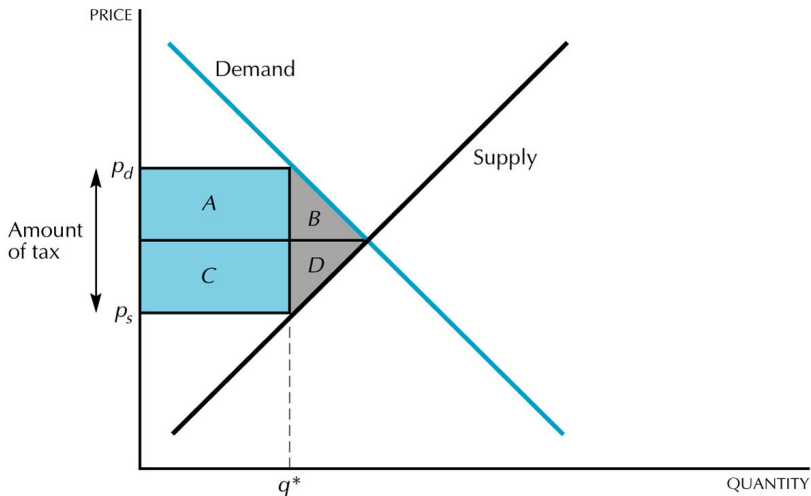
A flat supply – much of the tax can be passed along to demanders.

A steep supply – very little of the tax can be passed along.



Deadweight loss

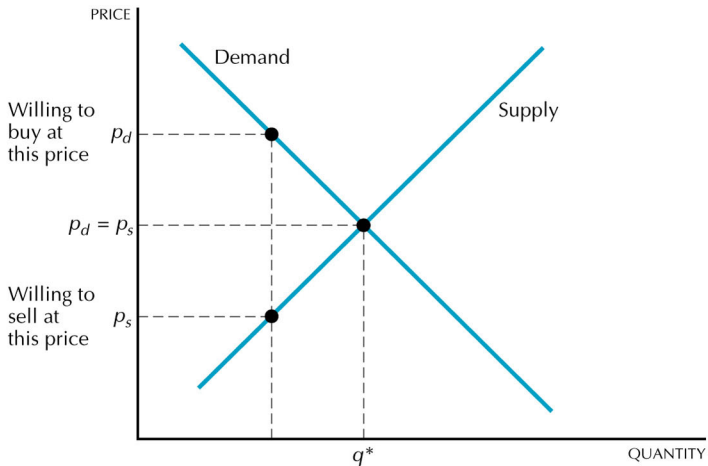
Deadweight loss of a tax – net loss in consumer's and producer's surplus due to a reduction in quantity (areas B + D in the graph).



Pareto efficiency

An economic situation is Pareto efficient if there is no way to make any person better off without hurting anybody else.

Perfectly competitive market is Pareto efficient at the quantity q^* .



APPLICATION: Waiting in line

Will waiting in line lead to a Pareto efficient allocation?

Is it possible that someone who waited for a ticket might be willing to sell it to someone who didn't wait in line?

Yes. Willingness to wait and to pay differ across the population.

Plus waiting in line is a form of deadweight loss—the people who wait in line pay a “price” but no one receives any benefits from the price they pay.



APPLICATION: President Obama and paparazzi

President Obama had a problem: paparazzi pursued his daughters.

Paparazzi were motivated by a high price (high demand in the media).

What would be your advice for him?

Reduce demand – the White house offers photographs of the presidential family for free.



What should you know?

- The effect of a change in price on welfare can be measured using CV, EV, or ΔCS .
- CV: What amount compensates the consumer for a completed change in price.
- EV: What amount is equivalent to the effect of a planned change in price.
- CS: The difference between the area below demand and the revenues.
- $CV = EV = \Delta CS$ for quasilinear preferences because they don't have IE. Not equal for other preferences.
- PS: The difference between the revenues and the area below supply.



What should you know? (cont'd)

- Market demand is a sum of individual demand curves.
- The more elastic the demand, the higher the MR at a given price.
- Unregulated market equilibrium is Pareto efficient – no way to make anyone better off without hurting anyone else.
- Tax is the difference between the demand and supply price.
- If a tax affects the quantity, the result is not Pareto efficient – there is DWL.

