Repayment Plan

The plan of how to repay a loan. For more detailed analysis is created so-called redemption table.
It is always true that the installment *annuity "a"* is divided into **two components:**

$$a = I + M$$

where:

I . . . paid interest
M . . . debt amortization

In practice, we will deal with two basic scenarios of repayment plan. First, the **annuity** will be **constant**. Second, the **amortization** will be **constant**.

Constant annuity:

The main task here will be to estimate the regular payment for the financial institution that lent us money. To do so we will use the already known concept of after paid pension. The logic is; first you will take a credit by a financial institution and after a specific time you provide the first installment. That means we will work with the formula of after paid pension (we just change the letter **B** for Budged to **D** for Debt:

$$D = a * \frac{1-\frac{1}{(1+r)^n}}{r}$$

$$a = \frac{D*r}{1-\frac{1}{(1+r)^n}}$$

here:

D . . . Initial debt
r . . . interest rate

Repayment table

	Annuity	Interest	Amortization	Debt
0				D_0
1	a	I_1	M_1	D_1
2	a	I_2	M_2	D_2
3	a	I_3	M_3	D_3
⋮	⋮	⋮	⋮	⋮
n	a	I_n	M_n	D_n

The column of amortization represents a **geometric series**. So, it allows to calculate any row of the amortization table and also it allows sum up how much of the initial debt was already amortized or how much was paid on the interest. And so:

$$\sum_{i=1}^{k} M_i = M_1 * \frac{q^k - 1}{q - 1}$$

quotient q corresponds $(1 + r)$.

and then

$$\sum_{i=1}^{k} I_i = k * a - \sum_{i=1}^{k} M_i$$

For the concrete row:

$$M_k = M_1 * q^{k-1}$$

so

$$I_k = a - M_k$$

or

$$I_k = D_{k-1} * r$$

and finally

$$D_k = \frac{I_{k+1}}{r}$$

Optional Homework

Please create an amortization plan for the following loan. You borrow from bank 1,800,000.00. The bank is requiring 7 % p. a. and calculates interest every month. You have to repay the total amount in 12 years. The payment period is every month (at the end of month).
How much you pay only on interest after 5 years? What will be the payment after 10 years *(row # 121)*?