Cost minimization and cost functions

Varian: Intermediate Microeconomics, 8e, 20 and 21

In this lecture you will learn

- how to define a cost function
- what the conditional demand for factor is
- what follows from revealed cost minimization
- what different cost functions look like
- how to measure cost inefficiency

Profit maximization and cost minimization

Profit maximization (last lecture) – what production plan maximizes the firm's profit (for a given technology and input and output prices).

Profit maximization and cost minimization

Profit maximization (last lecture) – what production plan maximizes the firm's profit (for a given technology and input and output prices).

Cost minimization – what combination of inputs minimizes the cost of producing a given output (for a given technology and input prices) – derivation of the **cost function**.

In the second step the firm chooses the profit-maximizing output (for a given cost function and demand).

In the rest of this lecture and the next 4 lectures we assume competitive input markets. $\Longrightarrow$ Prices of inputs ($\mathbf{w}$) are given.

Cost minimization

The firm chooses a combination of inputs that minimizes its costs of producing a given output (at given input prices and technology):

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2$$

$$\text{pro } f(x_1, x_2) = y$$

Cost minimization

The firm chooses a combination of inputs that minimizes its costs of producing a given output (at given input prices and technology):

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2$$

$$\text{pro } f(x_1, x_2) = y$$

Cost function $c(y)$ gives the minimum costs necessary for producing a given output y (at given prices and technology).

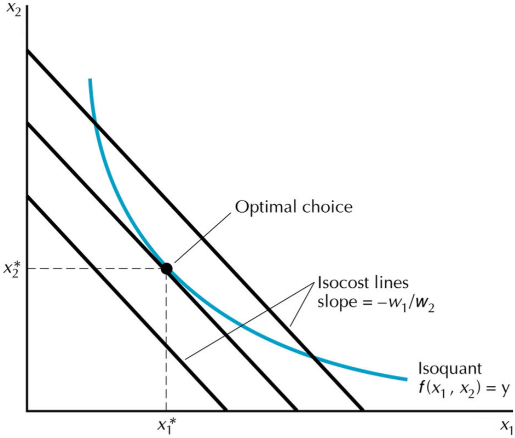
Isocost – all combinations of inputs x_1 and x_2 that correspond to a given level of costs C:

$$w_1 x_1 + w_2 x_2 = C \iff x_2 = \frac{C}{w_2} - \frac{w_1}{w_2} x_1$$

Cost minimization – graphical solution

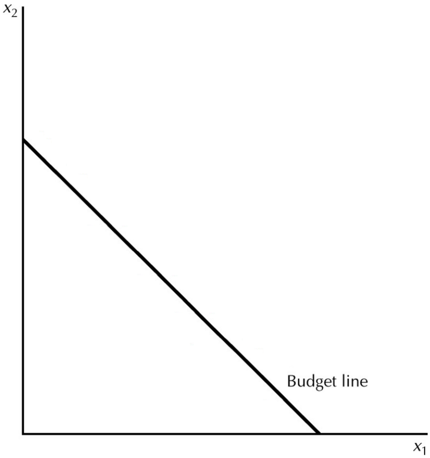
If the isoquant is monotonic, smooth and convex and we have the inner solution, then in the optimum holds:

$$\text{slope of the isoquant} = \mathrm{TRS}(x_1^*, x_2^*) = -\frac{w_1}{w_2} = \text{slope of the isocost}$$

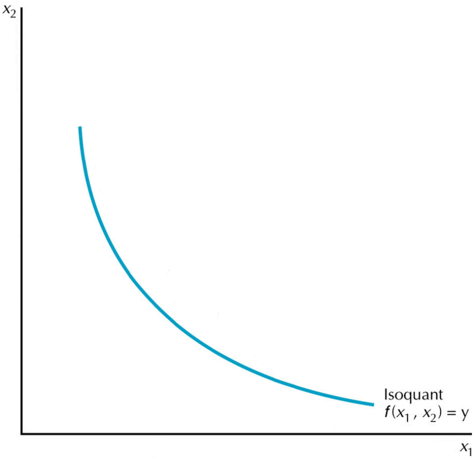


Firm optimum vs. consumer optimum

Consumer

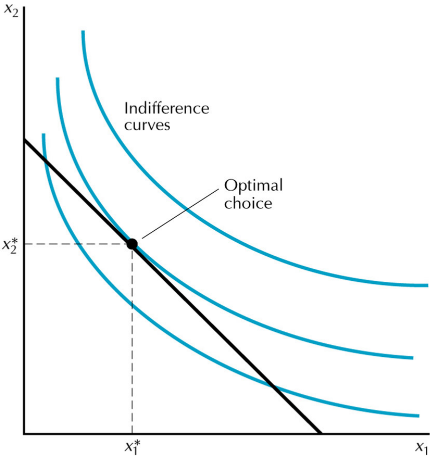


Firm

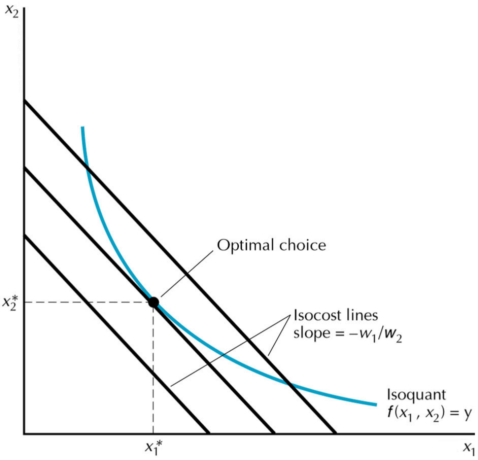


Firm optimum vs. consumer optimum

Consumer – the point on the BL with a maximum utility



Firm – the point on the isoquant corresponding to minimum costs



Conditional demand for inputs

Conditional demand for inputs – what quantity of input minimizes the costs of producing a given output

Conditional demand for inputs

Conditional demand for inputs – what quantity of input minimizes the costs of producing a given output

Difference between the demand and conditional demand for input:

- demand – what $\mathbf{x}$ maximizes the profit for $(p, \mathbf{w})$
- conditional demand – what $\mathbf{x}$ minimizes the costs of $(y, \mathbf{w})$

Deriving the cost function – convex isoquants

Production function: $y = \sqrt{x_1} + 3\sqrt{x_2}$
Input prices: $w_1 = 1$ a $w_2 = 1$

What are the conditional demand functions and the cost function?

Deriving the cost function – convex isoquants

Production function: $y = \sqrt{x_1} + 3\sqrt{x_2}$
Input prices: $w_1 = 1$ a $w_2 = 1$

What are the conditional demand functions and the cost function?

Monotonic, smooth and convex isoquant $\implies$ TRS $= -w_1/w_2$:

$$-\frac{\sqrt{x_2}}{3\sqrt{x_1}} = -1$$
$$x_2 = 9x_1$$

Deriving the cost function – convex isoquants

Production function: $y = \sqrt{x_1} + 3\sqrt{x_2}$
Input prices: $w_1 = 1$ a $w_2 = 1$

What are the conditional demand functions and the cost function?

Monotonic, smooth and convex isoquant $\implies$ TRS $= -w_1/w_2$:

$$-\frac{\sqrt{x_2}}{3\sqrt{x_1}} = -1$$
$$x_2 = 9x_1$$

By substituting back into the *pf*, we get the conditional demands:

$$x_1 = y^2/100 \quad \text{a} \quad x_2 = 9y^2/100$$

Deriving the cost function – convex isoquants

Production function: $y = \sqrt{x_1} + 3\sqrt{x_2}$
Input prices: $w_1 = 1$ a $w_2 = 1$

What are the conditional demand functions and the cost function?

Monotonic, smooth and convex isoquant $\implies$ TRS $= -w_1/w_2$:

$$-\frac{\sqrt{x_2}}{3\sqrt{x_1}} = -1$$
$$x_2 = 9x_1$$

By substituting back into the *pf*, we get the conditional demands:

$$x_1 = y^2/100 \quad \text{a} \quad x_2 = 9y^2/100$$

Cost function:

$$c(y) = w_1 x_1 + w_2 x_2 = 1 \times y^2/100 + 1 \times 9y^2/100 = y^2/10$$

Deriving the cost function – perfect complements

For a production of a 3D visualization (V) we need:

- 1 hour of labour (L)
- 2 hours of a computer (C)

Input prices: $w_L = 300$ and $w_C = 100$

What are the conditional demand functions and the cost function?

Deriving the cost function – perfect complements

For a production of a 3D visualization (V) we need:

- 1 hour of labour (L)
- 2 hours of a computer (C)

Input prices: $w_L = 300$ and $w_C = 100$

What are the conditional demand functions and the cost function?

Production function: $V = \min\{L, C/2\}$

Deriving the cost function – perfect complements

For a production of a 3D visualization (V) we need:

- 1 hour of labour (L)
- 2 hours of a computer (C)

Input prices: $w_L = 300$ and $w_C = 100$

What are the conditional demand functions and the cost function?

Production function: $V = \min\{L, C/2\}$

Conditional demand functions:

$$V = L = C/2$$

$$L = V \quad \text{and} \quad C = 2V$$

Deriving the cost function – perfect complements

For a production of a 3D visualization (V) we need:

- 1 hour of labour (L)
- 2 hours of a computer (C)

Input prices: $w_L = 300$ and $w_C = 100$

What are the conditional demand functions and the cost function?

Production function: $V = \min\{L, C/2\}$

Conditional demand functions:

$$V = L = C/2$$

$$L = V \quad \text{and} \quad C = 2V$$

Cost function:

$$c(V) = w_L \times L + w_C \times C = 300 \times V + 100 \times 2V = 500V$$

Deriving the cost function – perfect substitutes

Book (B) can be produced using

- 1/5 of an hour using a hi-tech printer (H)
- 1/3 of an hour using a standard printer (S)

Input prices: $w_H = 10$ and $w_S = 5$

What are the conditional demand functions and the cost function?

Deriving the cost function – perfect substitutes

Book (B) can be produced using

- $1/5$ of an hour using a hi-tech printer (H)
- $1/3$ of an hour using a standard printer (S)

Input prices: $w_H = 10$ and $w_S = 5$

What are the conditional demand functions and the cost function?

Production function: $B = 5H + 3S$

Deriving the cost function – perfect substitutes

Book (B) can be produced using

- 1/5 of an hour using a hi-tech printer (H)
- 1/3 of an hour using a standard printer (S)

Input prices: $w_H = 10$ and $w_S = 5$

What are the conditional demand functions and the cost function?

Production function: $B = 5H + 3S$

I use the cheaper technology – the cost of one book printed on

- the hi-tech printer is $w_H/5 = 2$
- the standard printer is $w_S/3 = 5/3$

The conditional demand functions are $H = 0$ and $S = B/3$

Deriving the cost function – perfect substitutes

Book (B) can be produced using

- $1/5$ of an hour using a hi-tech printer (H)
- $1/3$ of an hour using a standard printer (S)

Input prices: $w_H = 10$ and $w_S = 5$

What are the conditional demand functions and the cost function?

Production function: $B = 5H + 3S$

I use the cheaper technology – the cost of one book printed on

- the hi-tech printer is $w_H/5 = 2$
- the standard printer is $w_S/3 = 5/3$

The conditional demand functions are $H = 0$ and $S = B/3$

Cost function:

$$c(B) = w_H \times H + w_S \times S = 10 \times 0 + 5 \times B/3 = 5/3B$$

Revealed cost minimization

A cost-minimizing firm chooses a combination of inputs in order to produce a given output (at given input prices and technology) at costs that are at least as low as the costs of alternative combinations of inputs.

Revealed cost minimization – example

A firm produces output y using two different combinations of inputs:

- at input prices at time t (w_1^t, w_2^t) the firm chooses (x_1^t, x_2^t)
- at input prices at time s (w_1^s, w_2^s) the firm chooses (x_1^s, x_2^s)

Weak axiom of cost minimization (WACM): If a firm produces output y at minimum costs and technology hasn't changed between times t and s, then it holds that:

$$w_1^t x_1^t + w_2^t x_2^t \leq w_1^t x_1^s + w_2^t x_2^s \tag{1}$$

$$w_1^s x_1^s + w_2^s x_2^s \leq w_1^s x_1^t + w_2^s x_2^t \tag{2}$$

Revealed cost minimization – example (cont'd)

If we copy the equation (1) and multiply the equation (2) by -1, we get

$$w_1^t x_1^t + w_2^t x_2^t \leq w_1^t x_1^s + w_2^t x_2^s$$

$$-w_1^s x_1^t - w_2^s x_2^t \leq -w_1^s x_1^s - w_2^s x_2^s$$

Since both equations have $\leq$, also the sum of the equations must have $\leq$:

$$(w_1^t - w_1^s)x_1^t + (w_2^t - w_2^s)x_2^t \leq (w_1^t - w_1^s)x_1^s + (w_2^t - w_2^s)x_2^s$$

Rearranging this equation and substituting Δw_1 for $(w_1^t - w_1^s)$, Δx_1 for $(x_1^t - x_1^s)$, and so on, we find

$$\Delta w_1 \Delta x_1 + \Delta w_2 \Delta x_2 \leq 0.$$

Revealed cost minimization – example (cont'd)

What follows from the result $\Delta w_1 \Delta x_1 + \Delta w_2 \Delta x_2 \leq 0$?

E.g. if the price of factor 1 w_1 changes and the price of factor 2 w_2 remains constant, then

$$\Delta w_1 \Delta x_1 \leq 0.$$

It never holds that $\Delta w_1 > 0$ and $\Delta x_1 > 0$ or $\Delta w_1 < 0$ and $\Delta x_1 < 0$. $\Longrightarrow$ *The conditional factor demand of a competitive firm* can't be *increasing*.

APPLICATION: Costs and inefficiency

We can estimate cost functions from the input prices and output data.

Different cost functions in one industry – possible explanations:

- firms have different technologies
- firms do not minimize costs

APPLICATION: Costs and inefficiency

We can estimate cost functions from the input prices and output data.

Different cost functions in one industry – possible explanations:

- firms have different technologies
- firms do not minimize costs

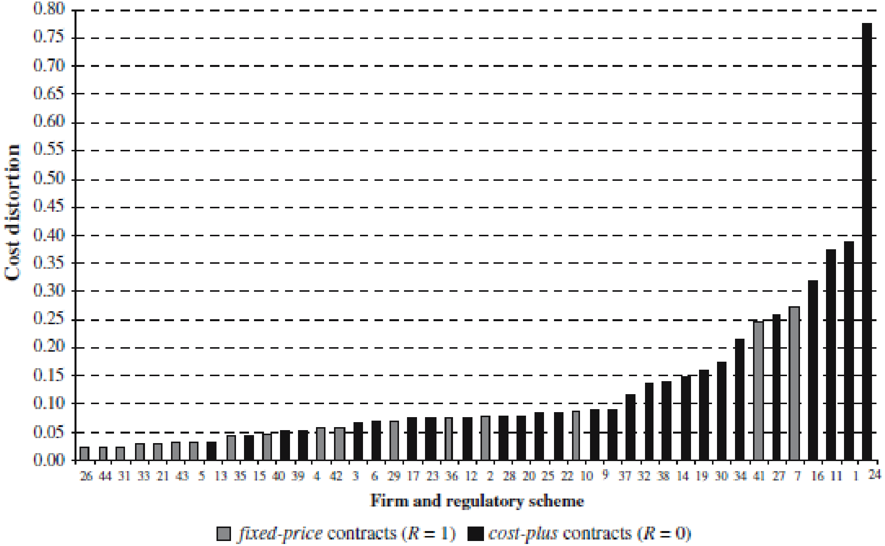
Piacenza (J Prod Anal, 2006) – the costs of Italian public transport firms is on average 11% above the minimum costs of producing the same output.

The inefficiency is influenced by the type of transport subsidy:

- Cost plus: the size of subsidy is a function of transport costs.
- Fixed price: transport firms have a subsidized, but fixed, price.

What type of subsidies generates a higher inefficiency?

APPLICATION: Costs and inefficiency (graph)



APPLICATION: Cost minimization in the US health sector

Before 1983: Medicare would reimburse a share of hospitals' capital and labor costs equal to Medicare patient-days/total patient-days.

After 1983: Capital costs paid as before, labor costs covered by a flat rate based on the patient's diagnosis (any additional labor cost covered fully by the hospital) = the isocost's slope changes.

What was the reaction of hospitals?

APPLICATION: Cost minimization in the US health sector

Before 1983: Medicare would reimburse a share of hospitals' capital and labor costs equal to Medicare patient-days/total patient-days.

After 1983: Capital costs paid as before, labor costs covered by a flat rate based on the patient's diagnosis (any additional labor cost covered fully by the hospital) = the isocost's slope changes.

What was the reaction of hospitals?

Acemoglu a Finkelstein (JPE, 2008): 10% increase in the K/L ratio. A bigger increase in hospitals with a higher share of Medicare patients.

Costs

Total costs: $c(y) = c_v(y) + F \; (+QF)$

- **variable costs** $c_v(y)$ – costs of variable inputs (SR and LR)
- **fixed costs** F = costs of fixed inputs (only SR): a constant for $y \geq 0$
- **quasifixed costs** QF = costs of quasifixed inputs (SR and LR)

$$QF = \begin{cases} \text{a constant} & \text{if } y > 0 \\ 0 & \text{if } y = 0 \end{cases}$$

Average costs:

$$AC(y) = \frac{c(y)}{y} = \frac{c_v(y)}{y} + \frac{F}{y} = AVC(y) + AFC(y)$$

Marginal costs:

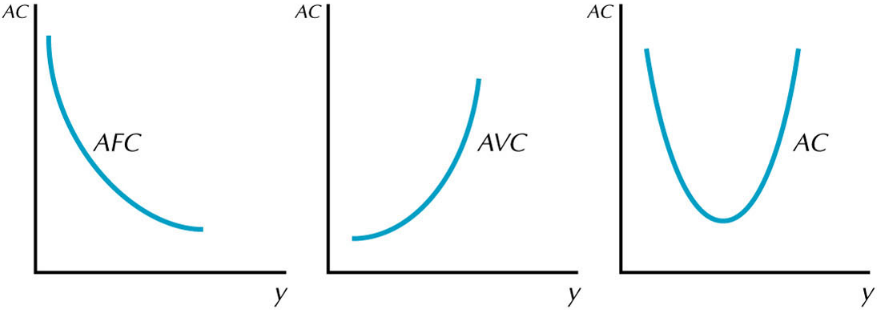
$$MC(y) = \frac{dc(y)}{dy}$$

Average costs

$AFC(y) = \frac{F}{y}$ – decreasing; the same fixed costs spread over a higher y

$AVC(y) = \frac{c_v(y)}{y}$ – increasing from a given y; limited by the fixed input

$AC(y) = AFC(y) + AVC(y)$ – typically U-shaped



Average and marginal costs

For discrete output $MC(y)$ and $AVC(y)$ are equal for $y = 1$:

$$MC(1) = \frac{c_v(1) + F - c_v(0) - F}{1} = \frac{c_v(1)}{1} = AVC(1)$$

Average and marginal costs

For discrete output $MC(y)$ and $AVC(y)$ are equal for $y = 1$:

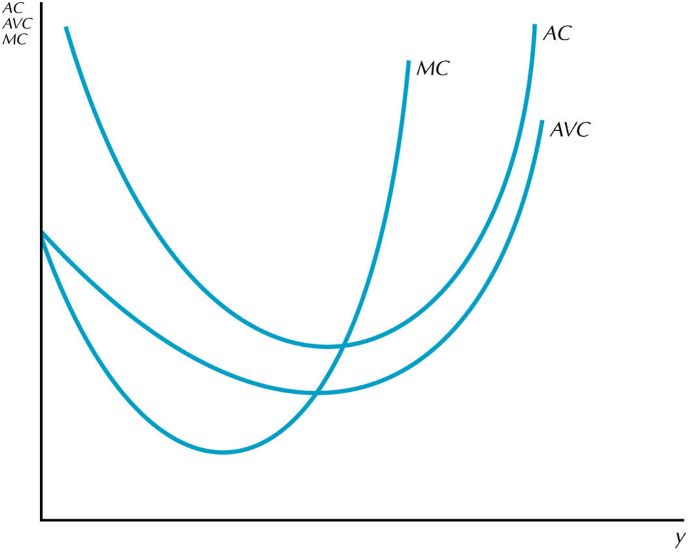
$$MC(1) = \frac{c_v(1) + F - c_v(0) - F}{1} = \frac{c_v(1)}{1} = AVC(1)$$

$MC(y)$ crosses the $AC(y)$ and $AVC(y)$ curves in their minimum:

$$AC'(y^*) = \left(\frac{c(y^*)}{y^*}\right)' = \frac{c'(y^*)y^* - c(y^*)}{y^{*2}} = 0 \iff c'(y^*) = \frac{c(y^*)}{y^*}$$

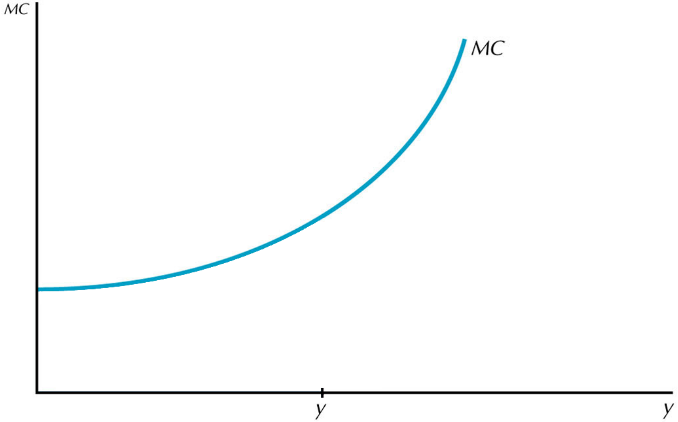
$$AVC'(\hat{y}) = \left(\frac{c_v(\hat{y})}{\hat{y}}\right)' = \frac{c_v'(\hat{y})\hat{y} - c_v(\hat{y})}{\hat{y}^2} = 0 \iff c_v'(\hat{y}) = \frac{c_v(\hat{y})}{\hat{y}}$$

Average and marginal costs (graph)



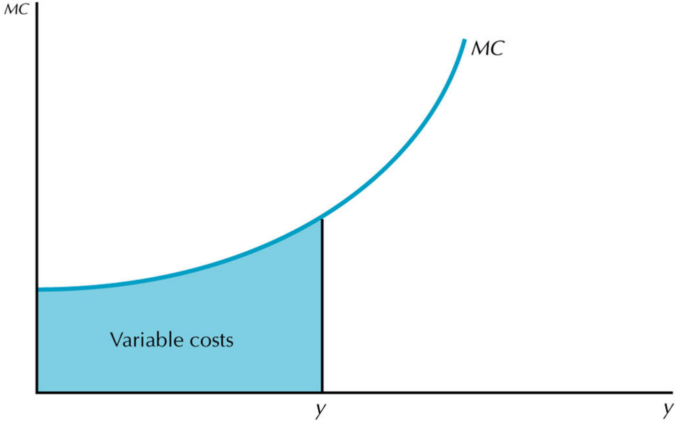
Marginal costs and total variable costs

Variable costs necessary for a production of y units of output $=$



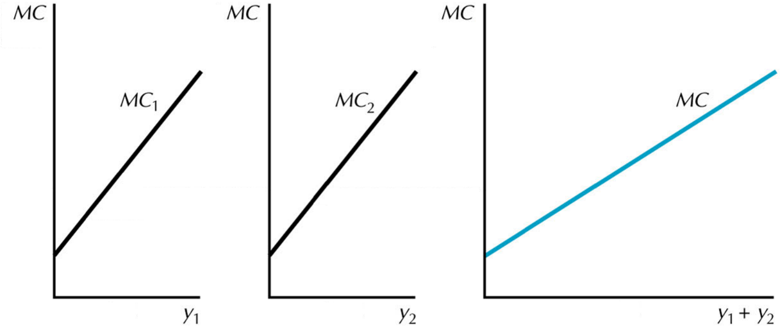
Marginal costs and total variable costs

Variable costs necessary for a production of y units of output $=$ the area below the MC curve for the output between 0 and y.



Example – marginal cost curves for two plants

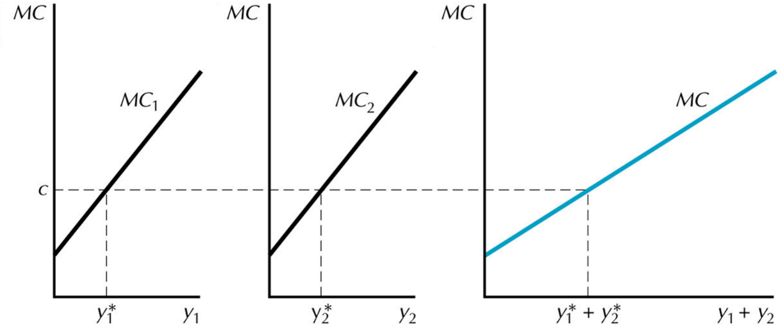
A firm has two plants with cost functions $c_1(y_1)$ a $c_2(y_2)$.
How to divide the production of y units between the plants?



Example – marginal cost curves for two plants

A firm has two plants with cost functions $c_1(y_1)$ a $c_2(y_2)$.
How to divide the production of y units between the plants?

Optimal outputs y_1^* and y_2^* are such that $MC_1(y_1^*) = MC_2(y_2^*) = c$.



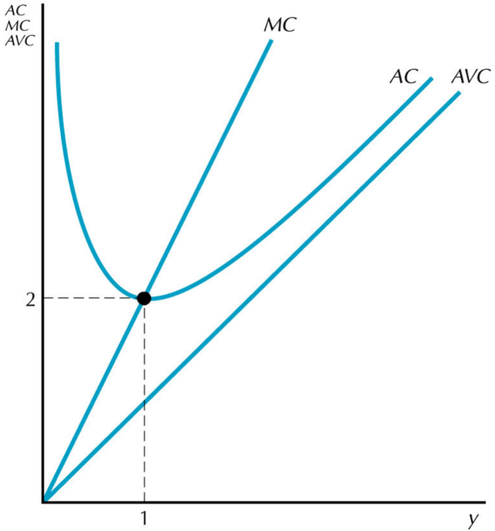
Numerical example – cost functions

Total costs:

- $c(y) = y^2 + 1$
- variable – $c_v(y) = y^2$
- fixed – $F = 1$

Average and marginal costs:

- $AFC(y) = 1/y$
- $AVC(y) = y^2/y = y$
- $AC(y) = y + 1/y$
- $MC(y) = 2y$



Long-run average costs (LAC)

If the quasifixed costs are 0 and the production function exhibits

- constant returns to scale, $LAC(y)$ is constant,
- increasing returns to scale, $LAC(y)$ is decreasing,
- decreasing returns to scale, $LAC(y)$ is increasing.

Why?

Long-run average costs (LAC)

If the quasifixed costs are 0 and the production function exhibits

- constant returns to scale, $LAC(y)$ is constant,
- increasing returns to scale, $LAC(y)$ is decreasing,
- decreasing returns to scale, $LAC(y)$ is increasing.

Why? If $t > 1$ and the production function has

- constant returns to scale, then

$$LAC(ty) = \frac{c(ty)}{ty} = \frac{t \cdot c(y)}{ty} = LAC(y).$$

- increasing returns to scale, then

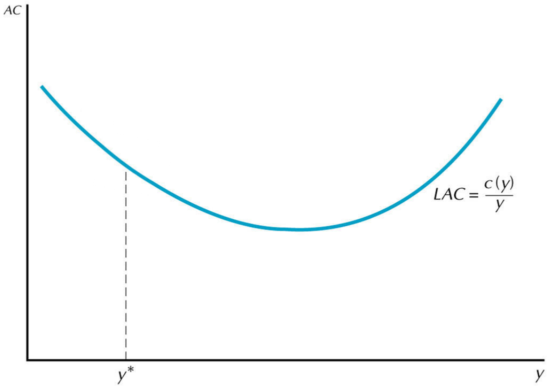
$$LAC(ty) = \frac{c(ty)}{ty} < \frac{t \cdot c(y)}{ty} = LAC(y).$$

- decreasing returns to scale, then

$$LAC(ty) = \frac{c(ty)}{ty} > \frac{t \cdot c(y)}{ty} = LAC(y).$$

Short-run and long-run average costs

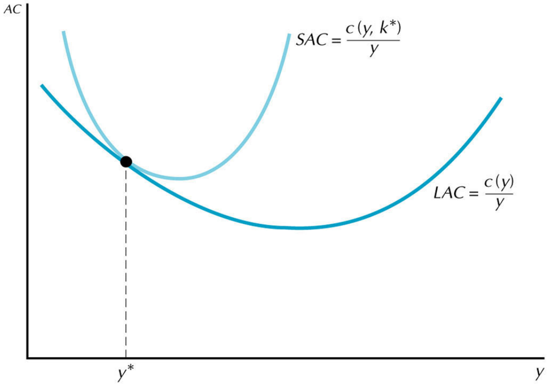
SR: for a fixed plant size k^*, the optimal output is y^*
LR: the firm chooses the optimal plant size for each output



Short-run and long-run average costs

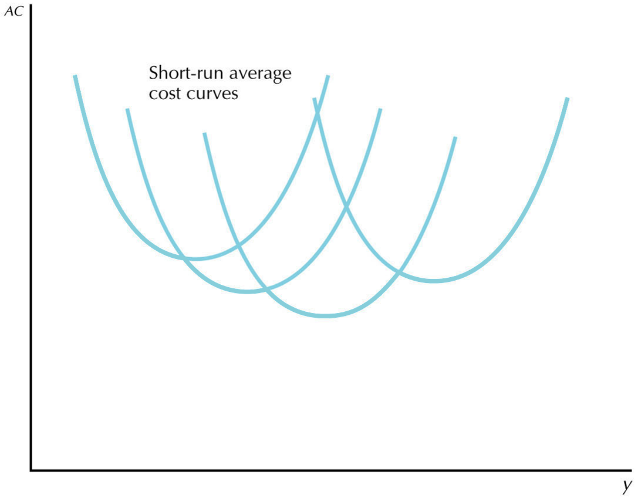
SR: for a fixed plant size k^*, the optimal output is y^*
LR: the firm chooses the optimal plant size for each output

For the output y^* holds: $SAC = LAC$
For all other outputs $y \neq y^*$ holds: $SAC > LAC$



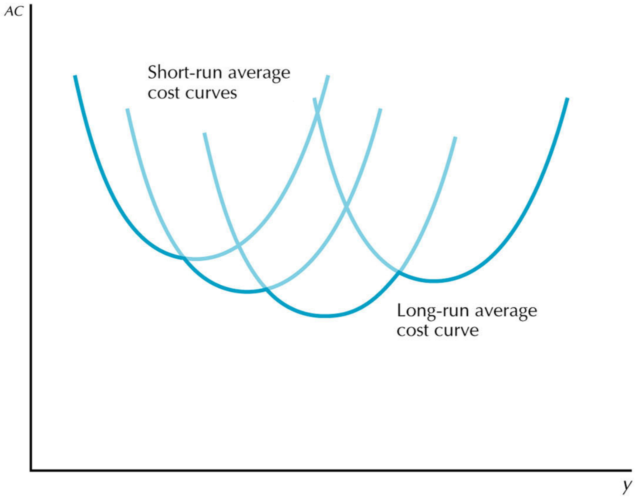
Discrete levels of plant size

The LAC curve (dark blue) if a firm chooses from 4 plant sizes:



Discrete levels of plant size

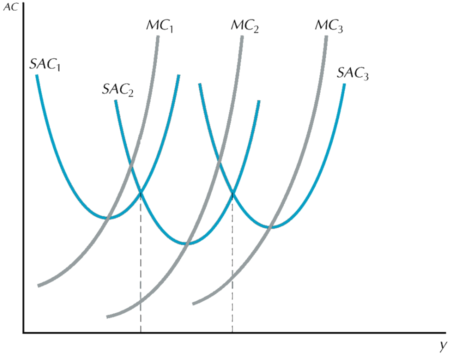
The LAC curve (dark blue) if a firm chooses from 4 plant sizes:



Long-run marginal costs (LMC)

The long-run marginal cost LMC curve:

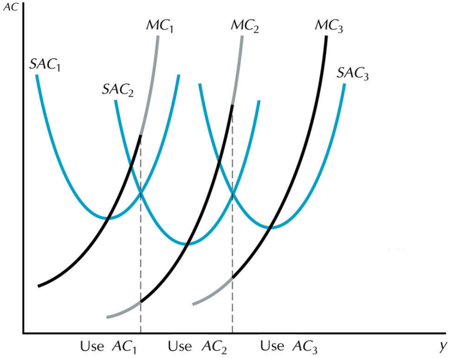
- left – a firm chooses among 3 plant sizes



Long-run marginal costs (LMC)

The long-run marginal cost LMC curve:

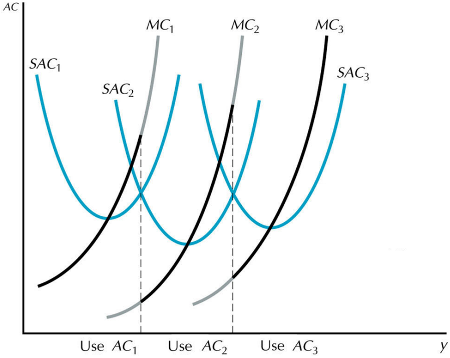
- left – a firm chooses among 3 plant sizes (black curve)

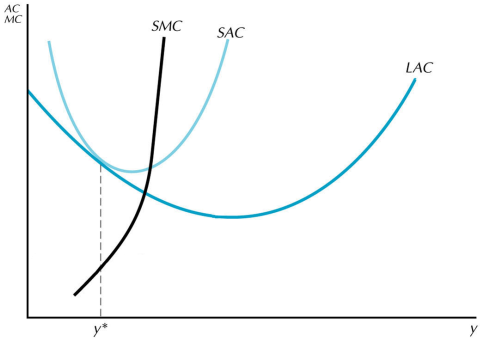


Long-run marginal costs (LMC)

The long-run marginal cost LMC curve:

- left – a firm chooses among 3 plant sizes (black curve)
- right – a firm can choose any continuous plant size

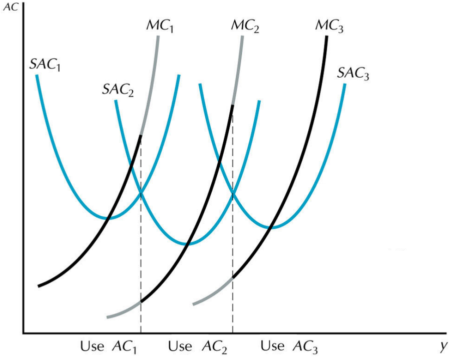


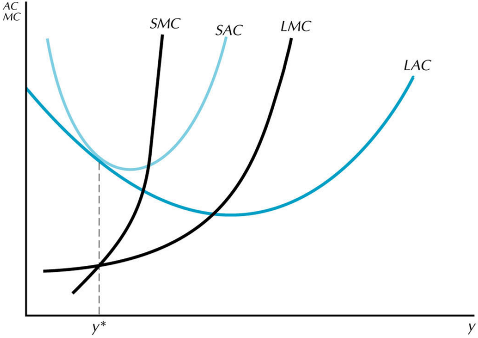


Long-run marginal costs (LMC)

The long-run marginal cost LMC curve:

- left – a firm chooses among 3 plant sizes (black curve)
- right – a firm can choose any continuous plant size





What should you know?

- Cost minimization – what combination of inputs minimizes costs of a given output (for a given technology and input prices).
- Cost function – the minimum costs necessary for producing a given output.
- Conditional demand for input – how much input minimizes the cost of production of a given output.

  X Demand for input – profit-maximizing firm buys such quantity of input that $w = pMP$.
- If a firm minimizes costs, its conditional demand function cannot be increasing.

What should you know? (cont'd)

- The average cost function AC is usually U-shaped, because AFC is decreasing and AVC increasing (beyond certain quantity).
- In the minimum, AC and AVC equal to MC.
- The area below MC from 0 to y equals $VC(y)$.
- The LAC curve is the lower envelope of the short-run average cost curves.
- Fixed cost: SR, can be positive for $y = 0$
  Quasificed cost: SR and LR, zero for $y = 0$