MPF_RRFI - Lecture 05

Historical Simulation and Extreme Value Theory (Chapter 13)

Methodology

- one-day VaR, confidence 99\%
- indenify risk factors
- calculate last 500 daily returns
- 99\% VaR is the 5th worst outcome
- for ES calculate average of losses that are worse than VaR (the worst four losses)
- Stressed VaR $\implies$ find a historical period of 251 days (250 consecutive returns) where the VaR was the worst

Accuracy of VaR

- standard error of historical VaR

$$\frac{1}{f(x)}\sqrt{\frac{(1-q)q}{n}}$$

- where q-th percentile of the distribution is estimated as x, n is the number of observations, $f(x)$ is probability density function
- normal distribution is not the best assumption (fat tails), alternative e.g. Pareto distribution

Extreme Value Theory

- describe the science of estimating the tails of a distribution
- used to improve VaR or ES estimates when we are interested in a very high confidence level
- it is a way of smoothing and extrapolating the tails of an empirical distribution

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Model-Building Approach (Chapter 14)

- variance-covariance approach
- in some cases faster than historical simulation approach
- extension of Martowitz portfolio theory (normality assumptions)

The Basic Methodology

- it uses VaR and ES equations from Chapter 12 (equation 12.1 and 12.2)

$$VaR = \mu + \sigma N^{-1}(X)$$

$$ES = \mu + \sigma \frac{e^{-Y^2/2}}{\sqrt{2\pi}(1-X)}$$

- we need to estimate the standard deviation
- using the assuption that changes are normally distributed calculate the selected percentile
- for multi-asset portfolio we need to know the correlation $\implies$ calculate standard deviation based on portfolio theory

Generalization

- assumption that the change in the value of the portfolio is linearly related to proportional changes in the risk factors so that

$$\Delta P = \sum_{i=1}^{n} \delta_i \Delta x_i$$

- where ΔP is the dollar change in the value of the whole portfolio in one day, Δx_i is the proportional change in the *i*th risk factor in one day, δ_i is a variation of the delta risk measure
- standard deviation for n assets

$$\sigma_P = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \delta_i \delta_j \sigma_i \sigma_j}$$

Extensions of the Basic Procedure

- stressed VaR and ES
- non-normal distributions
- Monte Carlo simulations for non-linear portfolios