

Linear programming-introduction

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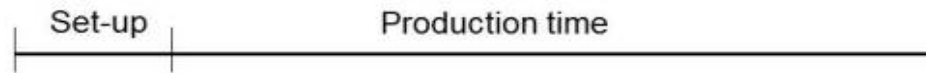
USE

- **Slitting and Levelling of material (coils, bars, sheets)**-Cutting material, trimming,...
- **Blending** - blending, diet, feeding rations for animals, ..
- **Transport problems** - material flow from stock to the destination and route planning - shortest route
- **Assignment of resources with limited capacities** - CCR
- **Sources** : Operation Management, Quality and Competitiveness in a global environment, Russel and Taylor (can be found easily in ESF library)

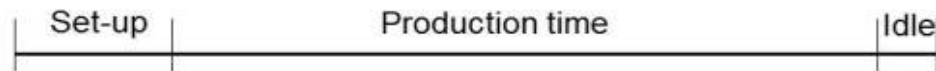
CCR –additional information

- There are 3 categories of resources from the point of view of capacity:
- Bottleneck
- CCR – Capacity Constraint Resource
- Non-CCR

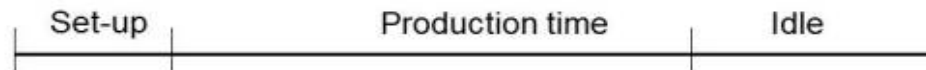
Bottleneck – demand on the machine **is higher than the available capacity**.
Works 24x7, the whole year around.





CCR (Capacity Constraint Resource) – according to the available time that you allow it to work, it becomes a trouble maker. The load bigger than 70%. The idle time is so little and unstable that in no time it can turn to Bottleneck.



Non-CCR – idle capacity includes some protective capacity.



Formulation of the simple model

	Product	Description	Work /hour	Material/pcs	Return/pcs
	Dish	x1	1	4	40
	Mug	x2	2	3	50

Which combination of products will have the greatest return at the limits of maximum production capacity type = **40** hours. Moreover, the amount of material that is limited to **120** kg of clay?

Note: A similar task in terms of flow was solved in the P&Q example (only valid for Czech student), where the limitation in resource B and with a maximum capacity of 2400 minutes)

Description x1 and x2 stands for variables, Material means e.g. 4 kg for one piece

Basic structures and used terminology

- We minimize our target function in the form of:

Target function
 $Z=Cx$

 $Z = c_1*x_1+c_2*x_2+.....+c_n*x_n$ with respect to the matrix of restrictive conditions:
 (in our case $c_1=40$ and $c_2=50$ which means return/pc)

$$A_{11}*x_1 + A_{12}*x_2+ ...+ A_{1n}*x_n \quad (<=>) \quad B_1$$

$$A_{22}*x_1 + A_{22}*x_2+ ...+ A_{2n}*x_n \quad (<=>) \quad B_2$$

$$A_{m1}*x_1 + A_{m2}*x_2+ ...+ A_{mn}*x_n \quad (<=>) \quad B_m$$

- It is a classical system of linear equations is $Ax=B$
- The solving of such a linear equation system, e.g. By use of GAUSS-JORDAN algorithm is not required if we will use **Excel Solver**.
- x_{ij} : decision variable = level of operation activity specified by this variable
- B_i : restrictive conditions , allowed deviations from the norm (in time and material)
- c_j : coefficient of the target function (in our case returns, meaning return 40 and 50)
- A_{ij} : restrictive coefficients: work and material for one unit (pcs) of the product

Example I (introduction to the problem – practical demonstration)

Product	Description	Work /hour	Material/pcs	Return/pcs
Dish	x1	1	4	40
Mug	x2	2	3	50

$$Z = c_1 * x_1 + c_2 * x_2 + \dots + c_n * x_n \text{ (classical equation from)}$$

Target function: $Z = 40 * x_1 + 50 * x_2$, which we must maximize

Maximal production capacity = 40 hours and Maximal quantity of material = 120 kg
(B1 and B2 in our mathematical expression)

Specifications of task restrictions by use of 2x2 matrix:

$$1 * x_1 + 2 * x_2 = 40 \text{ (work-no more than 40 hours)}$$

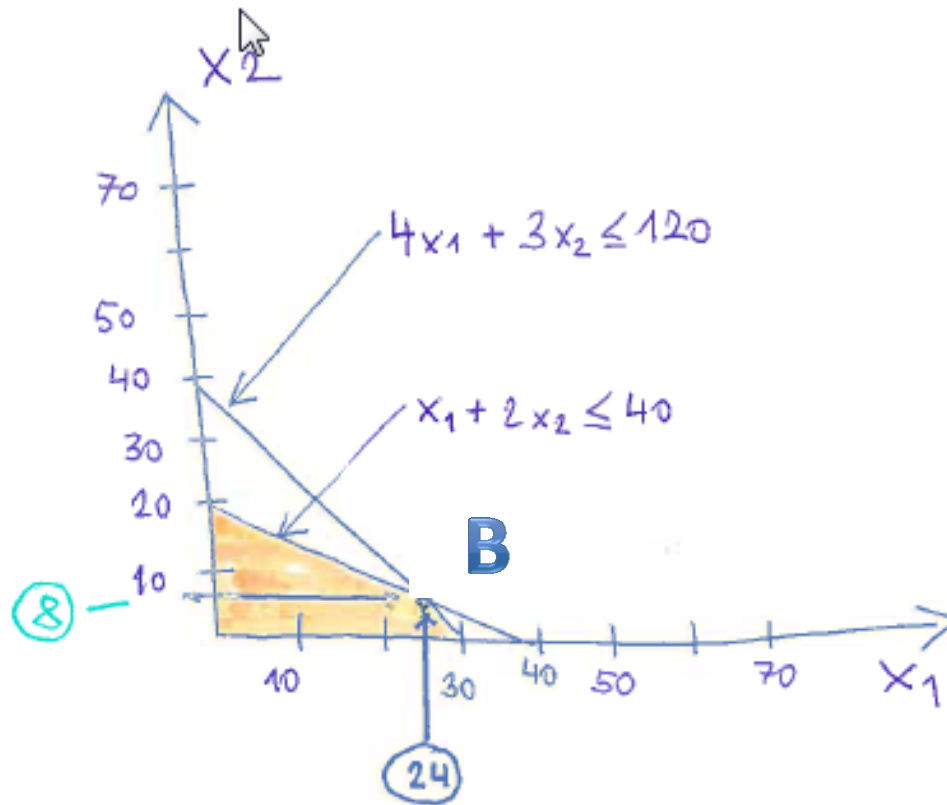
$$4 * x_1 + 3 * x_2 = 120 \text{ (material=kg of clay in our case) } \rightarrow x_1 = (40 - 2x_2) + 3x_2 = 120 \dots$$

Manual solving : $\rightarrow x_1 = 24$ a $x_2 = 8$ and after substitution of variables (24 pcs of Dish and 8 pcs of Mug)
in target function we will get

$$Z = 40 * 24 + 50 * 8 = 1360$$

(optimal Return meets the point B – see next slide)

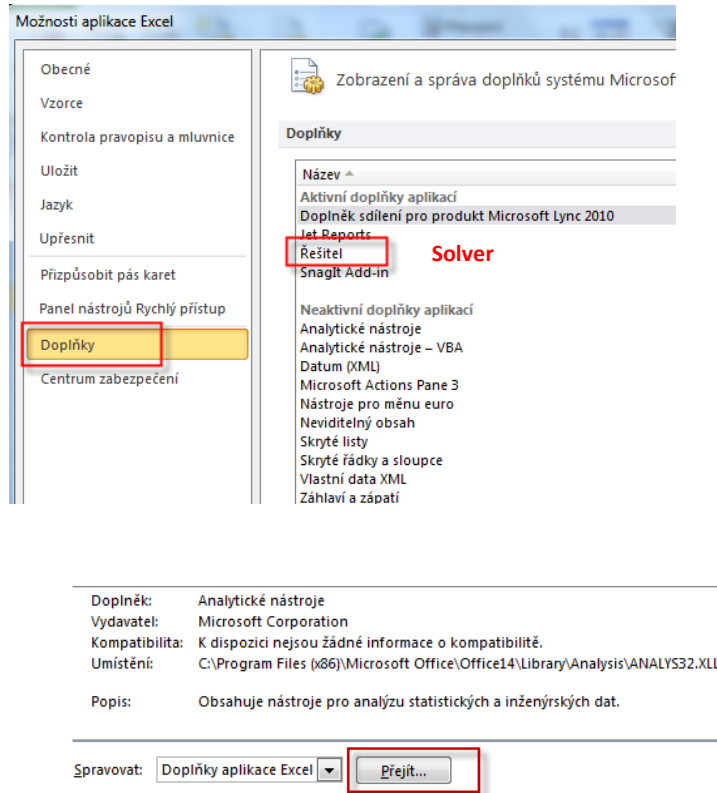
Graphical solution



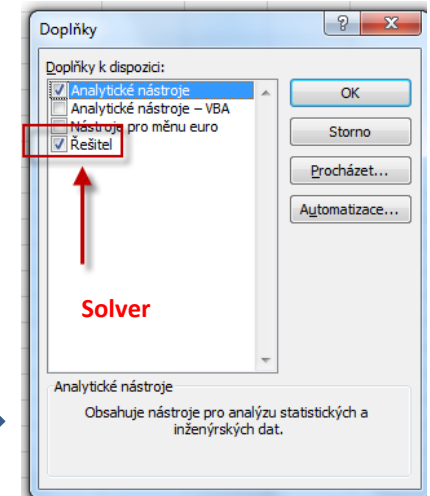
I apologize for the inappropriate graphic expression....

Use of Solver (Czech EXCEL)

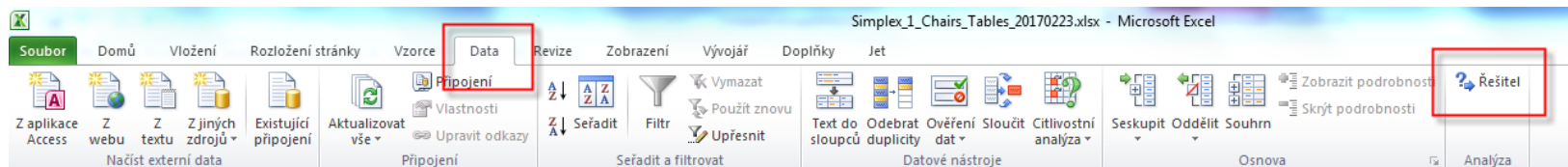
Complements Supplement



Excel setup



Solver



Use o solver (see actual Excel formulas on one of the next slides)

	Dish	Mug	Total	Capacity
Variables (x1, x2)	0	0	0	
Return	40	50	0	
Material	4	3	0	120
Work	1	2	0	40

$=D7*D6+E7*E6$
 $=D10*D6+E10*E6$
 $=D11*D6+E11*E6$

Assignment entered in table

Assignment

$x1 = \text{Dish}$, $x2 = \text{Mug}$, max 40 hour (B1), max 120 kg (B2)

Target function $Z = x1 * c1 + x2 * c2 = 40 * x1 + 50 * x2$

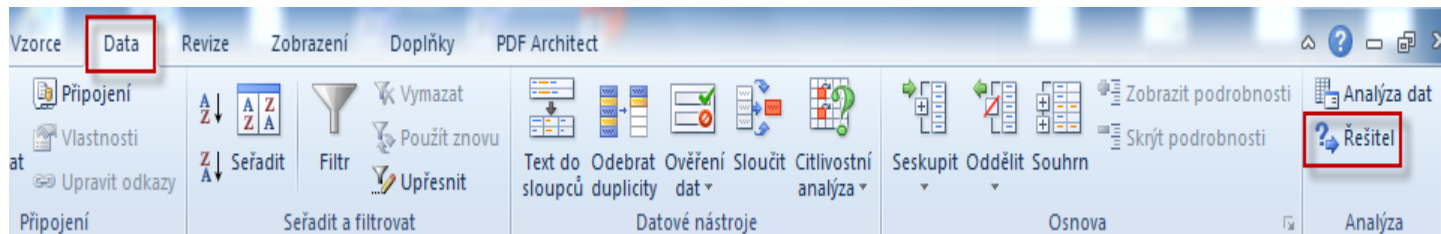
$4 * x1 + 3 * x2 = 120$ - capacity restrictions = max quantity of material = B1

$1 * x1 + 2 * x2 = 40$ - capacity restrictions by max work capacity = B2



Product	Description	Work /hour	Material/pcs	Return/pcs
Dish	x1	1	4	40
Mug	x2	2	3	50

Solver start



Use of Solver (Czech- not for MHP_AOPR)

	A	B	C	D	E	F	G
1							
2							
3			Miska	Hrnek	Total	Kapacita	
4		Proměnné x1,X2	0	0			
5		Přínos	40	50	0		
6							
7		Materiál	4	3	0	120	
8		Práce	1	2	0	40	
9							

$$E5 = =D7*D6+E7*E6$$

$$Z = x1 * c1 + x2 * c2 = 40 * x1 + 30 * x2$$

$$E7 = C7 * C4 + D7 * D4 = 4 * x1 + 3 * x2 = 120$$

$$E8 = C8 * C4 + D8 * D4 = x1 + 2 * x2 = 40$$

Parametry Řešitele

Účelová funkce:

\$E\$5

Hledat: Max

Min

Hodnota:

0

Proměnné modelu:

\$C\$4:\$D\$4

Omezující podmínky:

\$E\$7 <= \$F\$7

\$E\$8 <= \$F\$8

	Miska	Hrnek	Total	Kapacita
Proměnné x1,X2	24	8		
Přínos	40	50	1360	
Materiál	4	3	120	120
Práce	1	2	40	40

Use of solver (for MPH_AOPR)

F7 = $=D7*D6+E7*E6$

	A	B	C	D	E	F	G	H
1								
2								
3								
4								
5				Dish	Mug	Total	Capacity	
6			Variables X1, X2	0	0			
7			Return	40	50	0		
8								
9								
10			Material	4	3	0	120	
11			Work	1	2	0	40	
12								

$$Z = x1 * c1 + x2 * c2 = 40 * x1 + 30 * x2$$

$$F10 = D10 * D6 + E10 * E6 = 4 * x1 + 3 * x2 = 120$$

$$F11 = D11 * D6 + E11 * D6 = x1 + 2 * x2 = 40$$

Parametry Řešitele

Nastavit cíl: **Target** \$F\$7

Na: Max Min Hodnota: 0

Na základě změny proměnných buněk: **Variables** \$D\$6:\$E\$6

Omezující podmínky: **Restrictions**

\$F\$10 <= \$G\$10
\$F\$11 <= \$G\$11

Nastavit proměnné bez omezujících podmínek jako nezáporné

Vyberte metodu řešení: GRG Nonlinear

Metoda řešení
Modul GRG Nonlinear vyberte pro hladké nelineární problémy Řešitele. Modul LP Simplex zvolte pro lineární problémy Řešitele a modul Evolutionary pro nehladké problémy Řešitele.

Solve it

$=D10*D6+E10*E6$

$=D11*D6+E11*E6$



	Dish	Mug	Total	Capacity
Variables X1, X2	24	8		
Return	40	50	1360	
Material	4	3	120	120
Work	1	2	40	40

Využití Řešitele (use of Solver)

Microsoft Excel 15.0 Citlivostní sestava

List: [Simplex_1_Misky_Hrnky_Chairs_Tables_20170228.xlsx]List1

Sestava vytvořena: 9. 3. 2017 16:19:56

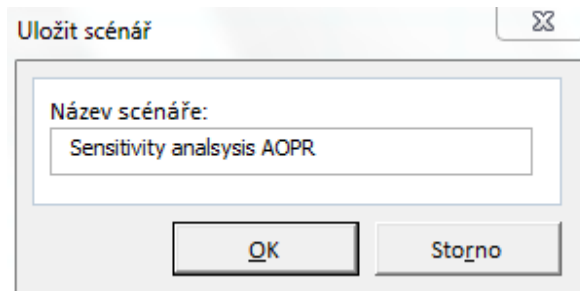
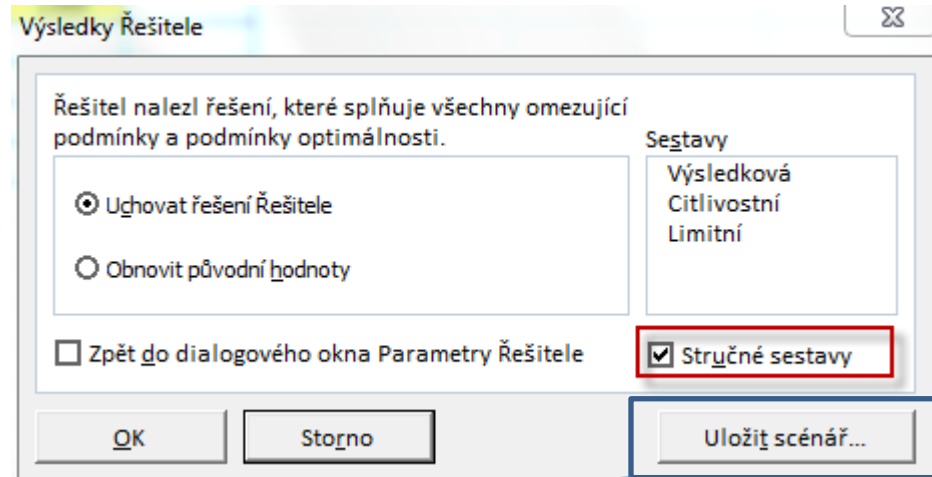
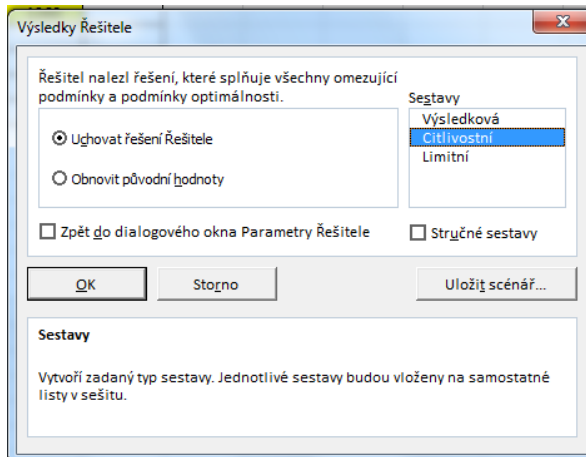
Proměnné

Levá strana omezující podmínky	Název	Konečná Hodnota	Redukovaná náklady	Účelová funkce koeficient	Povolený nárůst	Povolený pokles
\$C\$4	Proměnné x1,X2 Miska	24	0	40	26,66666667	15
\$D\$4	Proměnné x1,X2 Hrnek	8	0	50	30	20

Omezující podmínky

Levá strana omezující podmínky	Název	Konečná Hodnota	Stínová cena	Pravá strana omezující podmínky	Povolený nárůst	Povolený pokles
\$E\$7	Materiál Total	120	6	120	40	60
\$E\$8	Práce Total	40	16	40	40	10

Use of Solver (English)



New Excel List



Microsoft Excel 14.0 Citlivostní sestava
List: [LP_EXCEL_SOLVER USE_20171101.xlsx]List1
Sestava vytvořena: 2.11.2017 8:49:10

Proměnné buňky

Buňka	Název	Konečná Hodnota	Snížené Gradient
\$D\$6	Variables X1, X2 Dish	24	0
\$E\$6	Variables X1, X2 Mug	8	0

Omezující podmínky

Buňka	Název	Konečná Hodnota	Lagrangeův multiplikátor
\$F\$10	Material Total	120	6
\$F\$11	Work Total	40	16

Změna úlohy- jiné výnosy jiná omezení typu práce na dvou strojích a jejich kapacitní omezení

(Change of parameters- not necessary for MPH_AOPR !!!!!)

	Miska	Hrnek	Total	Kapacita
Proměnné x1,x2	0	0		
Přínos	40	50	0	
Stroj 1	7	5	0	200
Stroj 1	5	5	0	400



	Miska	Hrnek	Total	Kapacita
Proměnné x1,x2	0	40		
Přínos	40	50	2000	
Stroj 1	7	5	200	200
Stroj 1	5	5	200	400

Parametry Řešitele

Účejová funkce:

Hledat: Max Min Hodnota:

Proměnné modelu:

Omezující podmínky:

- \$E\$15 <= \$F\$15
- \$E\$16 <= \$F\$16



OK ?