Uncertainty

Varian: Intermediate Microeconomics, 8e, chapter 12

In this lecture you will learn

- how standard tools of consumer choice can be used for analysing decisions under risk and what is special about these decisions
- how to model different attitudes towards risk
- whether it is better to bet on favourites or outsiders
- what makes health insurance valuable
- what we are willing to pay in order to prevent catastrophes

What do we choose?

Probability distributions with different consumptions (= lotteries).

Probability distribution (lottery) = list of possible consumption bundles with probabilities that I get them

$$L = \{\pi_1, \pi_2, ..., \pi_n\},$$

where $\pi_n \geq 0$ is the probability to get bundle n where $\sum_n \pi_n = 1$.

Example:

I bet my last 100 CZK on a toss of a coin.
If I win, I have 200 CZK. If I lose, I have 0 CZK.

What does this lottery look like?

The lottery in this case is $L = \{\pi_1, \pi_2\} = \{1/2, 1/2\}$,
where result 1 is 200 CZK and result 2 is 0 CZK.

Contingent consumption

States of nature are different outcomes of some random event.

Examples:

- a bet (toss of a coin) – 2 states of nature: head, tail
- car insurance – 2 states of nature: car stolen, car not stolen

For simplicity we will study the decisions of consumers

- facing one random event with a few states of nature,
- with consumption measured in monetary units.

Contingent consumption plan is a specification of what will be consumed in each different state of nature—each different outcome of one random process.

Difference to lottery: Contingent consumption plan shows only consumption and not probabilities.

Example – insurance

A consumer plans to spend 35 000. Her car will be destroyed in an accident with probability 1% – the damage of 10 000.

Her contingent consumption plan is $(c_b, c_g) = (25\,000, 35\,000)$:

- a bad state of nature b occurs with probability 1%
- a good state of nature g occurs with probability 99%

Insurance offers a way to change this probability distribution.
If she pays a premium of γK she gets an insurance payment of K.

The consumer chooses between the following consumption plans:

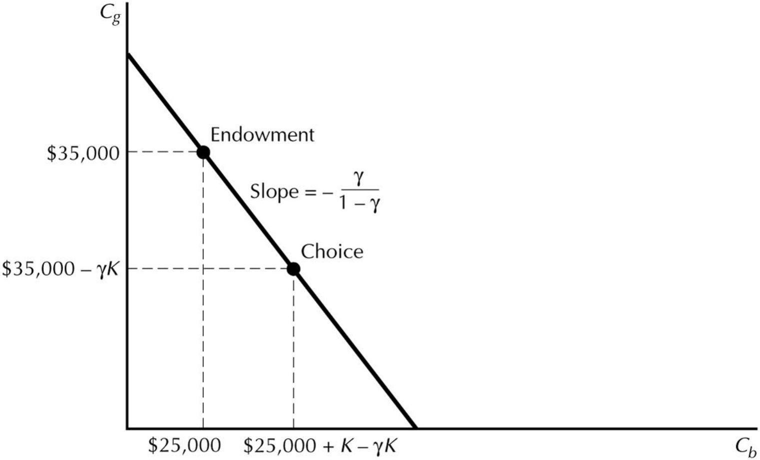
$$(c_b, c_g) = (25\,000 + K - \gamma K, 35\,000 - \gamma K) \tag{1}$$

By eliminating K from (1) we get the budget line (BL):

$$c_g = 35\,000 + \frac{\gamma}{(1-\gamma)} 25\,000 - \frac{\gamma}{(1-\gamma)} c_b$$

Example – insurance (cont'd)

Budget line (BL): $c_g = \underbrace{35\,000 + \frac{\gamma}{(1-\gamma)}25\,000}_{\text{vertical intercept}} - \underbrace{\frac{\gamma}{(1-\gamma)}}_{\text{slope of BL}} c_b.$



Example – insurance (cont'd)

For the choice of the contingent consumption plan we can use the consumer theory we have developed in previous lectures:

- *budget constraint* is given (e.g. by insurance choice)
- *preferences* defined over different consumption plans

The consumer chooses the best consumption plan she can afford.

What is the optimal premium K? It depends on preferences.

E.g. on the consumer's attitudes towards risk:

- If she is conservative, she chooses a high K.
- If she likes risk, she might not buy any insurance.

Before we continue our insurance example, we explain

1. how preferences under risk are represented by utility functions,
2. what the properties of these functions are,
3. how to use utility functions to represent attitudes to risk.

Representing preferences using utility functions

Choice under uncertainty does add a special structure to the problem. How a person values consumption in different states will depend on the probabilities that the states occur.

E.g. the probability of an accident π influences my *marginal rate of substitution*. The higher the π, the more of c_g I am willing to sacrifice for an additional unit of c_b (a more expensive insurance).

The utility function for consumption in states 1 and 2 is given by

$$u(c_1, c_2, \pi_1, \pi_2)$$

where

- c_1 and c_2 is consumption in states 1 and 2,
- π_1 and π_2 are probabilities of states 1 and 2.

Examples of utility functions

- Perfect substitutes:

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1 + \pi_2 c_2$$

$\pi_1 c_1 + \pi_2 c_2$ is the **expected value** of a given event.

- Cobb-Douglas utility function:

$$u(c_1, c_2, \pi_1, \pi_2) = c_1^{\pi_1} c_2^{\pi_2}$$

Or sometimes a more convenient monotonic transformation:

$$\ln u(c_1, c_2, \pi_1, \pi_2) = \pi_1 \ln c_1 + \pi_2 \ln c_2$$

Von Neumann-Morgenstern utility function

Von Neumann-Morgenstern utility function is given by

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2),$$

where $v(c_1)$ and $v(c_2)$ are utilities in individual states of nature.

In examples on the previous slide:

- perfect substitutes: $v(c) = c$
- Cobb-Douglas utility function: $v(c) = \ln c$

This function is also called the **expected utility function** – $u(c_1, c_2, \pi_1, \pi_2)$ equals to the **expected utility** of consumption in individual states of nature $\pi_1 v(c_1) + \pi_2 v(c_2)$.

Positive affine transformation

Consumer preferences represented by the *expected utility function*, which has the **additive form** described above.

Any monotonic transformation describes the same preferences, but the additive form representation is especially convenient.

E.g. the functions $\pi_1 \ln c_1 + \pi_2 \ln c_2$ and $c_1^{\pi_1} c_2^{\pi_2}$ describe the same Cobb-Douglas preferences but $c_1^{\pi_1} c_2^{\pi_2}$ does not have the additive form.

Positive affine transformation $t(u)$ – a type of monotonic transformation that preserves the expected utility property:

$$t(u) = au + b \text{ where } a > 0.$$

A positive affine transformation simply means multiplying by a positive number a and adding a constant b.

Why is expected utility reasonable?

Let us have a random event with 3 states of nature:

- my house burns down with probability π_f – consumption c_f
- my house does not burns down with probability π_n – consumption c_n
- I sell the house this year with probability π_s – consumption c_s

Under uncertainty *only one* state of nature is actually going to occur.
$\implies$ There is a natural *independence* among different states.

The independence is well represented by the additive utility function:

$$u(c_f, c_n, c_s, \pi_f, \pi_n, \pi_s) = \pi_f v(c_f) + \pi_n v(c_n) + \pi_s v(c_s)$$

MRS between c_f and c_n is independent from c_s:

$$\text{MRS}_{fn} = -\frac{\pi_f \frac{\partial v(c_f)}{\partial c_f}}{\pi_n \frac{\partial v(c_n)}{\partial c_n}}$$

A comparison to the decision-making under certainty

My preferences for 3 goods (tee, coffee, milk) $= (t, c, m)$ can be represented by a utility function

$$u(t, c, m) = 2t + cm.$$

The marginal rate of substitution between t and c is

$$\text{MRS}_{tc} = -\frac{2}{m}.$$

The MRS between tee and coffee depends on the quantity of milk m.

Decision-making under certainty: one can consume combinations of goods *at the same time*. $\Longrightarrow$ We cannot *a priori* exclude any functional forms of the utility function.

SUPPLEMENT: Independence assumption

Preferences can be represented by the expected utility function only if the independence assumption holds.

Preference relation $\succeq$ satisfies the **independence assumption** if for all triples of lotteries L, L' a L'' and for the parameter $\alpha \in (0, 1)$ it holds that

$$L \succeq L'$$

if and only if

$$\alpha L + (1 - \alpha)L'' \succeq \alpha L' + (1 - \alpha)L''.$$

<u>In other words:</u>

If we mix any two lotteries with a third one, preferences between the two lotteries will stay the same (are not influenced by the third one).

SUPPLEMENT: Example of a choice under risk

Peter can go for a dinner to a restaurant A, or to a restaurant B.
There are three different food qualities: good G, average A, or bad B.

Restaurant A has a good cook who often has a bad day:
With probability 50% Peter gets G and with probability 50% B.

The cook in restaurant B is more average:
With probability 90% Peter gets A and with probability 10% B.

What restaurant does Peter choose? It depends on preferences.

Let us assume that Peter's preferences are represented by such an expected utility function that he chooses restaurant A, i.e.:

$$0.5v(G) + 0.5v(B) > 0.9v(A) + 0.1v(B) \tag{2}$$

SUPPLEMENT: Example of a choice under risk (cont'd)

Does Peter choose differently if he finds out that in both restaurants Jamie Oliver prepares a perfect food P for him with probability 50%?

No. This information increases the expected utility from restaurants A and B by the same amount. If (2) holds, then it must also hold that:

$$\frac{1}{2}v(P) + \frac{1}{2}\Big(0.5v(G) + 0.5v(B)\Big) > \frac{1}{2}v(P) + \frac{1}{2}\Big(0.9v(A) + 0.1v(B)\Big)$$

The independence assumption says that if both restaurants offer P with the same probability, Peter's choice does not change.

The independence assumption sounds reasonable. $\implies$ It seams reasonable to represent preferences under uncertainty using the expected utility function.

SUPPLEMENT: TV competition

Suppose that you have won two weekly editions of a TV competition and each time you were given a choice between two lotteries:

1st week:

1. 100% – 500 000 CZK
2. 1% – 0 CZK
   10% – 1 000 000 CZK
   89% – 500 000 CZK

2nd week:

1. 11% – 500 000 CZK
   89% – 0 CZK
2. 10% – 1 000 000 CZK
   90% – 0 CZK

SUPPLEMENT: TV competition – Allais paradox

If the independence assumption holds, the consumer should have chosen the same option in both weeks, either 1 or 2. Why?

The choices 1 and 2 in both weeks are the same:

		Probabilities		
		1/100	10/100	89/100
1st week	1	500 000	500 000	500 000
	2	0	1 000 000	500 000
2nd week	1	500 000	500 000	0
	2	0	1 000 000	0

By mixing $0.89 \times 500\,000$ (1st week) and 0.89×0 (2nd week) to both lotteries 1 and 2, we get the choices from the previous slide.

In this situation people usually violate the independence assumption (Allais paradox).

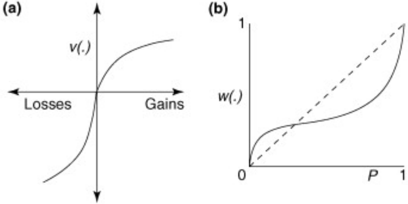
SUPPLEMENT: Prospect theory

Prospect theory (Kahneman and Tversky, Econometrica, 1979) = the most cited behavioral alternative to the expected utility theory that is able to explain i.a. the Allais paradox.

Utility from a lottery (x_1, x_2, π_1, π_2):

$$V(x_1, x_2, p_1, p_2) = w(\pi_1)v(x_1) + w(\pi_2)v(x_2)$$

- **value function** $v(\cdot)$ – S-shaped around reference + loss aversion
- **weighting function** $w(\cdot)$ – Inverted S = people underweight high and overweight high probabilities; 0% and 100% are perceived correctly



SUPPLEMENT: Explaining Allais paradox

Choices leading to Allais paradox:

1st week:

1. 100% – 500 000 CZK
2. 1% – 0 CZK
   10% – 1 000 000 CZK
   89% – 500 000 CZK

2nd week:

1. 11% – 500 000 CZK
   89% – 0 CZK
2. 10% – 1 000 000 CZK
   90% – 0 CZK

Choices in the 1st week 1 and 2nd week 2 (violation of the independence assumption) are in line with prospect theory if

$$\frac{w(0.1)}{1 - w(0.89)} < \frac{v(500\,000)}{v(1\,000\,000)} < \frac{w(0.1)}{w(0.11)}.$$

An intuitive explanation:

- 1st week: 1 – 89% underweighted vs. 100% perceived correctly
- 2nd week: 2 – 10 and 11% overweighted to a similar extent

Attitudes toward risk

The consumer has a wealth of \$10,

- with probability 50 % she wins \$5,
- with probability 50 % she loses \$5.

Her wealth has

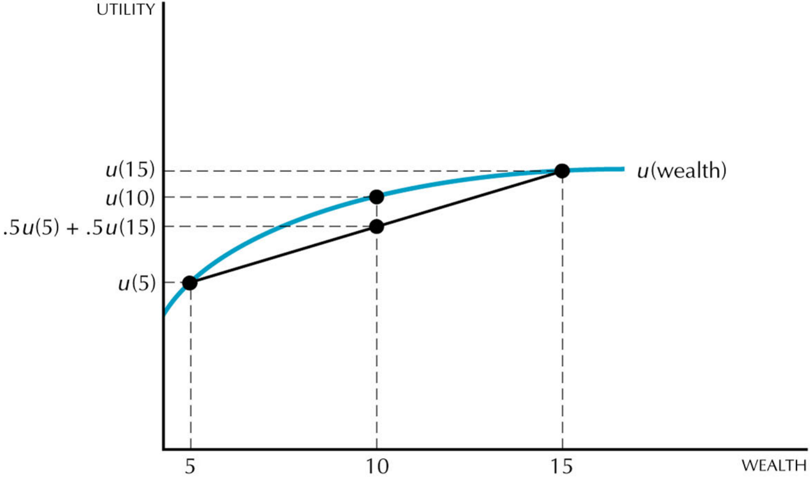
- the expected value $EV = 0.5 \times 5 + 0.5 \times 15 = 10$,
- the expected utility $EU = 0.5 \times u(5) + 0.5 \times u(15)$.

The consumer

- is **risk averse** if $u(EV) > EU$ – a concave $u(c)$,
- is **risk seeking** if $u(EV) < EU$ – a convex $u(c)$,
- is **risk neutral** if $u(EV) = EU$ – a linear $u(c)$.

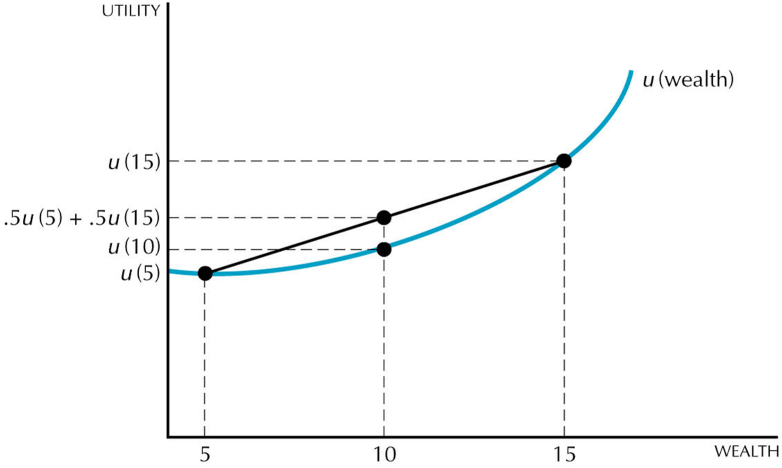
Risk aversion

A concave utility function $\implies$ $u(EV) > EU$



Risk seeking

A convex utility function $\implies u(EV) < EU$



Attitudes toward risk – specific utility functions

The consumer with the initial wealth of \$10

- wins \$5 with probability 50%,
- looses \$5 with probability 50%.

What is her attitude toward risk for the following utility functions?

- Utility function $u(c) = \sqrt{c}$:
  $u(EV) = \sqrt{EV} = \sqrt{10} = 3.16$
  $EU = 0.5 \times \sqrt{5} + 0.5 \times \sqrt{15} = 3.05$
  $u(EV) > EU \implies$ The consumer is risk-averse.

- Utility function $u(c) = c^2$:
  $u(EV) = EV^2 = 10^2 = 100$
  $EU = 0.5 \times 5^2 + 0.5 \times 15^2 = 125$
  $u(EV) < EU \implies$ The consumer is risk-seeking.

Certainty equivalent

Certainty equivalent (CE) = what is the (minimal) certain amount of money I am willing to exchange for a given lottery.

Example:
Consumer's wealth is 10.
A lottery: $L = \{\pi_1, \pi_2\} = \{0.5, 0.5\}$, where $c_1 = 15$ a $c_2 = 5$
Expected utility function: $u = \pi_1\sqrt{c_1} + \pi_2\sqrt{c_2}$
What is the certainty equivalent and the expected value?

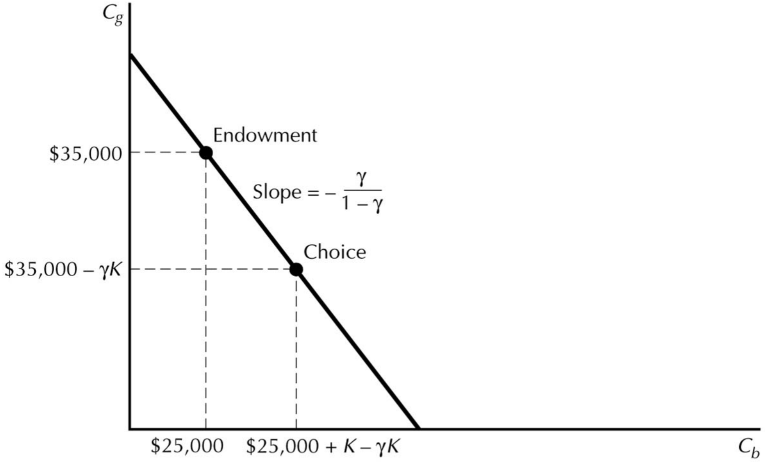
Utility of the lottery: $u_L = EU = 0.5 \times \sqrt{15} + 0.5 \times \sqrt{5} = 3.05$
Utility from CE equals to u_L: $u_L = \sqrt{CE} \iff CE = u_L^2 = 9.33$
The expected value: $EV = 0.5 \times 15 + 0.5 \times 5 = 10$

If:

- $CE < EV$, the consumer is **risk-averse**.
- $CE > EV$, the consumer is **risk-seeking**.
- $CE = EV$, the consumer is **risk-neutral**.

Example – choice of the optimal insurace (graph)

Linie rozpočtu (BL): $c_g = 35\,000 + \frac{\gamma}{(1-\gamma)} 25\,000 - \underbrace{\frac{\gamma}{(1-\gamma)}}_{\text{sklon BL}} c_b$



Example – fair insurance

Consumption in a bad state: $c_b = 25\,000 + K - \gamma K$
Consumption in a good state: $c_g = 35\,000 - \gamma K$
Probability of the bad state (accident) is π.
We assume that the insurer offer the fair insurance.

What is the optimal insurance premium for a risk-averse consumer?

Fair insurance – the insurer chooses such a premium ratio γ so that its profit is zero: $\gamma K - \pi K = 0 \iff \gamma = \pi$.

By substituting $\gamma = \pi$ into the equation

$$MRS = -\frac{\pi \frac{\Delta u(c_b)}{\Delta c_b}}{(1-\pi)\frac{\Delta u(c_g)}{\Delta c_g}} = -\frac{\gamma}{1-\gamma}$$

we get

$$\frac{\Delta u(c_b)}{\Delta c_b} = \frac{\Delta u(c_g)}{\Delta c_g}.$$

Example – fair insurance (cont'd)

Marginal utility of consumption has to be the same in both states.
A risk-averse consumer has a diminishing MU of consumption.

If e.g. $c_b < c_g$, than it would have to hold: $\frac{\Delta u(c_b)}{\Delta c_b} > \frac{\Delta u(c_g)}{\Delta c_g}$

If we want to get $\frac{\Delta u(c_b)}{\Delta c_b} = \frac{\Delta u(c_g)}{\Delta c_g}$, then it must hold:

$$c_b = c_g$$

$$25\,000 + K - \gamma K = 35\,000 - \gamma K$$

$$K = 10\,000$$

<u>Conclusion:</u>

If a risk-averse consumer faces a fair insurance, she fully insures.

Numerical example – insurance

Consumption in a bad state: $c_b = 25\,000 + K - \gamma K = 25\,000 + 0.9K$
Consumption in a good state: $c_g = 35\,000 - \gamma K = 35\,000 - 0.1K$
Utility function: $u(c_b, c_g, \pi_b, \pi_g) = 0.1 \ln c_b + 0.9 \ln c_g$

What is the optimal insurance payment K?

We solve the equation c_g for K:

$$K = (35\,000 - c_g)/0.1 = 350\,000 - c_g/0.1$$

We substitute into c_b:

$$c_b = 25\,000 + 0.9(350\,000 - c_g/0.1)$$

The budget line:

$$c_b + 9c_g = 340\,000$$

Numerical example – insurance (cont'd)

We look for the bundle, at which MRS = the slope of BL:

$$\text{MRS} = -\frac{\gamma}{1-\gamma} \quad \left(\text{or MRS} = -\frac{p_b}{p_g}\right)$$

$$-\frac{0.1c_g}{0.9c_b} = -\frac{0.1}{0.9}$$
$$c_b = c_g$$

By substituting into the budget line we get:

$$c_g + 9c_g = 340\,000$$

$$c_g = 34\,000$$

The optimal insurance payment:

$$K = (35\,000 - c_g)/0.1 = 10\,000\ \$$$

APPLICATION: Diversification

The investor has \$100 which she can invest in

- firm S (sun glasses) – price of 1 share $p_S = \$10$
- firm U (umbrellas) – price of 1 share $p_U = \$10$

Summer will be rainy with 50% and sunny with 50% probability:

- rainy: $p_U = \$20$ and $p_S = \$5$
- sunny: $p_U = \$5$ a $p_S = \$20$

What should a risk-averse investor do?

Two options:

- \$100 in firm U, contingent consumption plan $(c_U, c_S) = (200, 50)$, $EV = 0.5 \times 200 + 0.5 \times 50 = 125$ \$,
- 50 \$ in each firm, contingent consumption plan $(c_U, c_S) = (125, 125)$, $EV = 0.5 \times 125 + 0.5 \times 125 = 125$ \$.

Diversification reduces the risk, but the EV remains the same.

As long as asset price movements are not *perfectly* positively correlated, there will be some gains from diversification.

APPLICATION: Risk spreading

A village: 1 000 risk-averse farmers with a wealth 3 500 000 CZK
The risk of fire: 1 % (houses far apart = the risk is independent)
The cost of fire: 1 000 000 CZK

How do farmer insure if a standard insurance is not available?

Risk spreading: Fire victims get 1 000 CZK from all farmers.

On average 10 farmers are affected – farmer's expected wealth

- if she is not affected: $3\,500\,000 - 10 \times 1\,000 = 3\,490\,000$ CZK
- if she is affected: $2\,500\,000 + 990 \times 1\,000 = 3\,490\,000$ CZK

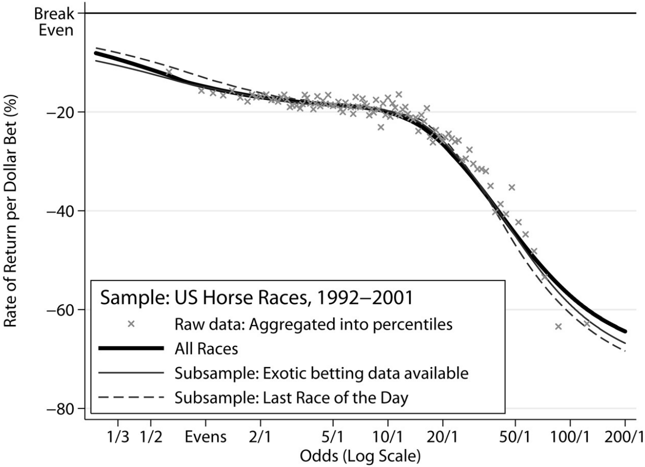
The expected wealth of the farmer without insurance:

$$0.99 \times 3\,500\,000 + 0.01 \times 2\,500\,000 = 3\,490\,000 \text{ CZK}$$

The expected wealth is the same, but the risk is lower.

APPLICATION: Favourite-longshot bias (FLB)

Is it more profitable to bet on favourites or outsiders? On favourites.



Source: Snowberg a Wolfers, JPE, 2010, Fig. 1

APPLICATION: Favourite-longshot bias (FLB) (cont'd)

Snowberg and Wolfers (JPE, 2010) study FLB in the US racetracks.

Two explanations why people prefer betting on outsiders:

1. neoclassical approach – risk-seeking:

   higher odd for the outsider $\Longrightarrow$ higher risk (riskier is better)

2. behavioral approach – people misperceive probabilities:

   overweighting low probabilities of outsiders and a possible effect of underweighting of high probabilities of favourites

Racetrack odds in the US correspond more to the behavioral explanation.

Still an open question.

CASE: The insurance value of Medicare

Medicare (health insurance to people older than 65) was introduced in 1965 – the largest expansion of health insurance in the 20th century.

Finkelstein and McKnight (J Publ. Econ., 2008) study the effect of the expansion: Medicare has almost no effect on mortality.

But Medicare reduces cash expenditures – for the quartile of people with the highest health costs reduced cash expenditures by 40%.

For risk-averse patients it is valuable. According to Finkelstein and McKnight the insurance value equals to 2/5 of the cost of Medicare.

APPLICATION: Insurance against global warming

What percentage of your yearly income is equivalent to eliminating the risk of a catastrophe that happens once every 1,000 years at random, and when it occurs, there is a 1% chance you will be killed?

Becker, Murphy and Topel (BEJEAP, 2010) estimate that we should be willing to pay 4.5% of the income. If the event happened once in every 100 years, then 36% of the income.

Why that much?

1. Death is permenent and bad for utility.
2. People are risk-averse.
3. We expect the growth in income in future, so are willing to pay a big share of today's income to secure our growing future incomes.

Weitzman (RES, 2009) estimates 1% probability that global warming raises the temperature by more than 20° Celsius = a huge problem.

Others view it more optimistically.

What should you know?

- We can use the tools of consumer choice also for studying the decision-making under uncertainty.
- The difference to decision-making under certainty (utility function can have any form) is that due to the independence of the states of nature we can use a utility function with an additive form =
- = Expected utility function: $EU = \pi_1 v(c_1) + \pi_2 v(c_2)$
- Expected value: $EV = \pi_1 c_1 + \pi_2 c_2$
- Certainty equivalent CE = willingness to pay for a lottery
- The consumer is
  - risk averse: concave $v(c)$, $u(EV) > EU$, $EV > CE$
  - risk seeking: convex $v(c)$, $u(EV) < EU$, $EV < CE$
  - risk neutral: linear $v(c)$, $u(EV) = EU$, $EV = CE$
- If a risk-averse consumer faces fair insurance, she fully insures.