KARUSH-KHUN-TUCKER PODMÍNKY

KKT

MIN $f(x)$ $\quad i=1,\dots,n$

$g_i(x)\le 0$ } OMEZ. VE TVARU NEROV.

$h_j(x)=0 \quad j=1,\dots,m$

$f(x)+\sum_{i=1}^{m}\mu_i g_i(x)+\sum_{j=1}^{m} z_j h_j(x)=L(x,\mu,z)$

$L'_x(x,\mu,z)=0$

$(g_i(x)=0)$

$h_j(x)=0$

$\boxed{\mu_i\cdot g_i(x)=0\ \forall i}$

$\mu_i\ge 0$

MAX $f(x)$

$g_i(x)\le 0$

$h_j(x)=0$

$f(x)-\sum_{i=1}^{m}\mu_i g_i(x)-\sum_{j=1}^{m} z_j h_j(x)=L(x,\mu,z)$

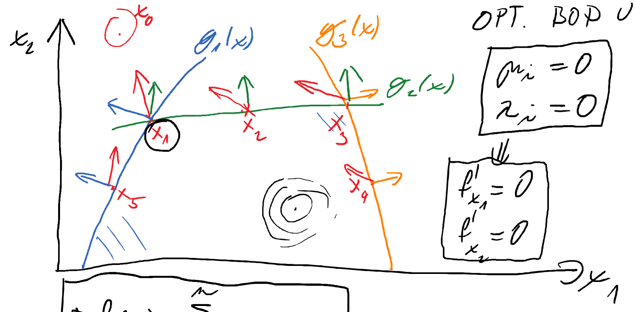
$L'_x(\dots)=0$

$(g_i(x)=0)$ ← JEDNO Z NICH PLATÍ

$h_j(x)=0$

$\boxed{\mu_i\cdot g_i(x)=0}$ $\quad g_i(x)\ne 0 \Rightarrow \mu_i=0$ NEJSEM NA H.

$\mu_i\ge 0$



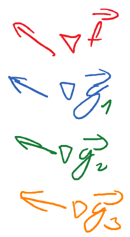
OPT. BOD UVNITŘ.

$\boxed{\begin{matrix}\mu_i=0\\ z_i=0\end{matrix}}$

⇓

$\boxed{\begin{matrix}f'_{x_1}=0\\ f'_{x_2}=0\end{matrix}}$

„OPT BOD MIMO"



$\boxed{\nabla f(x)=\sum_{i=1}^{n}\mu_i\nabla g_i}$

$\mu_i\cdot g_i(x)=0$

I $g_1(x)=0,\ \mu_1\ne 0$ $\quad$ II. $g_{1,2}=0$

$g_{2,3}(x)\le 0$

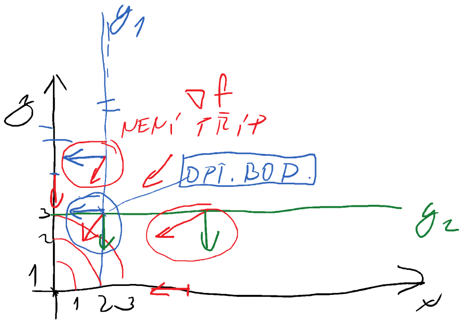
1.) MAX. $\boxed{\begin{matrix}1-x^2-y^2\\ x\ \overset{\le}{\ge}\ 2\\ y\ \overset{\le}{\ge}\ 3\end{matrix}}$ $\quad \boxed{\begin{matrix}2-x\le 0\\ 3-y\le 0\end{matrix}}$ $\quad \begin{matrix}g_1\le 0\\ g_2\le 0\end{matrix}$

$L(x,y,\mu_i)=1-x^2-y^2-\mu_1(2-x)-\mu_2(3-y)$

$\boxed{\begin{matrix}L'_x=-2x+\mu_1=0\\ L'_y=-2y+\mu_2=0\end{matrix}}$ $\quad \begin{matrix}x=0\\ y=0\end{matrix}$

$\boxed{\begin{matrix}2x=+\mu_1\\ 2y=\mu_2\end{matrix}}$

$\boxed{\begin{matrix}\mu_1\cdot(2-x)=0\\ \mu_2(3-y)=0\end{matrix}}$ $\quad \begin{matrix}\Rightarrow x=2\\ \Rightarrow y=3\end{matrix}$

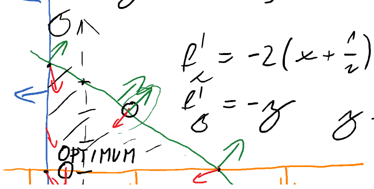


2. MAX $-(x+\frac{1}{2})^2-\frac{1}{2}y^2$

g_1 $x+y\ge 4 \Rightarrow 0\ge 4-x-y$

g_2 $x\ge -1 \Rightarrow 0\ge -1-x$

g_3 $y\ge +1 \Rightarrow 0\ge 1-y$



$f'_x=-2(x+\frac{1}{2}) \quad = x=-\frac{1}{2}$

$f'_y=-y \quad y=0$

$-(x+\frac{1}{2})^2-\frac{1}{2}y^2-\mu_1(4-x-y)-\mu_2(-1-x)-\mu_3(1-y)$

$f'_x=-2(x+\frac{1}{2})+\mu_1+\mu_2=0$

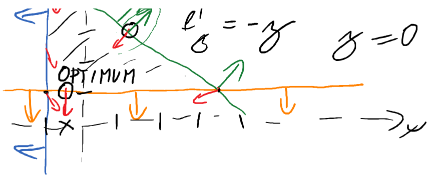
$f'_y=-y+\mu_1+\mu_3=0$

$\mu_1(4-x-y)=0$

$\mu_2(-1-x)=0$

$\mu_3(1-y)=0$

I. $\mu_1\ne 0;\ \mu_2=0,\ \mu_3=0$ $\quad$ II $\mu_3\ne 0;\ \mu_1=0,\ \mu_2=0$



$f'_y = -y \qquad y = 0$

$\mu_3(1-y) = 0$

I. $\mu_1 \neq 0$; $\mu_2 = 0$, $\mu_3 = 0$

$\boxed{4 - x - y = 0}$

$-2(x + \frac{1}{2}) + \mu_1 = 0 \Rightarrow x = \frac{1}{4}\mu_1 = \boxed{\frac{4}{5}}$

$-y + \mu_1 = 0 \Rightarrow y = \mu_1 = \boxed{\frac{16}{5}}$

$4 - x - y = 0$

$4 - \frac{1}{4}\mu_1 - \mu_1 = 0$

$[\frac{4}{5}, \frac{16}{5}] \qquad \frac{5}{4}\mu_1 = 4 \Rightarrow \mu_1 = \frac{16}{5}$

$-(\frac{4}{5} + \frac{1}{2})^2 - \frac{1}{2}(\frac{16}{5})^2 = -\frac{81}{100} - \frac{128}{5}$

II $\mu_3 \neq 0$; $\mu_1 = 0$, $\mu_2 = 0$

$1 - y = 0$

$x = -\frac{1}{2}$

$-2(x + \frac{1}{2}) = 0$

$-y + \mu_3 = 0$

$1 - y = 0$

$\boxed{y = 1}$

$\boxed{[-\frac{1}{2}, 1]}$

$-(-\frac{1}{2} + \frac{1}{2})^2 - \frac{1}{2}(1)^2 = -\frac{1}{2}$

3. MAX $x + y - e^x - e^{x+y}$ NEMÁ STAC. B.

$e^{-x} - y \leq 0$

$y - \frac{1}{2} \leq 0$

$f'_x = 1 - e^x - e^{x+y} = 0 \qquad 1 - e^{-y} - e^0 = 0$

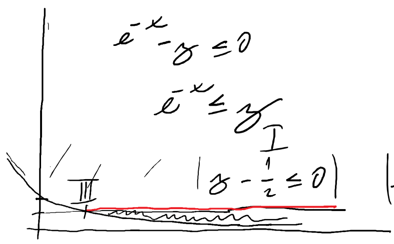
$f'_y = 1 - e^{x+y} = 0 \qquad 1 - e^{-y} - 1 = 0$

$x + y = 0 \qquad e^{-y} = 0$ NEMÁ Ř.

$x = -y$

$e^{-x} - y \leq 0$

$e^{-x} \leq y$



I $|y - \frac{1}{2} \leq 0|$ II. $|e^{-x} - y \leq 0|$

$L'_x = 1 - e^x - e^{x+y} + \mu_1 \cdot e^{-x} = 0$

$L'_y = 1 - e^{x+y} + \mu_1 - \mu_2 = 0$

$\mu_1(e^{-x} - y) = 0$

$\mu_2(y - \frac{1}{2}) = 0$

II

$\mu_1 \neq 0 \qquad y = e^{-x}$

$\mu_2 = 0$

$1 - e^x - e^{x + e^{-x}} + \mu_1 e^{-x} = 0$

$1 - e^{x + e^{-x}} + \mu_1 = 0$

$y = \frac{1}{2} \qquad 1 - e^x - e^{x + \frac{1}{2}} = 0$

$\mu_1 = 0 \qquad 1 - e^{x + \frac{1}{2}} - \mu_2 = 0$

$-e^x - e^x \cdot e^{\frac{1}{2}} = -1$

$e^x(-1 - e^{\frac{1}{2}}) = -1$

$e^x = \frac{1}{1 + e^{\frac{1}{2}}}$

$\boxed{x = \ln\left(\frac{1}{1 + e^{\frac{1}{2}}}\right)}$

4.

MAX $-x^2 - y^2 - x - \mu_1(x^2 + y^2 - 1)$

$f'_x = -2x - 1$

$f'_y = -2y$

$y = 0$

$x = \frac{1}{2}$

$\boxed{\frac{1}{4} + 0 \leq 1}$

$L'_x = -2x - 1 - 2x\mu_1 = 0$

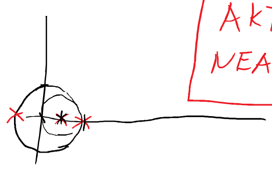
$L'_y = -2y - 2y\mu_1 = 0$

$\mu_1(x^2 + y^2 - 1) = 0$

$\mu_1 \geq 0$

AKTIVNÍ OMEZENÍ $\Rightarrow \mu \neq 0$

NEAKTIVNÍ — II — $\Rightarrow \mu = 0$



5. b) $f = xy + x + y$

$f' = x + 1 = 0 \Rightarrow [x = -1$

5. b) $f = xy + x + y$

$x^2 + y^2 - 2 \leq 0$

$\mu_2 \quad x + y - 1 \leq 0$

$f'_x = y + 1 = 0 \Rightarrow \boxed{y = -1}$

$f'_y = x + 1 = 0 \Rightarrow \boxed{x = -1}$

$1 + 1 - 2 \leq 0$ ✓

$-1 - 1 - 1 \leq 0$ ✓