Binary dependent variables

LECTURE 8

09.12.2022

Lecture Outline

- The linear probability model
- Nonlinear probability models
  - Probit
  - Logit
- Brief introduction of maximum likelihood estimation
- Interpretation of coefficients in logit and probit models

Introduction

- So far the dependent variable (Y) has been continuous:
  - Average hourly earnings
  - Birth weight of babies
- What if Y is binary?
  - Y = get into college, or not; X = parental income.
  - Y = person smokes, or not; X = cigarette tax rate, income.
  - Y = mortgage application is accepted, or not; X = race, income, house characteristics, marital status ...

The linear probability model

- Multiple regression model with continuous dependent variable

$$Y_i = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki} + u_i$$

- The coefficient β_j can be interpreted as the change in Y associated with a unit change in X_j

- We will now discuss the case with a binary dependent variable

- We know that the expected value of a binary variable Y is

$$E[Y] = 1 \cdot Pr(Y = 1) + 0 \cdot Pr(Y = 0) = Pr(Y = 1)$$

- In the multiple regression model with a binary dependent variable we have

$$E[Y_i | X_{1i}, \cdots, X_{ki}] = Pr(Y_i = 1 | X_{1i}, \cdots, X_{ki})$$

- It is therefore called the **linear probability model.**

Mortgage applications

Example:

- Most individuals who want to buy a house apply for a mortgage at a bank.
- Not all mortgage applications are approved.
- What determines whether or not a mortgage application is approved or denied?
- During this lecture we use a subset of the Boston HMDA data ($N = 2380$)
  - a data set on mortgage applications collected by the Federal Reserve Bank in Boston

Variable	**Description**	**Mean**	**SD**
deny	$=$ 1if mortgage application is denied	0.120	0.325
pi_ratio	anticipated monthly loan payments / monthly income	0.331	0.107
black	$=$ 1if applicant is black, $=$ 0 if applicant is white	0.142	0.350

Mortgage applications

- Does the payment to income ratio affect whether or not a mortgage application is denied?

```
. regress deny pi_ratio, robust

Linear regression                               Number of obs =      2380
                                                F( 1, 2378)   =     37.56
                                                Prob > F      =    0.0000
                                                R-squared     =    0.0397
                                                Root MSE      =    .31828
```

deny	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
pi_ratio	.6035349	.0984826	6.13	0.000	.4104144	.7966555
_cons	-.0799096	.0319666	-2.50	0.012	-.1425949	-.0172243

- The estimated OLS coefficient on the payment to income ratio equals $\widehat{\beta_1} = 0.6$
- The estimated coefficient is significantly different from 0 at a 1% significance level.
- How should we interpret $\widehat{\beta_1}$?

The linear probability model

- The conditional expectation equals the probability that $Y_i = 1$ conditional on $X_{1i}, \cdots, X_{ki}$:

$$E[Y_i|X_{1i}, \cdots, X_{ki}] = Pr(Y_i = 1|X_{1i}, \cdots, X_{ki}) = \beta_0 + \beta_1 X_{1i} + \cdots \beta_k X_{ki}$$

- The population coefficient β_j equals the change in the probability that $Y_i = 1$ associated with a unit change in X_j.

$$\frac{\partial Pr(Y_i = 1|X_{1i}, \cdots, X_{ki})}{\partial X_j} = \beta_j$$

In the mortgage application example:

- $\widehat{\beta_1} = 0.6$
- A change in the payment to income ratio by 1 is estimated to increase the probability that the mortgage application is denied by 0.60.
- A change in the payment to income ratio by 0.10 is estimated to increase the probability that the application is denied by 6% (0.10*0.60*100).

The linear probability model

Assumptions are the same as for general multiple regression model:

- $E(u_i|X_{1i}, X_{2i}, \ldots, X_{ki}) = 0$
- Big outliers are unlikely
- No perfect multicollinearity.

Advantages of the linear probability model:

- Easy to estimate
- Coefficient estimates are easy to interpret

Disadvantages of the linear probability model

- Predicted probability can be above 1 or below 0!
- Error terms are heteroskedastic

The linear probability model: heteroskedasticity

$$Y_i = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki} + u_i$$

- The variance of a Bernoulli random variable:

$$Var(Y) = Pr(Y = 1) \quad (1 - Pr(Y = 1))$$

- We can use this to find the conditional variance of the error term

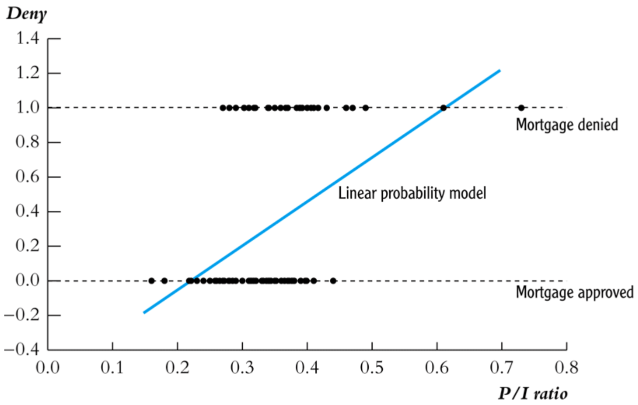
$$\begin{aligned} Var(u_i|X_{1i},\ldots,X_{ki}) &= Var(Y_i - (\beta_0 + \beta_1 X_{1i} + \ldots + \beta_k X_{ki})|\, X_{1i},\ldots,X_{ki}) \\ &= Var(Y_i|X_{1i},\ldots,X_{ki}) \\ &= Pr(Y_i = 1|\, X_{1i},\cdots,X_{ki}) \times (1 - Pr(Y_i = 1|\, X_{1i},\cdots,X_{ki})) \\ &= (\beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}) \times (1 - \beta_0 - \beta_1 X_{1i} - \cdots - \beta_k X_{ki}) \\ &\neq \sigma_u^2 \end{aligned}$$

- Solution: Always use heteroskedasticity robust standard errors when estimating a linear probability model!

The linear probability model: shortcomings

In the linear probability model the predicted probability can be below 0 or above 1!

Example: linear probability model, HMDA data
Mortgage denial v. ratio of debt payments to income (P/I ratio) in a subset of the HMDA data set (*n* = 127)



Nonlinear probability models

- Probabilities cannot be less than 0 or greater than 1
- To address this problem we will consider nonlinear probability models

$$Pr(Y_i = 1) = G(Z)$$

$$\text{with } Z = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}$$

$$\text{and} \quad 0 \leq G(Z) \leq 1$$

- We will consider 2 nonlinear functions

1. Probit

$$G(Z) = \Phi(Z)$$

2. Logit

$$G(Z) = \frac{1}{1 + e^{-Z}}$$

Probit

Probit regression models the probability that $Y = 1$

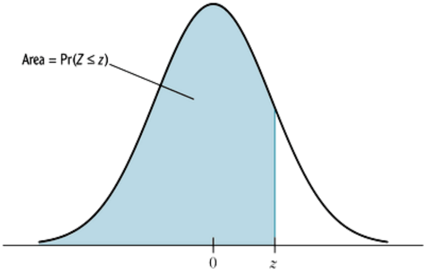
- Using the cumulative standard normal distribution function $\Phi(Z)$
- evaluated at $Z = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}$
- since $\Phi(z) = Pr(Z \leq z)$ we have that the predicted probabilities of the probit model are between 0 and 1

Example

- Suppose we have only 1 regressor and $Z = -2 + 3X_1$
- We want to know the probability that $Y = 1$ when $X_1 = 0.4$
- $z = -2 + 3 \cdot 0.4 = -0.8$
- $Pr(Y = 1) = Pr(Z \leq -0.8) = \Phi(-0.8)$

Probit

TABLE 1 The Cumulative Standard Normal Distribution Function, $\Phi(z)$ = Pr(Z ″ z)



z	Second Decimal Value of z									
	0	1	2	3	4	5	6	7	8	9
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121

$$Pr(Y = 1) = Pr(Z \leq -0.8) = \Phi(-0.8) = 0.2119$$

Logit

Logit regression models the probability that $Y = 1$

- Using the cumulative standard logistic distribution function

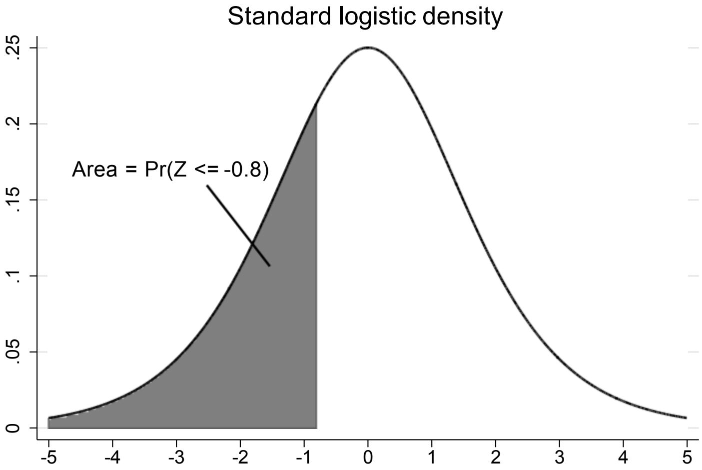
$$F(Z) = \frac{1}{1 + e^{-Z}}$$

- evaluated at $Z = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}$
- since $F(z) = Pr(Z \leq z)$ we have that the predicted probabilities of the probit model are between 0 and 1

Example

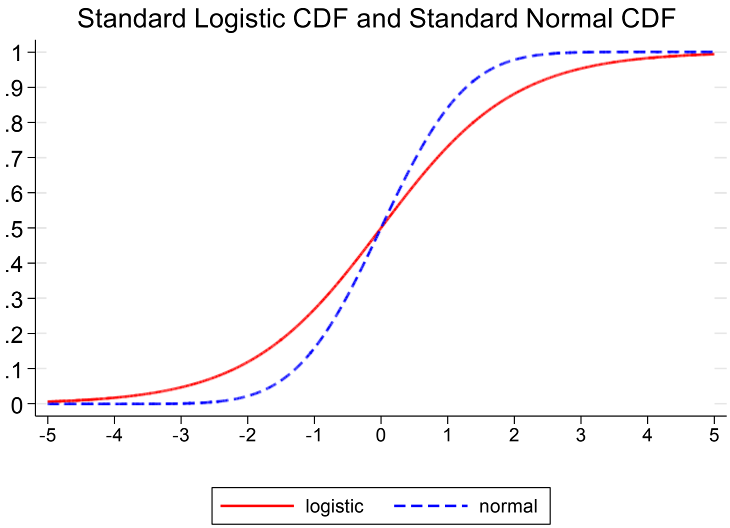
- Suppose we have only 1 regressor and $Z = -2 + 3X_1$
- We want to know the probability that $Y = 1$ when $X_1 = 0.4$
- $z = -2 + 3 \cdot 0.4 = -0.8$
- $Pr(Y = 1) = Pr(Z \leq -0.8) = F(-0.8)$

Logit



- $Pr(Y = 1) = Pr(Z \leq -0.8) = \frac{1}{1+e^{0.8}} = 0.31$

Logit & probit



How to estimate logit and probit models

- In previous lectures we discussed regression models that are nonlinear in the independent variables
  - these models can be estimated by OLS
- Logit and Probit models are nonlinear in the coefficients $\beta_0, \beta_1, \cdots, \beta_k$
  - these models can't be estimated by OLS
- The method used to estimate logit and probit models is Maximum Likelihood Estimation (MLE).
- The MLE are the values of $(\beta_0, \beta_1, \cdots, \beta_k)$ that best describe the full distribution of the data.

Maximum likelihood estimation

- The **likelihood function** is the joint probability distribution of the data, treated as a function of the unknown coefficients.
- The **maximum likelihood estimator (MLE)** are the values of the coefficients that maximize the likelihood function.
- MLE's are the parameter values "most likely" to have produced the data.

Lets start with a special case: The MLE with no X

- We have n i.i.d. observations $Y_1, \ldots, Y_n$ on a binary dependent variable
- Y is a Bernoulli random variable
- There is only 1 unknown parameter to estimate:
  - The probability $\boldsymbol{p}$ that $Y = 1$,
  - which is also the mean of Y

Maximum likelihood estimation (Optional)

Step 1: write down the likelihood function, the joint probability distribution of the data

- Y_i is a Bernoulli random variable we therefore have

$$Pr(Y_i = y) = Pr(Y_i = 1)^y \cdot (1 - Pr(Y_i = 1))^{1-y} = p^y(1-p)^{1-y}$$

  - $Pr(Y_i = 1) = p^1(1 - p)^0 = p$
  - $Pr(Y_i = 0) = p^0(1 - p)^1 = 1 - p$

- $Y_1, \ldots, Y_n$ are i.i.d, the joint probability distribution is therefore the product of the individual distributions

$$\begin{aligned} Pr(Y_1 = y_1, \ldots . Y_n = y_n) &= Pr(Y_1 = y_1) \times \ldots \times Pr(Y_n = y_n) \\ &= \left[p^{y_1}(1-p)^{1-y_1}\right] \times \ldots \times \left[p^{y_n}(1-p)^{1-y_n}\right] \\ &= p^{(y_1+y_2+\ldots+y_n)}(1-p)^{n-(y_1+y_2+\ldots+y_n)} \end{aligned}$$

Maximum likelihood estimation (Optional)

We have the likelihood function:

$$f_{Bernouilli}(p;\ Y_1 = y_1, \ldots . Y_n = y_n) = p^{\sum y_i}(1-p)^{n-\sum y_i}$$

Step 2: Maximize the likelihood function w.r.t *p*

- Easier to maximize the logarithm of the likelihood function

$$ln\left(f_{Bernouilli}(p;\ Y_1 = y_1, \ldots . Y_n = y_n)\right) = \left(\sum_{i=1}^{n} y_i\right) \cdot ln\,(p) + \left(n - \sum_{i=1}^{n} y_i\right) ln\,(1-p)$$

- Since the logarithm is a strictly increasing function, maximizing the likelihood or the log likelihood will give the same estimator.

Maximum likelihood estimation (Optional)

- Taking the derivative w.r.t *p* gives

$$\frac{d}{dp} ln\left(f_{Bernouilli}(p;\ Y_1 = y_1, \ldots . Y_n = y_n)\right) = \frac{\sum_{i=1}^{n} y_i}{p} - \frac{n - \sum_{i=1}^{n} y_i}{1-p}$$

- Setting to zero and rearranging gives

$$\begin{aligned} (1-p) \times \sum_{i=1}^{n} y_i &= p \times (n - \sum_{i=1}^{n} y_i) \\ \sum_{i=1}^{n} y_i - p\sum_{i=1}^{n} y_i &= n \cdot p - p\sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} y_i &= n \cdot p \end{aligned}$$

- Solving for *p* gives the MLE

$$\widehat{p}_{MLE} = \frac{1}{n}\sum_{i=1}^{n} y_i = \overline{Y}$$

MLE of the probit model (Optional)

Step 1: write down the likelihood function

$$
\begin{aligned}
Pr(Y_1 = y_1, \ldots . Y_n = y_n) &= Pr(Y_1 = y_1) \times \ldots \times Pr(Y_n = y_n) \\
&= \left[p_1^{y_1}(1 - p_1)^{1-y_1}\right] \times \ldots \times \left[p_n^{y_n}(1 - p_n)^{1-y_n}\right]
\end{aligned}
$$

- so far it is very similar as the case without explanatory variables except that p_i depends on $X_{1i}, \ldots, X_{ki}$

$$p_i = \Phi\left(X_{1i}, \ldots, X_{ki}\right) = \Phi\left(\beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}\right)$$

- substituting for p_i gives the likelihood function:

$$
\begin{aligned}
&\left[\Phi\left(\beta_0 + \beta_1 X_{11} + \cdots + \beta_k X_{k1}\right)^{y_1}\left(1 - \Phi\left(\beta_0 + \beta_1 X_{11} + \cdots + \beta_k X_{k1}\right)\right)^{1-y_1}\right] \times \ldots \\
&\times \left[\Phi\left(\beta_0 + \beta_1 X_{1n} + \cdots + \beta_k X_{kn}\right)^{y_n}\left(1 - \Phi\left(\beta_0 + \beta_1 X_{1n} + \cdots + \beta_k X_{kn}\right)\right)^{1-y_n}\right]
\end{aligned}
$$

MLE of the probit model (Optional)

Also with obtaining the MLE of the probit model it is easier to take the logarithm of the likelihood function

Step 2: Maximize the log likelihood function

$$ln\left[f_{probit}\left(\beta_0, \ldots, \beta_k;\ Y_1, \ldots, Y_n | X_{1i}, \ldots, X_{ki}, i = 1, \ldots, n\right)\right]$$

$$= \sum_{i=1}^{n} Y_i ln\left[\Phi\left(\beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}\right)\right]$$

$$+ \sum_{i=1}^{n}(1 - Y_i) ln\left[1 - \Phi\left(\beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}\right)\right]$$

w.r.t $\beta_0, \ldots, \beta_1$

- There is no simple formula for the probit MLE, the maximization must be done using numerical algorithm on a computer.

MLE of the logit model (Optional)

Step 1: write down the likelihood function

$$Pr(Y_1 = y_1, \ldots . Y_n = y_n) \quad = \quad \left[p_1^{y_1}(1-p_1)^{1-y_1}\right] \times \ldots \times \left[p_n^{y_n}(1-p_n)^{1-y_n}\right]$$

- very similar to the Probit model but with a different function for p_i

$$p_i = 1 / \left[1 + e^{-(\beta_0+\beta_1 X_{1i}+\ldots+\beta_k X_{ki})}\right]$$

Step 2: Maximize the log likelihood function w.r.t $\beta_0, \ldots, \beta_1$

$$ln\left[f_{logit}\left(\beta_0, \ldots, \beta_k;\; Y_1, \ldots, Y_n | \; X_{1i}, \ldots, X_{ki}, i = 1, \ldots, n\right)\right]$$

$$= \quad \sum_{i=1}^{n} Y_i ln\left(1 / \left[1 + e^{-(\beta_0+\beta_1 X_{1i}+\ldots+\beta_k X_{ki})}\right]\right)$$

$$+ \sum_{i=1}^{n}(1 - Y_i) ln\left(1 - \left(1 / \left[1 + e^{-(\beta_0+\beta_1 X_{1i}+\ldots+\beta_k X_{ki})}\right]\right)\right)$$

- There is no simple formula for the logit MLE, the maximization must be done using numerical algorithm on a computer.

Probit: mortgage applications

```
. probit deny pi_ratio

Iteration  0:  log  likelihood  =     -872.0853
Iteration  1:  log  likelihood  =    -832.02975
Iteration  2:  log  likelihood  =    -831.79239
Iteration 3: log likelihood =        -831.79234

Probit regression                                Number of obs   =       2380
                                                 LR chi2(    1)      =   80.59
                                                 Prob > chi2     =     0.0000
Log likelihood = -831.79234                        Pseudo R2       =   0.0462
```

deny	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
pi_ratio	2.967907	.3591054	8.26	0.000	2.264073	3.67174
_cons	-2.194159	.12899	-17.01	0.000	-2.446974	-1.941343

- The estimated MLE coefficient on the payment to income ratio equals $\widehat{\beta_1} = 2.97$
- The estimated coefficient is positive and significantly different from 0 at a 1% significance level.
- How should we interpret $\widehat{\beta_1}$?

Probit: mortgage applications

The estimate of β_1 in the probit model CANNOT be interpreted as the change in the probability that $Y_i = 1$ associated with a unit change in X_1!!

- In general the effect on Y of a change in X is the expected change in Y resulting from the change in X
- Since Y is binary the expected change in Y is the change in the probability that $Y = 1$

In the probit model the predicted change the probability that the mortgage application is denied when the payment to income ratio increases from

0.10 to 0.20:

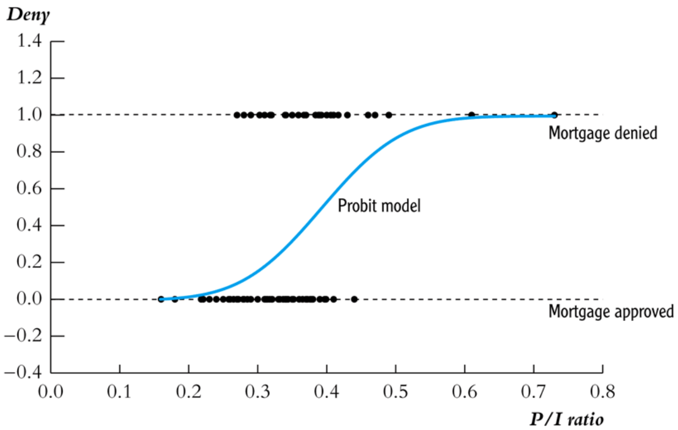
$$\triangle \widehat{Pr(Y_i = 1)} = \Phi\left(-2.19 + 2.97 \cdot 0.20\right) - \Phi\left(-2.19 + 2.97 \cdot 0.10\right) = 0.0495$$

0.30 to 0.40:

$$\triangle \widehat{Pr(Y_i = 1)} = \Phi\left(-2.19 + 2.97 \cdot 0.40\right) - \Phi\left(-2.19 + 2.97 \cdot 0.30\right) = 0.0619$$

Probit: mortgage applications

Predicted values in the probit model:



- All predicted probabilities are between 0 and 1!

Logit: mortgage applications

```
. logit deny pi_ratio

Iteration  0:  log  likelihood =    -872.0853
Iteration  1:  log  likelihood =   -830.96071
Iteration  2:  log  likelihood =   -830.09497
Iteration  3:  log  likelihood =   -830.09403
Iteration 4: log likelihood =      -830.09403

Logistic regression                      Number of obs   =        2380
                                         LR chi2(    1)      =   83.98
                                         Prob > chi2     =      0.0000
Log likelihood = -830.09403                Pseudo R2       =    0.0482
```

deny	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
pi_ratio	5.884498	.7336006	8.02	0.000	4.446667	7.322328
_cons	-4.028432	.2685763	-15.00	0.000	-4.554832	-3.502032

- The estimated MLE coefficient on the payment to income ratio equals $\widehat{\beta_1} = 5.88$
- **The estimated coefficient is positive and significantly different from 0 at a 1% significance level.**
- **How should we interpret $\widehat{\beta_1}$?**

Logit: mortgage applications

Also in the Logit model:

The estimate of β_1 CANNOT be interpreted as the change in the probability that $Y_i = 1$ associated with a unit change in X_1!!

In the logit model the predicted change the probability that the mortgage application is denied when the payment to income ratio increases from

0.10 to 0.20:

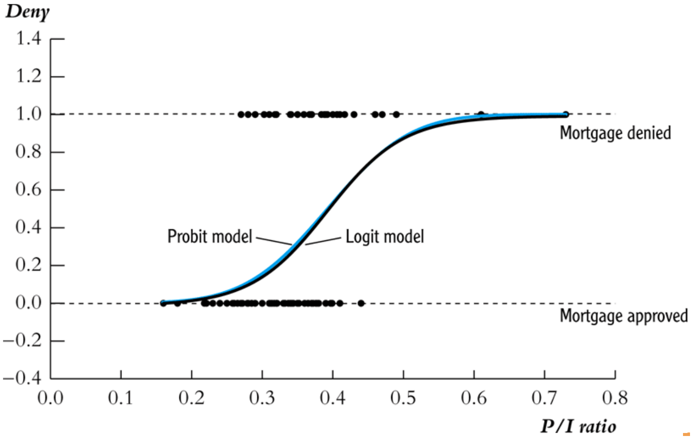
$$\triangle \widehat{Pr(Y_i = 1)} = \left(1/1 + e^{-(-4.03+5.88 \cdot 0.20)}\right) - \left(1/1 + e^{-(-4.03+5.88 \cdot 0.10)}\right) = 0.023$$

0.30 to 0.40:

$$\triangle \widehat{Pr(Y_i = 1)} = \left(1/1 + e^{-(-4.03+5.88 \cdot 0.40)}\right) - \left(1/1 + e^{-(-4.03+5.88 \cdot 0.30)}\right) = 0.063$$

Logit: mortgage applications

The predicted probabilities from the probit and logit models are very close in these HMDA regressions:



Probit & Logit with multiple regressors

- We can easily extend the Logit and Probit regression models, by including additional regressors
- Suppose we want to know whether white and black applications are treated differentially
- Is there a significant difference in the probability of denial between black and white applicants conditional on the payment to income ratio?
- To answer this question we need to include two regressors
  - P/I ratio
  - Black

Probit with multiple regressors

```
Probit regression                               Number of obs =        2380
                                                LR chi2(    2)     =  149.90
                                                Prob > chi2      =    0.0000
Log likelihood =  -797.13604                     Pseudo R2        =    0.0859
```

deny	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
black	.7081579	.0834327	8.49	0.000	.5446328	.8716831
pi_ratio	2.741637	.3595888	7.62	0.000	2.036856	3.446418
_cons	-2.258738	.129882	-17.39	0.000	-2.513302	-2.004174

- To say something about the size of the impact of race we need to specify a value for the payment to income ratio
- Predicted denial probability for a white application with a P/I-ratio of 0.3 is

$$\Phi(-2.26 + 0.71 \cdot 0 + 2.74 \cdot 0.3) = 0.0749$$

- Predicted denial probability for a black application with a P/I-ratio of 0.3 is

$$\Phi(-2.26 + 0.71 \cdot 1 + 2.74 \cdot 0.3) = 0.2327$$

- Difference is 15.8%

Logit with multiple regressors

```
Logistic regression                               Number of obs  =      2380
                                                  LR chi2(    2)      =    152.78
                                                  Prob > chi2      =    0.0000
Log likelihood =  -795.69521                        Pseudo R2        =    0.0876
```

deny	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
black	1.272782	.1461983	8.71	0.000	.9862385	1.559325
pi_ratio	5.370362	.7283192	7.37	0.000	3.942883	6.797841
_cons	-4.125558	.2684161	-15.37	0.000	-4.651644	-3.599472

- To say something about the size of the impact of race we need to specify a value for the payment to income ratio
- Predicted denial probability for a white application with a P/I-ratio of 0.3 is

$$1/1 + e^{-(-4.13+5.37\cdot 0.30)} = 0.075$$

- Predicted denial probability for a black application with a P/I-ratio of 0.3 is

$$1/1 + e^{-(-4.13+5.37\cdot 0.30+1.27)} = 0.224$$

- Difference is 14.8%

LPM, Probit & Logit

Table 1: Mortgage denial regression using the Boston HMDA Data

Dependent variable: deny = 1 if mortgage application is denied, = 0 if accepted

regression model	LPM	Probit	Logit
black	0.177*** (0.025)	0.71*** (0.083)	1.27*** (0.15)
P/I ratio	0.559*** (0.089)	2.74*** (0.44)	5.37*** (0.96)
constant	-0.091*** (0.029)	-2.26*** (0.16)	-4.13*** (0.35)
difference Pr(deny=1) between black and white applicant when P/I ratio=0.3	17.7%	15.8%	14.8%

Threats to internal and external validity

Both for the Linear Probability as for the Probit & Logit models we have to consider threats to

1. Internal validity

- Is there omitted variable bias?
- Is the functional form correct?
  - Probit model: is assumption of a Normal distribution correct?
  - Logit model: is assumption of a Logistic distribution correct?
- Is there measurement error?
- Is there sample selection bias?
- is there a problem of simultaneous causality?

2. External validity

- These data are from Boston in 1990-91.
- Do you think the results also apply today, where you live?

Distance to college & probability of obtaining a college degree

Linear regression

Number of obs	=	3796
F(1, 3794)	=	15.77
Prob > F	=	0.0001
R-squared	=	0.0036
Root MSE	=	.44302

college	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
dist	-.012471	.0031403	-3.97	0.000	-.0186278	-.0063142
_cons	.2910057	.0093045	31.28	0.000	.2727633	.3092481

Probit regression

Log likelihood = -2204.8977

Number of obs	=	3796
LR chi2(1)	=	14.48
Prob > chi2	=	0.0001
Pseudo R2	=	0.0033

college	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
dist	-.0407873	.0109263	-3.73	0.000	-.0622025	-.0193721
_cons	-.5464198	.028192	-19.38	0.000	-.6016752	-.4911645

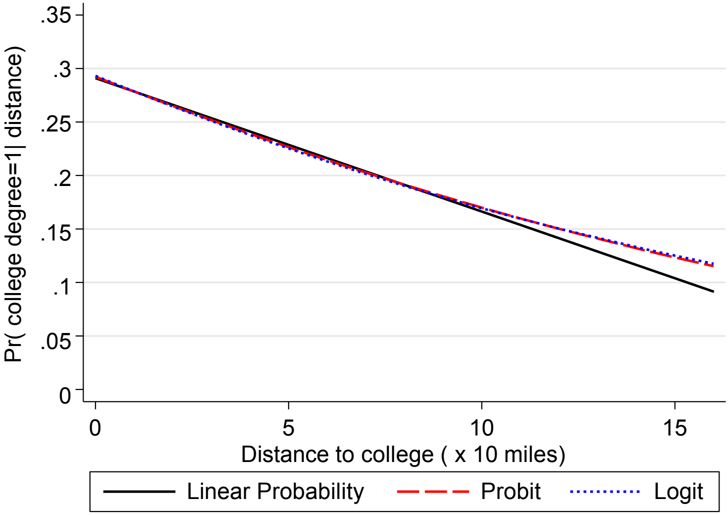
Logistic regression

Log likelihood = -2204.8006

Number of obs	=	3796
LR chi2(1)	=	14.68
Prob > chi2	=	0.0001
Pseudo R2	=	0.0033

college	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
dist	-.0709896	.0193593	-3.67	0.000	-.1089332	-.033046
_cons	-.8801555	.0476434	-18.47	0.000	-.9735349	-.786776

Distance to college & probability of obtaining a college degree



- The 3 different models produce very similar results.

Summary

- If Y_i is binary, then $E(Y_i|X_i) = Pr(Y_i = 1|X_i)$
- Three models:

1. linear probability model (linear multiple regression)
2. probit (cumulative standard normal distribution)
3. logit (cumulative standard logistic distribution)

- LPM, probit, logit all produce predicted probabilities
- Effect of ΔX is a change in conditional probability that $Y = 1$
- For logit and probit, this depends on the initial X
- Probit and logit are estimated via maximum likelihood
  - Coefficients are normally distributed for large n
  - Large-n hypothesis testing, conf. intervals is as usual