MPF_RRFI - Lecture 04

Value at Risk and Expected Shortfall (Chapter 12)

Value at risk (VaR) and expected shortfall (ES) are attempts to provide a single num- ber that summarizes the total risk in a portfolio.

- VaR $\implies$ We are X percent certain that we will not lose more than V dollars in time T.
- VaR asks the question: "How bad can things get?"
- ES asks: "If things do get bad, what is the expected loss?
- ES is the expected loss during time T conditional on the loss being greater than the VaR.

Coherent Risk Measures

Conditions:

1. Monotonicity
2. Translation Invariance
3. Homogeneity
4. Subadditivity

- VaR satisfies the first three conditions.
- ES satisfies all four conditions $\implies$ coherent risk measure

Choice of Parameters and Back-Testing

- selected time horizon and confidence level depends on application of VaR
- it is difficult to estimate VaR for the very high confidence level
- back-testing measures the accuracy of VaR or ES in the real world, much more complicated for ES
- if we assume normality of returns:

$$VaR = \mu + \sigma N^{-1}(X)$$

$$ES = \mu + \sigma \frac{e^{-Y^2/2}}{\sqrt{2\pi}(1-X)}$$

- where X is the confidence level, $N^{-1}(.) = Y$ is the cumulative normal distribution
- VaR and ES are usually calculated for one time period and then extended:

$$\text{T-day VaR} = \text{1-day VaR} \times \sqrt{T}$$

$$\text{T-day ES} = \text{1-day ES} \times \sqrt{T}$$

- in the presence of autocorrelation we have to adjust $\sqrt{T}$

$$\sqrt{T + 2(T-1)\pi + 2(T-2)\pi^2 + 2(T-3)\pi^3 + \ldots 2\pi^{T-1}}$$