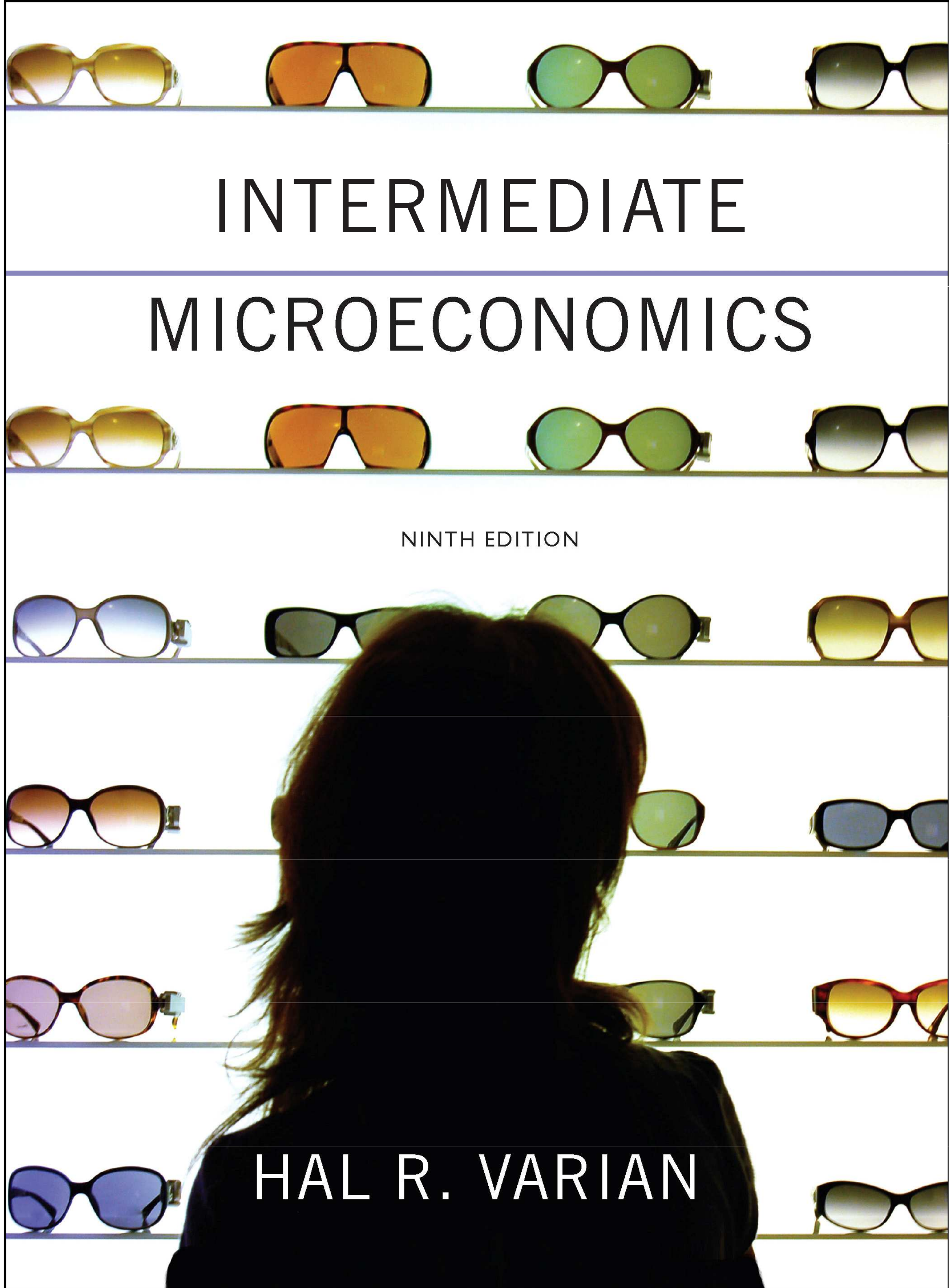



# Chapter 6

# Demand

# Properties of Demand Functions

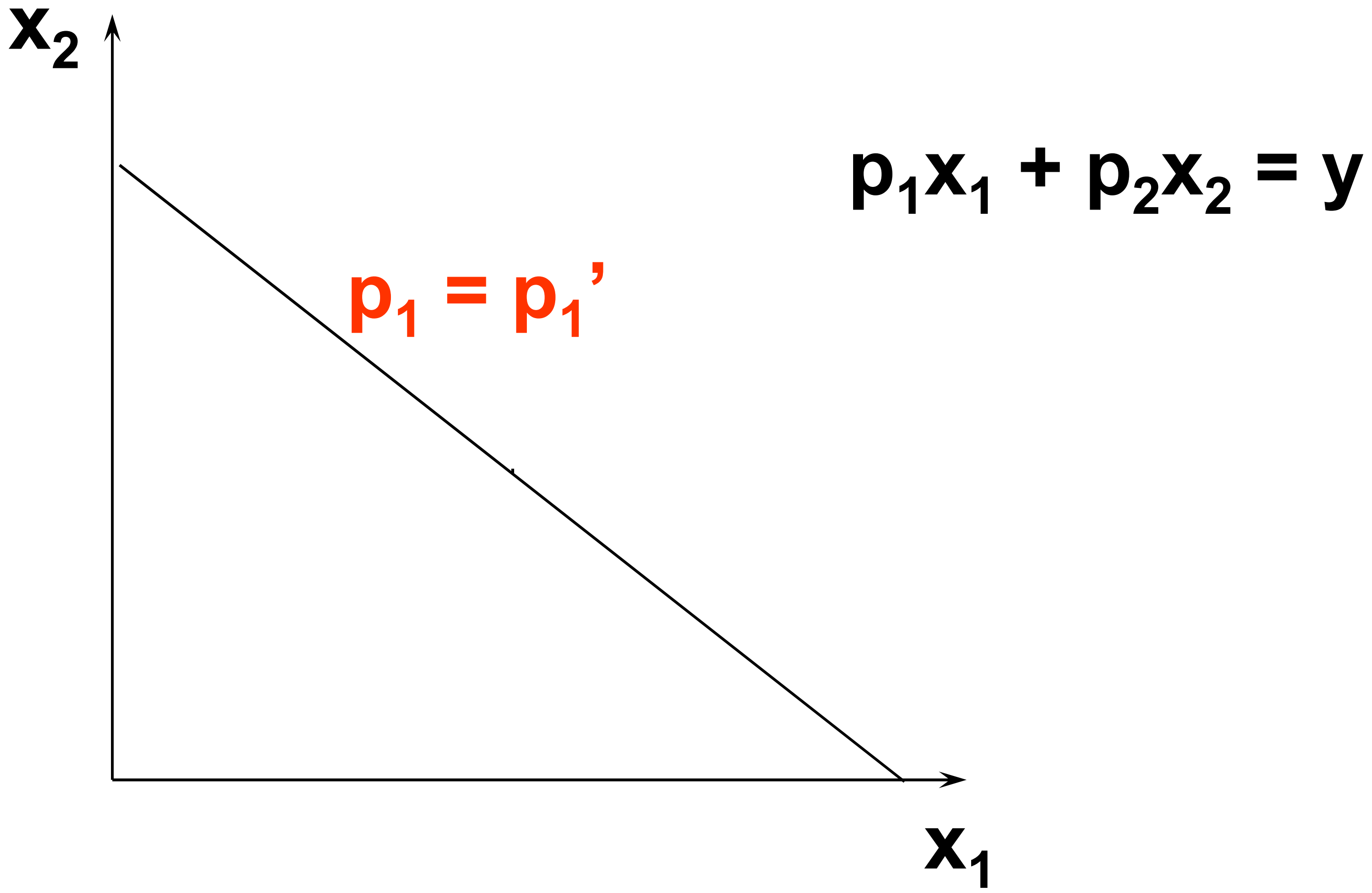
- **Comparative statics analysis of ordinary demand functions -- the study of how ordinary demands $x_1^*(p_1,p_2,y)$ and $x_2^*(p_1,p_2,y)$ change as prices $p_1$, $p_2$ and income y change.**

# Own-Price Changes

- **How does $x_1^*(p_1,p_2,y)$ change as $p_1$ changes, holding $p_2$ and y constant?**
- **Suppose only $p_1$ increases, from $p_1$' to $p_1$'' and then to $p_1$'''.**

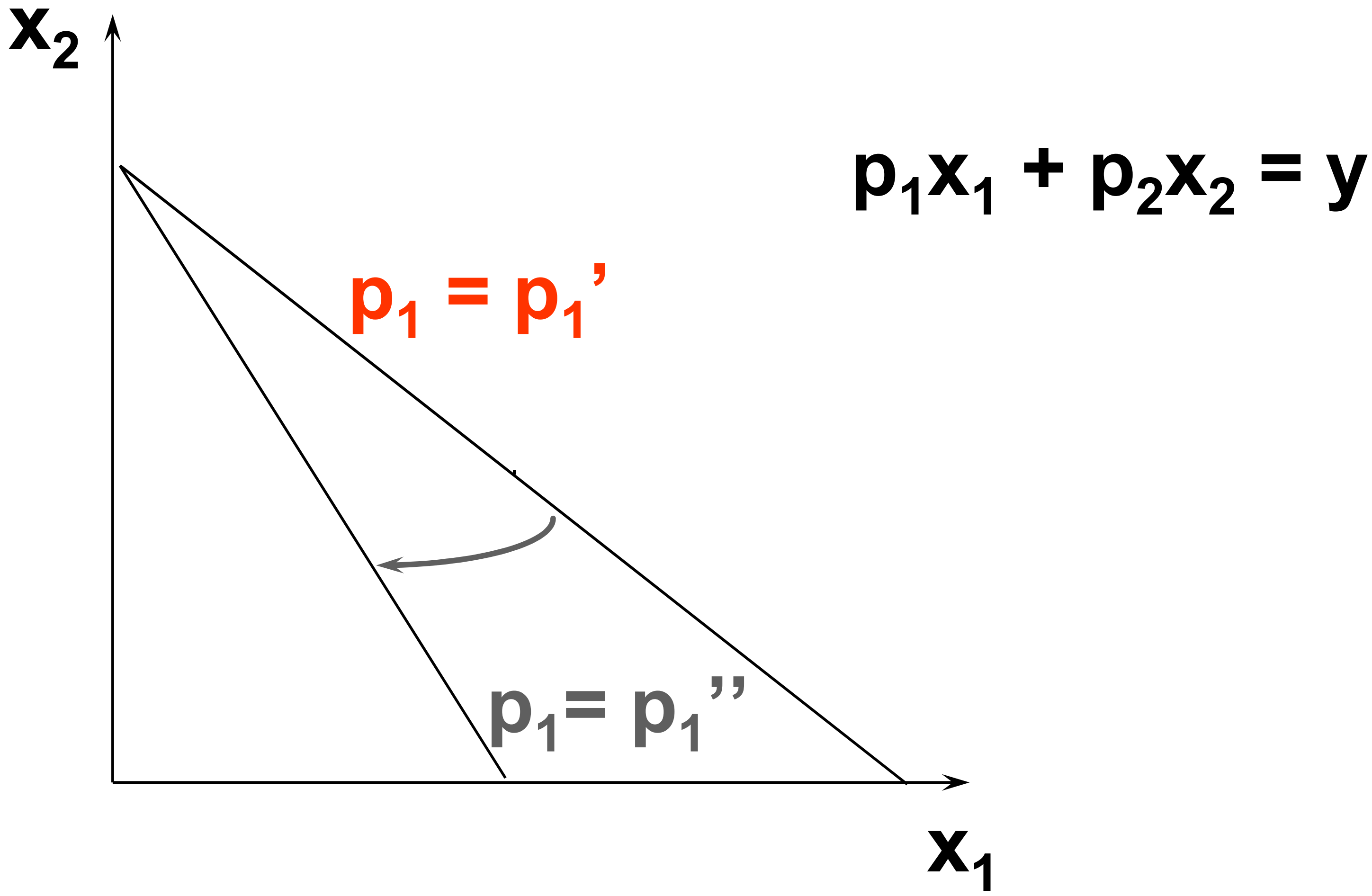
# Own-Price Changes

**Fixed $p_2$ and y.**



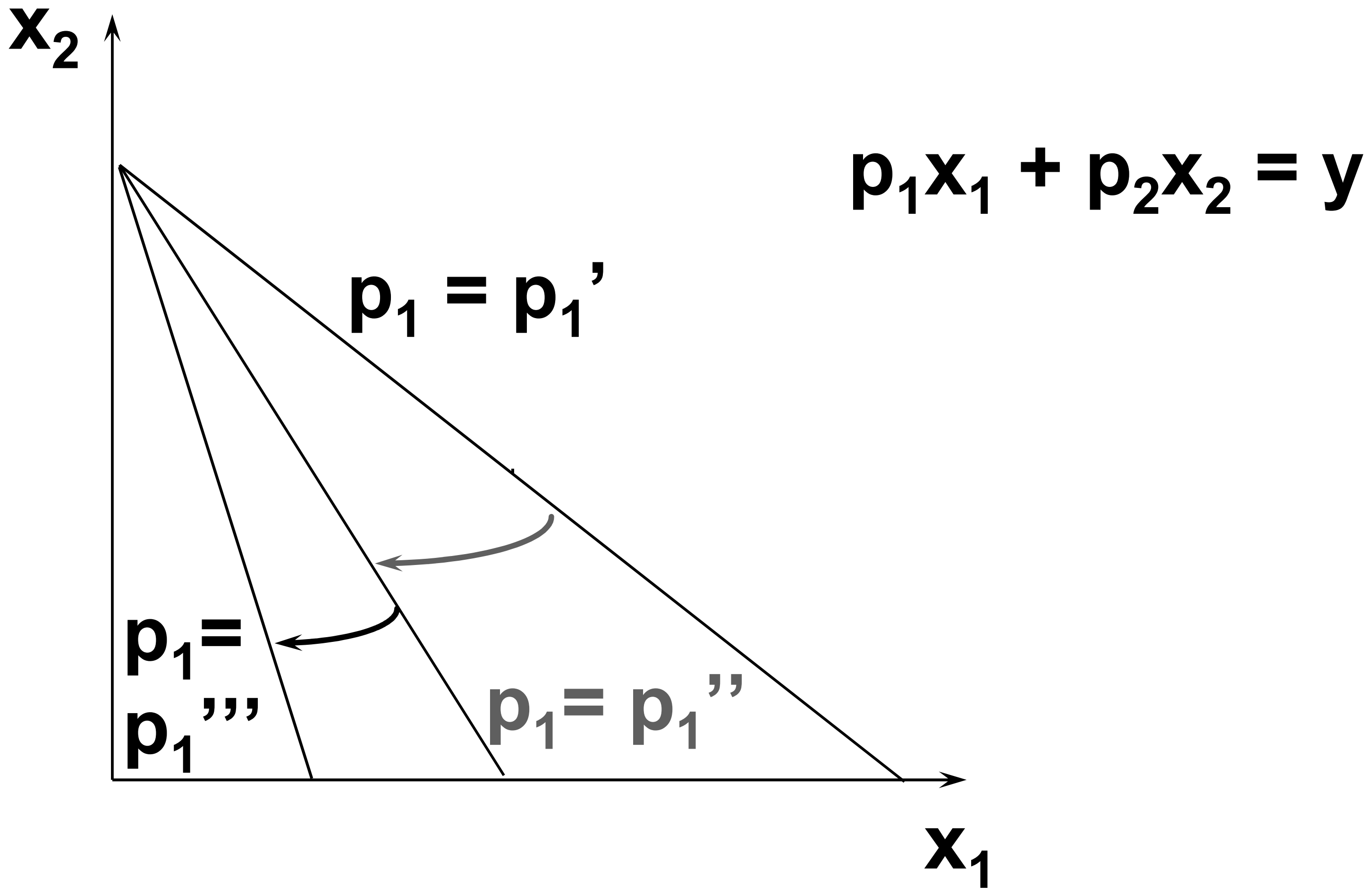
# Own-Price Changes

**Fixed $p_2$ and y.**



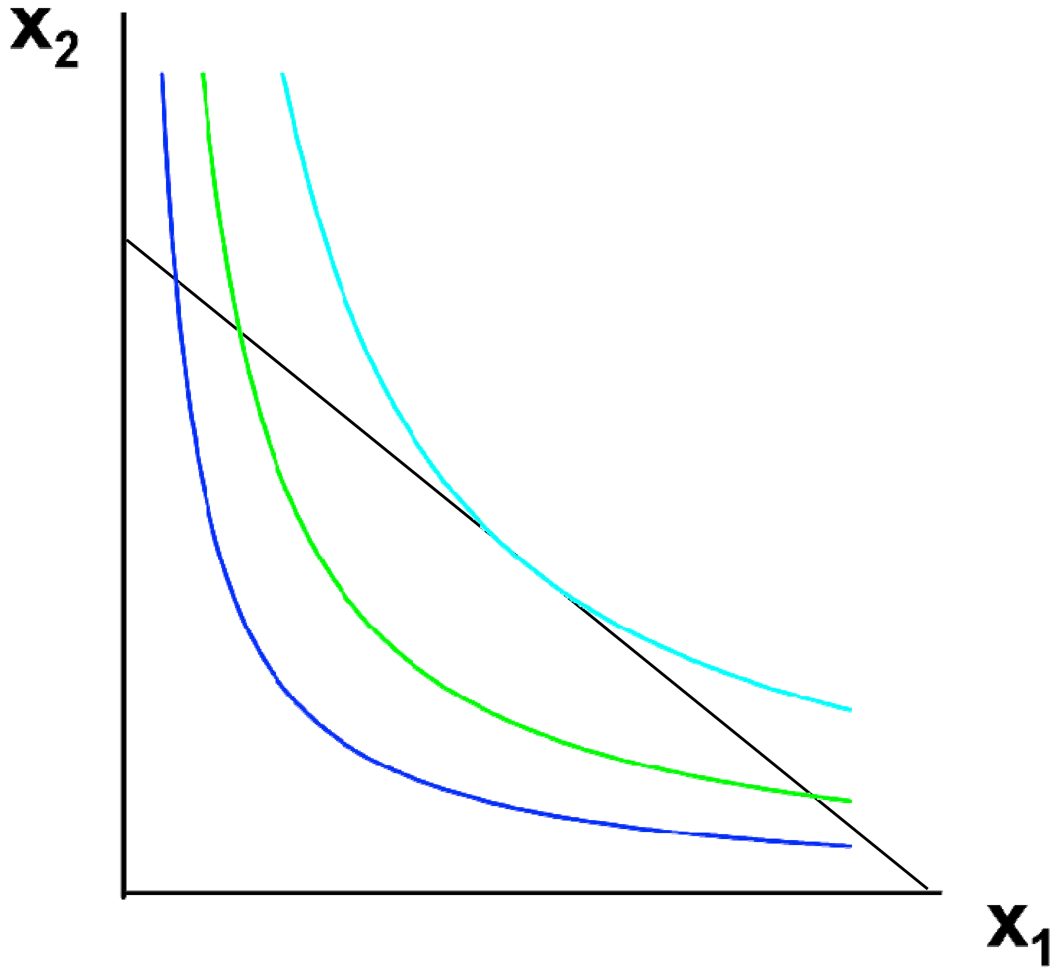
# Own-Price Changes

**Fixed $p_2$ and y.**



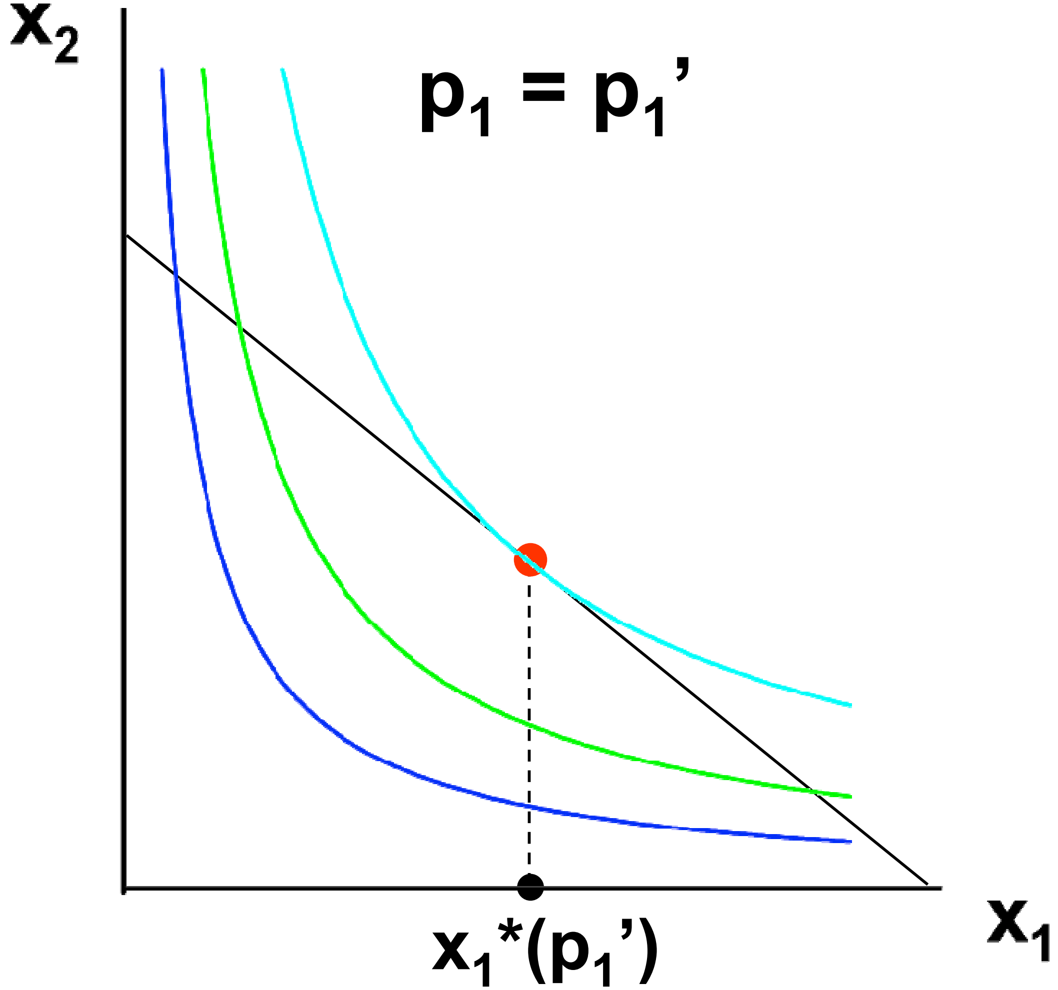
# Own-Price Changes

**Fixed $p_2$ and y.**

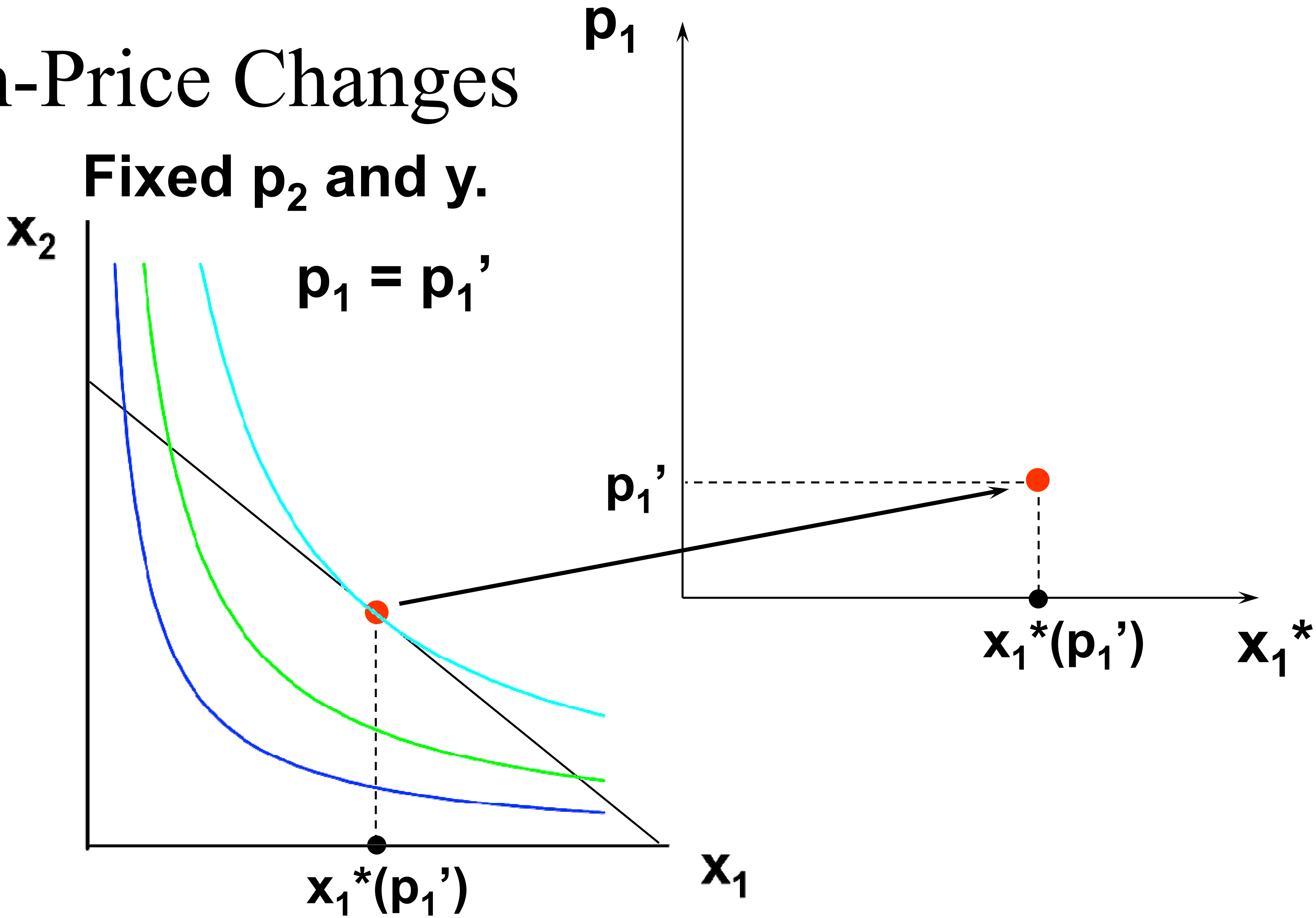


# Own-Price Changes

**Fixed $p_2$ and y.**

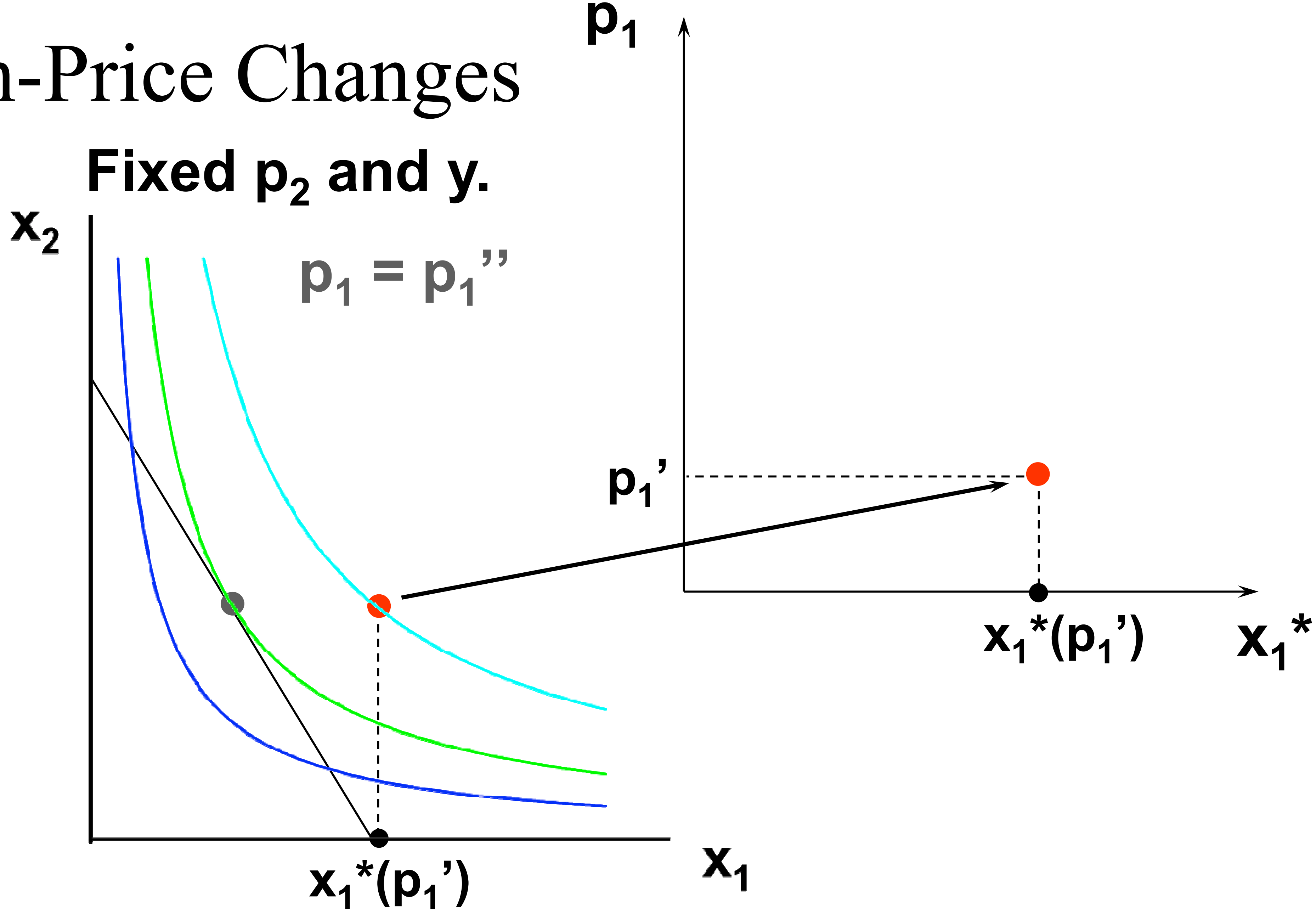


# Own-Price Changes

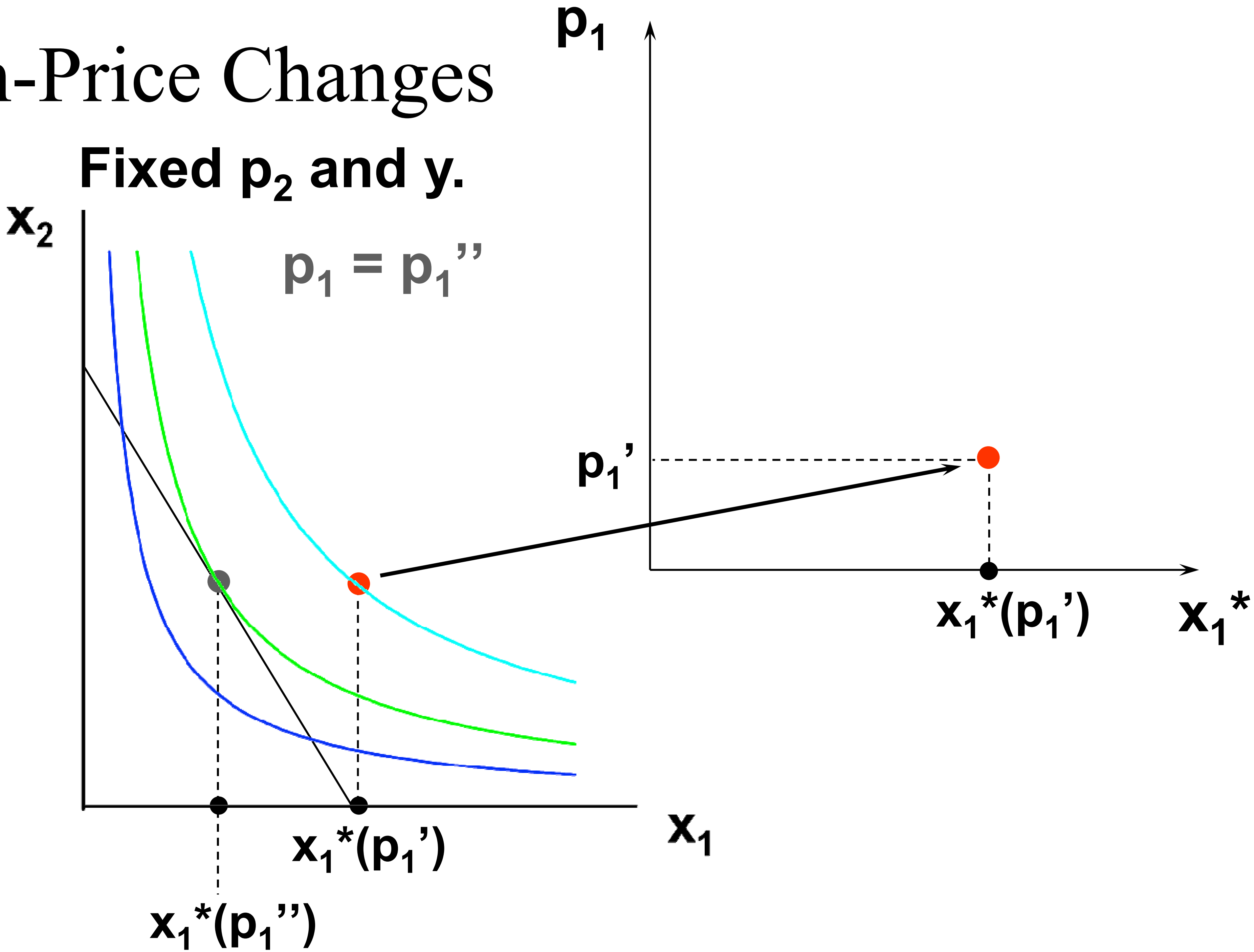


# Own-Price Changes

**Fixed $p_2$ and y.**

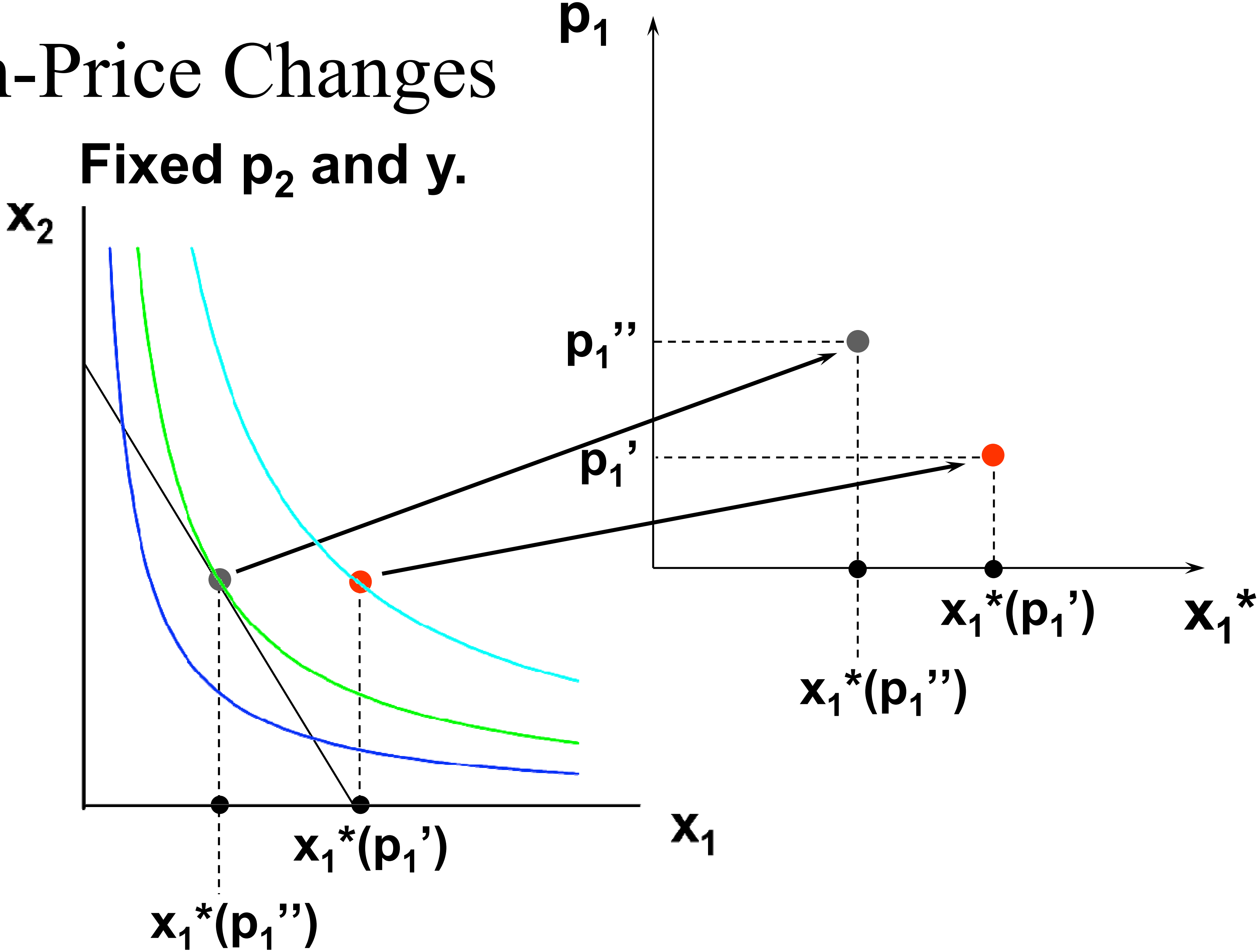


# Own-Price Changes

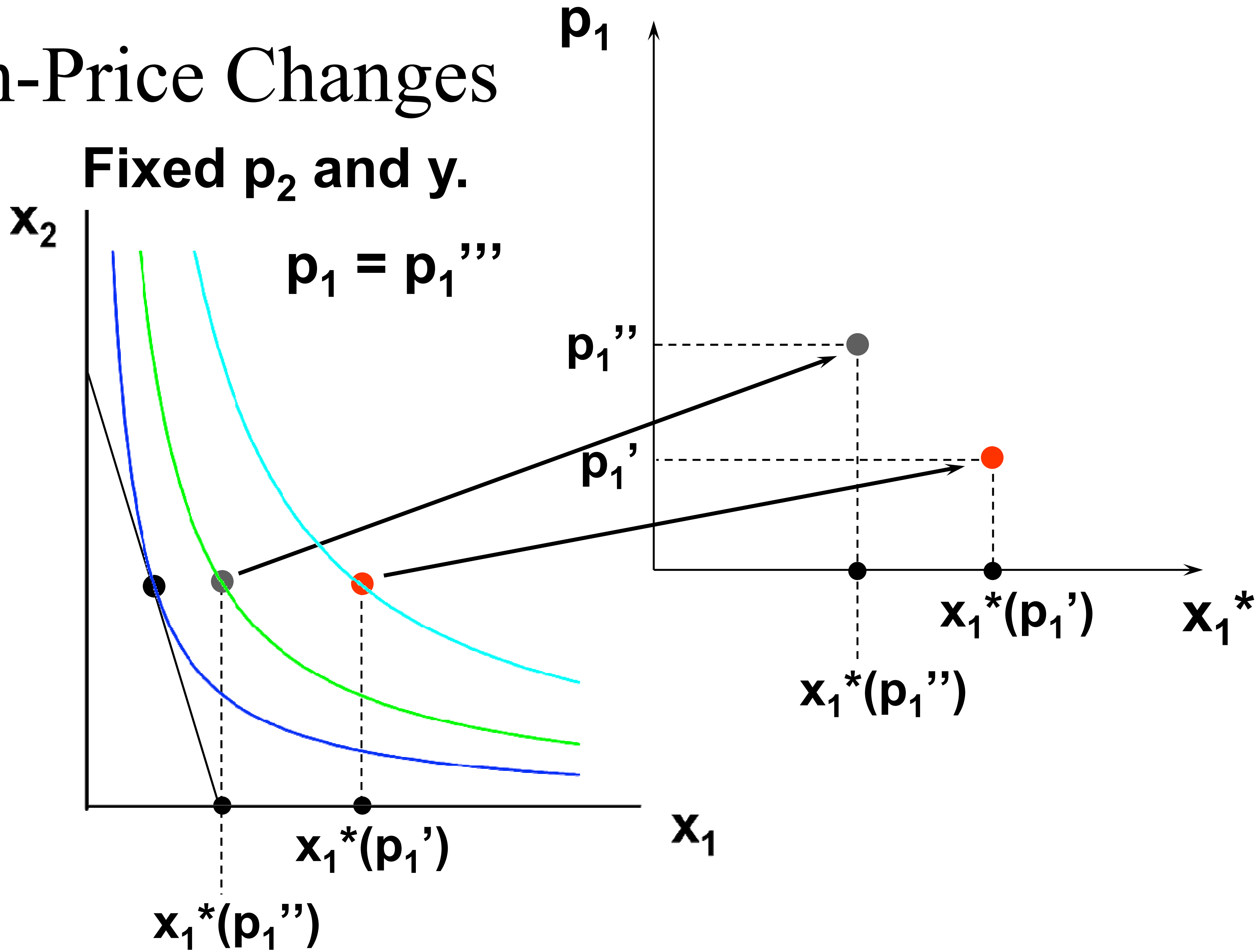


# Own-Price Changes

**Fixed $p_2$ and y.**



# Own-Price Changes



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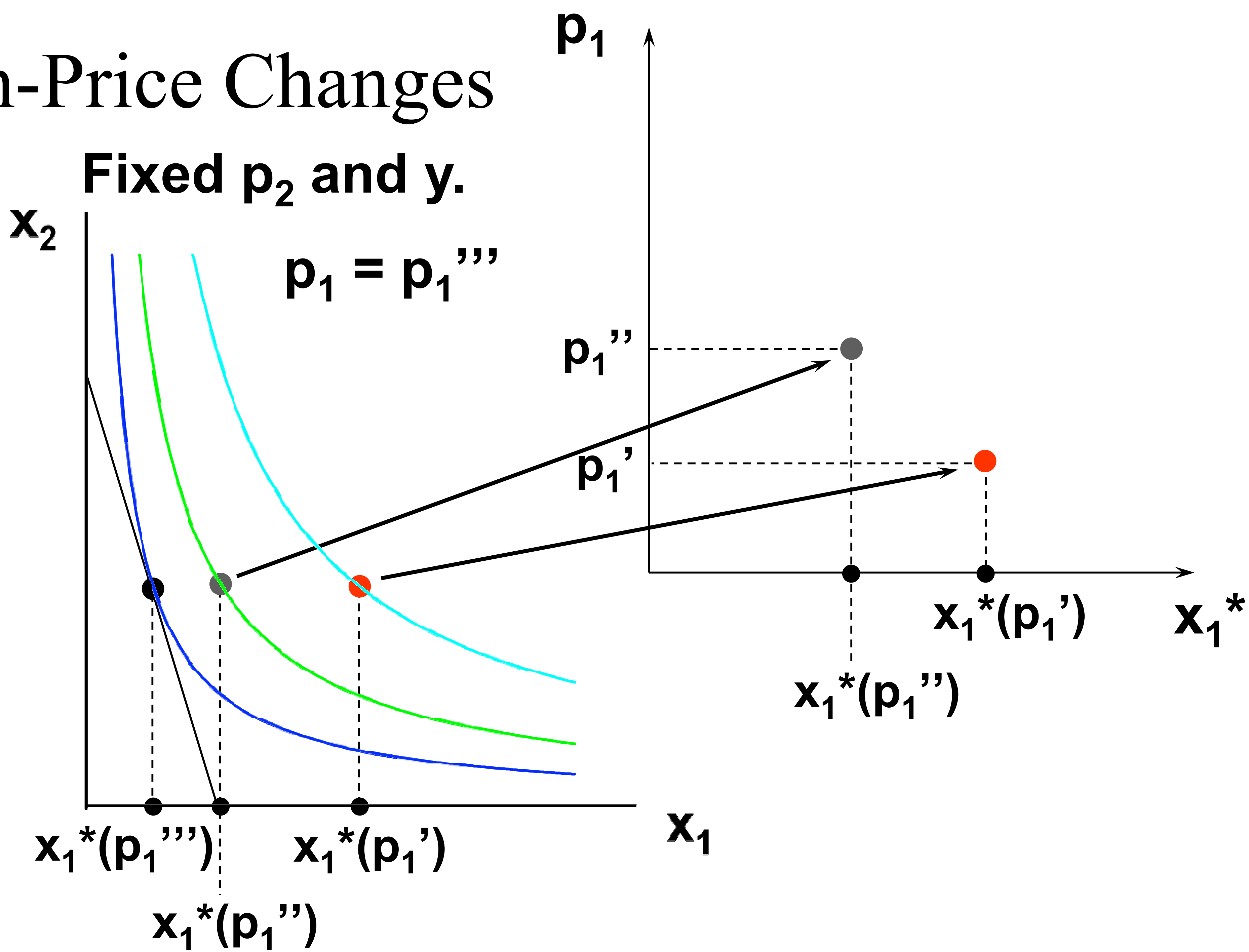

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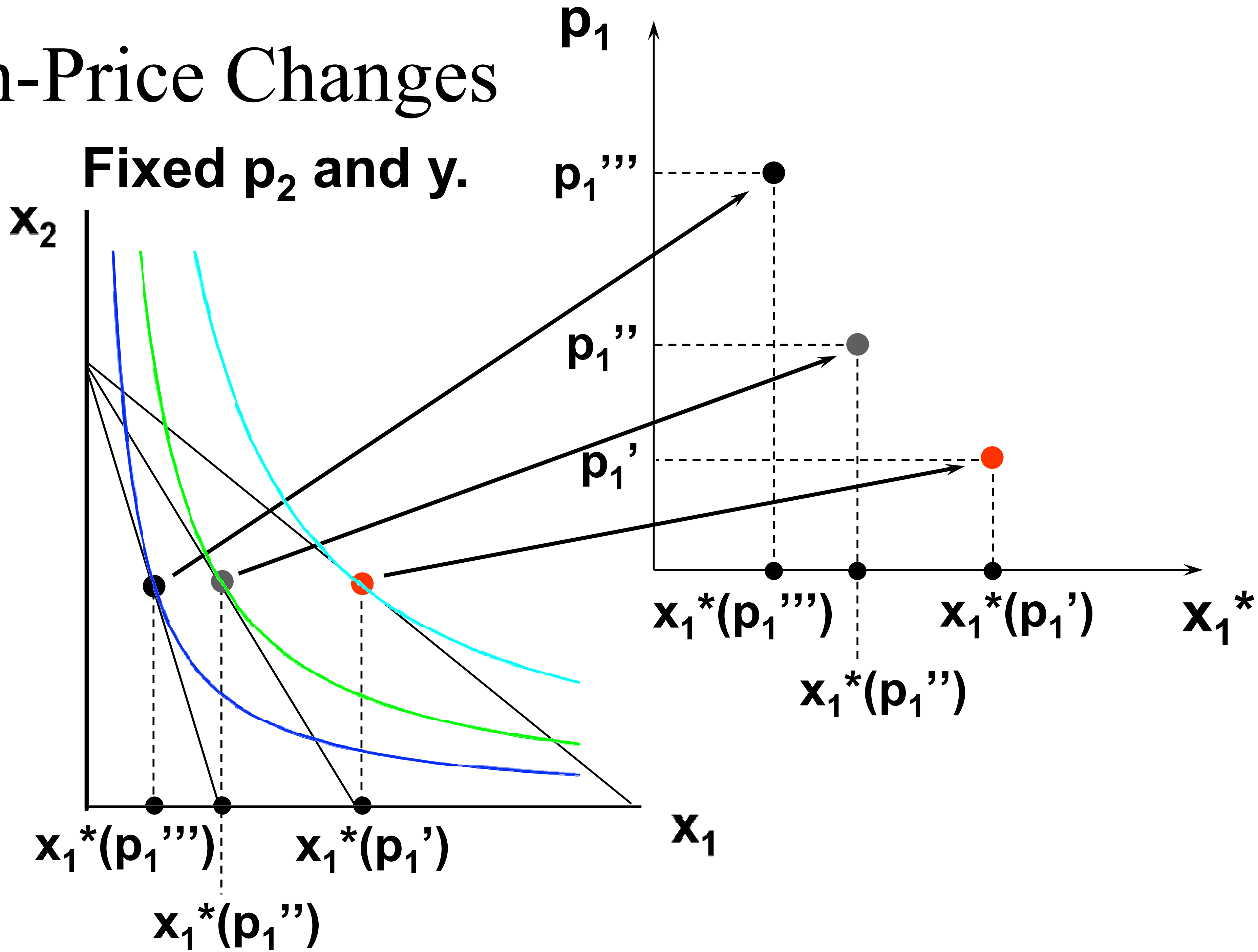

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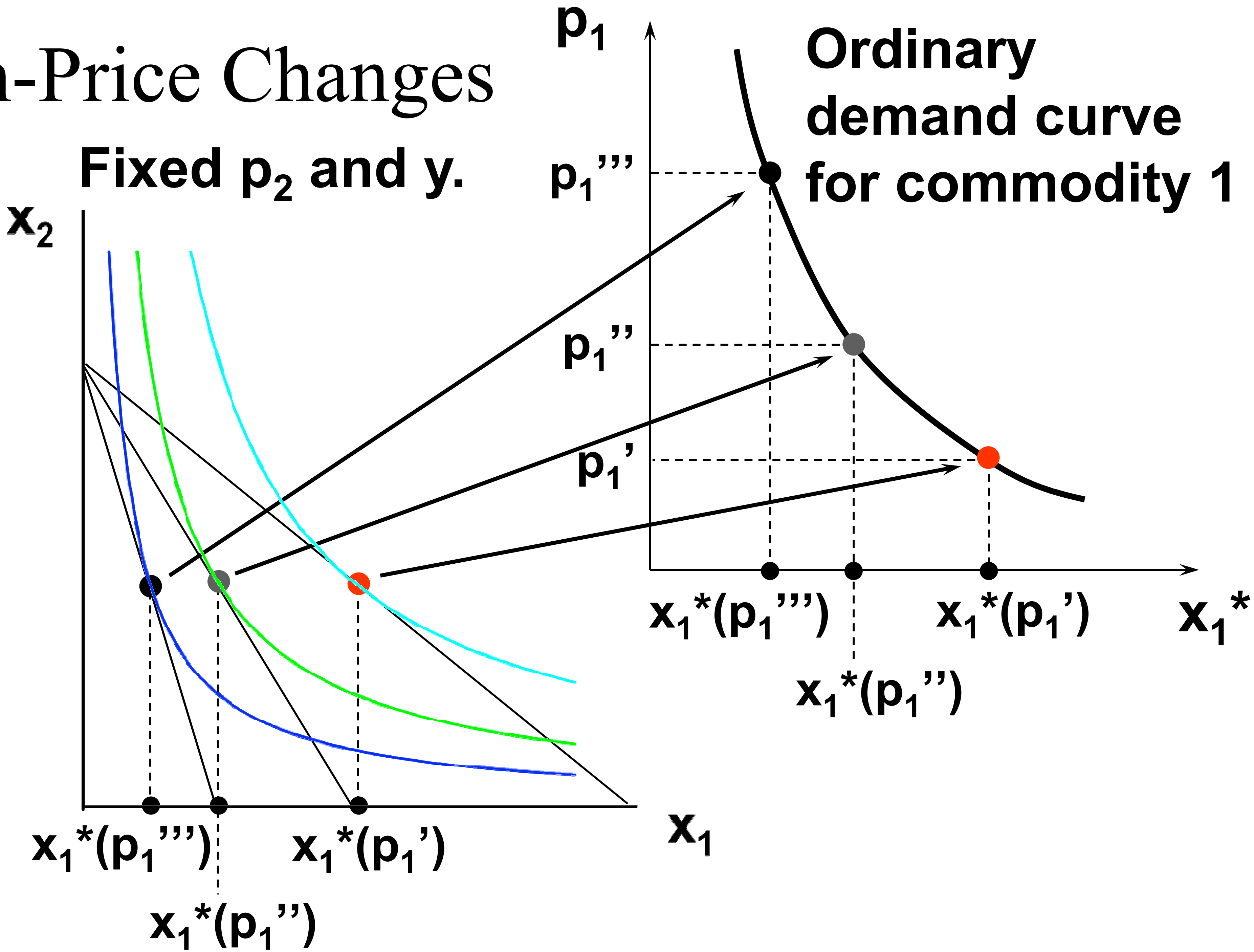

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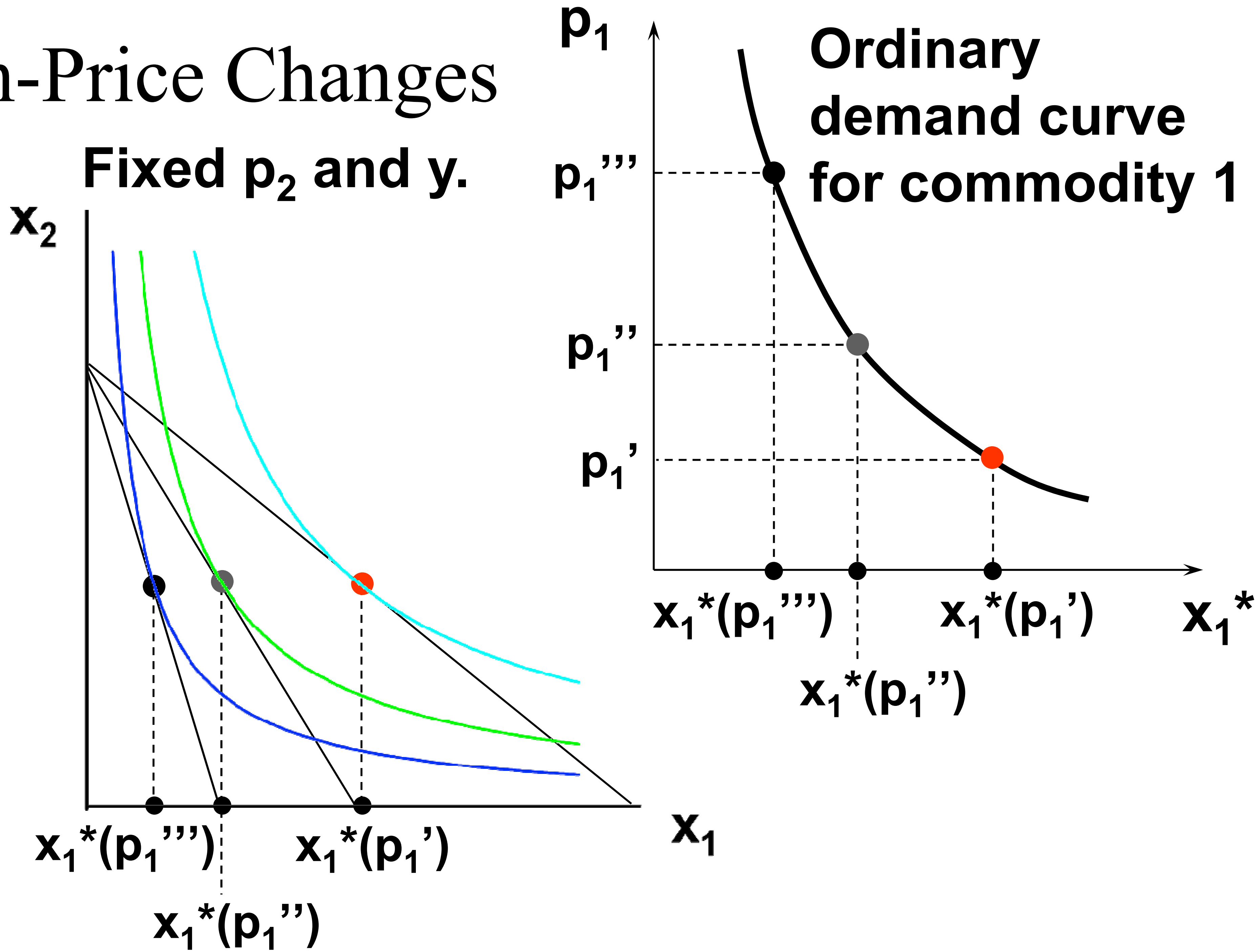
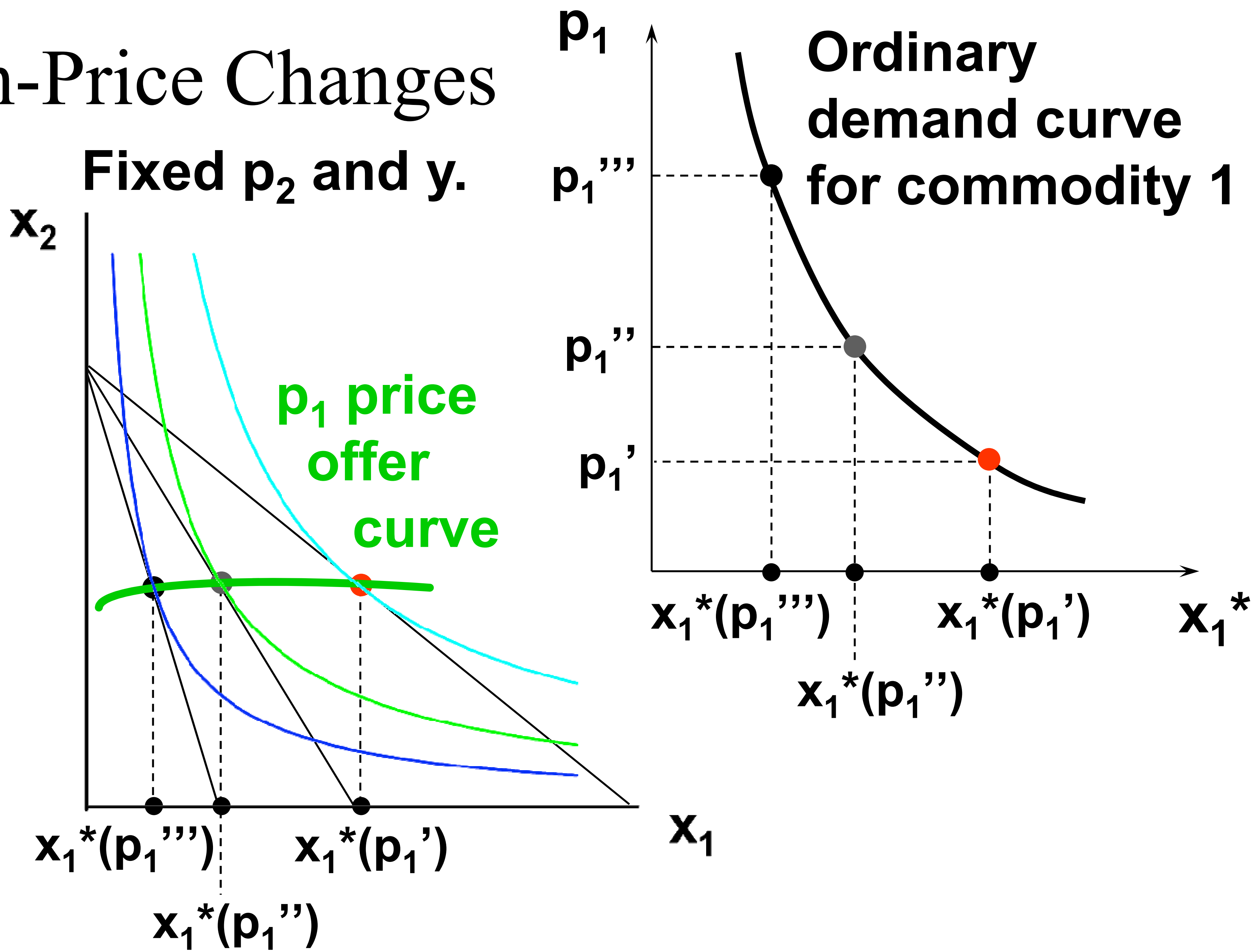


# Own-Price Changes



# Own-Price Changes

- **The curve containing all the utility-maximizing bundles traced out as $p_1$ changes, with $p_2$ and y constant, is the $p_1$- price offer curve.**
- **The plot of the $x_1$-coordinate of the $p_1$- price offer curve against $p_1$ is the ordinary demand curve for commodity 1.**

# Own-Price Changes

- What does a $p_1$ price-offer curve look like for Cobb-Douglas preferences?

# Own-Price Changes

- What does a $p_1$ price-offer curve look like for Cobb-Douglas preferences?
- Take

$$U(x_1, x_2) = x_1^a x_2^b.$$

Then the ordinary demand functions for commodities 1 and 2 are

# Own-Price Changes

$$x_1^*(p_1, p_2, y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

$$x_2^*(p_1, p_2, y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

**Notice that $x_2^*$ does not vary with $p_1$ so the $p_1$ price offer curve is**

# Own-Price Changes

$$x_1^*(p_1, p_2, y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

$$x_2^*(p_1, p_2, y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

**Notice that $x_2$* does not vary with $p_1$ so the $p_1$ price offer curve is flat**

# Own-Price Changes

$$x_1^*(p_1, p_2, y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

$$x_2^*(p_1, p_2, y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

**Notice that $x_2$* does not vary with $p_1$ so the $p_1$ price offer curve is flat and the ordinary demand curve for commodity 1 is a**

# Own-Price Changes

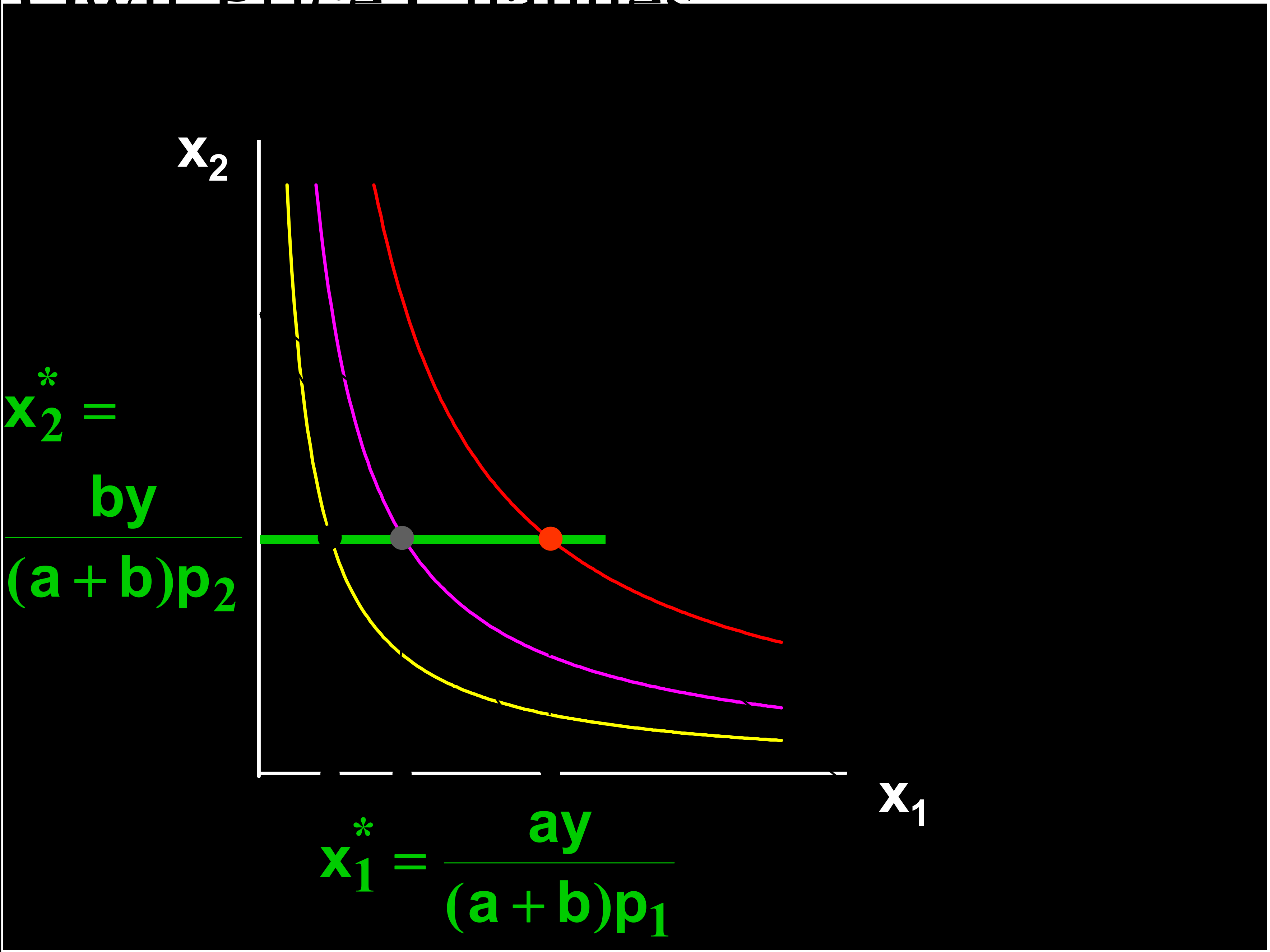
$$x_1^*(p_1,p_2,y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

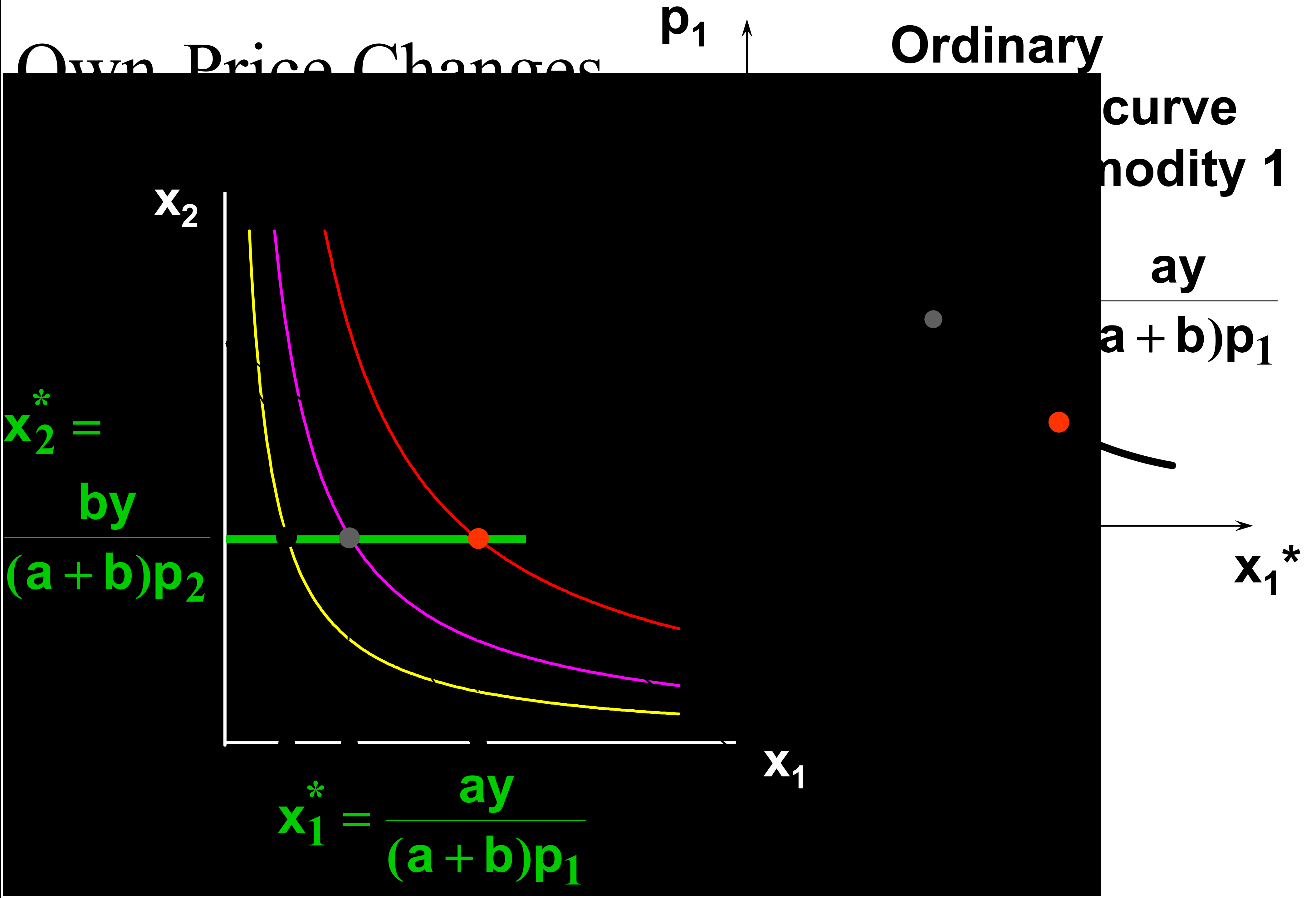
$$x_2^*(p_1,p_2,y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

**Notice that $x_2$* does not vary with $p_1$ so the $p_1$ price offer curve is flat and the ordinary demand curve for commodity 1 is a rectangular hyperbola.**

# Own-Price Changes



# Own Price Changes



# Own-Price Changes

- What does a $p_1$ price-offer curve look like for a perfect-complements utility function?

# Own-Price Changes

- What does a $p_1$ price-offer curve look like for a perfect-complements utility function?

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

Then the ordinary demand functions for commodities 1 and 2 are

# Own-Price Changes

$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1 + p_2}.$$

# Own-Price Changes

$$x_1^*(p_1, p_2, y) = x_2^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}.$$

**With $p_2$ and y fixed, higher $p_1$ causes smaller $x_1$* and $x_2$*.**

# Own-Price Changes

$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1+p_2}.$$

**With $p_2$ and y fixed, higher $p_1$ causes smaller $x_1$* and $x_2$*.**

**As** $p_1 \to 0, \quad x_1^* = x_2^* \to \dfrac{y}{p_2}.$

# Own-Price Changes

$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1 + p_2}.$$

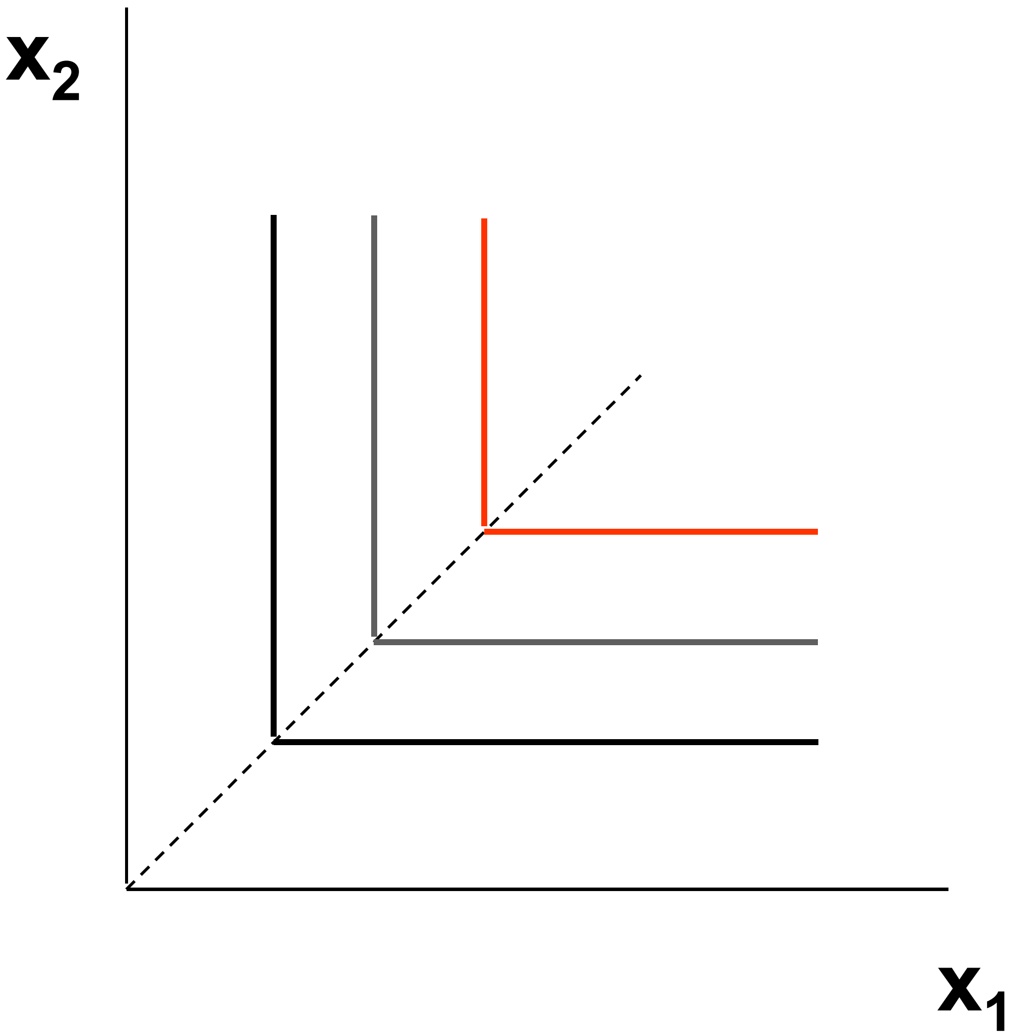
**With $p_2$ and y fixed, higher $p_1$ causes smaller $x_1$* and $x_2$*.**

**As** $p_1 \to 0, \quad x_1^* = x_2^* \to \frac{y}{p_2}.$

**As** $p_1 \to \infty, \quad x_1^* = x_2^* \to 0.$

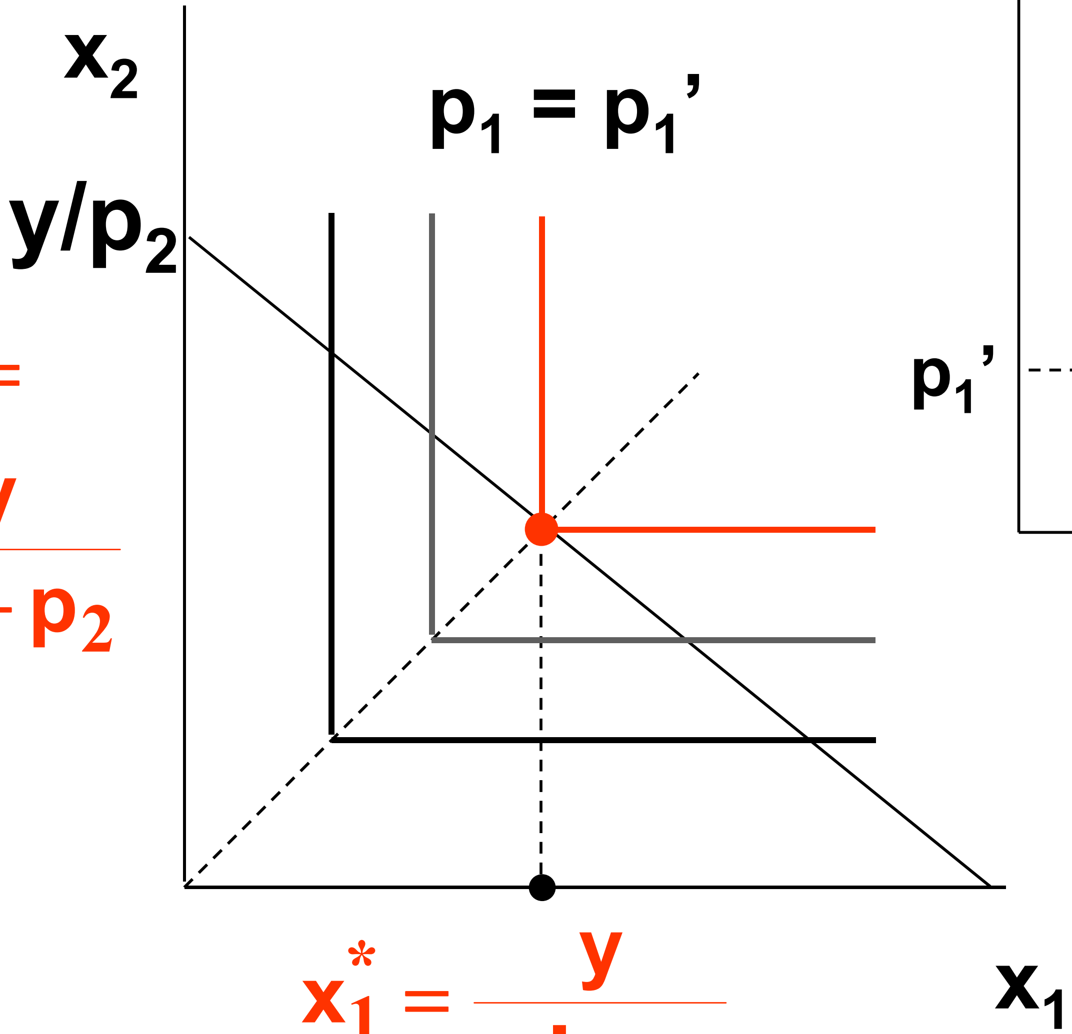
# Own-Price Changes

**Fixed $p_2$ and y.**



# Own-Price Changes

**Fixed $p_2$ and y.**




$$x_2^* = \frac{y}{p_1' + p_2}$$

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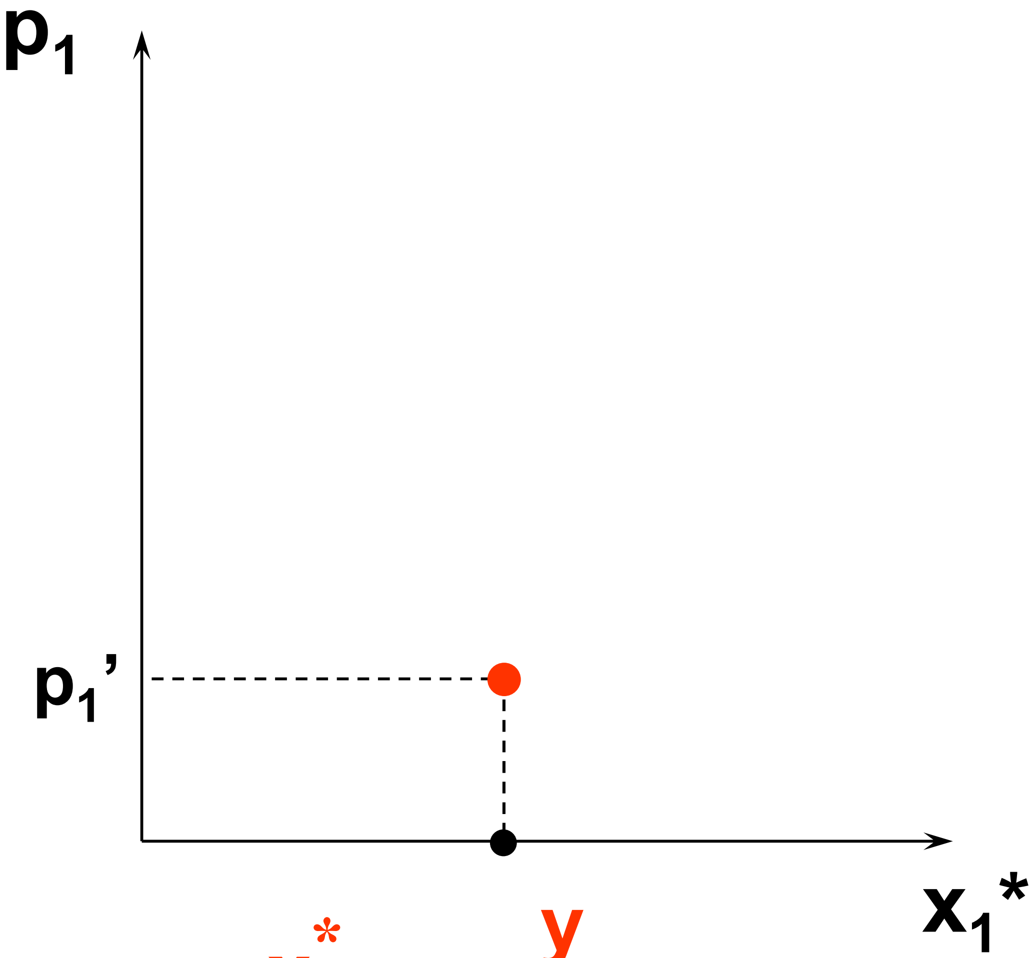
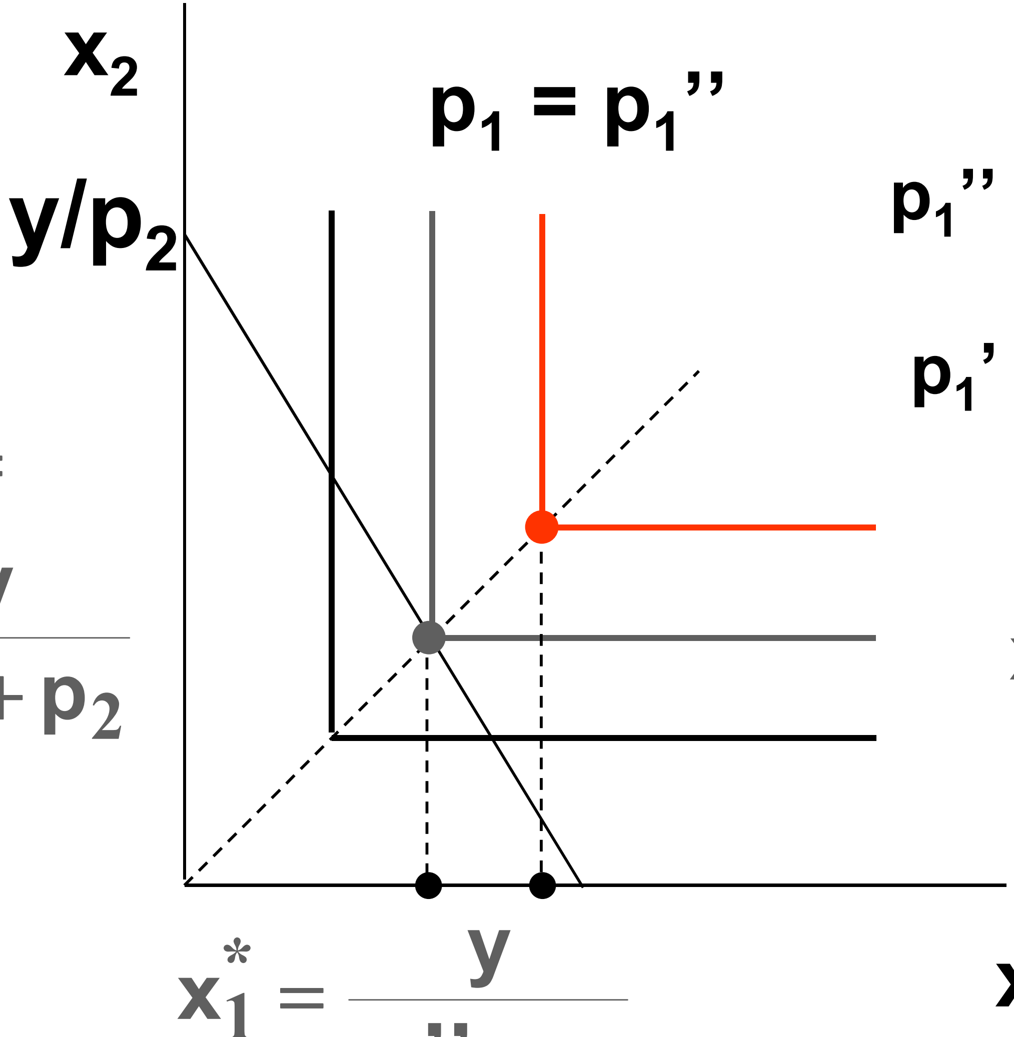



$$x_1^* = \frac{y}{p_1' + p_2}$$

# Own-Price Changes

**Fixed $p_2$ and y.**

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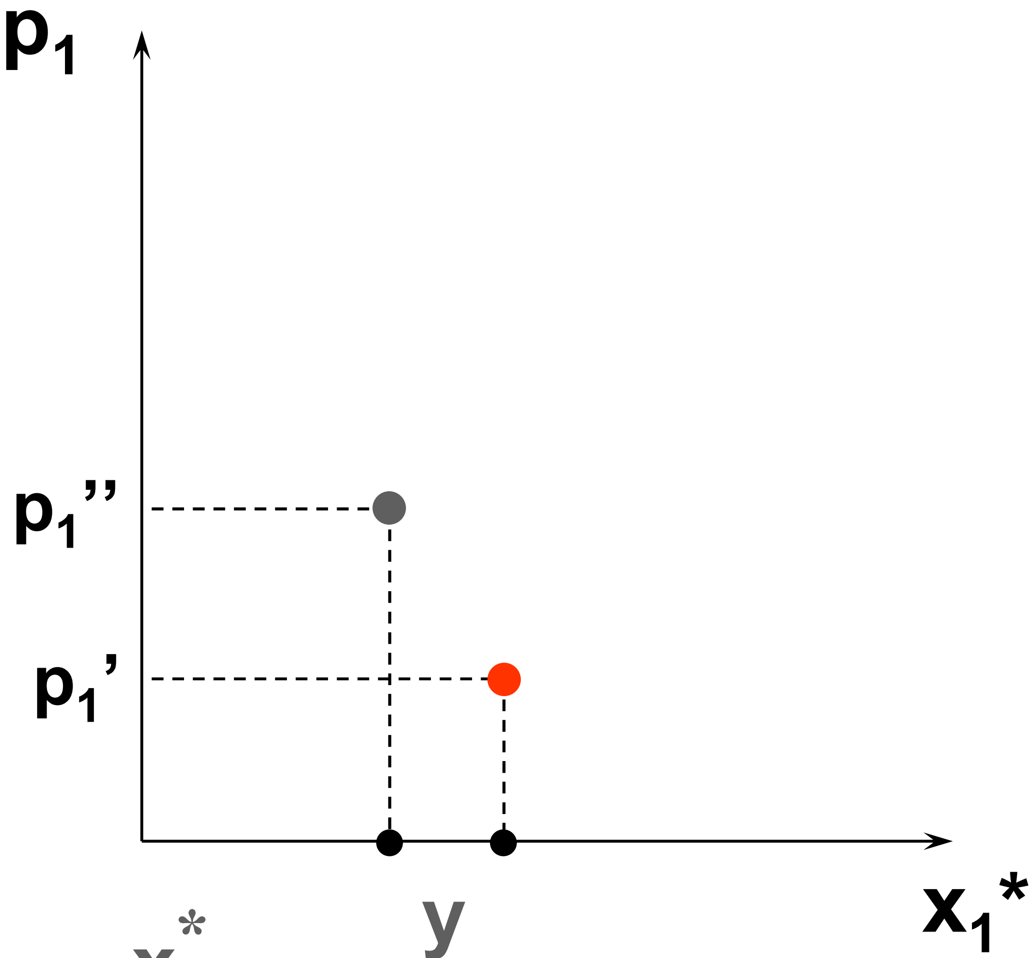
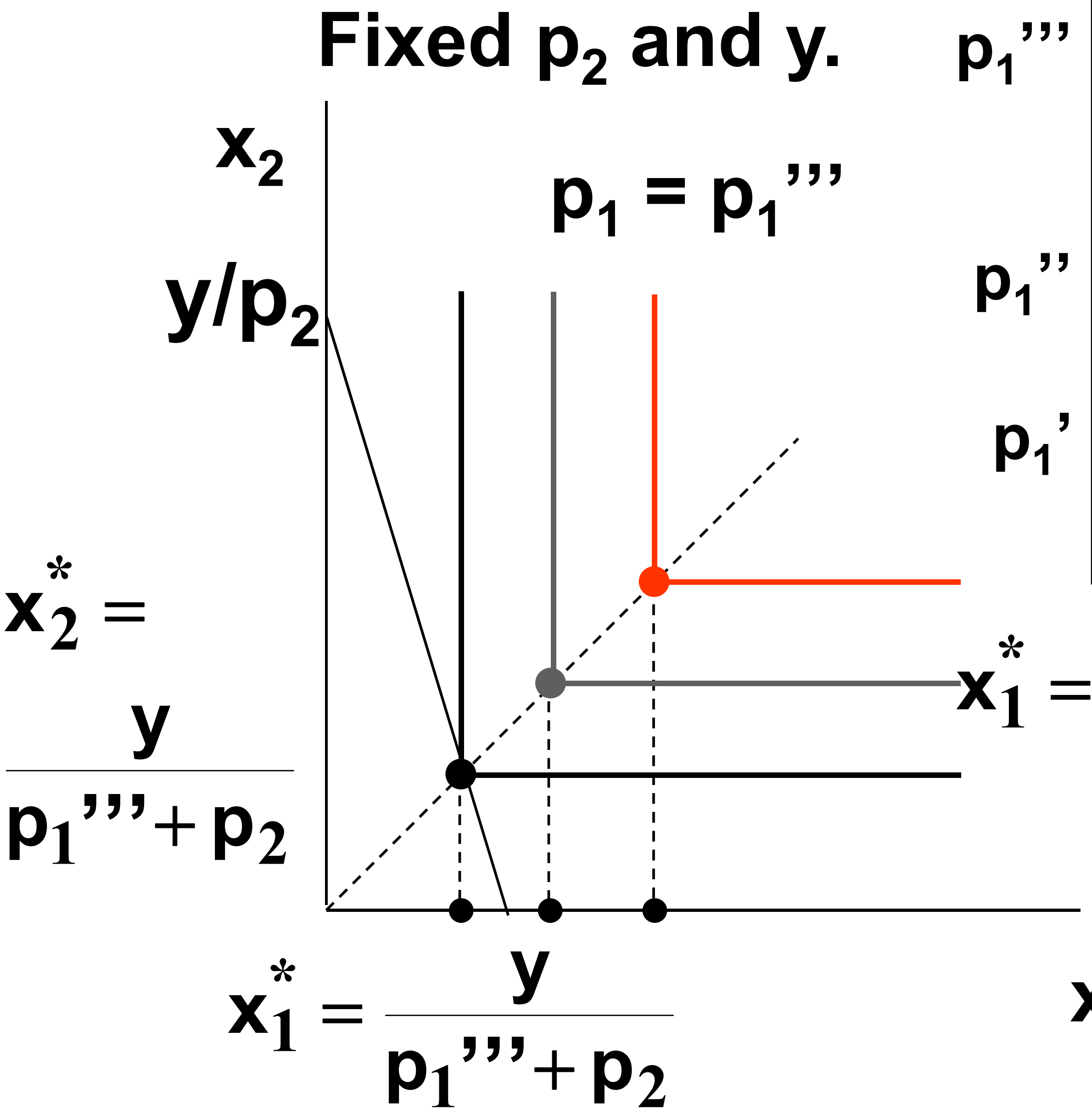
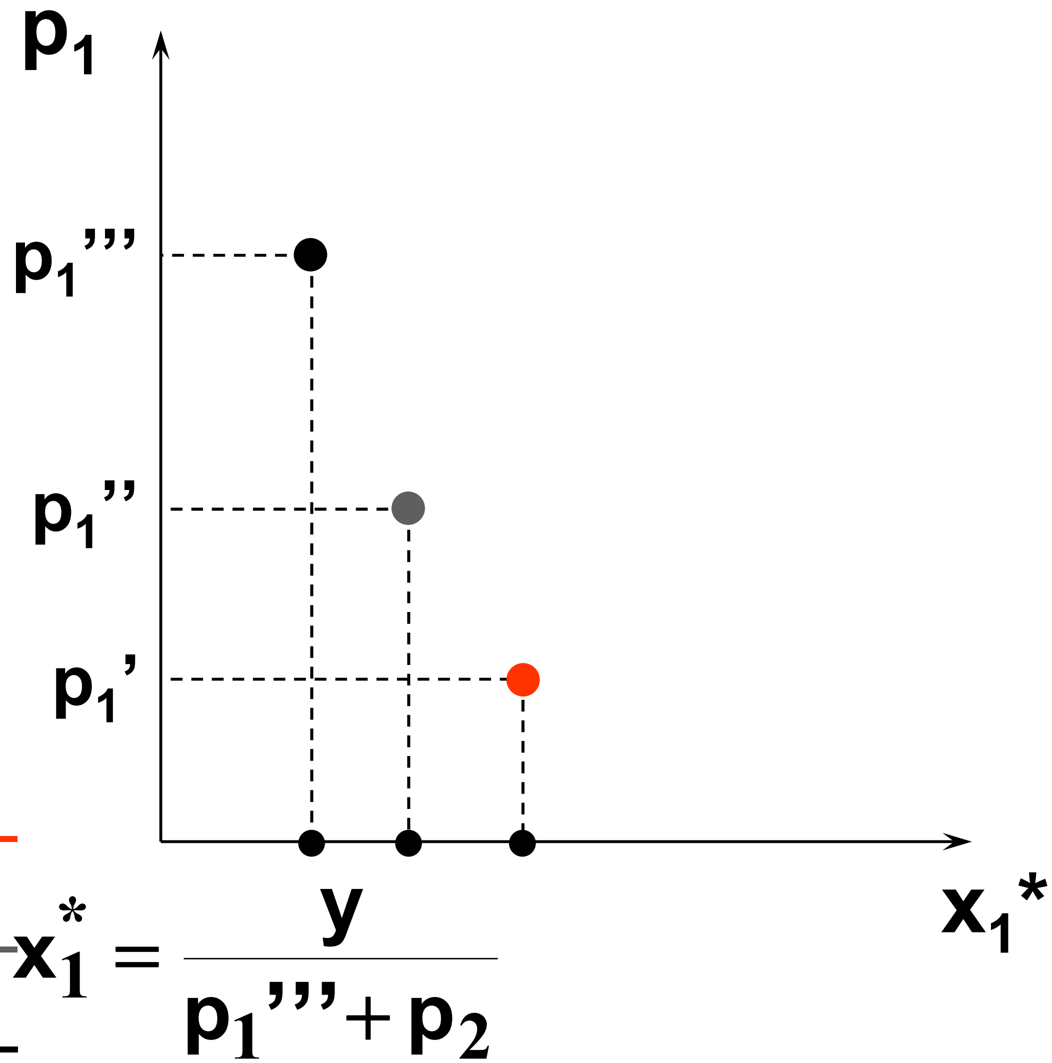



$$x_1^* = \frac{y}{p_1'' + p_2}$$

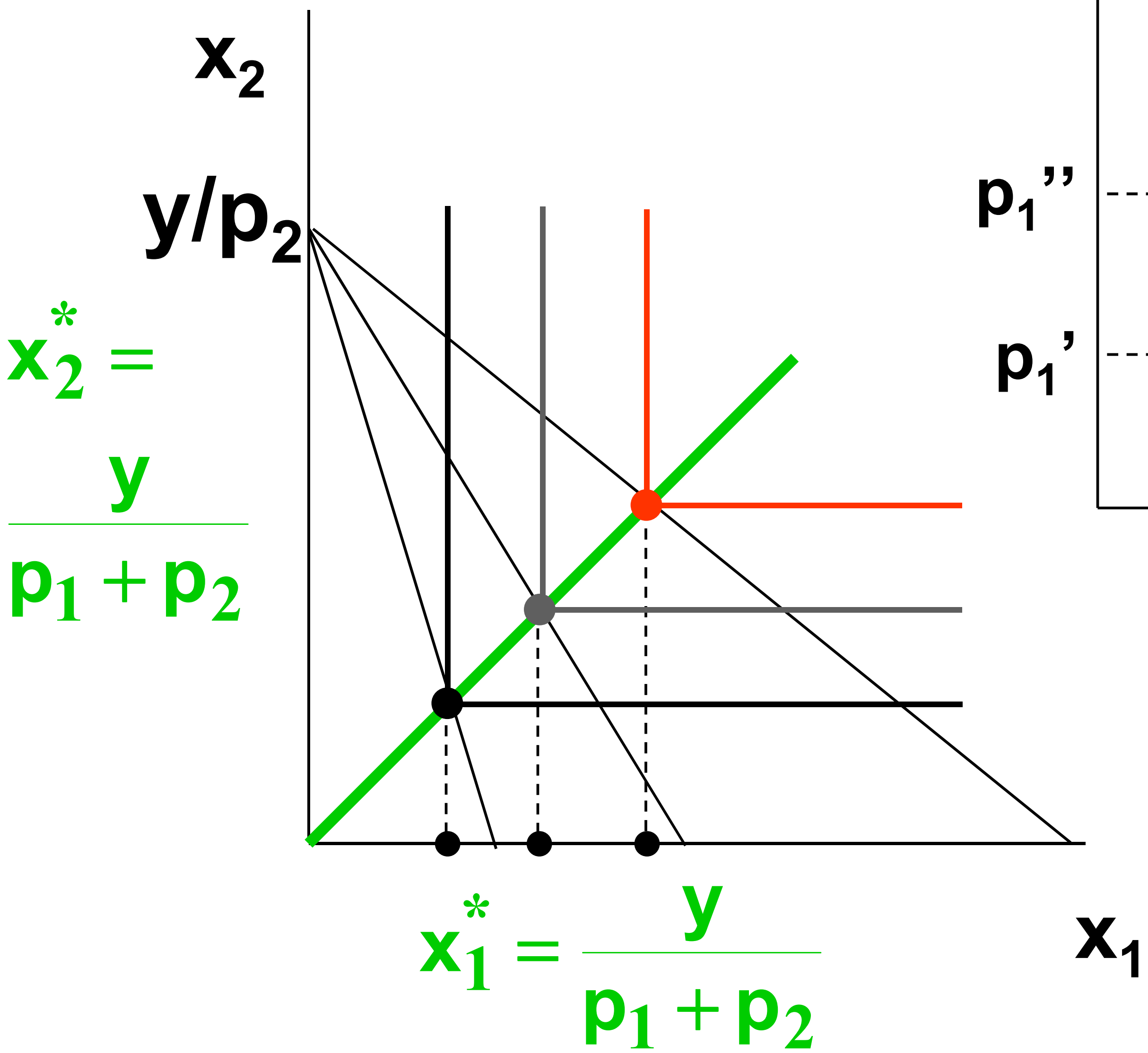
# Own-Price Changes



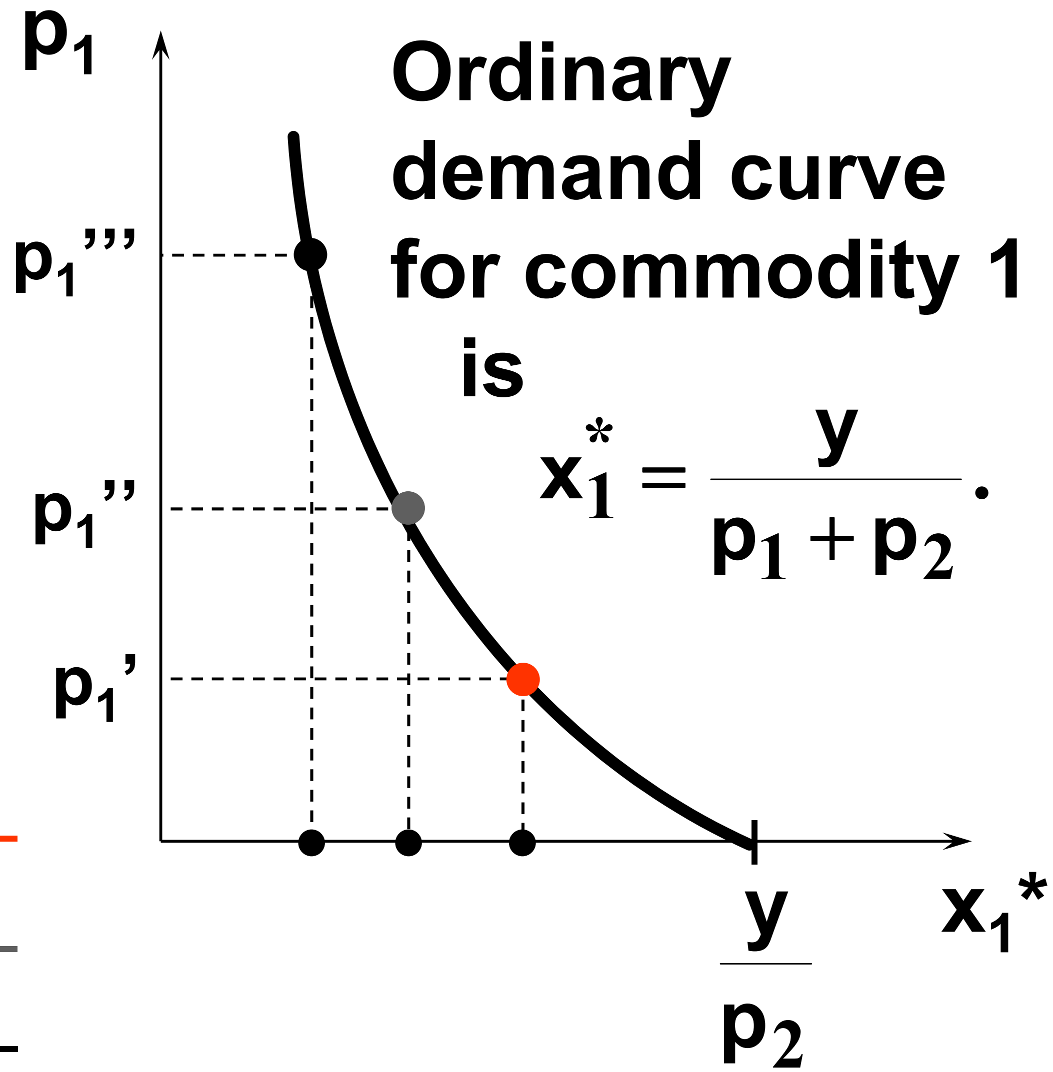
# Own-Price Changes

**Fixed $p_2$ and y.**




**Ordinary demand curve for commodity 1 is**

$$x_1^* = \frac{y}{p_1 + p_2}.$$



# Own-Price Changes

- **What does a $p_1$ price-offer curve look like for a perfect-substitutes utility function?**

$$U(x_1, x_2) = x_1 + x_2.$$

**Then the ordinary demand functions for commodities 1 and 2 are**

# Own-Price Changes

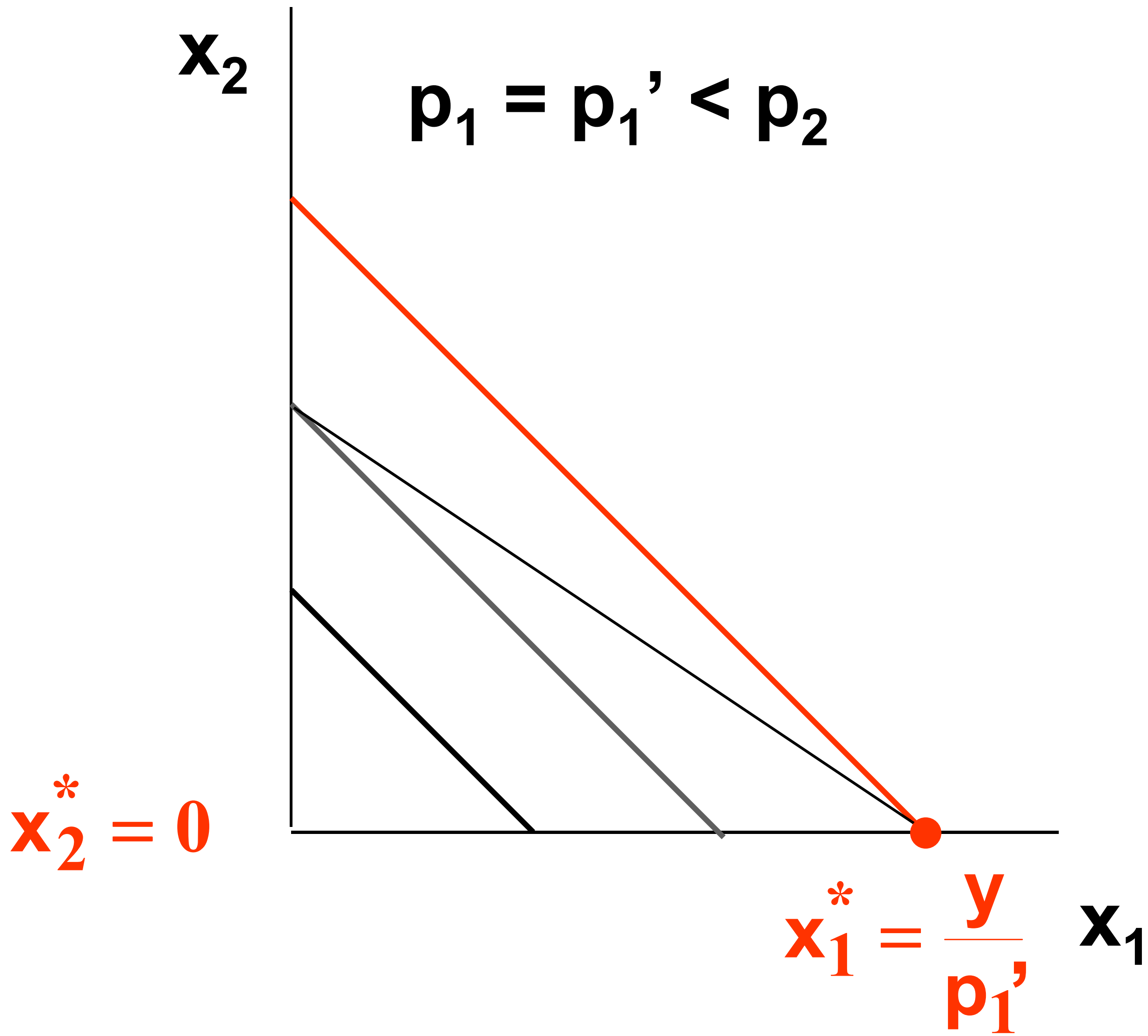
$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & \text{, if } p_1 > p_2 \\ y/p_1 & \text{, if } p_1 < p_2 \end{cases}$$

and

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & \text{, if } p_1 < p_2 \\ y/p_2 & \text{, if } p_1 > p_2. \end{cases}$$

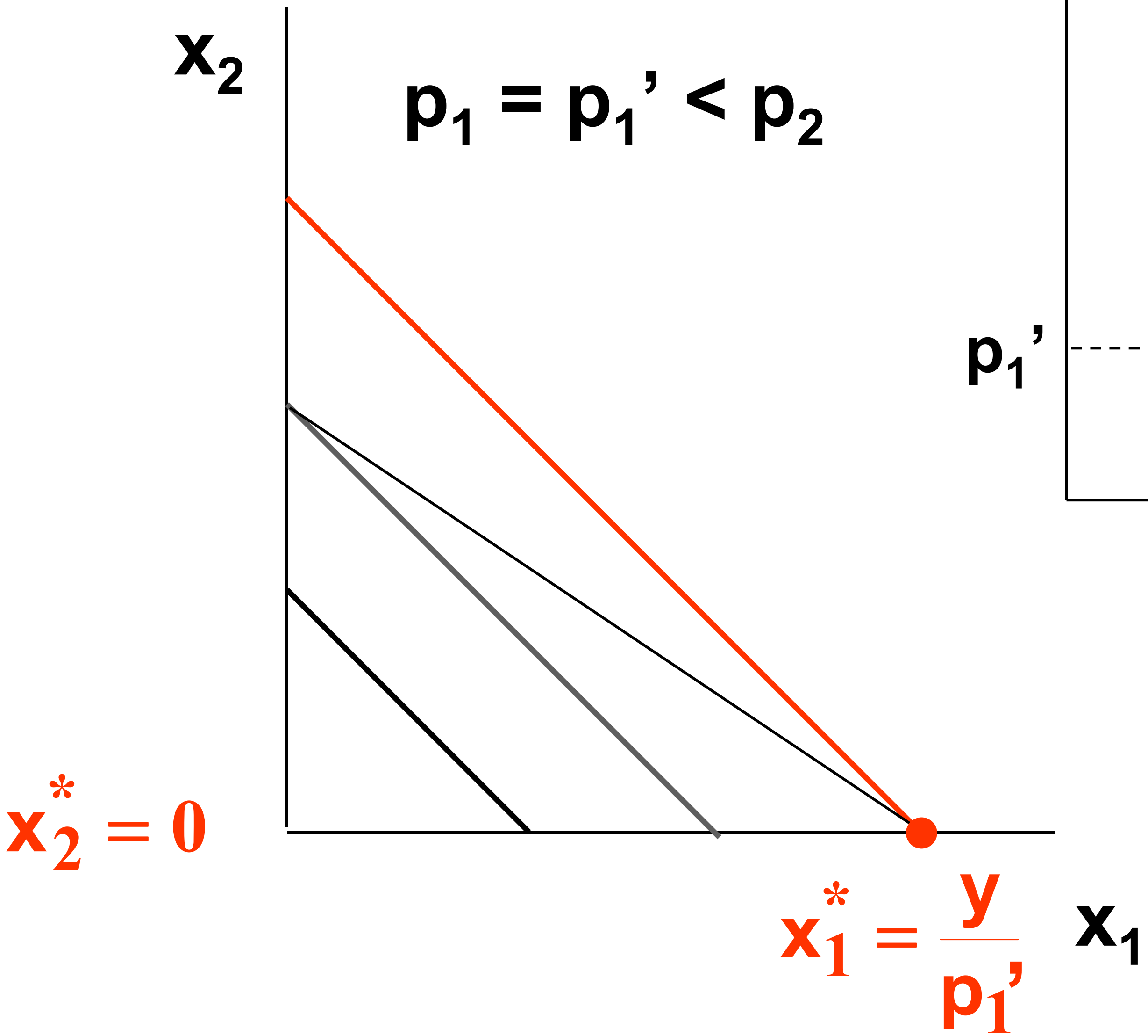
# Own-Price Changes

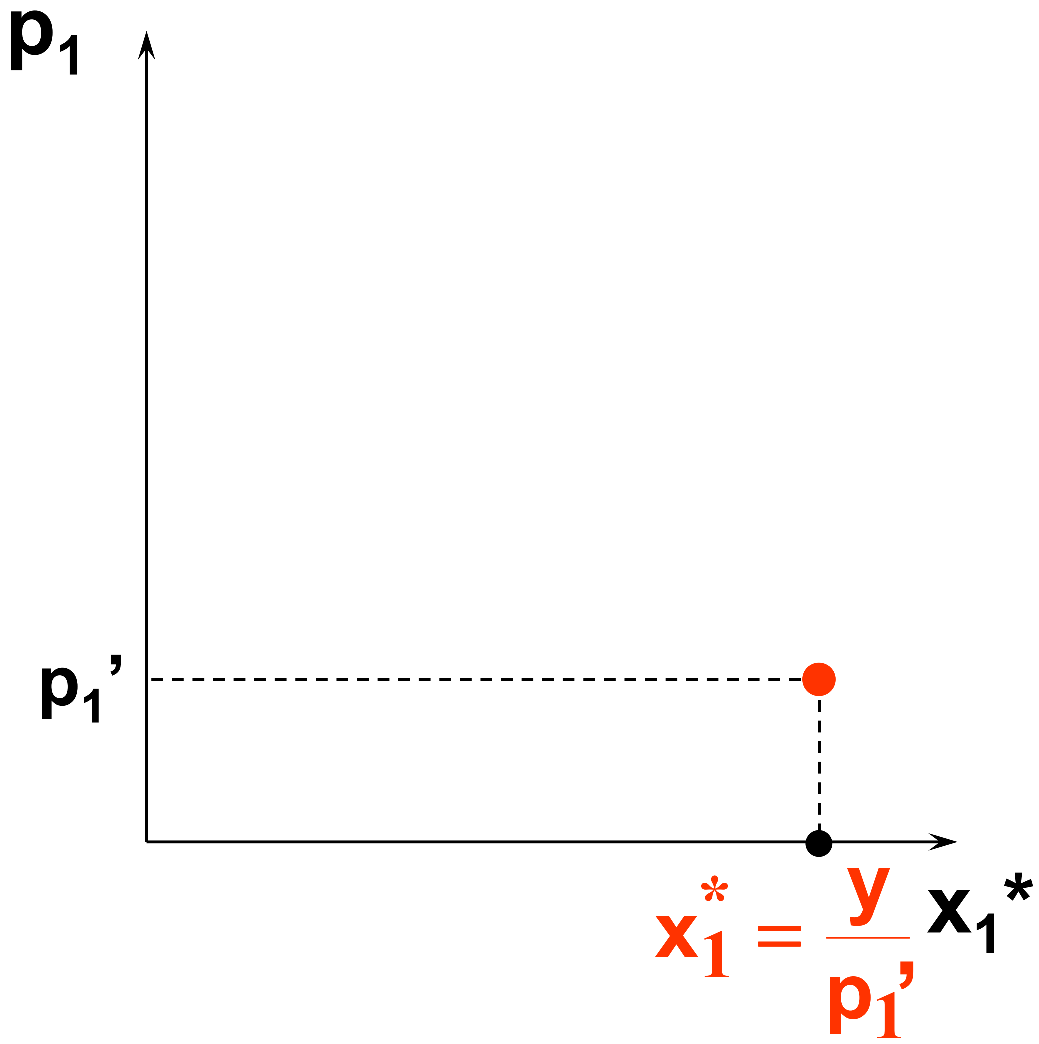
**Fixed $p_2$ and y.**



# Own-Price Changes

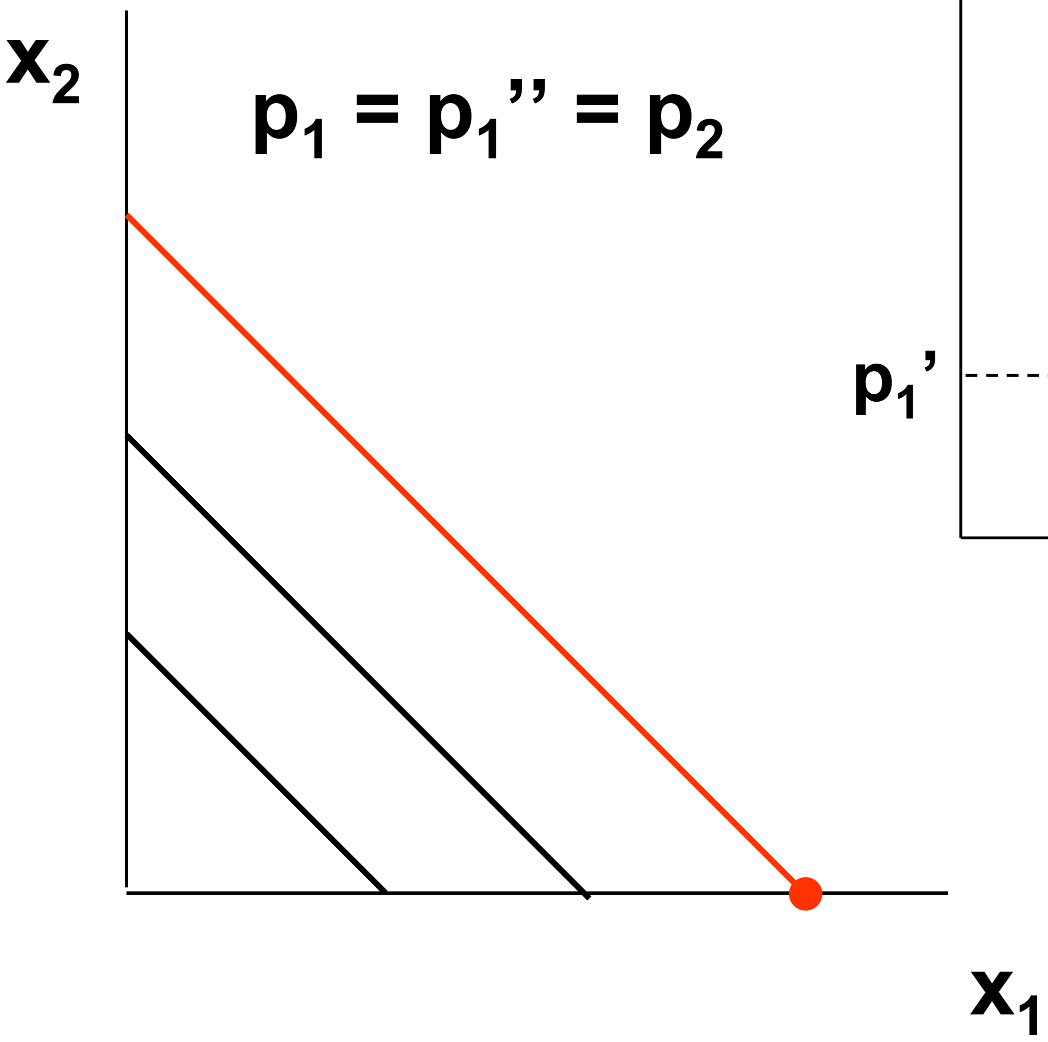
**Fixed $p_2$ and y.**

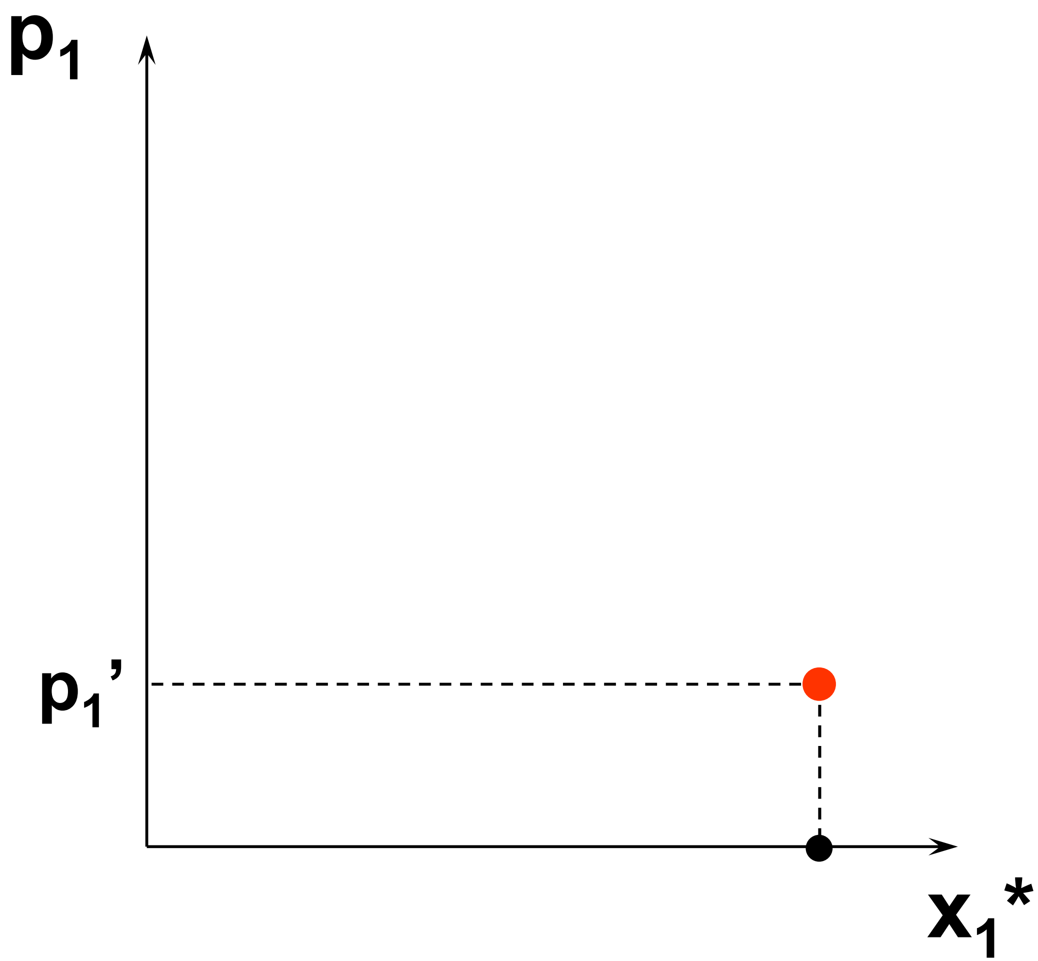


# Own-Price Changes

**Fixed $p_2$ and y.**



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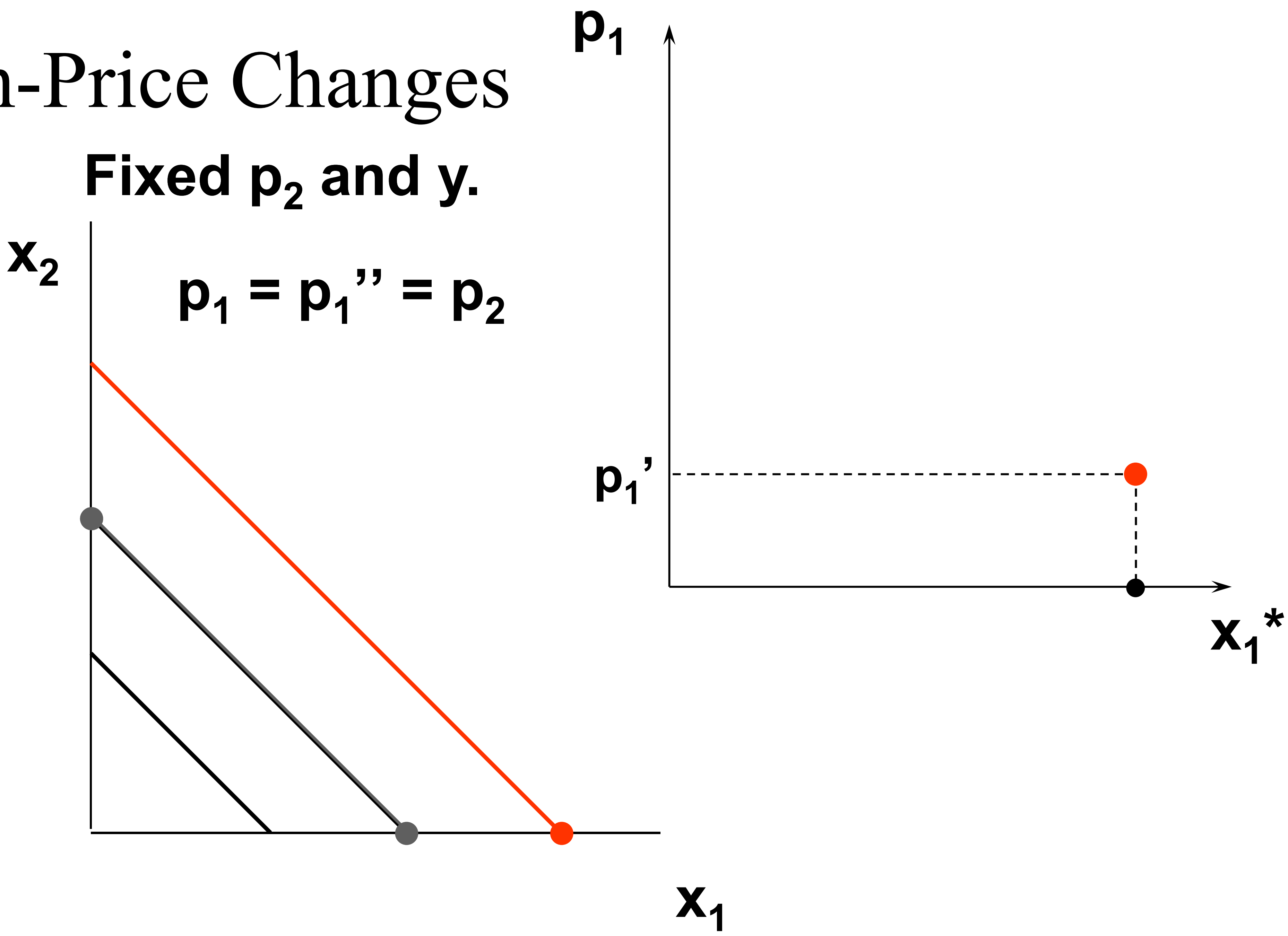

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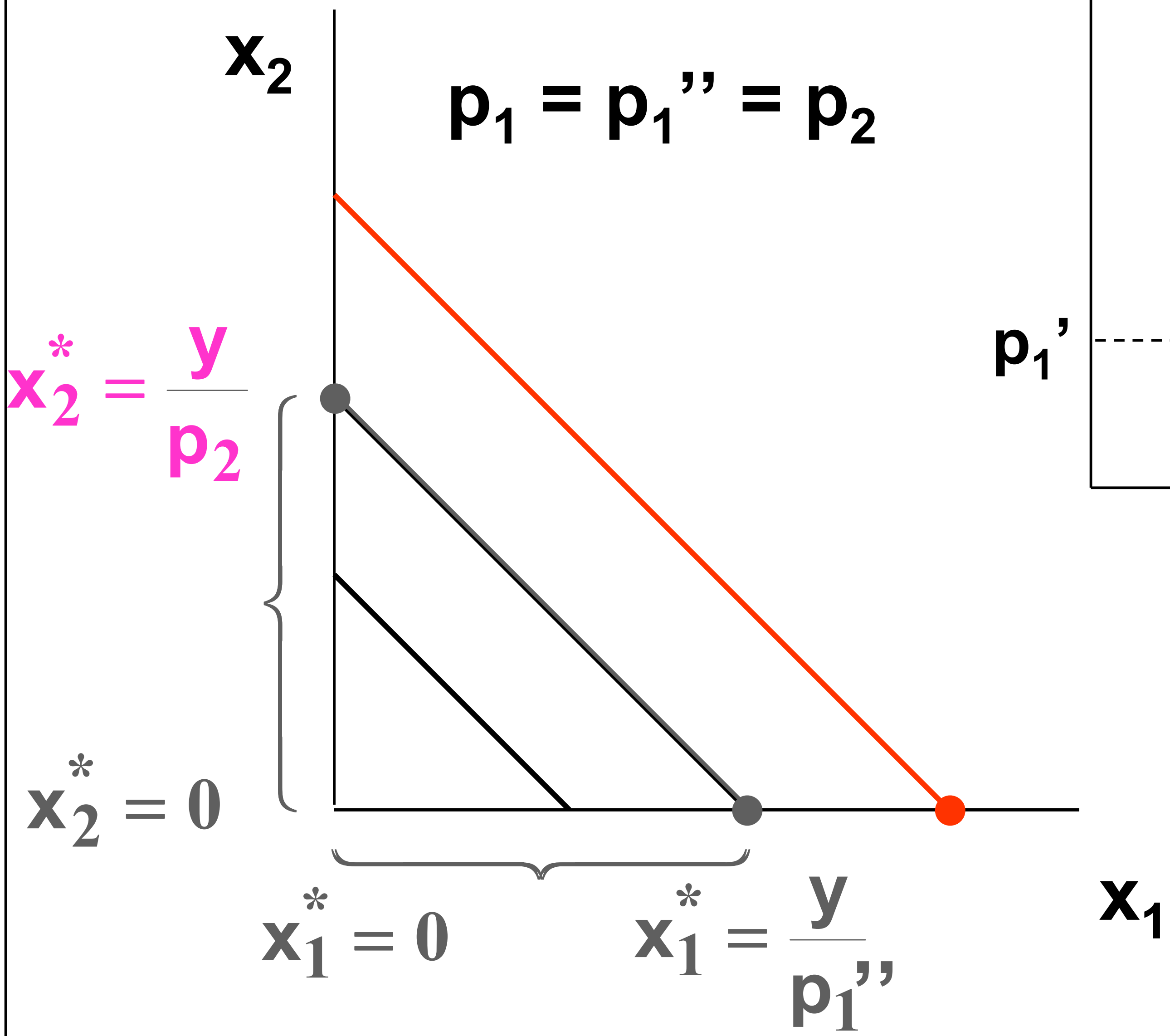


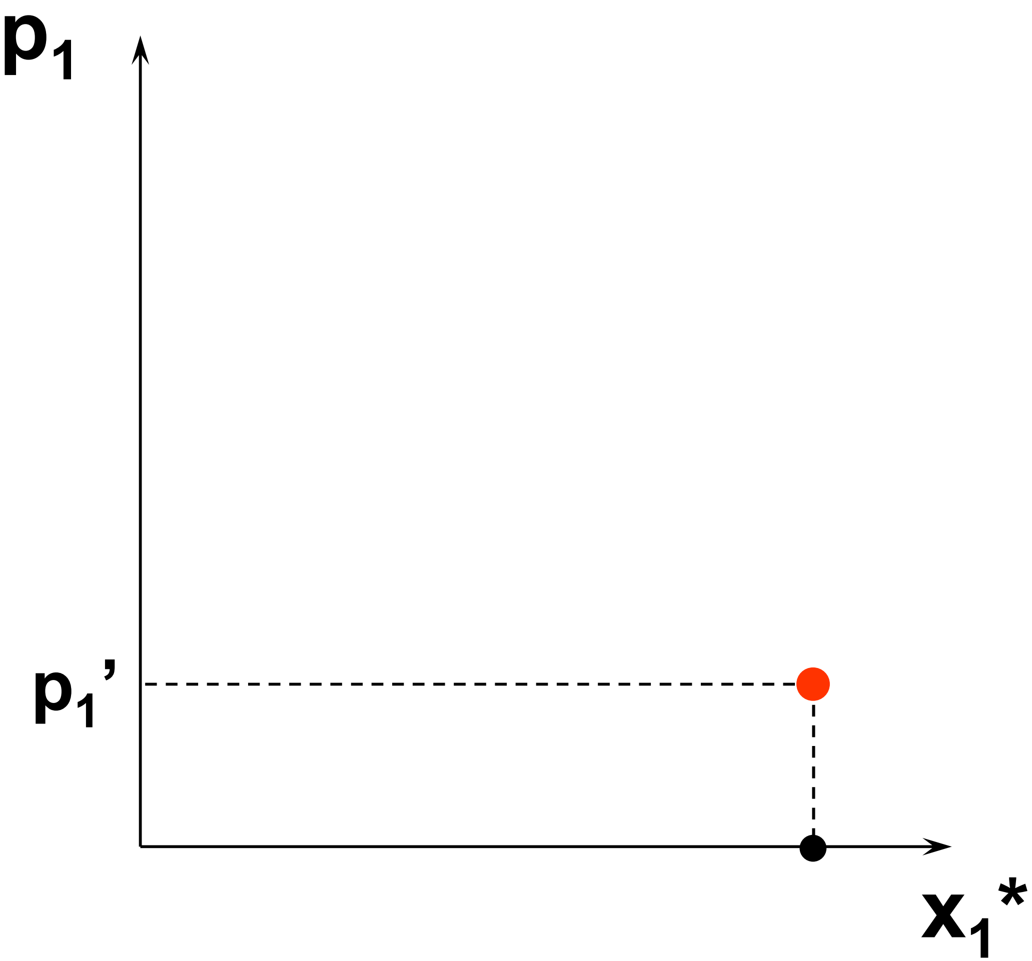

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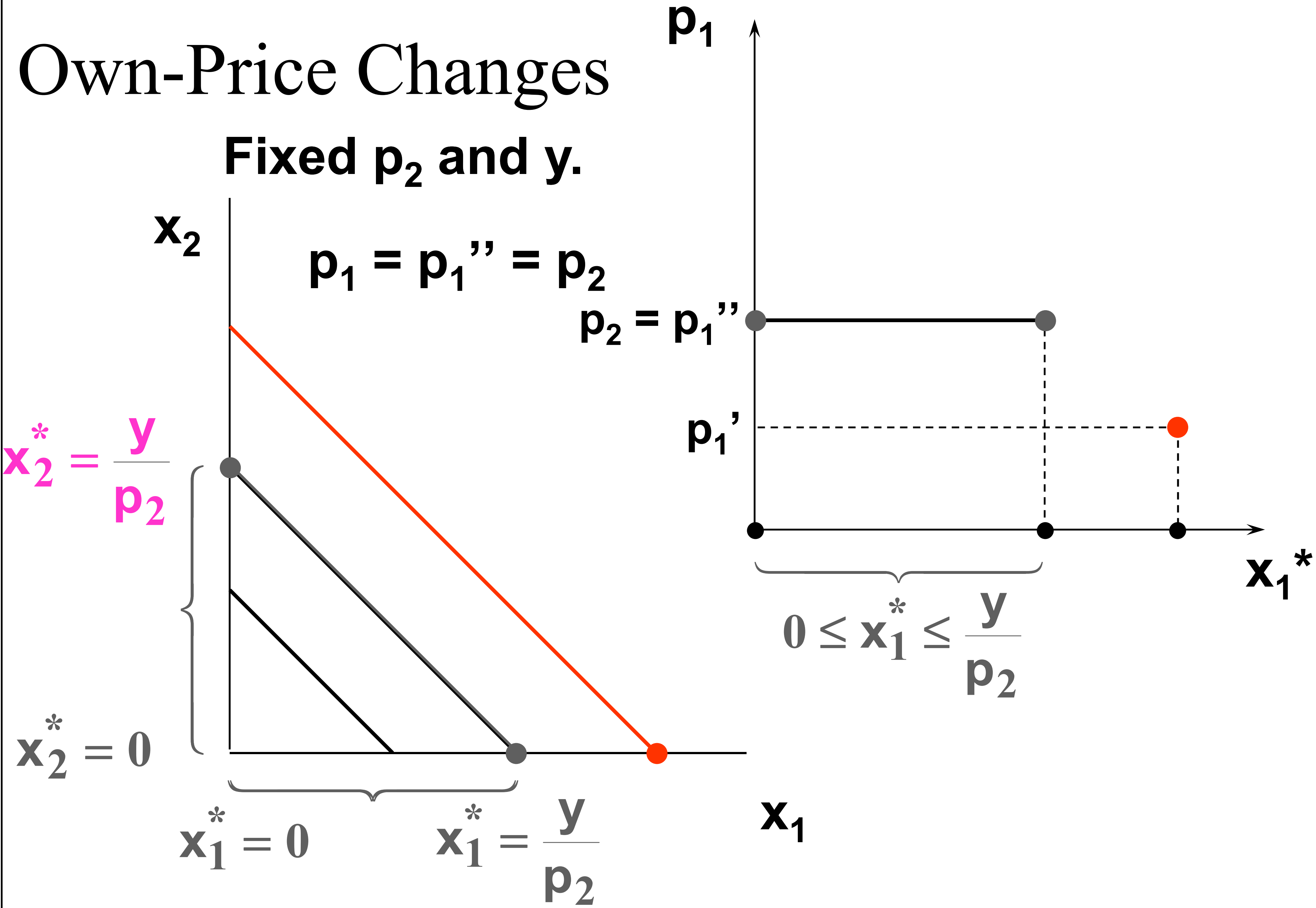

# Own-Price Changes

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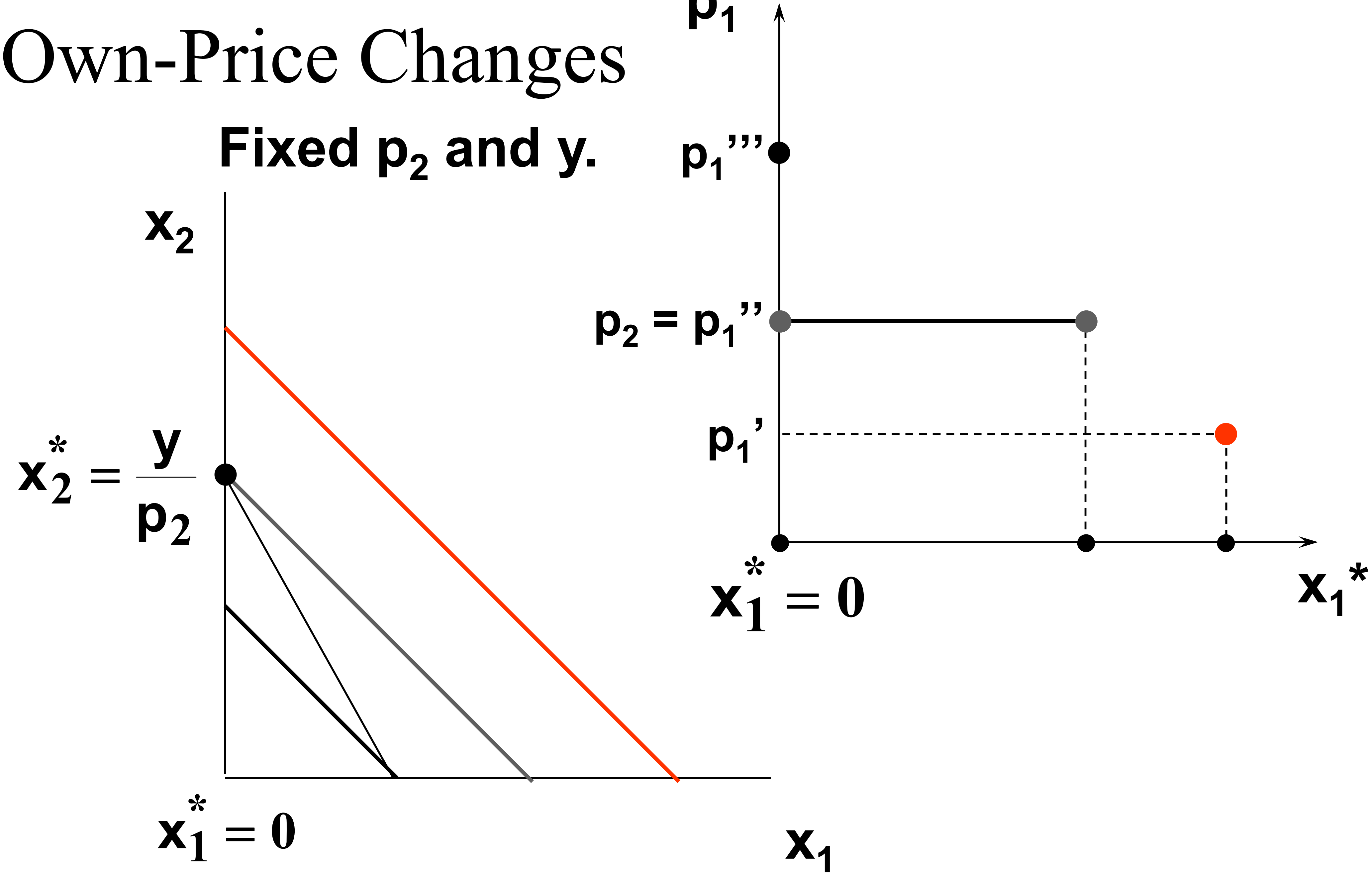
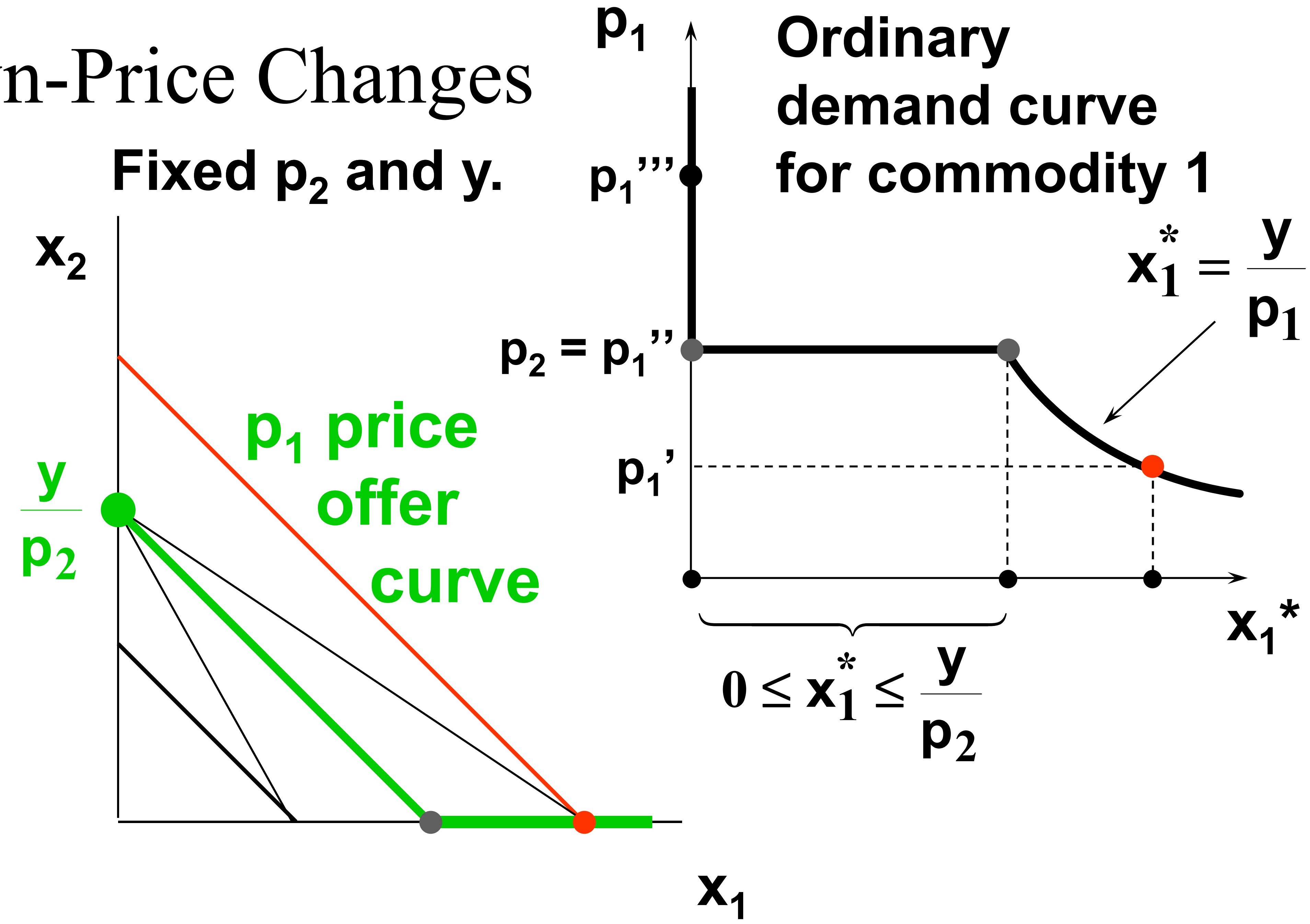


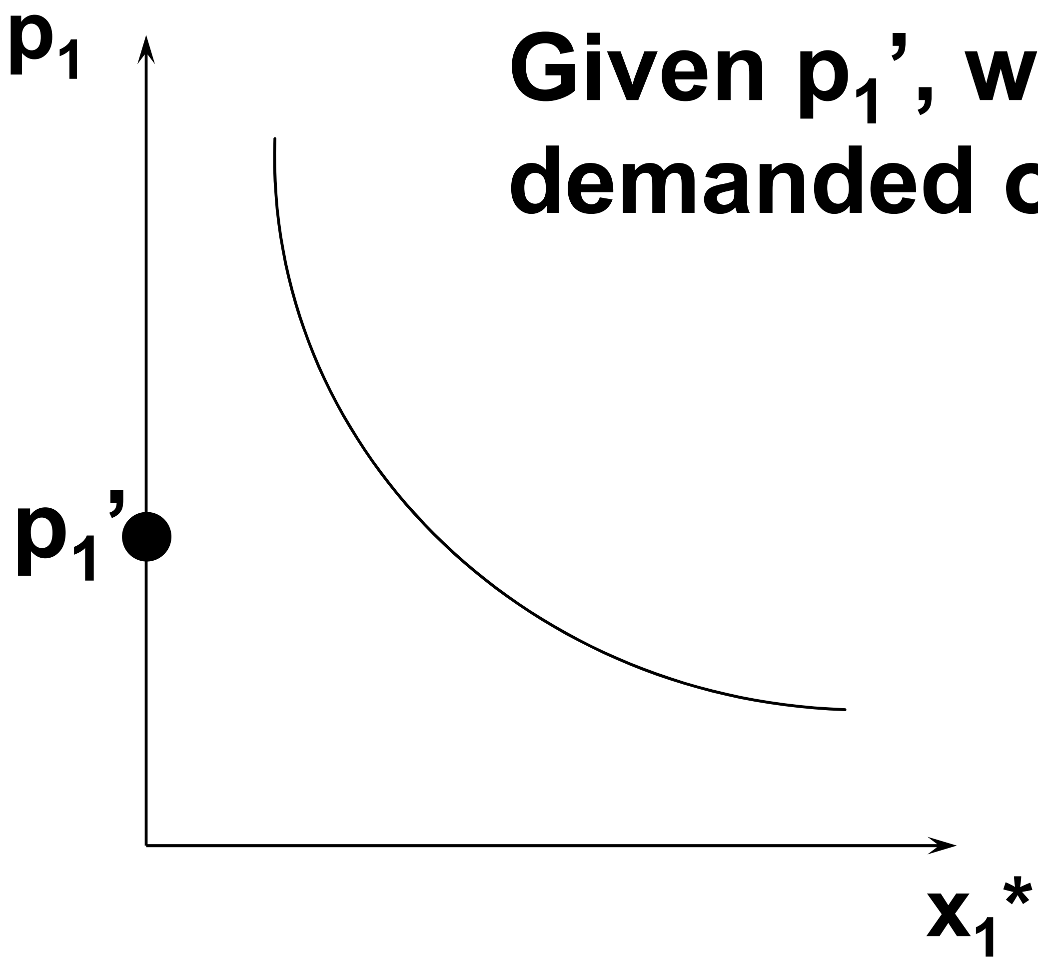
# Own-Price Changes



# Own-Price Changes

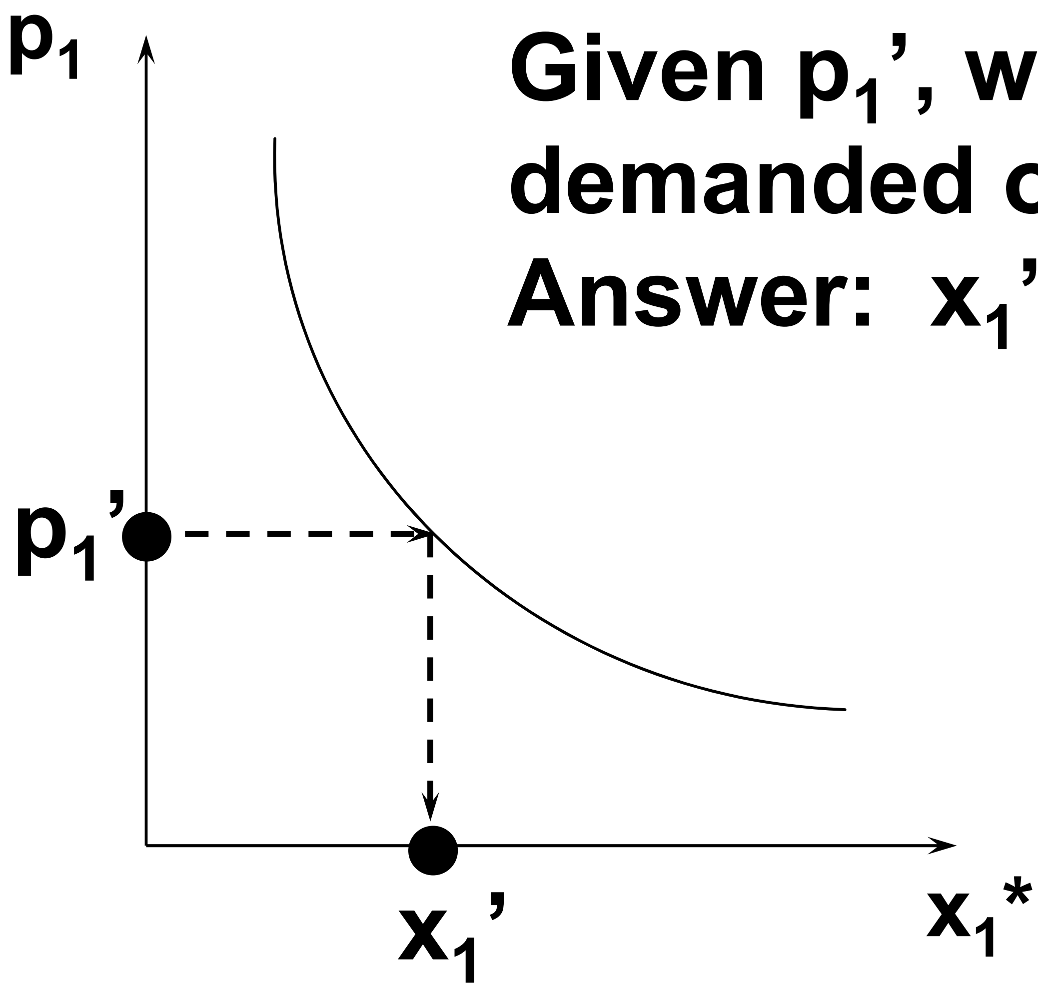
- **Usually we ask "Given the price for commodity 1 what is the quantity demanded of commodity 1?"**
- **But we could also ask the inverse question "At what price for commodity 1 would a given quantity of commodity 1 be demanded?"**

# Own-Price Changes



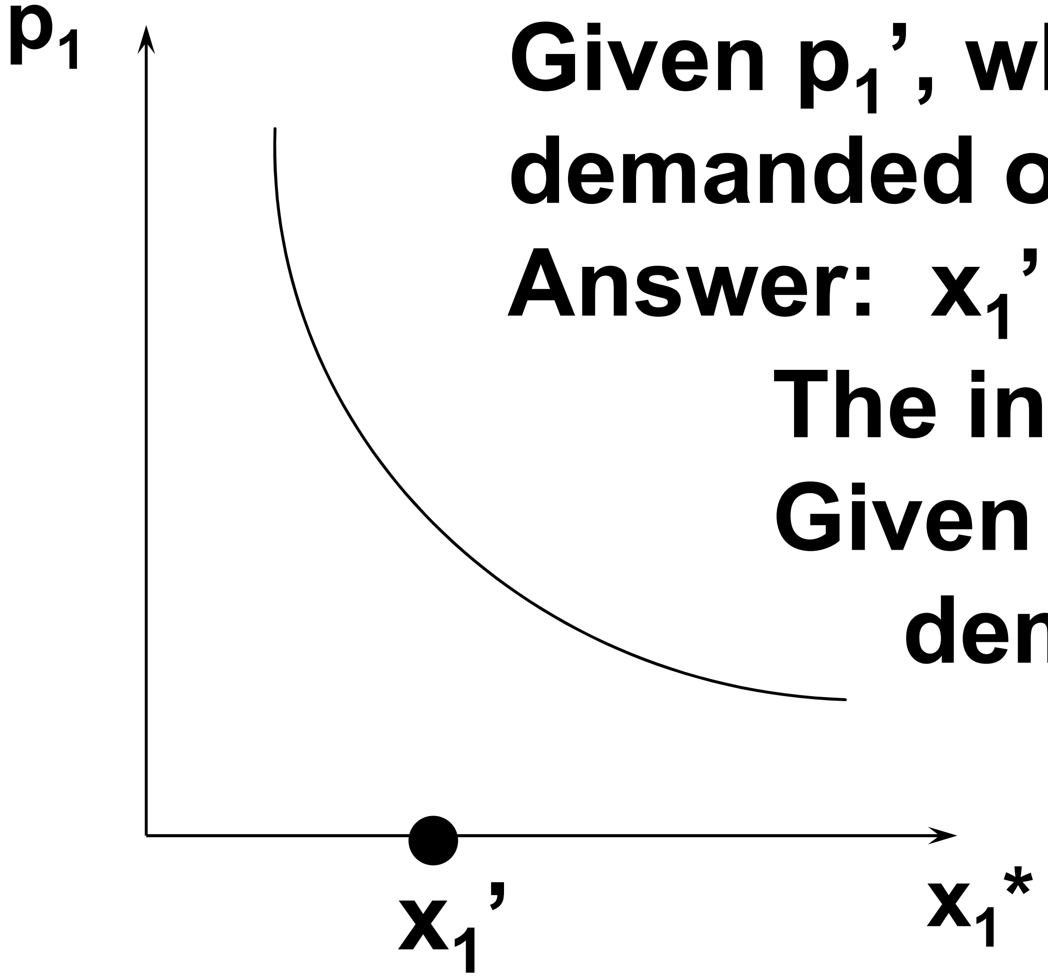

**Given $p_1'$, what quantity is demanded of commodity 1?**

# Own-Price Changes




**Given $p_1$', what quantity is demanded of commodity 1?**
**Answer: $x_1$' units.**

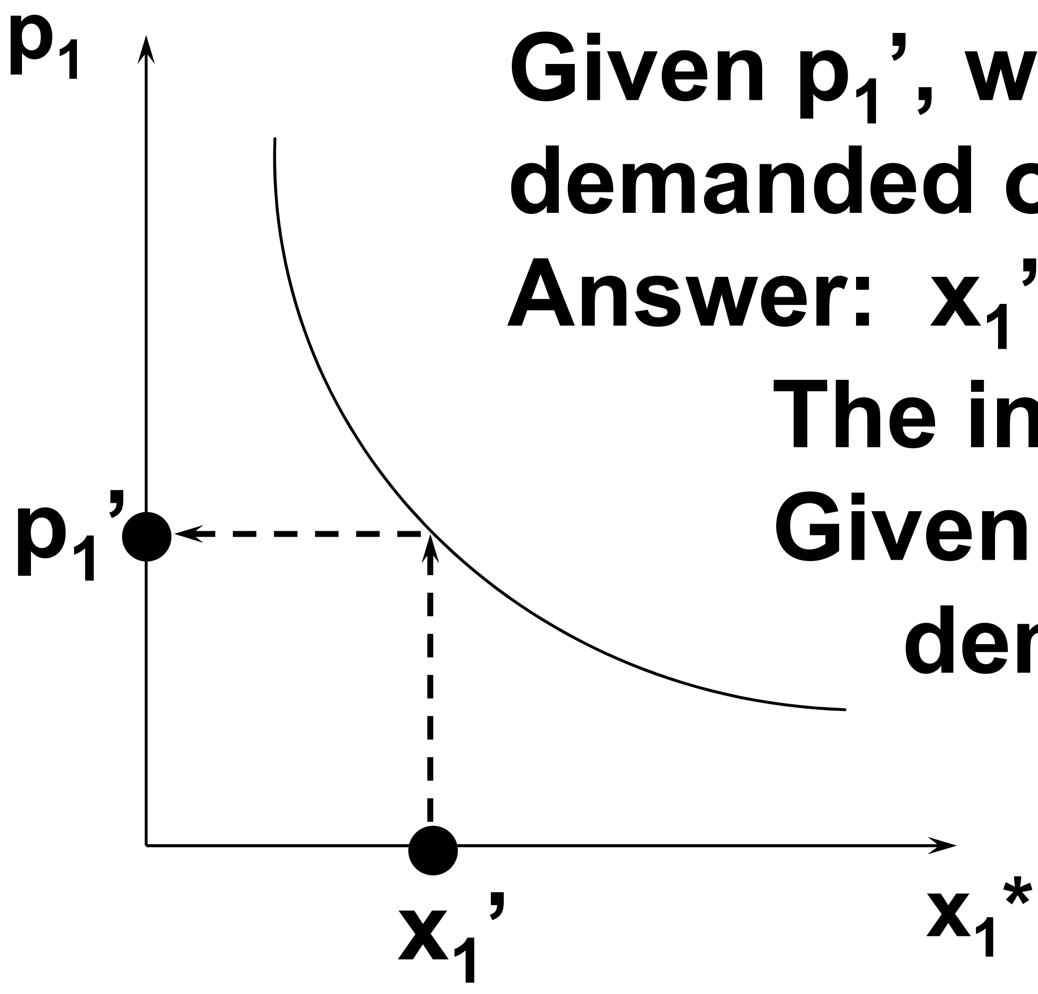
# Own-Price Changes




**Given $p_1'$, what quantity is demanded of commodity 1?**
**Answer: $x_1'$ units.**

**The inverse question is: Given $x_1'$ units are demanded, what is the price of commodity 1?**

# Own-Price Changes




**Given $p_1'$, what quantity is demanded of commodity 1?**
**Answer: $x_1'$ units.**

**The inverse question is: Given $x_1'$ units are demanded, what is the price of commodity 1?**
**Answer: $p_1'$**

# Own-Price Changes

- **Taking quantity demanded as given and then asking what must be price describes the inverse demand function of a commodity.**

# Own-Price Changes

**A Cobb-Douglas example:**

$$x_1^* = \frac{ay}{(a+b)p_1}$$

**is the ordinary demand function and**

$$p_1 = \frac{ay}{(a+b)x_1^*}$$

**is the inverse demand function.**

# Own-Price Changes

**A perfect-complements example:**

$$x_1^* = \frac{y}{p_1 + p_2}$$

**is the ordinary demand function and**

$$p_1 = \frac{y}{x_1^*} - p_2$$

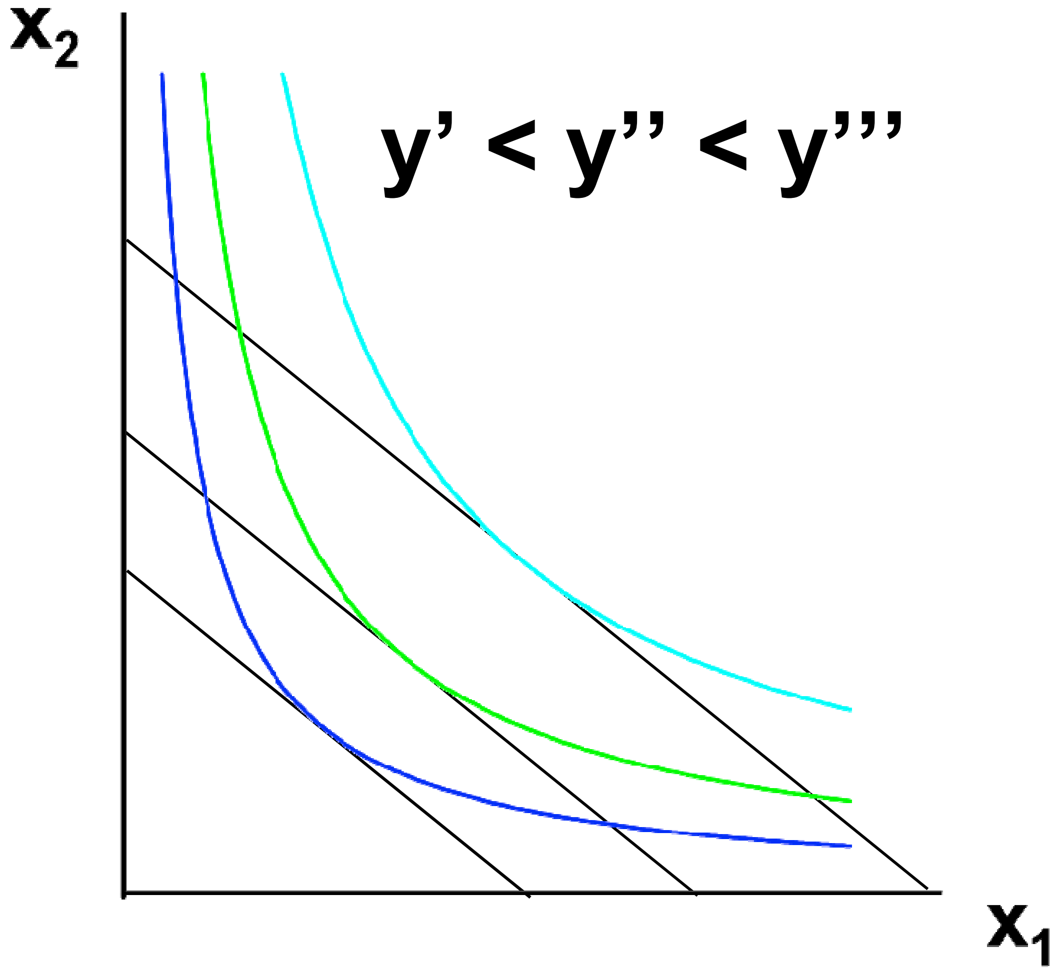
**is the inverse demand function.**

# Income Changes

- **How does the value of $x_1^*(p_1,p_2,y)$ change as y changes, holding both $p_1$ and $p_2$ constant?**

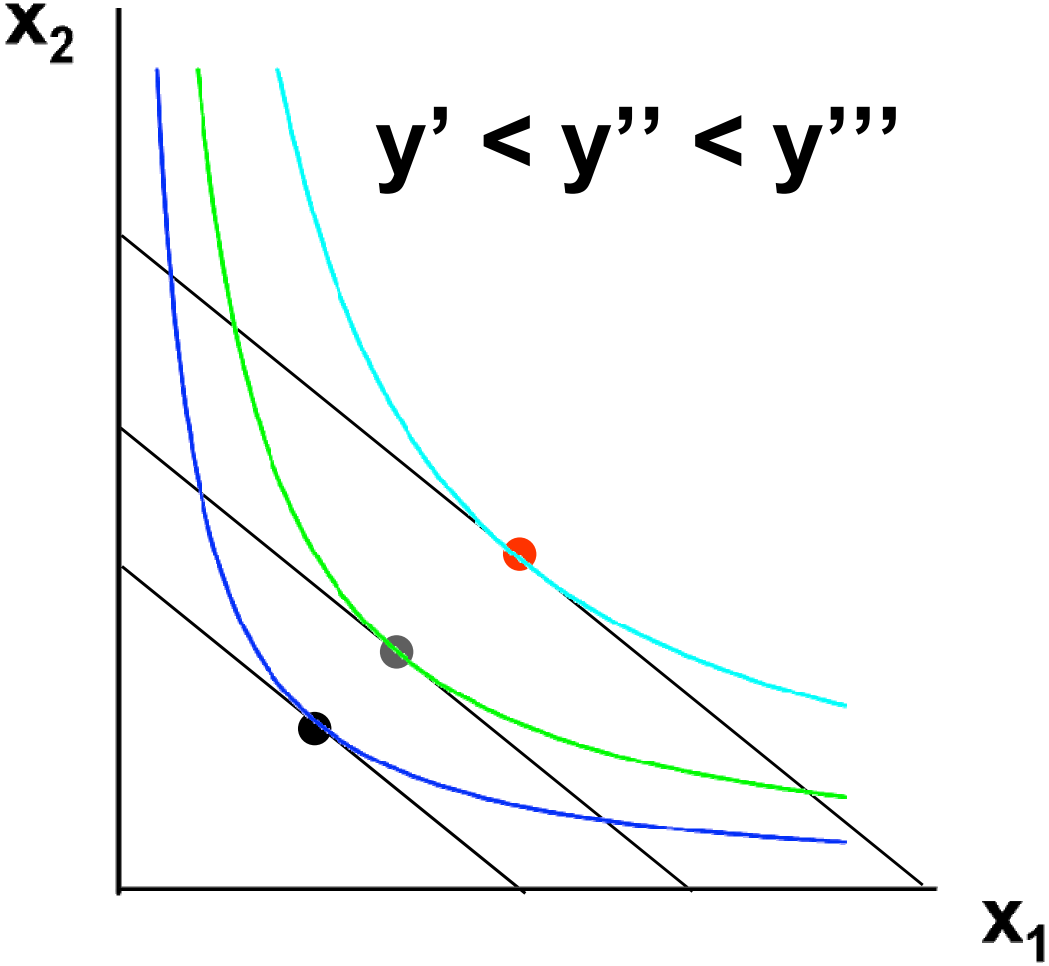
# Income Changes

**Fixed $p_1$ and $p_2$.**



# Income Changes

**Fixed $p_1$ and $p_2$.**



# Income Changes

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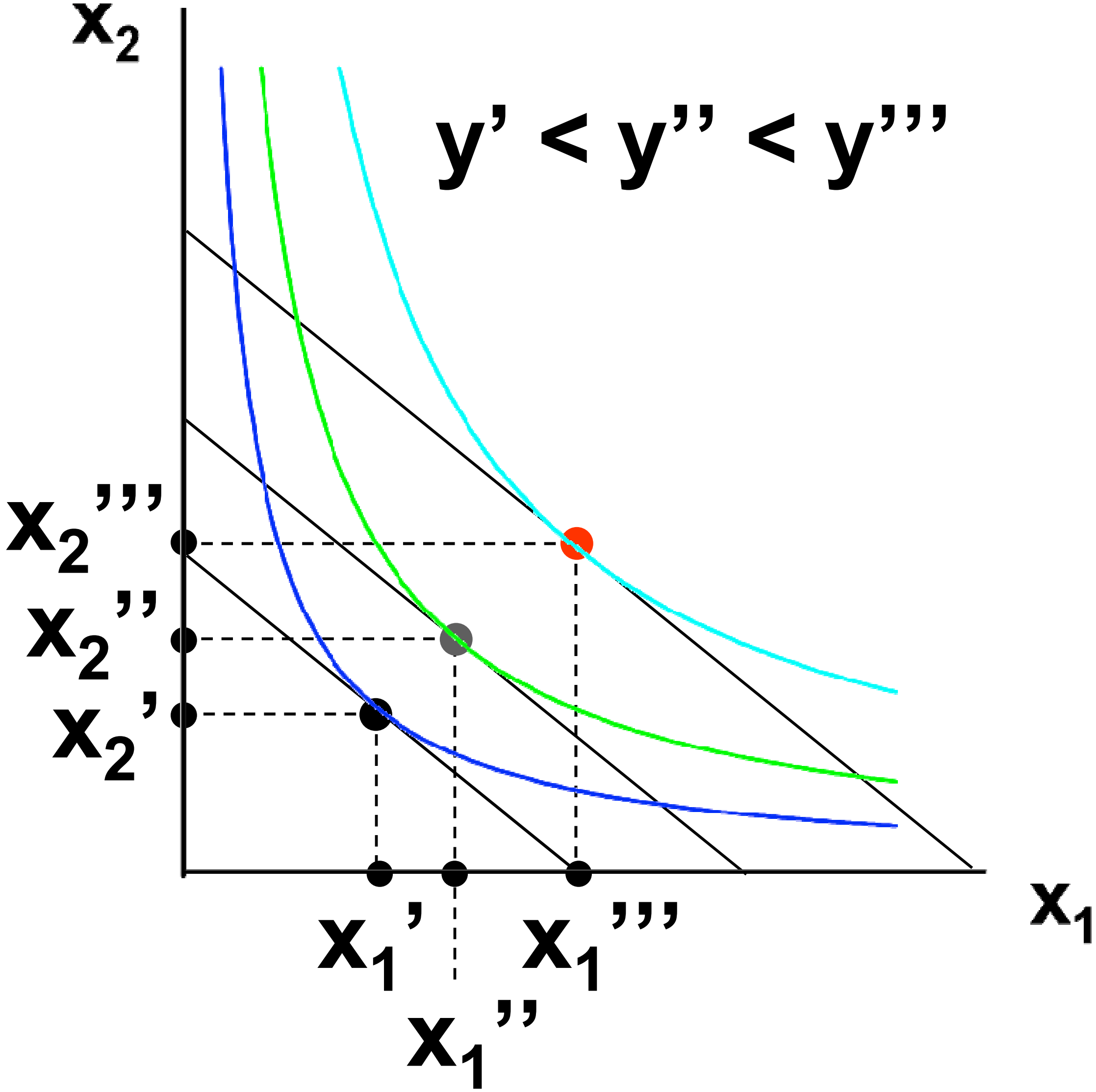
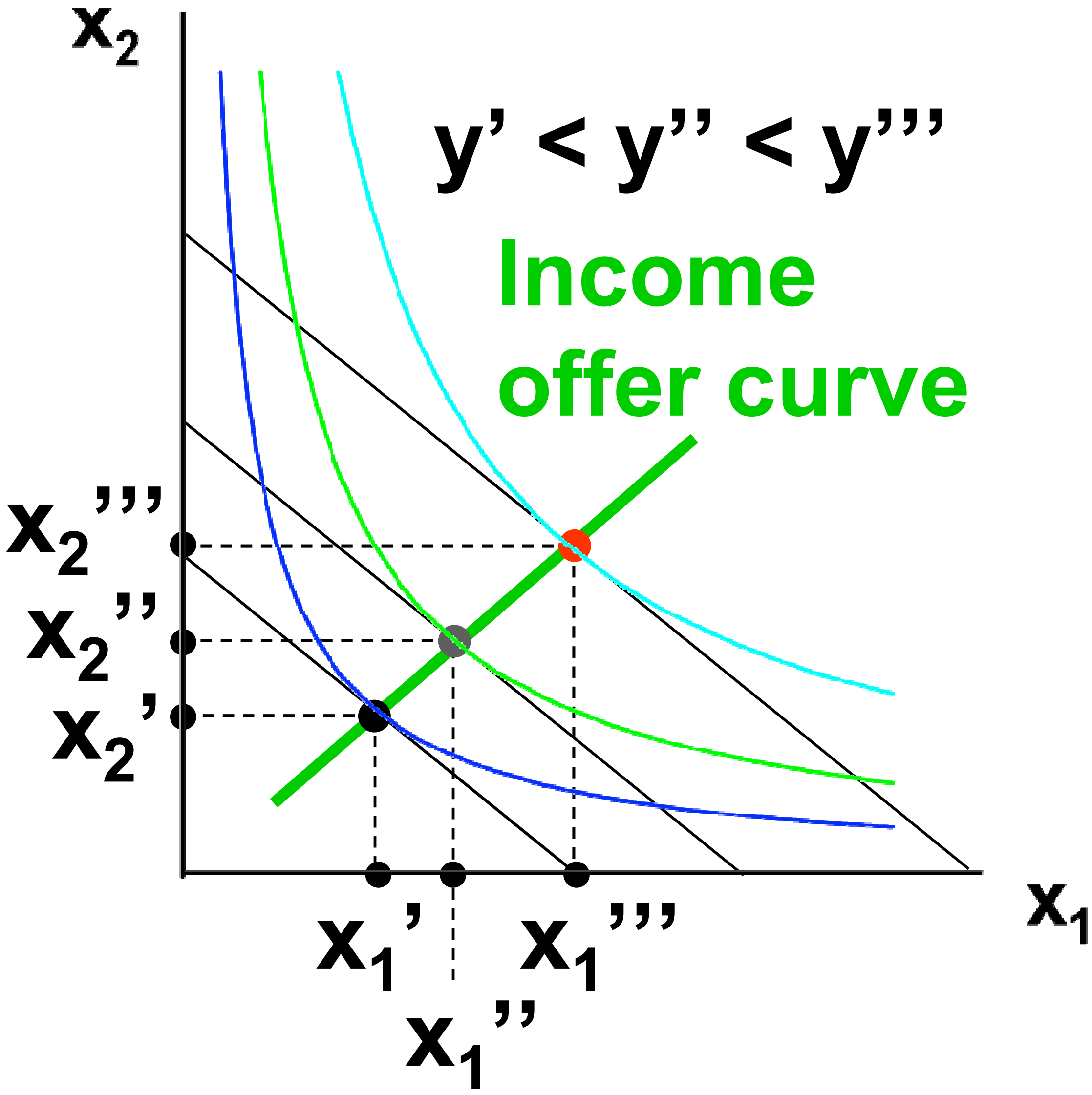


# Income Changes

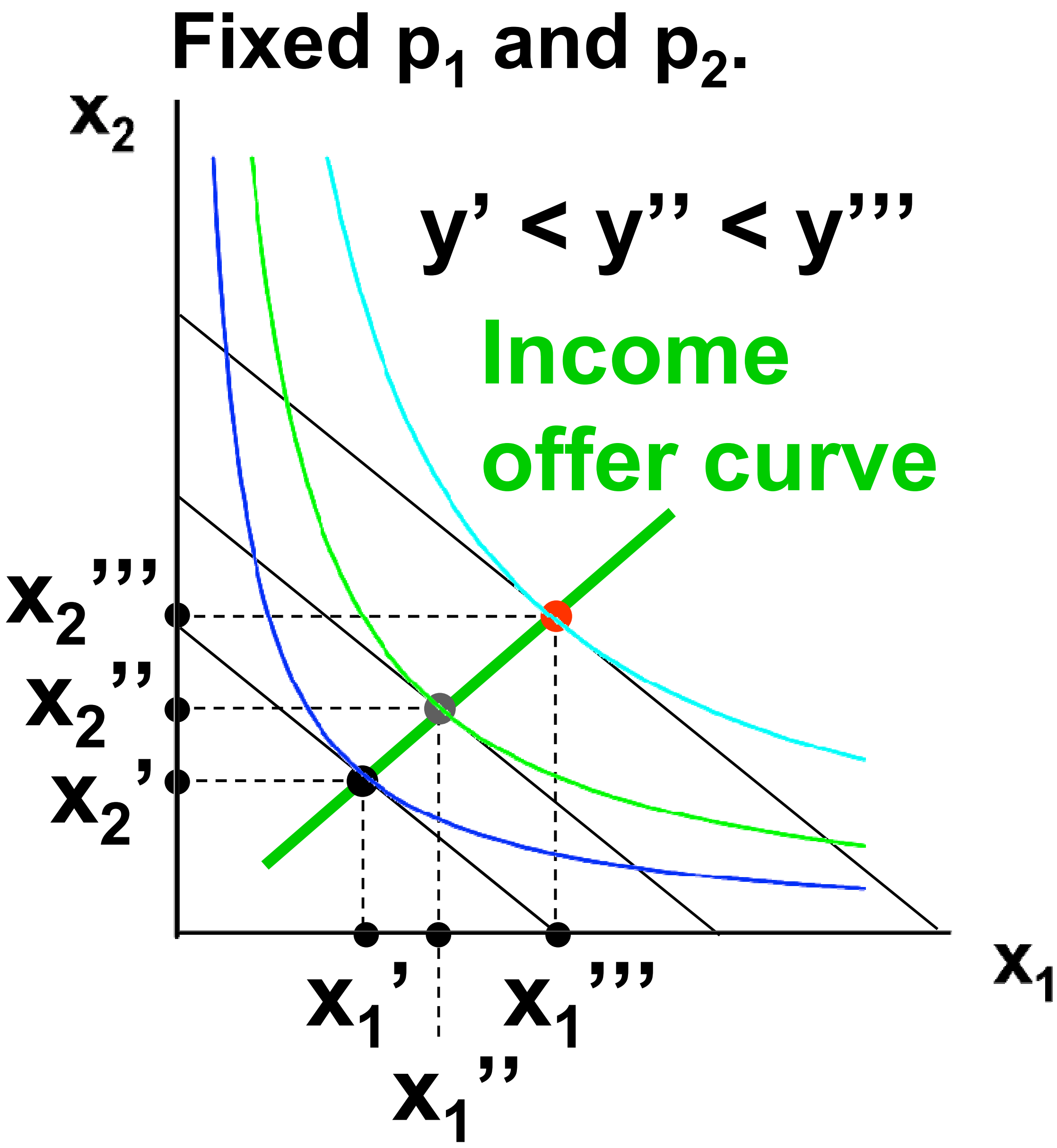
**Fixed $p_1$ and $p_2$.**



# Income Changes

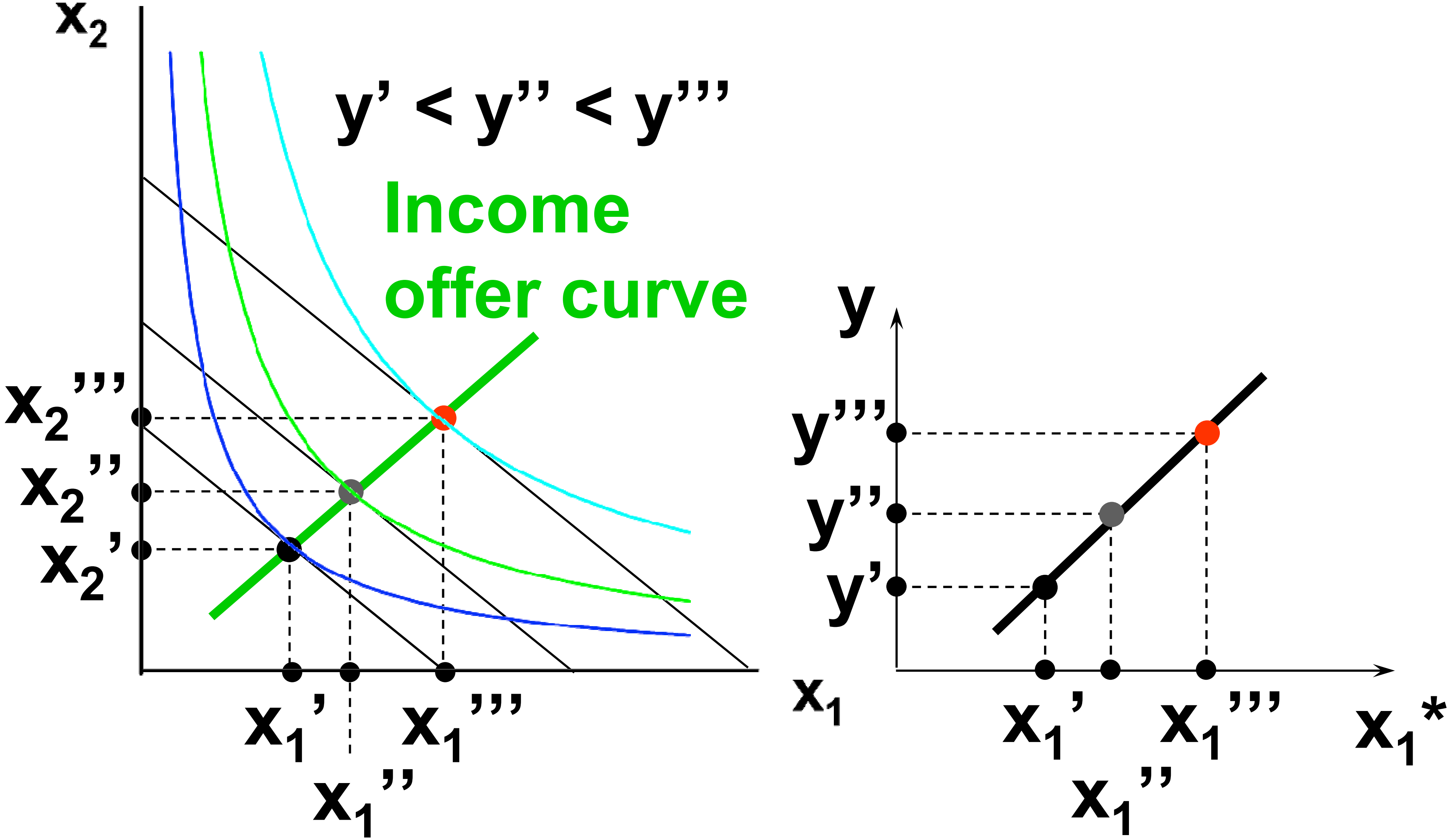
- **A plot of quantity demanded against income is called an Engel curve.**

# Income Changes



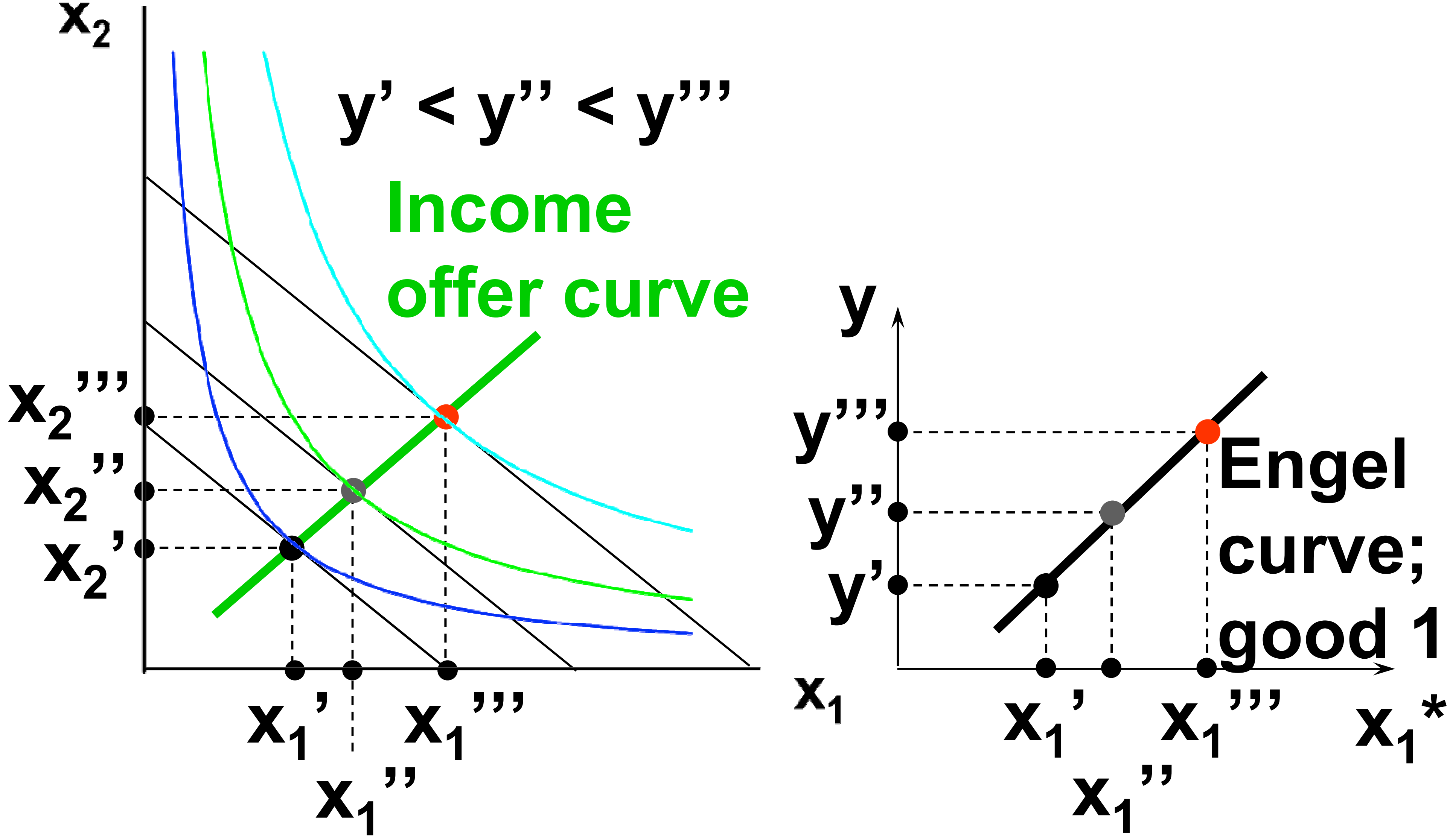
# Income Changes

**Fixed $p_1$ and $p_2$.**

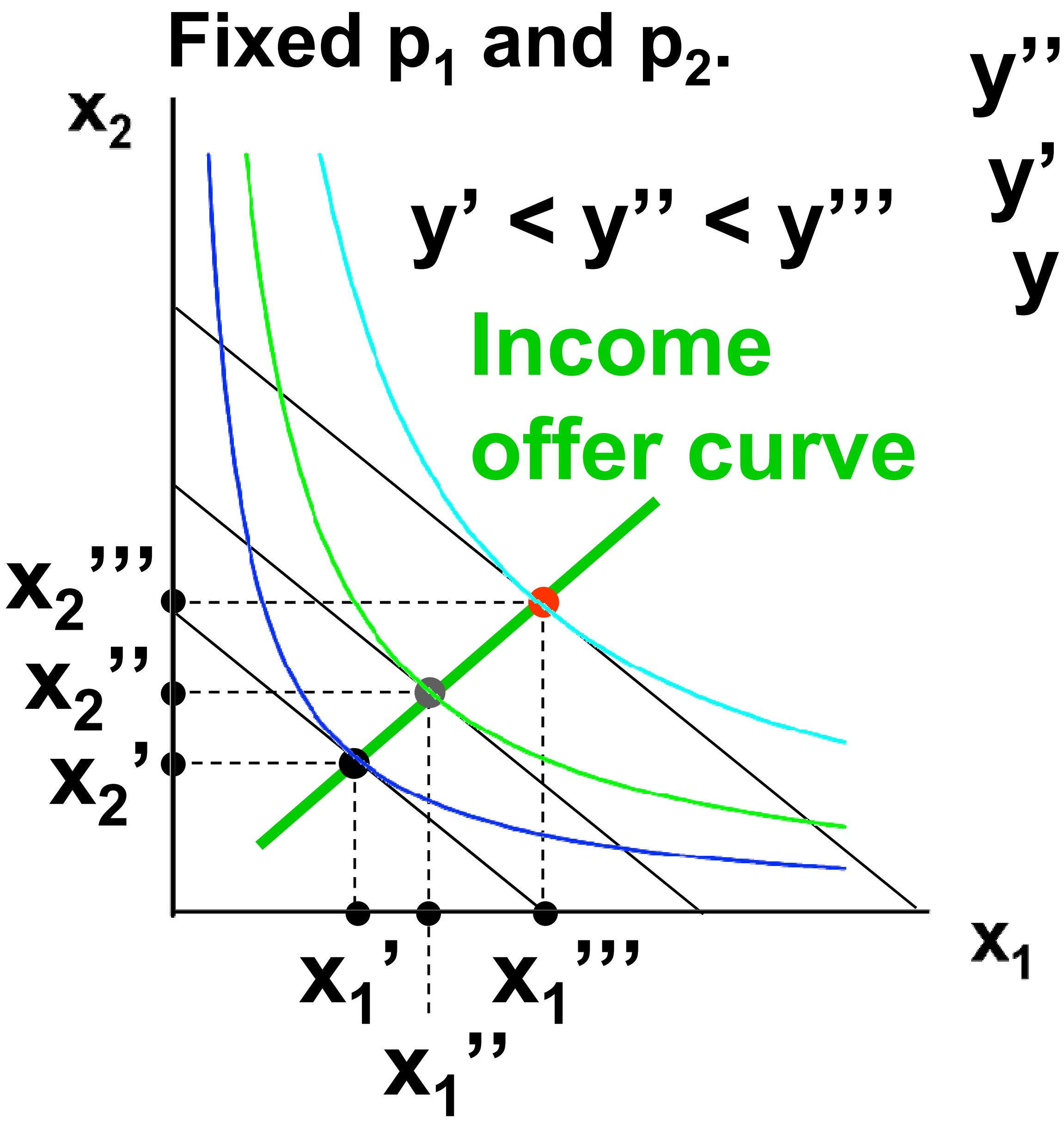


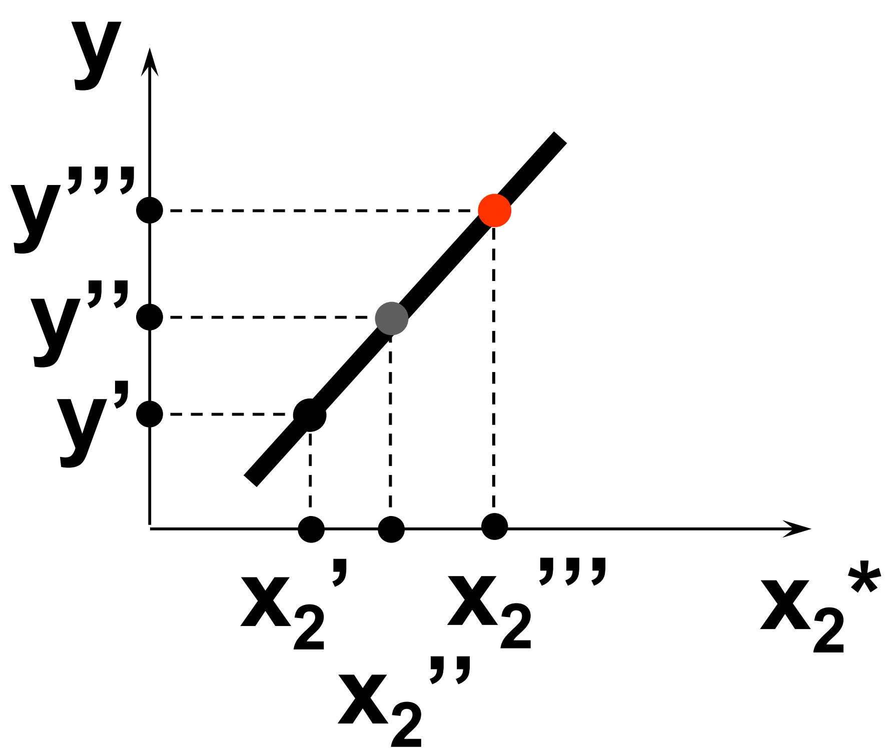
# Income Changes

**Fixed $p_1$ and $p_2$.**



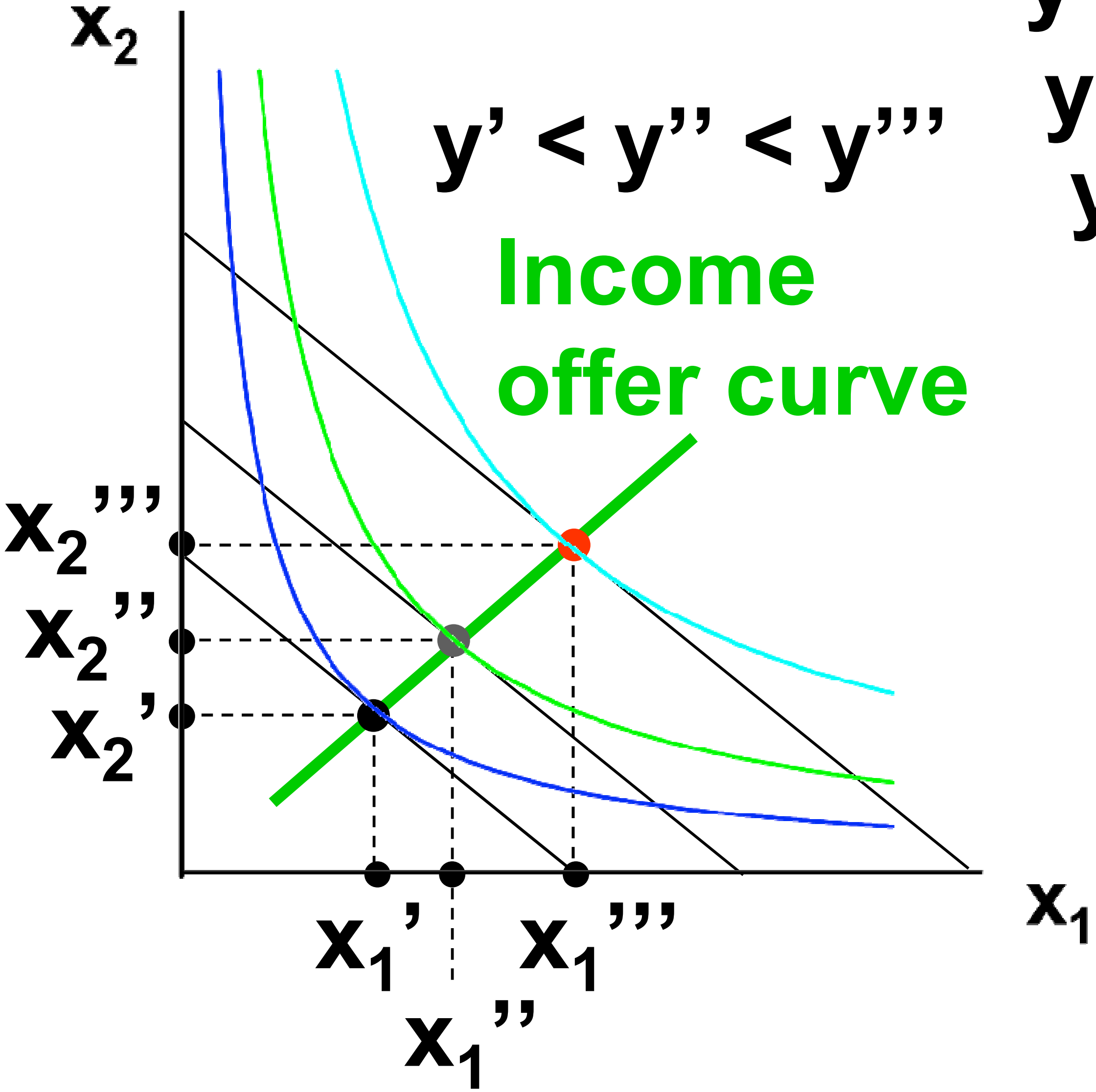
# Income Changes

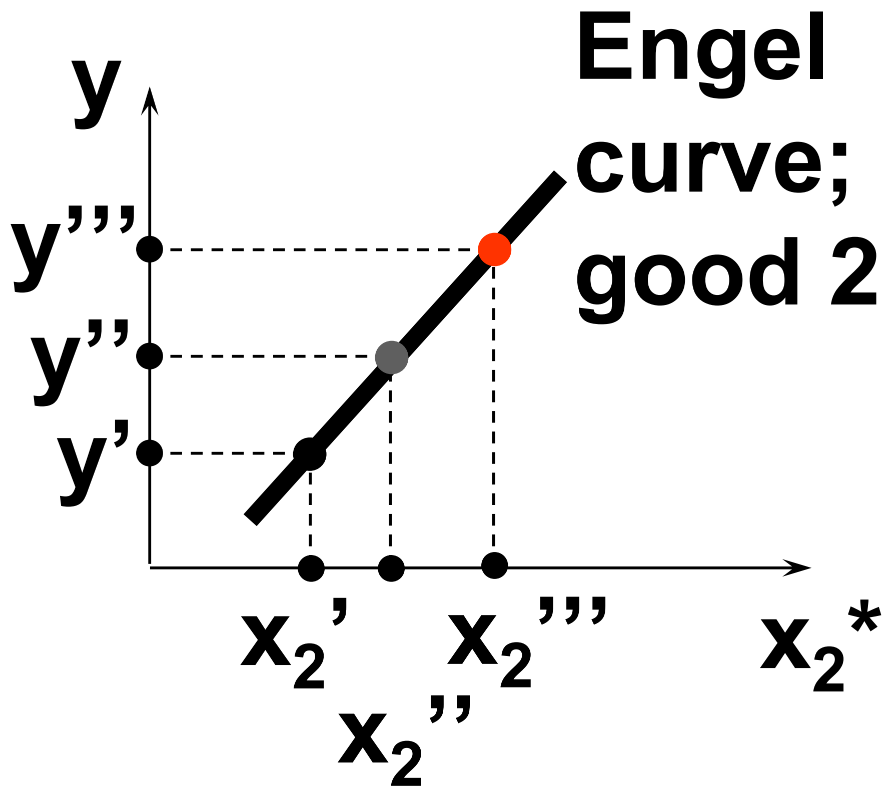


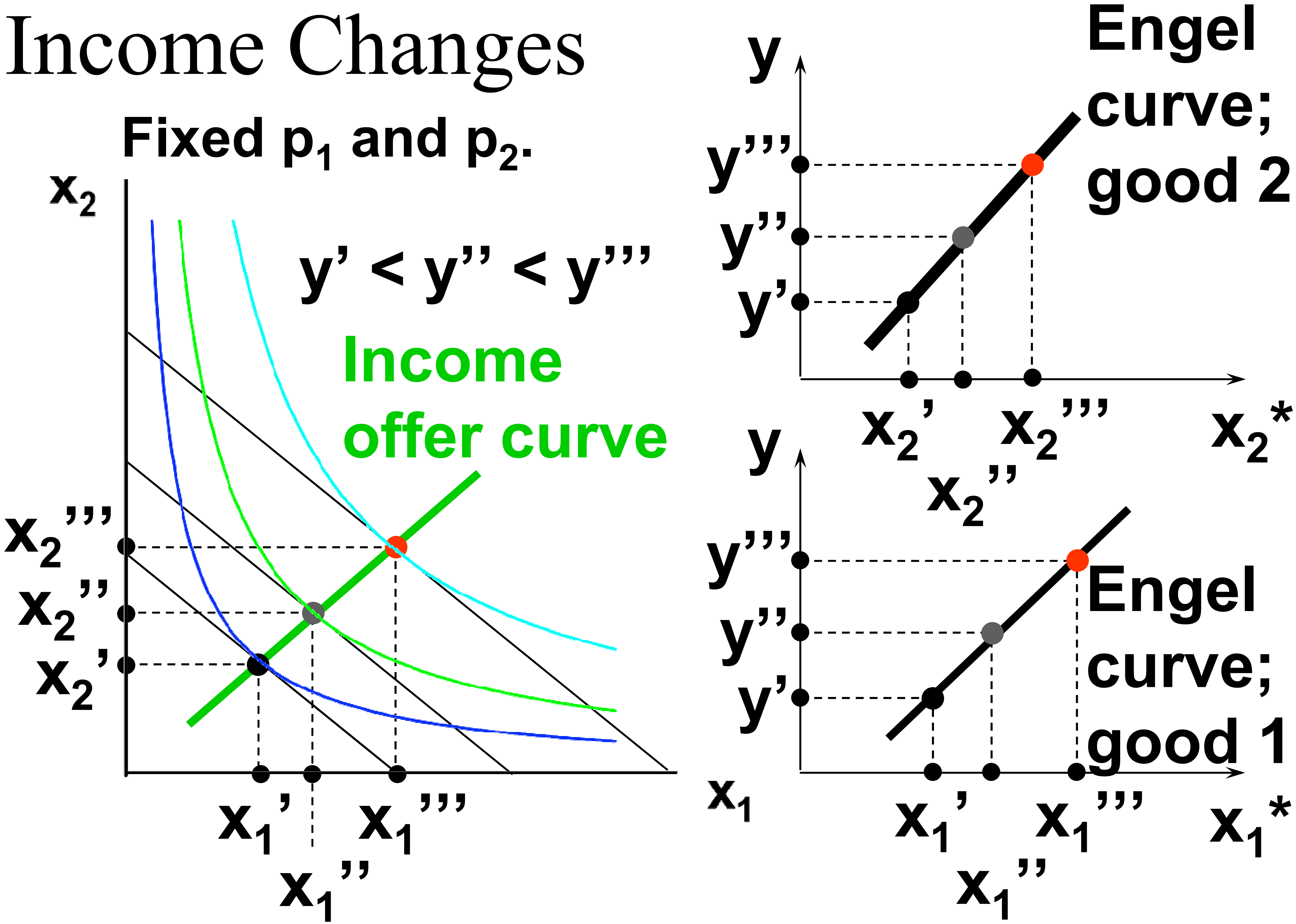
# Income Changes

**Fixed $p_1$ and $p_2$.**



# Income Changes



# Income Changes and Cobb-Douglas Preferences

- **An example of computing the equations of Engel curves; the Cobb-Douglas case.**

$$U(x_1, x_2) = x_1^a x_2^b.$$

- **The ordinary demand equations are**

$$x_1^* = \frac{ay}{(a+b)p_1}; \quad x_2^* = \frac{by}{(a+b)p_2}.$$

# Income Changes and Cobb-Douglas Preferences

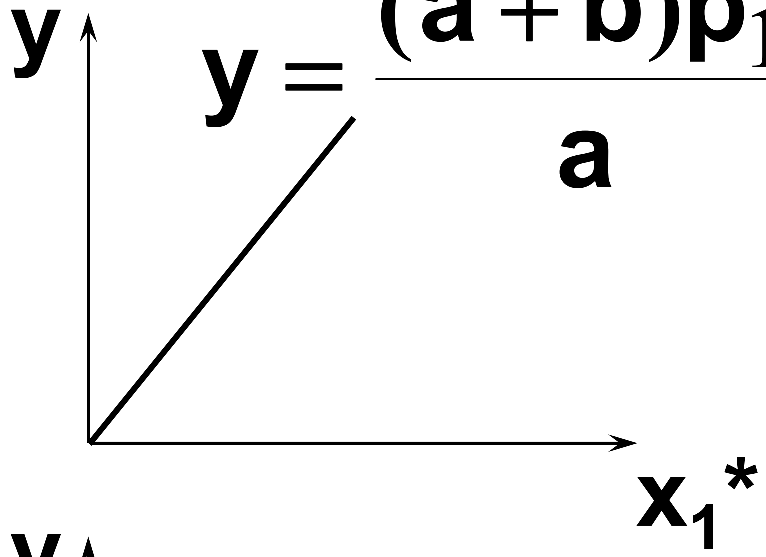
$$x_1^* = \frac{ay}{(a+b)p_1}; \quad x_2^* = \frac{by}{(a+b)p_2}.$$

**Rearranged to isolate y, these are:**

$$y = \frac{(a+b)p_1}{a} x_1^*$$ **Engel curve for good 1**

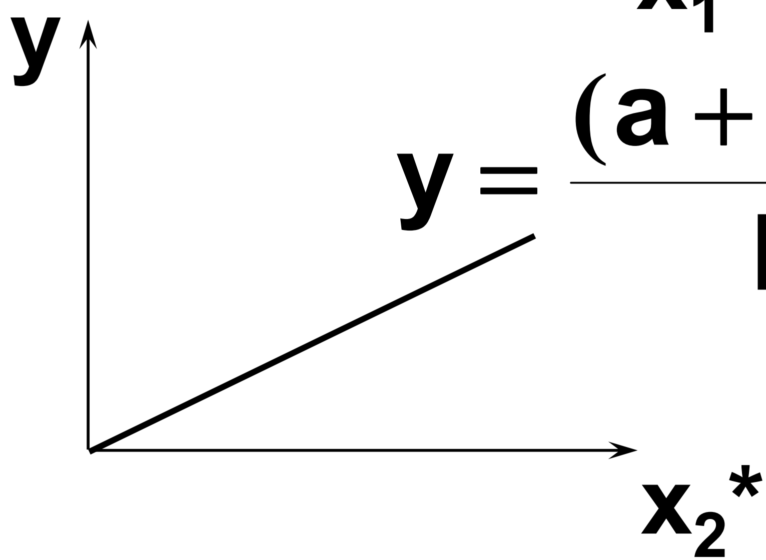
$$y = \frac{(a+b)p_2}{b} x_2^*$$ **Engel curve for good 2**

# Income Changes and Cobb-Douglas Preferences




$$y = \frac{(a+b)p_1}{a} x_1^*$$

**Engel curve for good 1**




$$y = \frac{(a+b)p_2}{b} x_2^*$$

**Engel curve for good 2**

# Income Changes and Perfectly-Complementary Preferences

- **Another example of computing the equations of Engel curves; the perfectly-complementary case.**

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

- **The ordinary demand equations are**

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

# Income Changes and Perfectly-Complementary Preferences

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

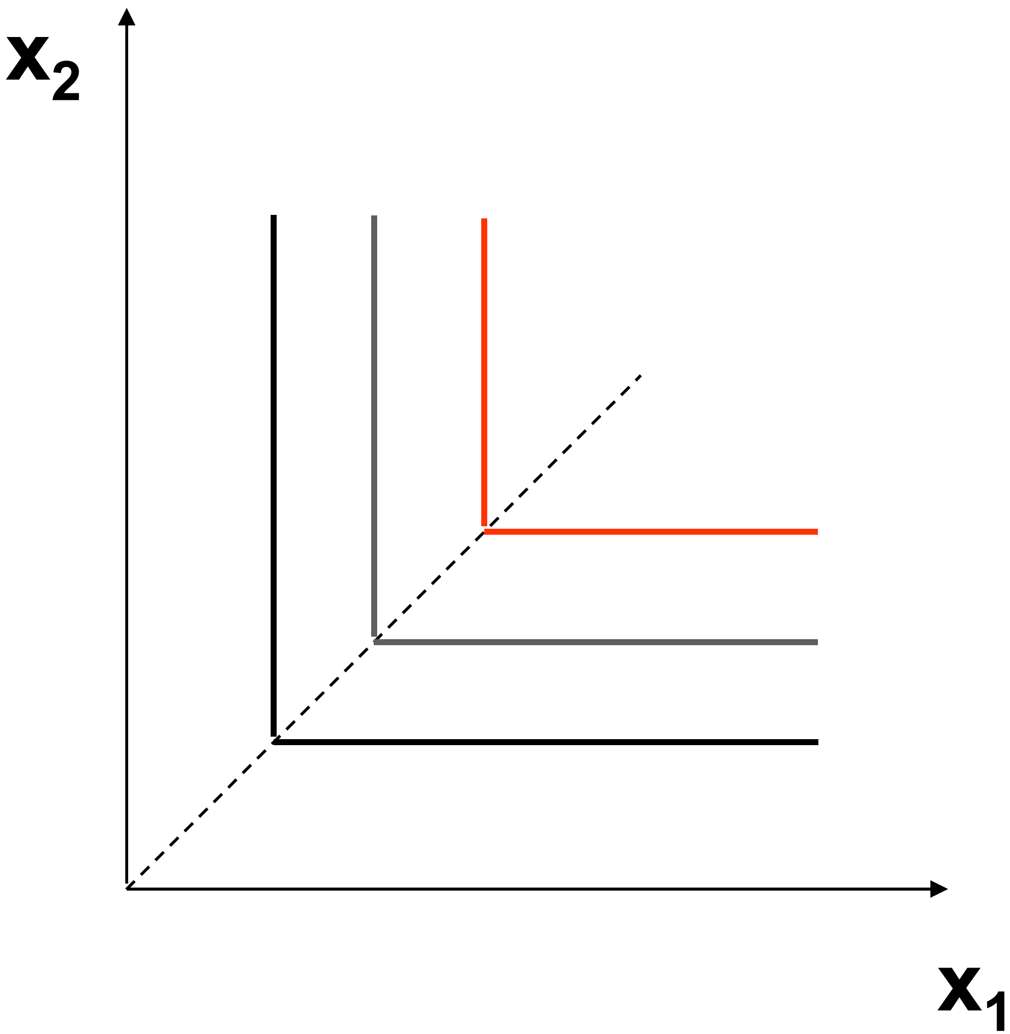
**Rearranged to isolate y, these are:**

$$y = (p_1 + p_2)x_1^*$$ **Engel curve for good 1**

$$y = (p_1 + p_2)x_2^*$$ **Engel curve for good 2**

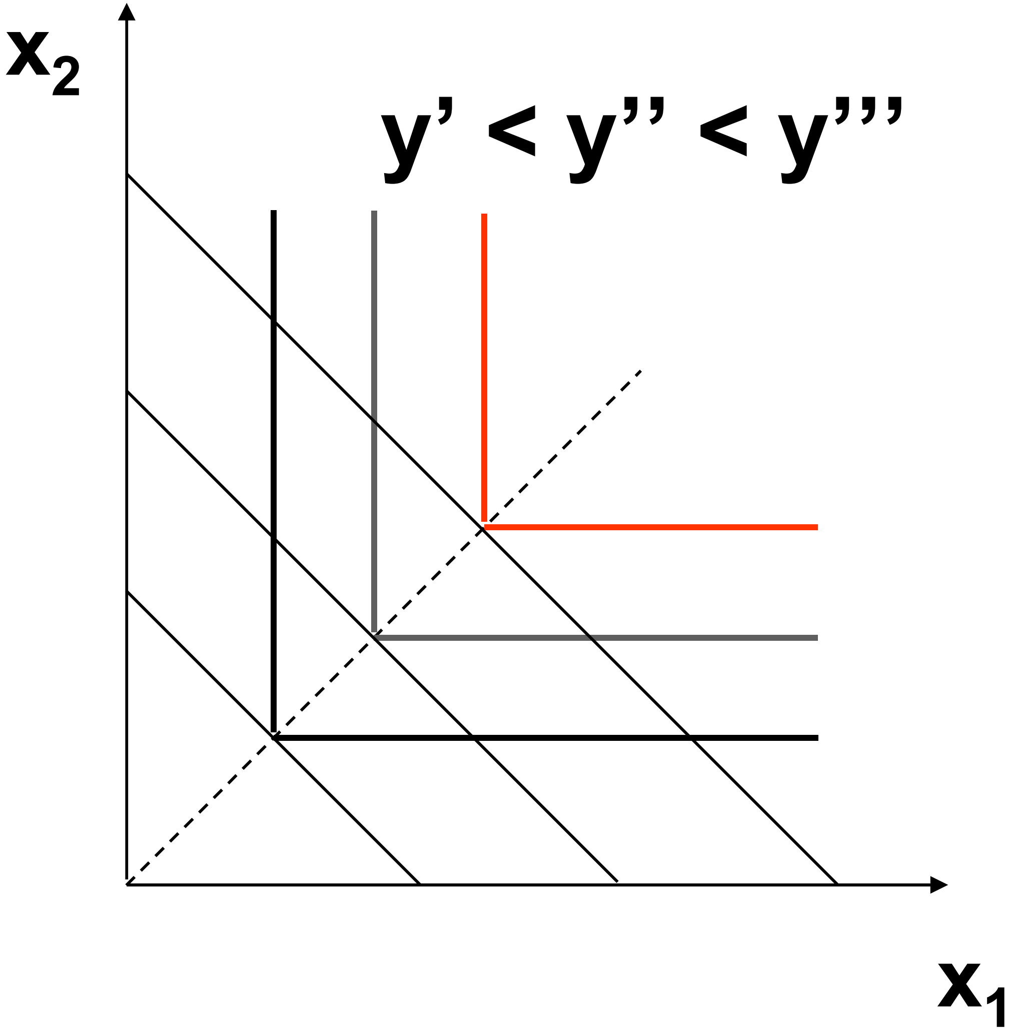
# Income Changes

**Fixed $p_1$ and $p_2$.**

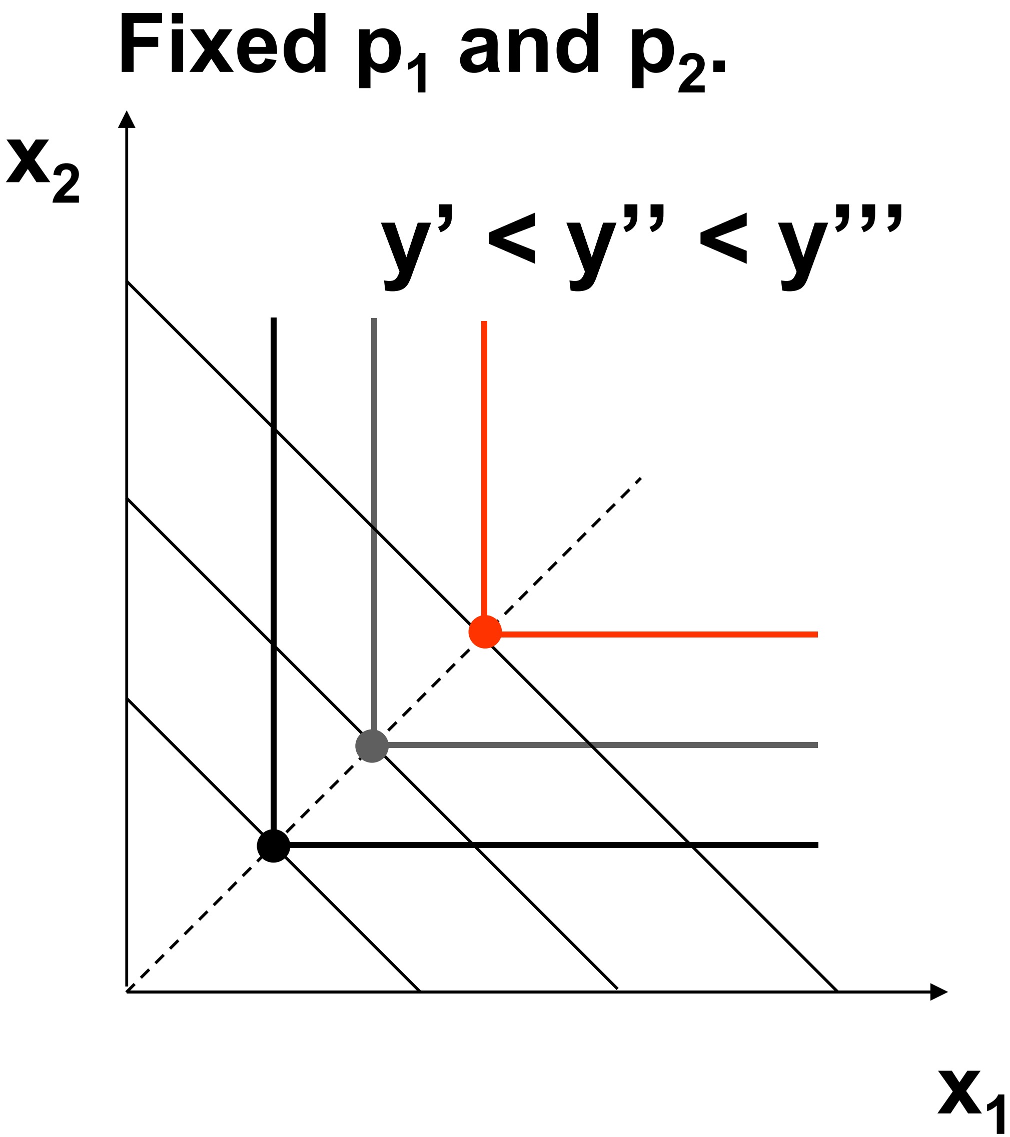


# Income Changes

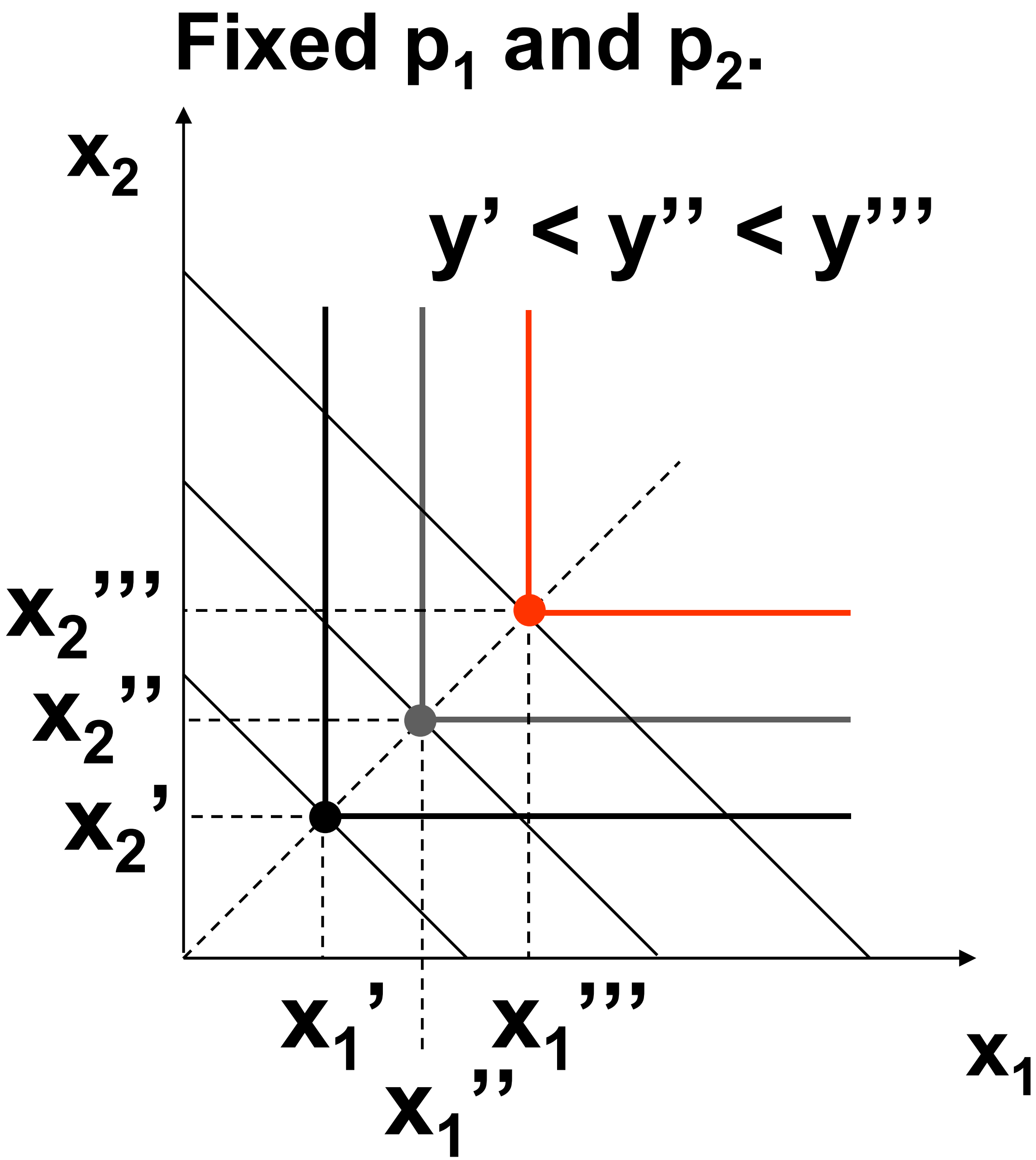
**Fixed $p_1$ and $p_2$.**



# Income Changes

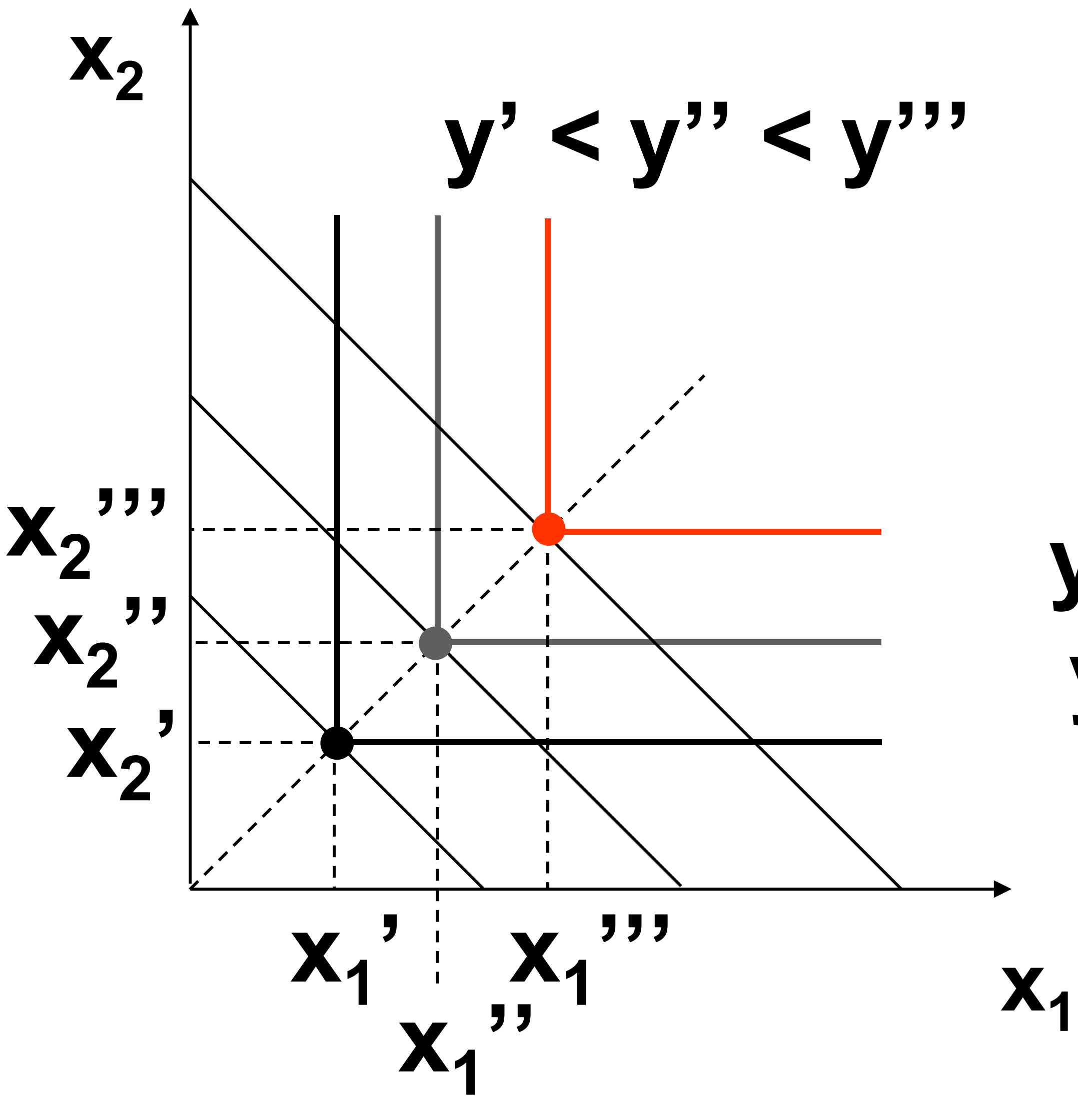


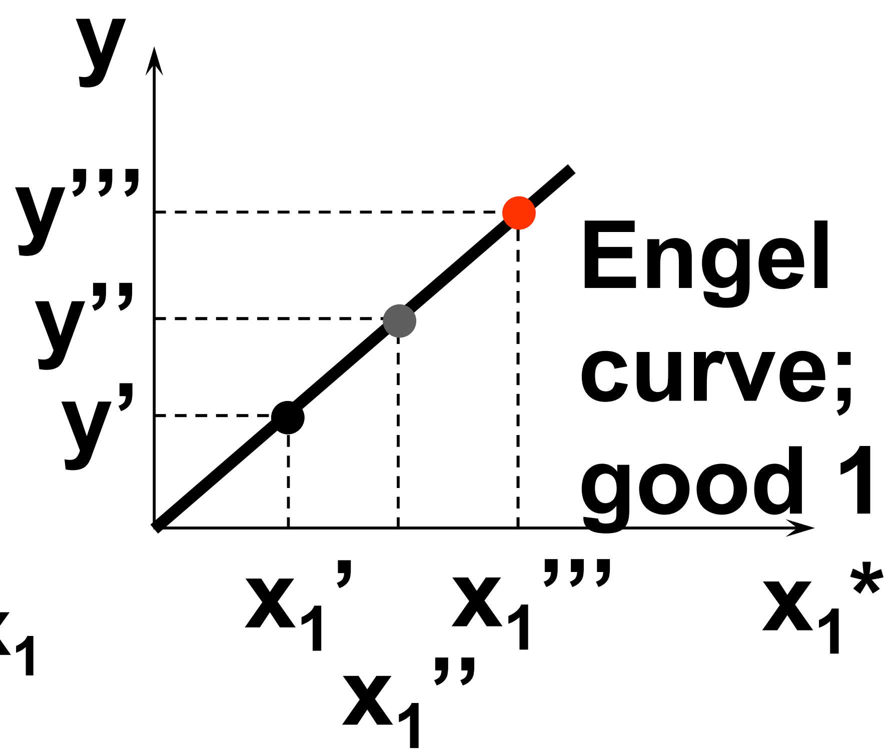
# Income Changes



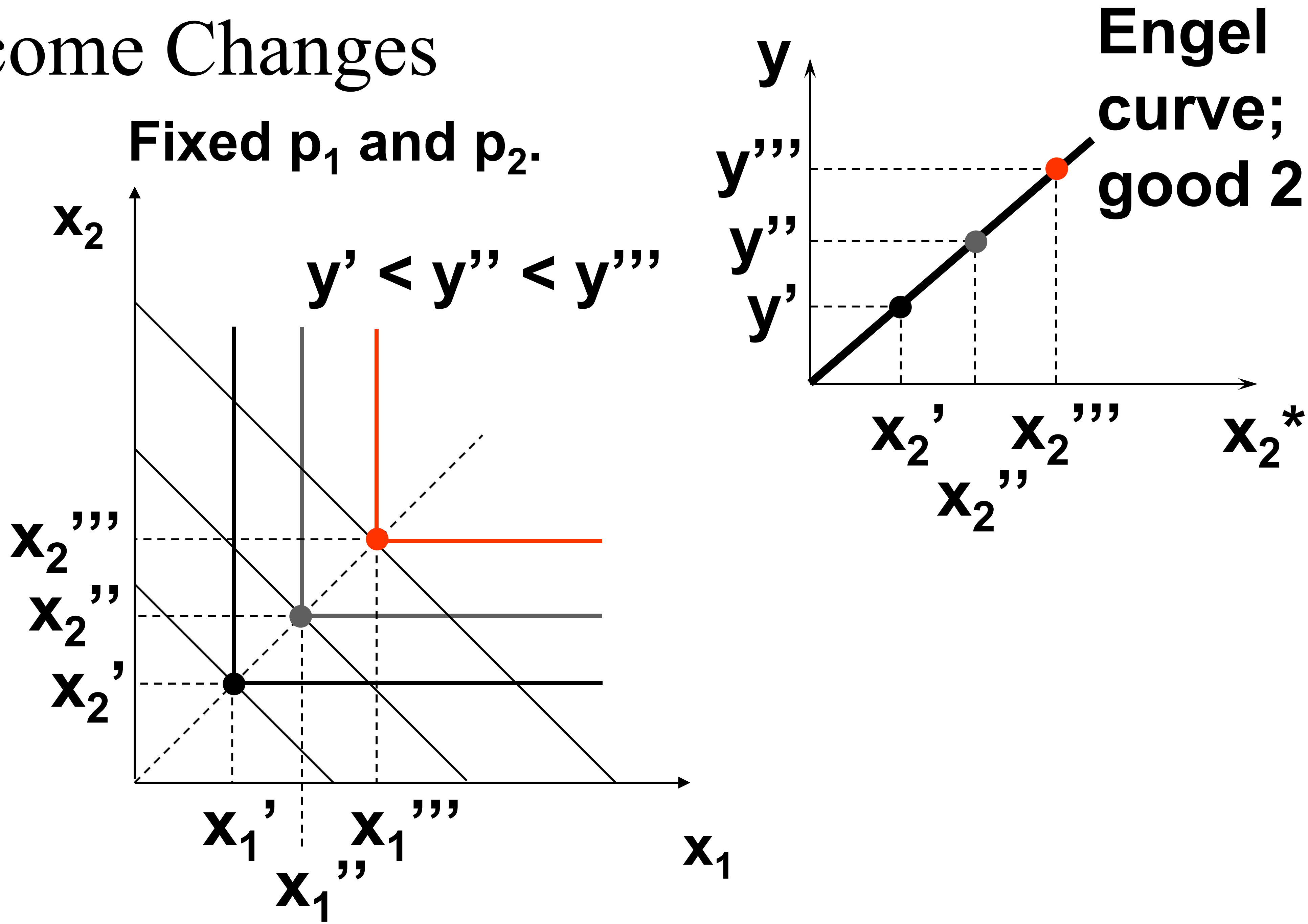
# Income Changes

**Fixed $p_1$ and $p_2$.**

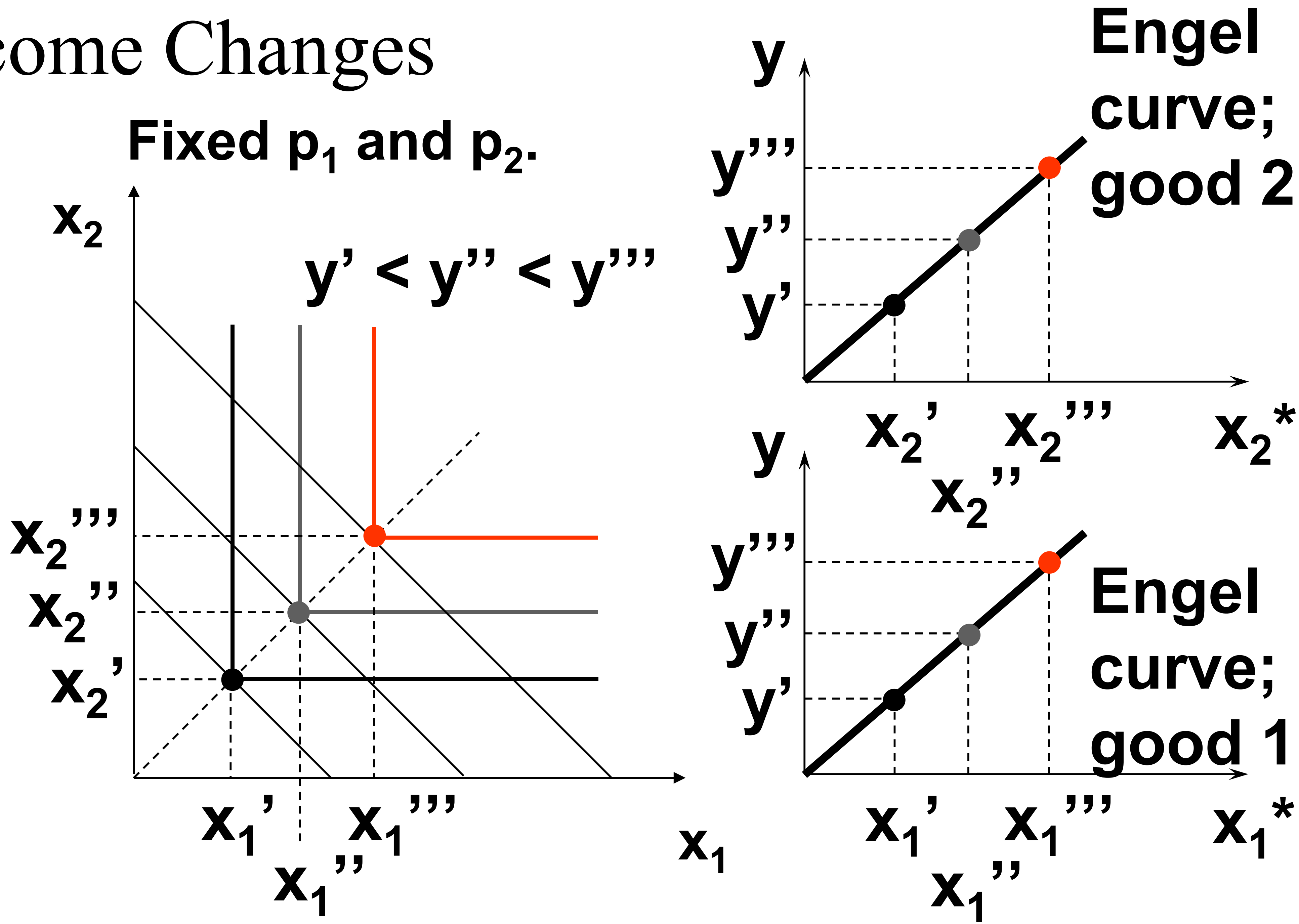


# Income Changes



# Income Changes

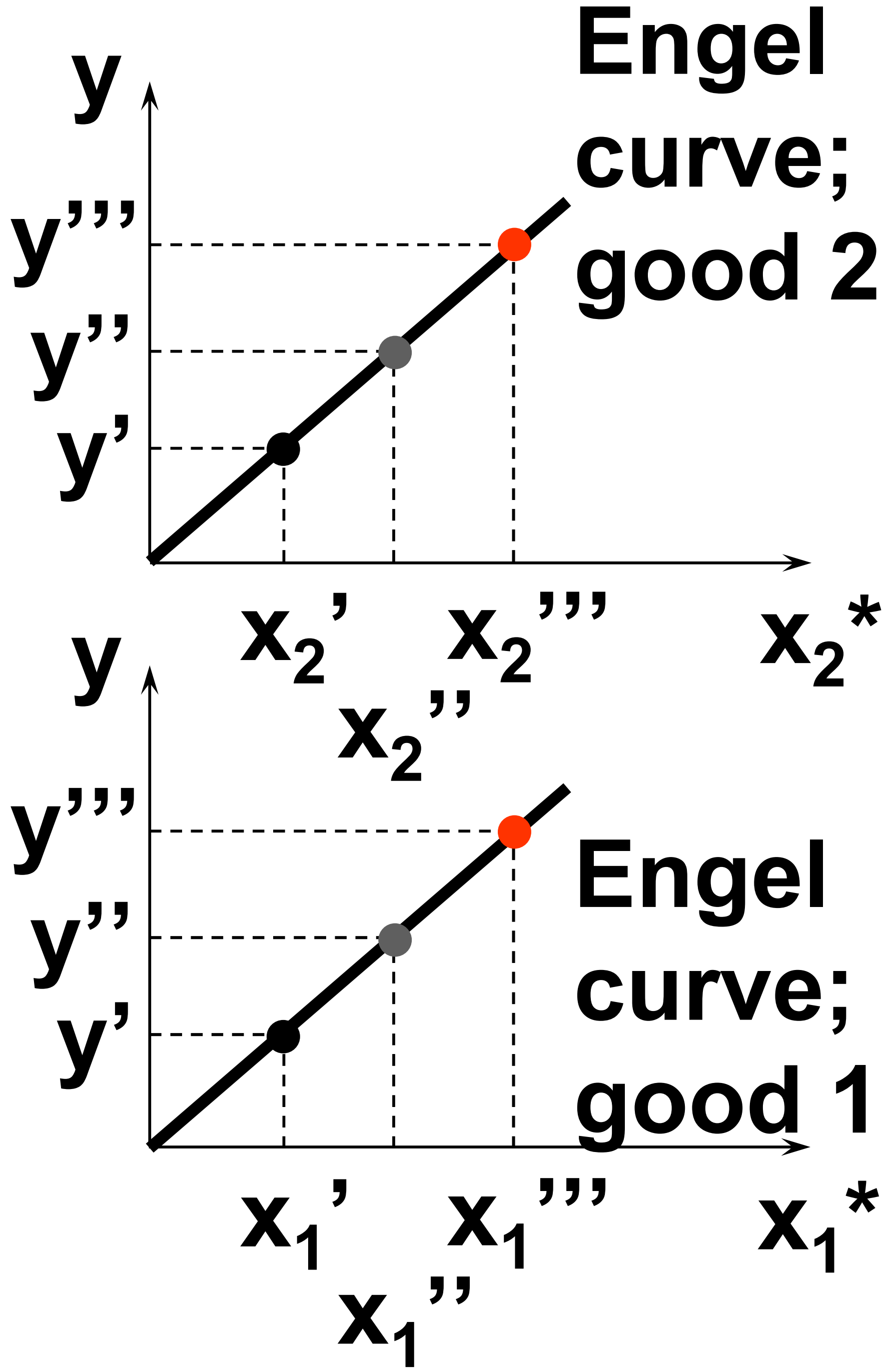


# Income Changes

**Fixed $p_1$ and $p_2$.**

$$y = (p_1 + p_2)x_2^*$$

$$y = (p_1 + p_2)x_1^*$$



# Income Changes and Perfectly-Substitutable Preferences

- **Another example of computing the equations of Engel curves; the perfectly-substitution case.**

$$U(x_1, x_2) = x_1 + x_2.$$

- **The ordinary demand equations are**

# Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1,p_2,y) = \begin{cases} 0 & \text{, if } p_1 > p_2 \\ y/p_1 & \text{, if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1,p_2,y) = \begin{cases} 0 & \text{, if } p_1 < p_2 \\ y/p_2 & \text{, if } p_1 > p_2. \end{cases}$$

# Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ y / p_1 & , \text{if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 < p_2 \\ y / p_2 & , \text{if } p_1 > p_2. \end{cases}$$

**Suppose $p_1 < p_2$. Then**

# Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1,p_2,y) = \begin{cases} 0 & \text{, if } p_1 > p_2 \\ y/p_1 & \text{, if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1,p_2,y) = \begin{cases} 0 & \text{, if } p_1 < p_2 \\ y/p_2 & \text{, if } p_1 > p_2. \end{cases}$$

**Suppose $p_1 < p_2$. Then $x_1^* = \dfrac{y}{p_1}$ and $x_2^* = 0$**

# Income Changes and Perfectly-Substitutable Preferences

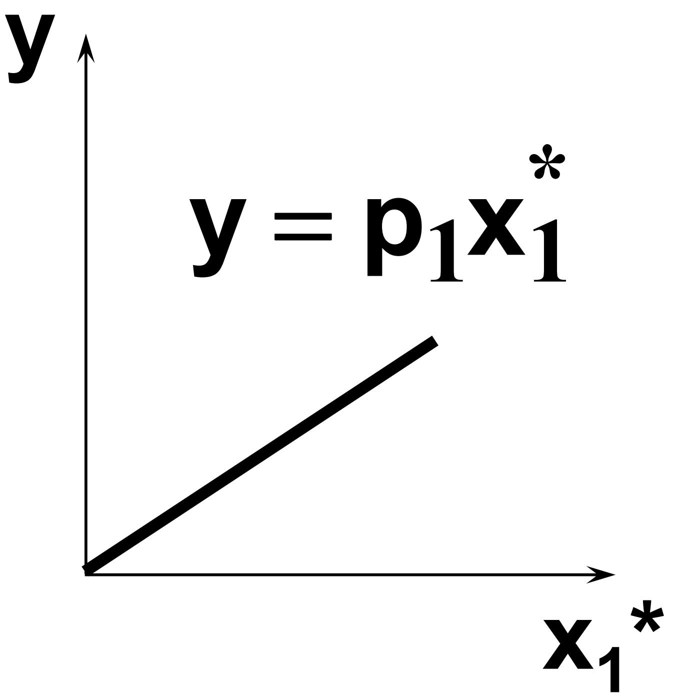
$$x_1^*(p_1,p_2,y) = \begin{cases} 0 & \text{, if } p_1 > p_2 \\ y/p_1 & \text{, if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1,p_2,y) = \begin{cases} 0 & \text{, if } p_1 < p_2 \\ y/p_2 & \text{, if } p_1 > p_2. \end{cases}$$

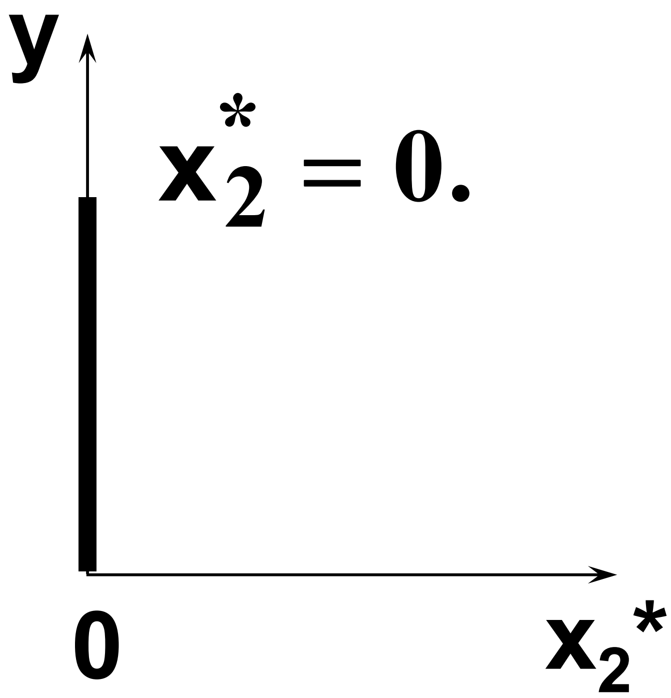
Suppose $p_1 < p_2$. Then $x_1^* = \dfrac{y}{p_1}$ and $x_2^* = 0$

➡ $y = p_1 x_1^*$ and $x_2^* = 0.$

# Income Changes and Perfectly-Substitutable Preferences




Engel curve
for good 1




Engel curve
for good 2

# Income Changes

- In every example so far the Engel curves have all been straight lines? Q: Is this true in general?
- A: No. Engel curves are straight lines if the consumer's preferences are homothetic.

# Homotheticity

- **A consumer's preferences are homothetic if and only if**

$$(x_1,x_2) \prec (y_1,y_2) \Leftrightarrow (kx_1,kx_2) \prec (ky_1,ky_2)$$

**for every k > 0.**

- **That is, the consumer's MRS is the same anywhere on a straight line drawn from the origin.**

# Income Effects -- A Nonhomothetic Example

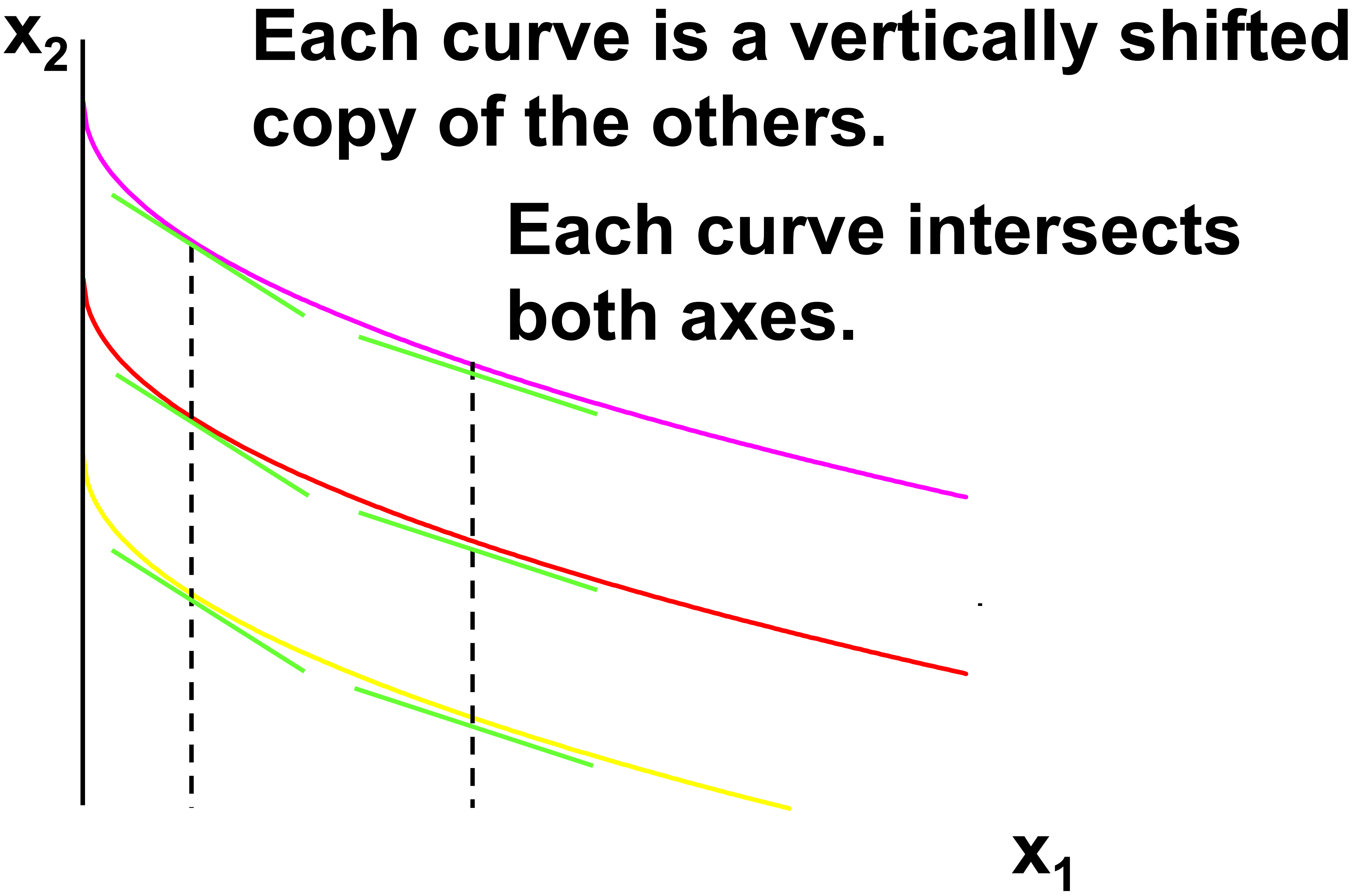
- Quasilinear preferences are not homothetic.

$$U(x_1, x_2) = f(x_1) + x_2.$$

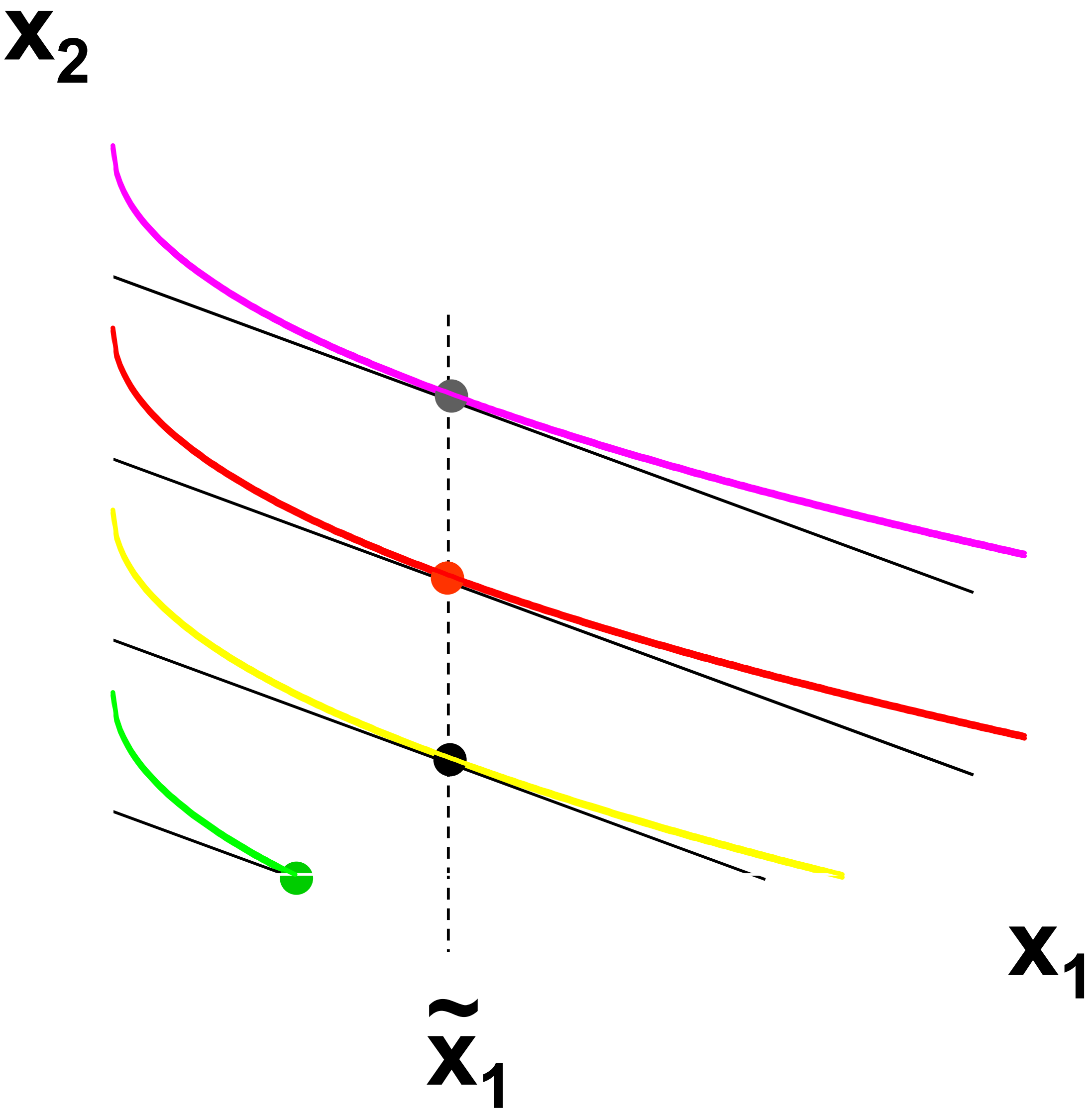
- For example,

$$U(x_1, x_2) = \sqrt{x_1} + x_2.$$

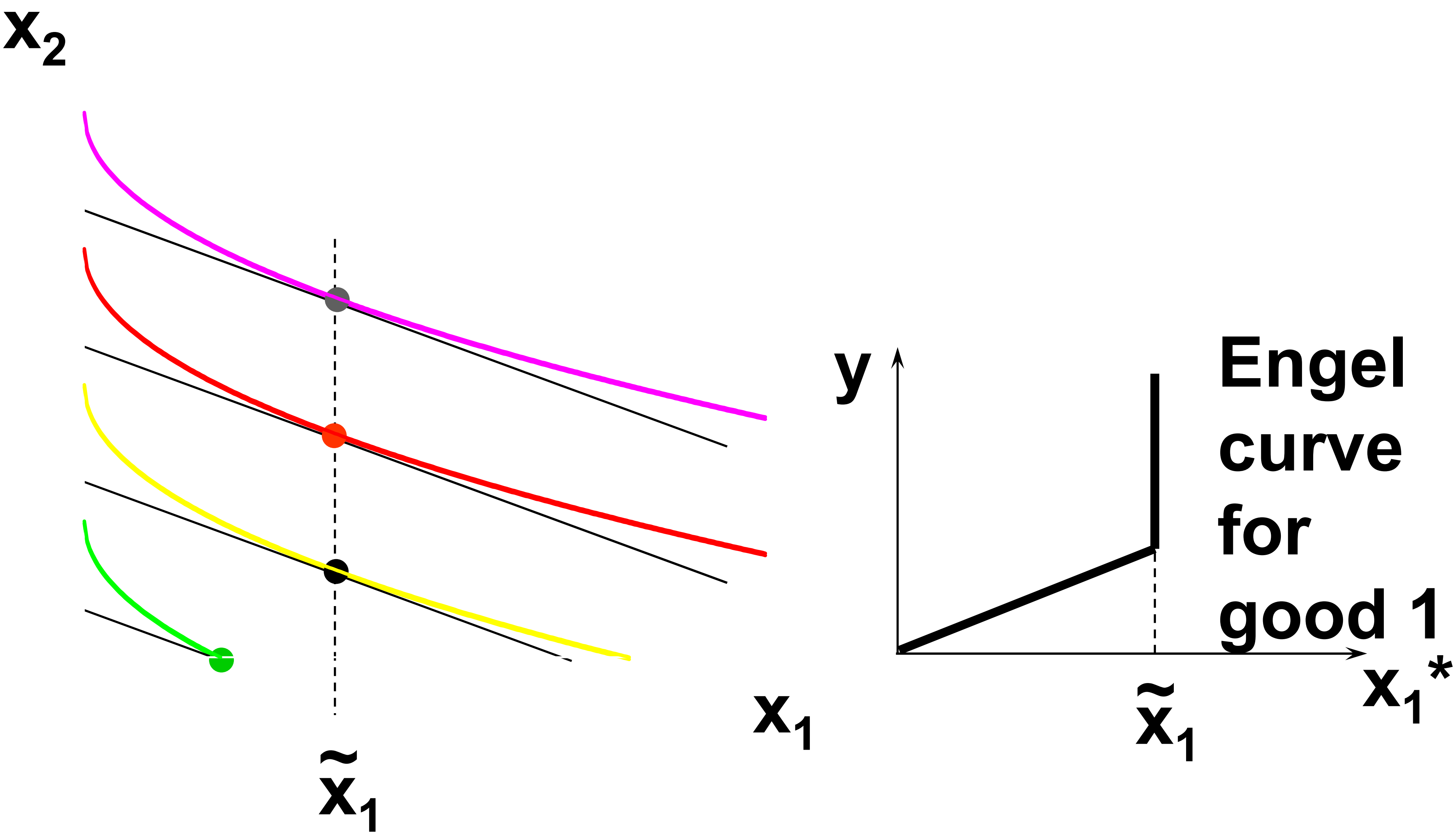
# Quasi-linear Indifference Curves



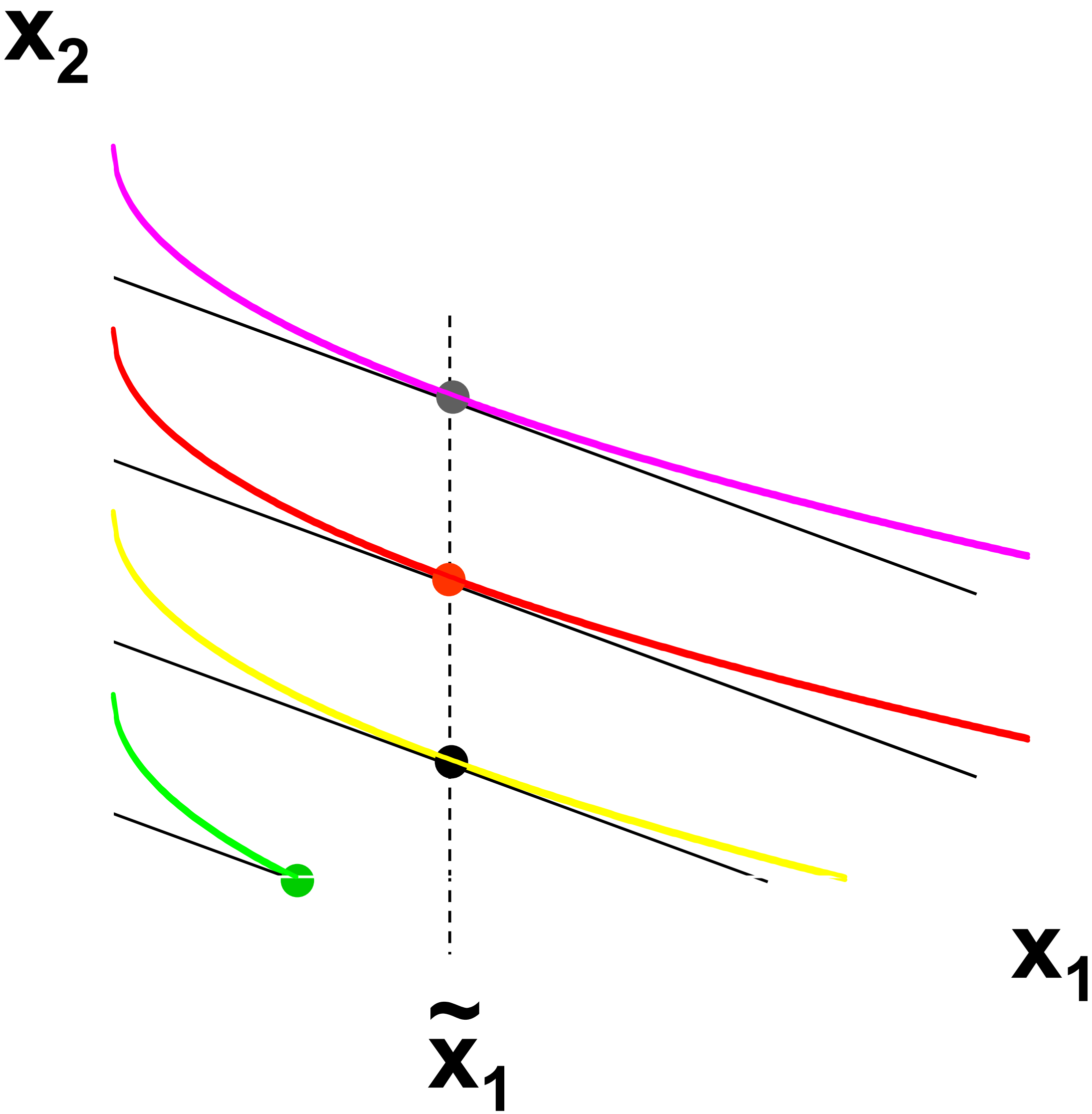
# Income Changes; Quasilinear Utility

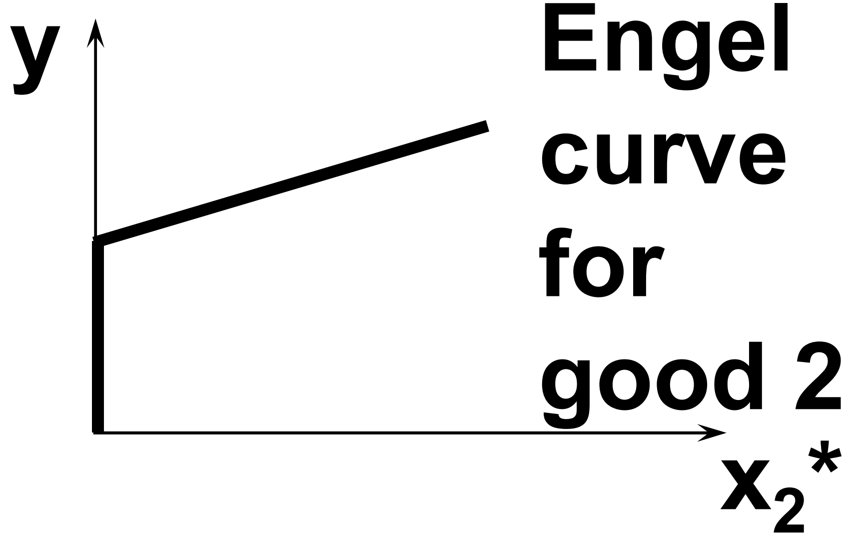


# Income Changes; Quasilinear Utility

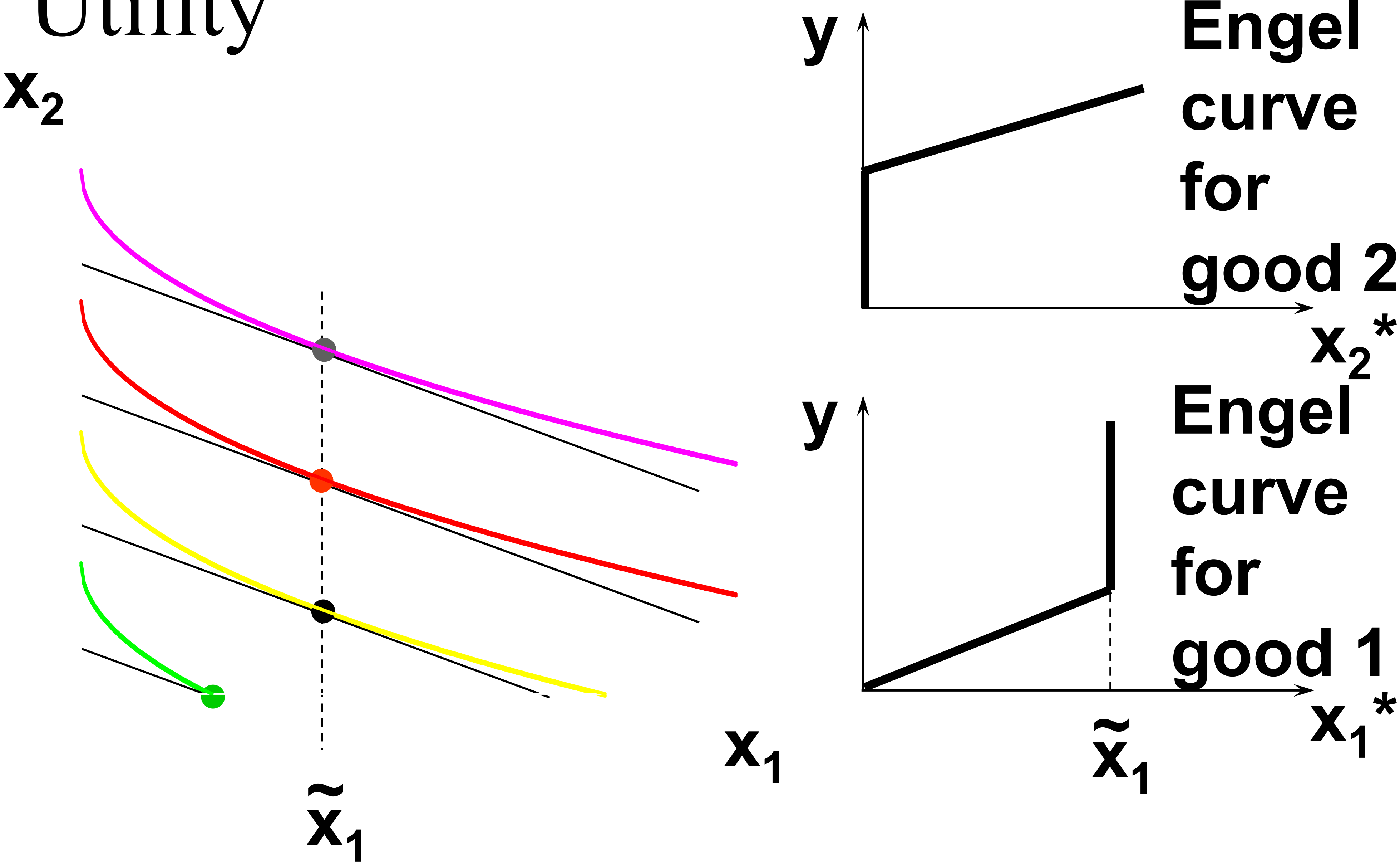


# Income Changes; Quasilinear Utility



# Income Changes; Quasilinear Utility



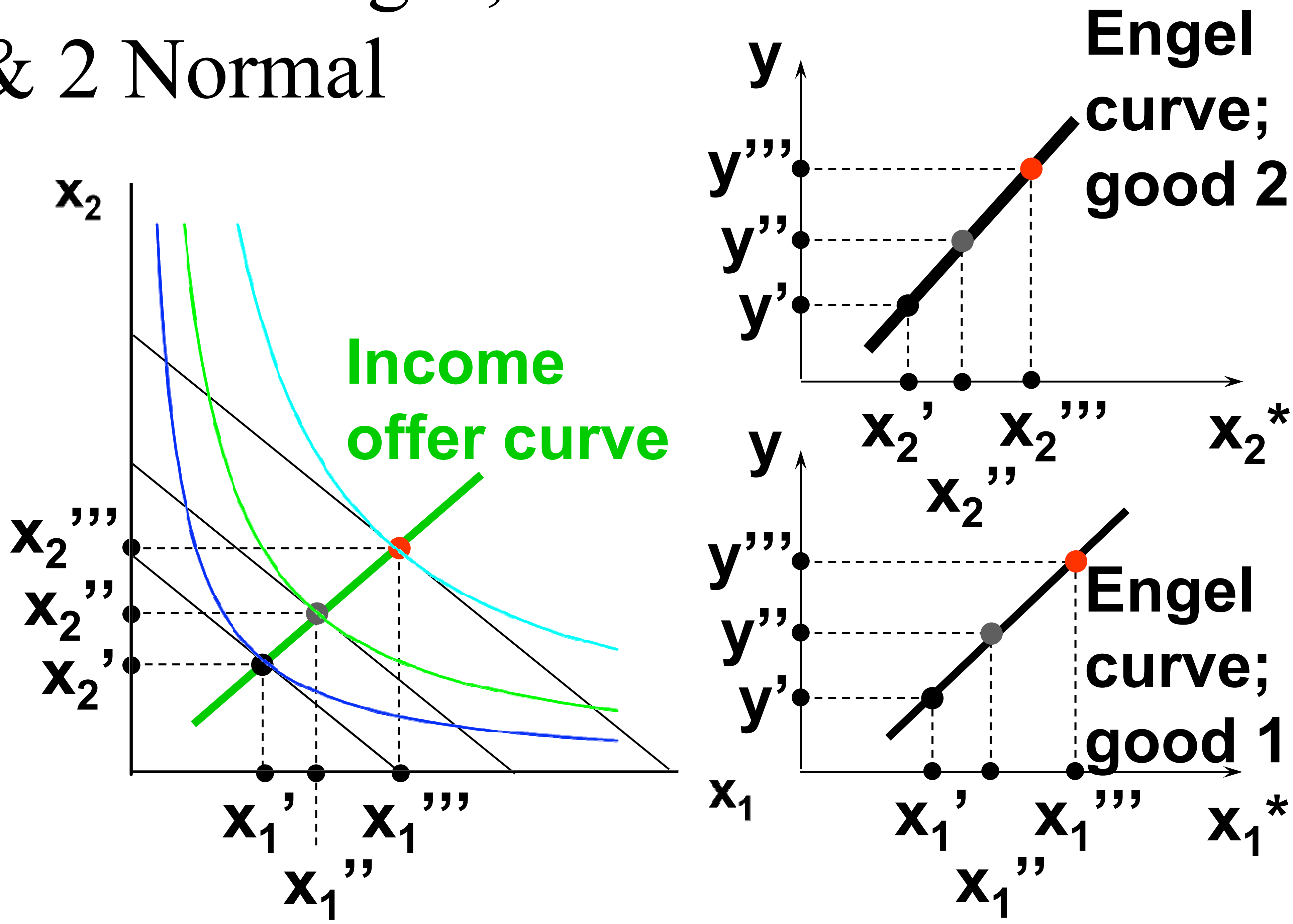
# Income Effects

- A good for which quantity demanded rises with income is called normal.
- Therefore a normal good's Engel curve is positively sloped.

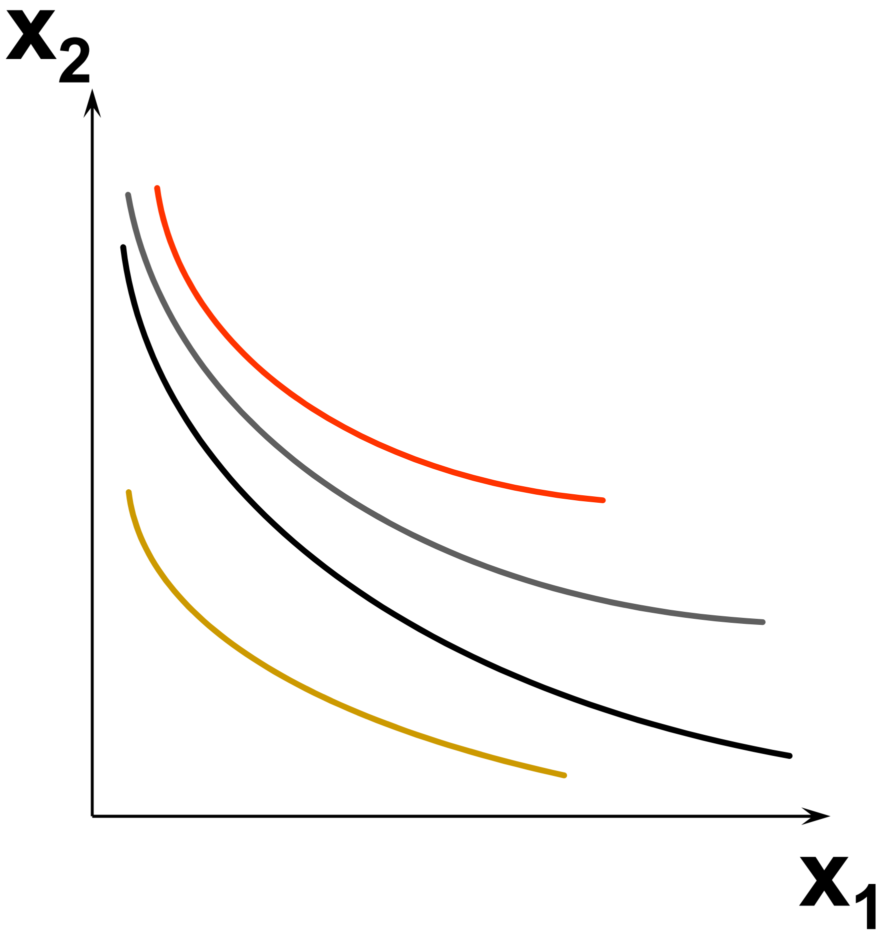
# Income Effects

- **A good for which quantity demanded falls as income increases is called income inferior.**
- **Therefore an income inferior good's Engel curve is negatively sloped.**

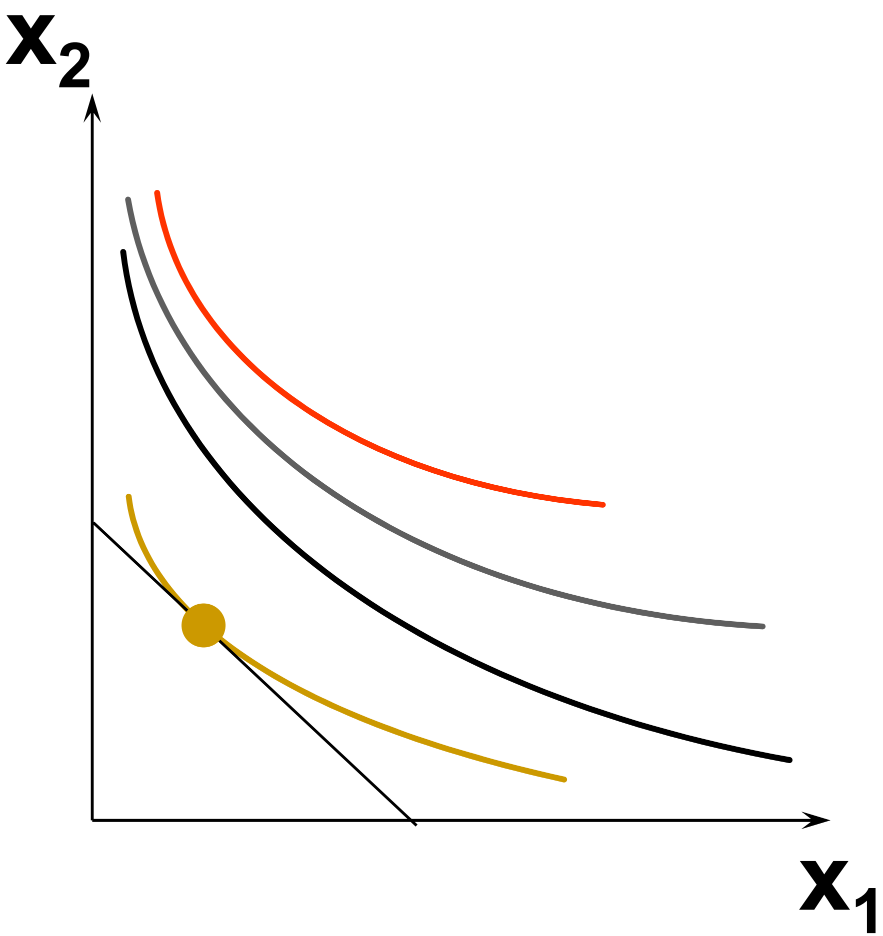
# Income Changes; Goods 1 & 2 Normal



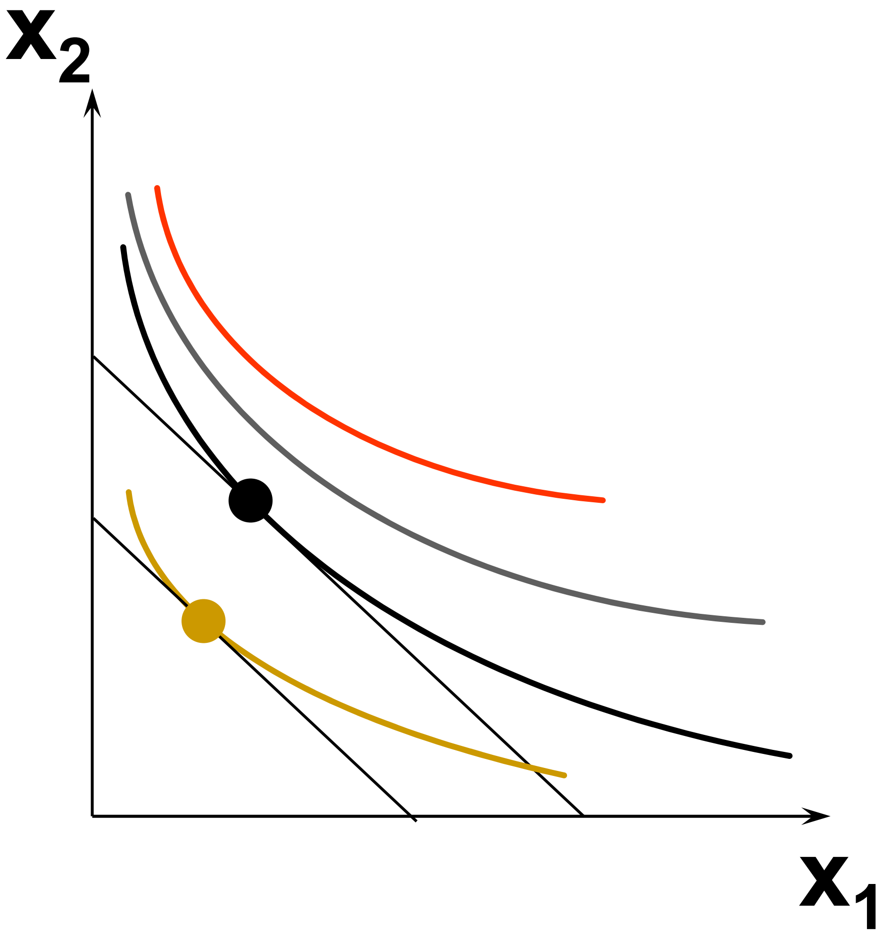
# Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



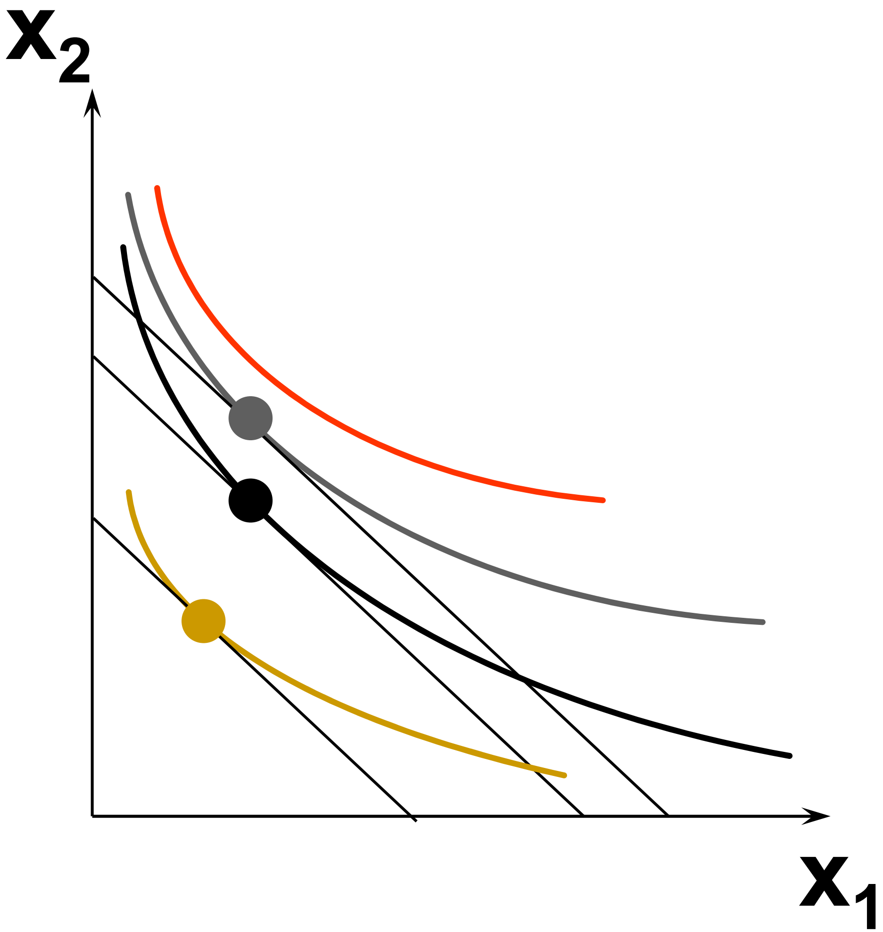
# Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



# Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



# Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



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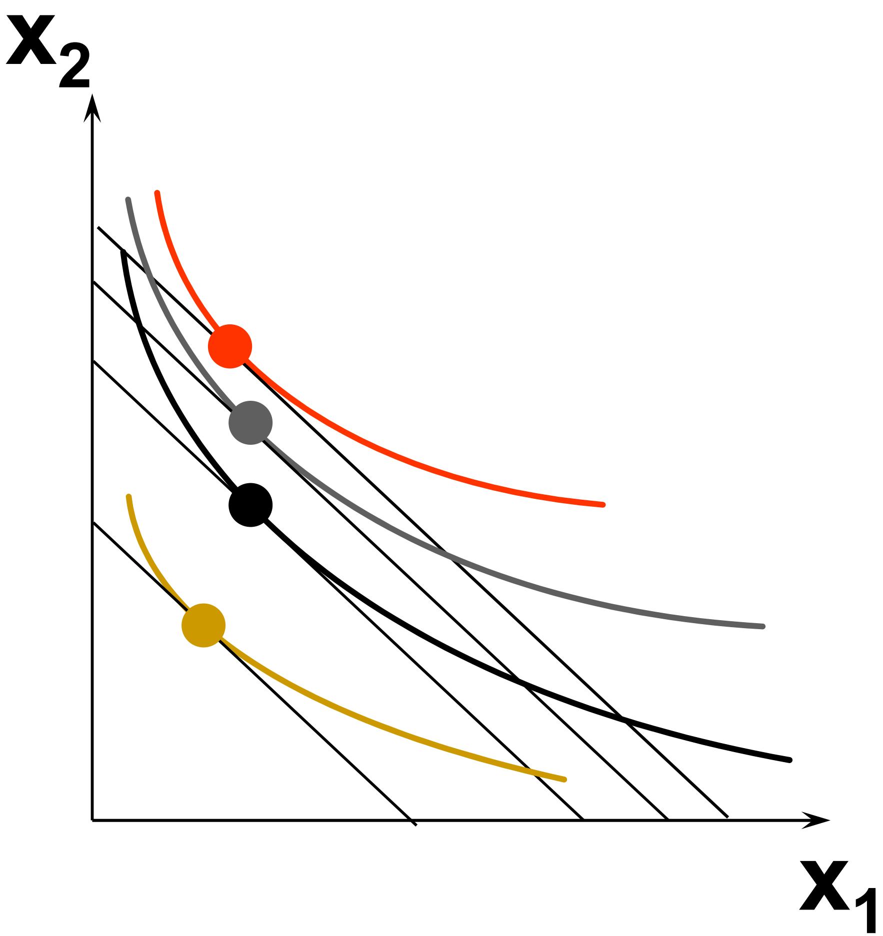
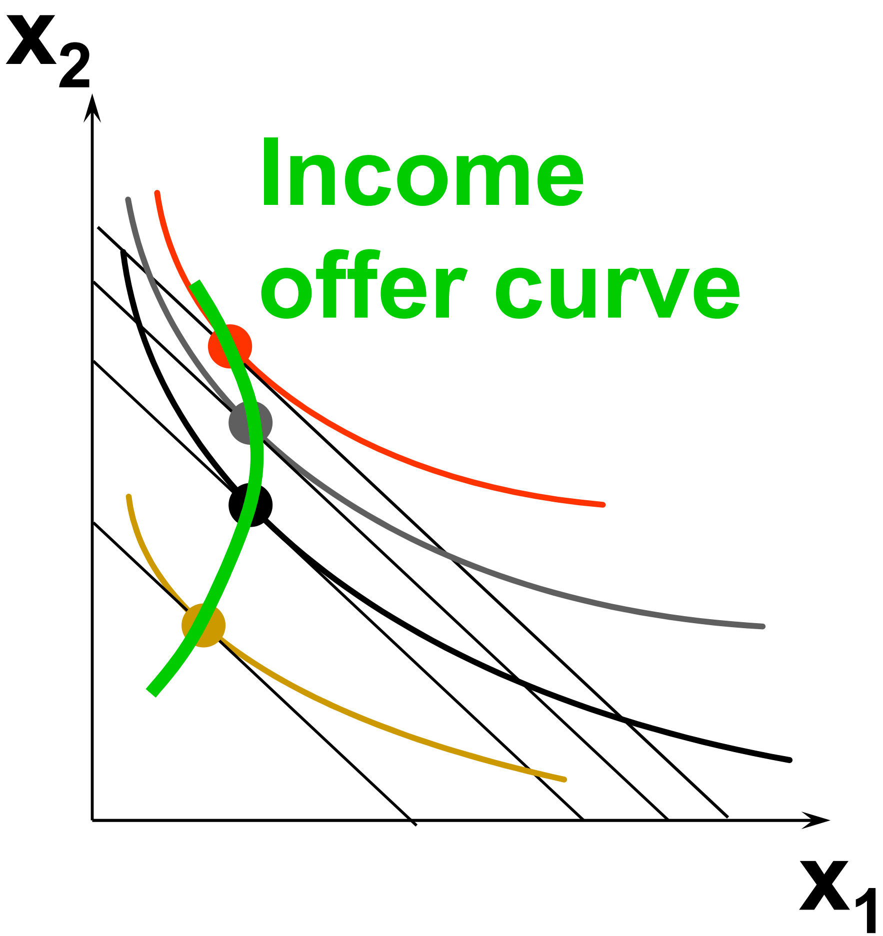
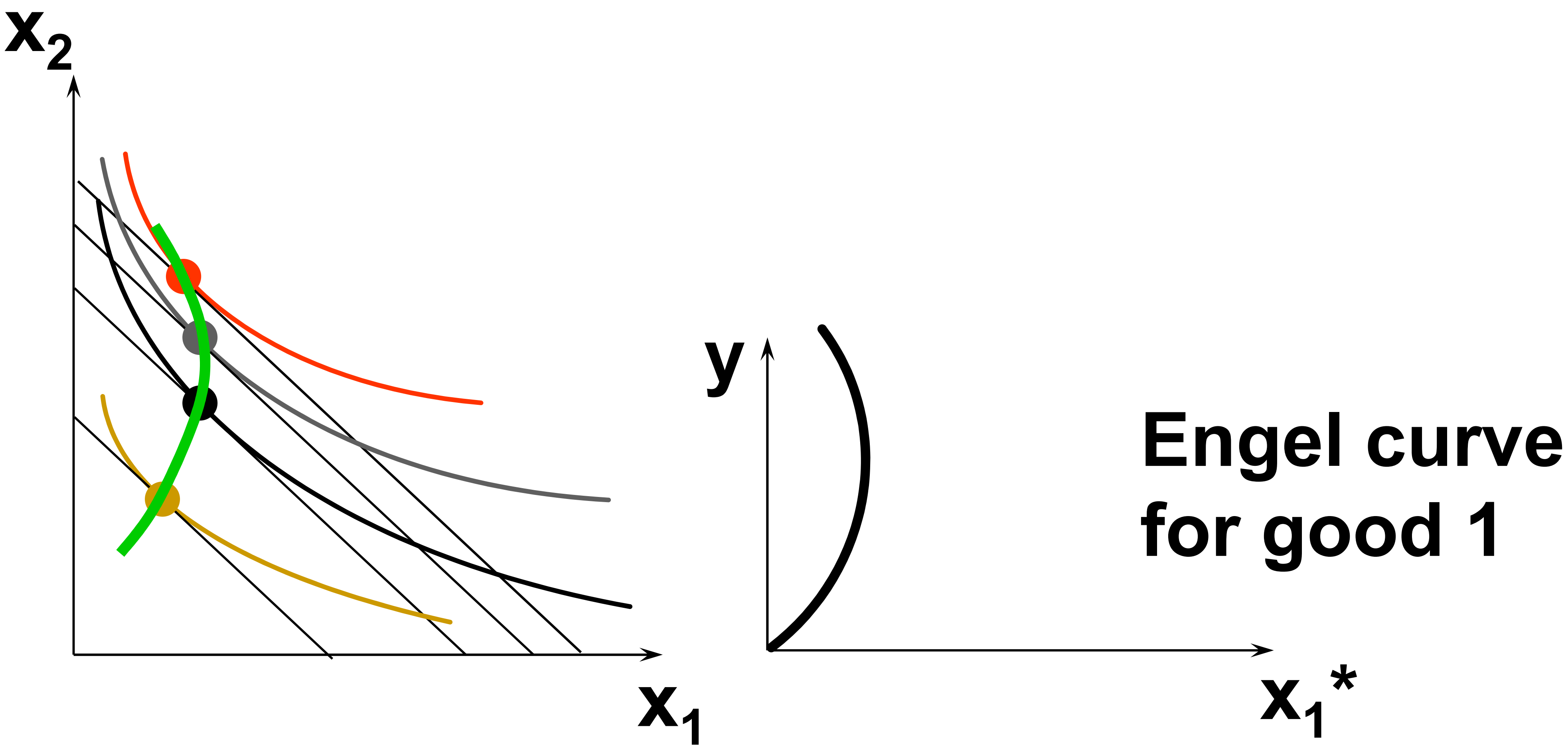


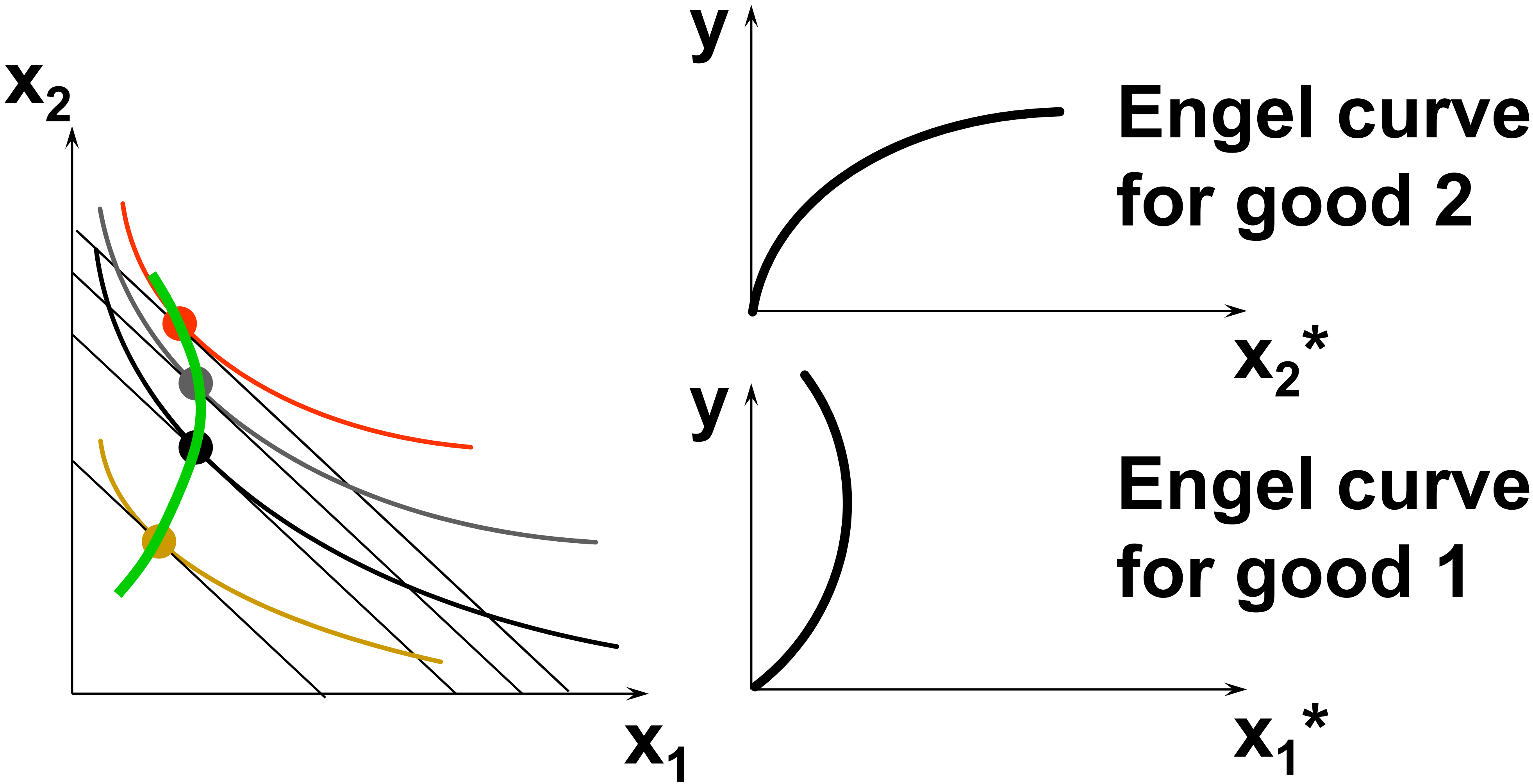
# Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



# Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



# Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior

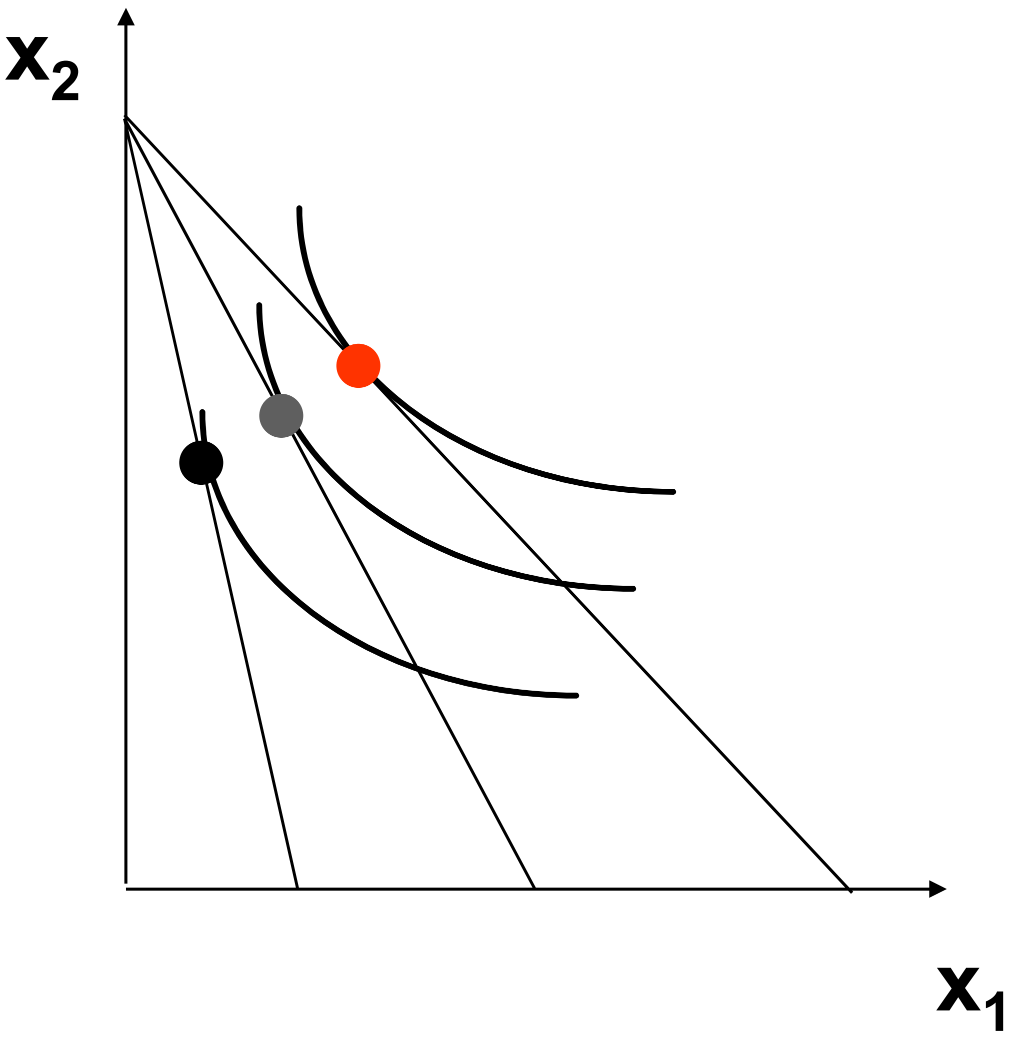


# Ordinary Goods

- A good is called ordinary if the quantity demanded of it always increases as its own price decreases.

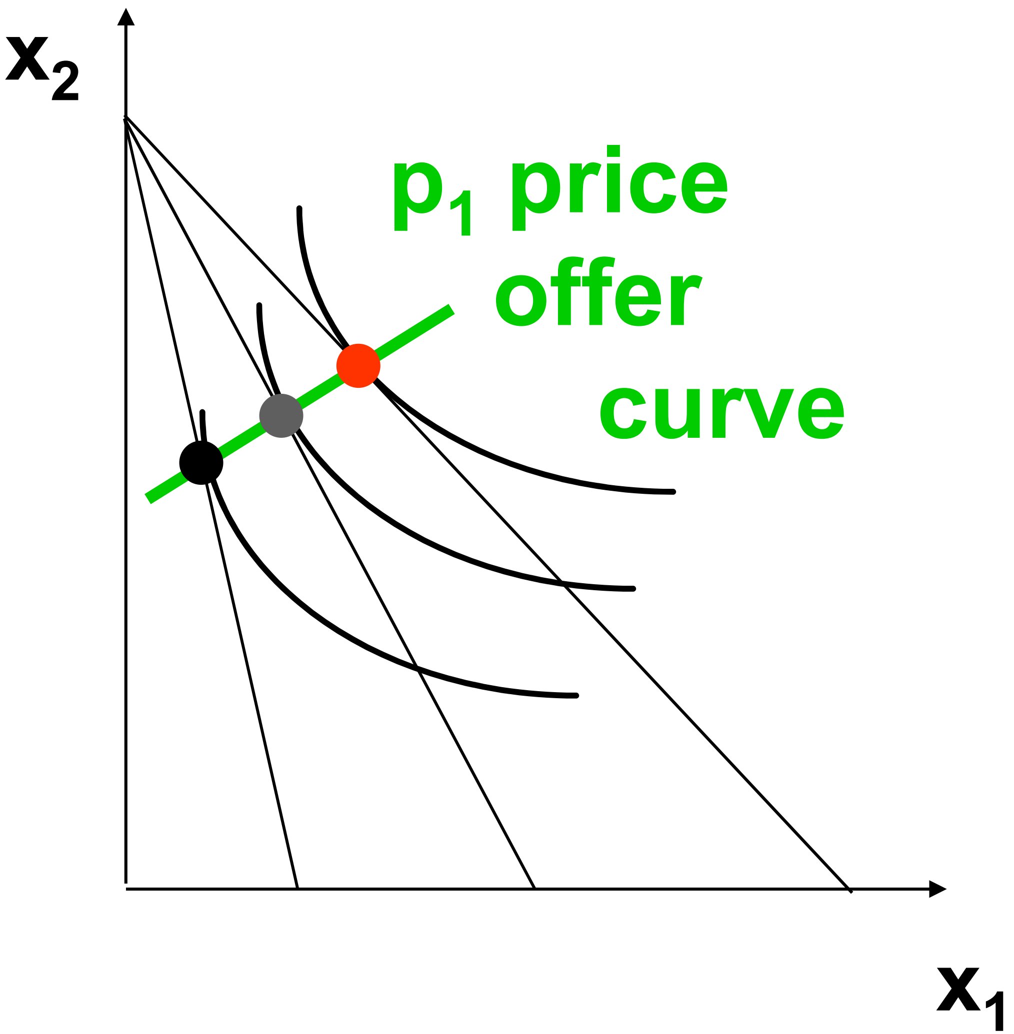
# Ordinary Goods

**Fixed $p_2$ and y.**

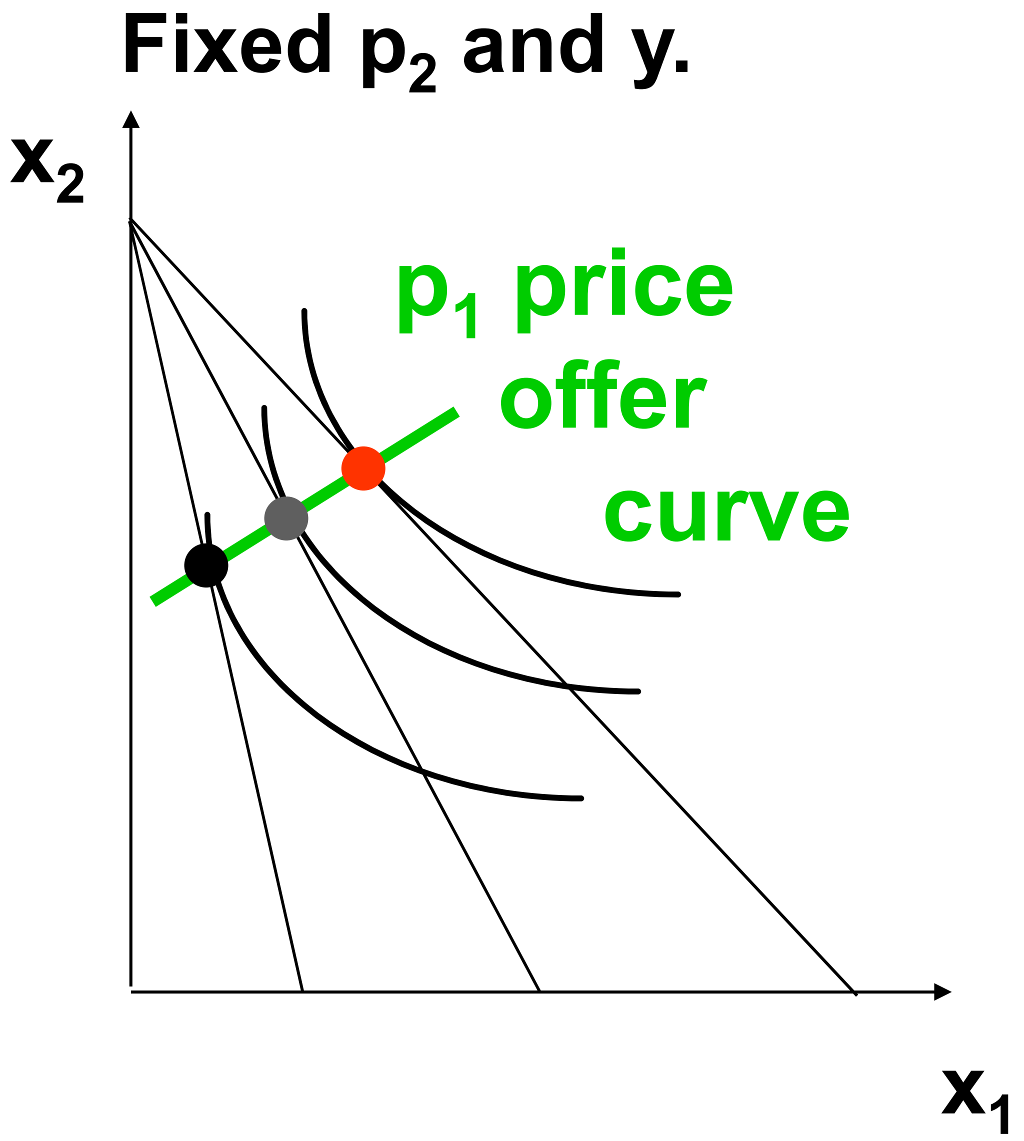


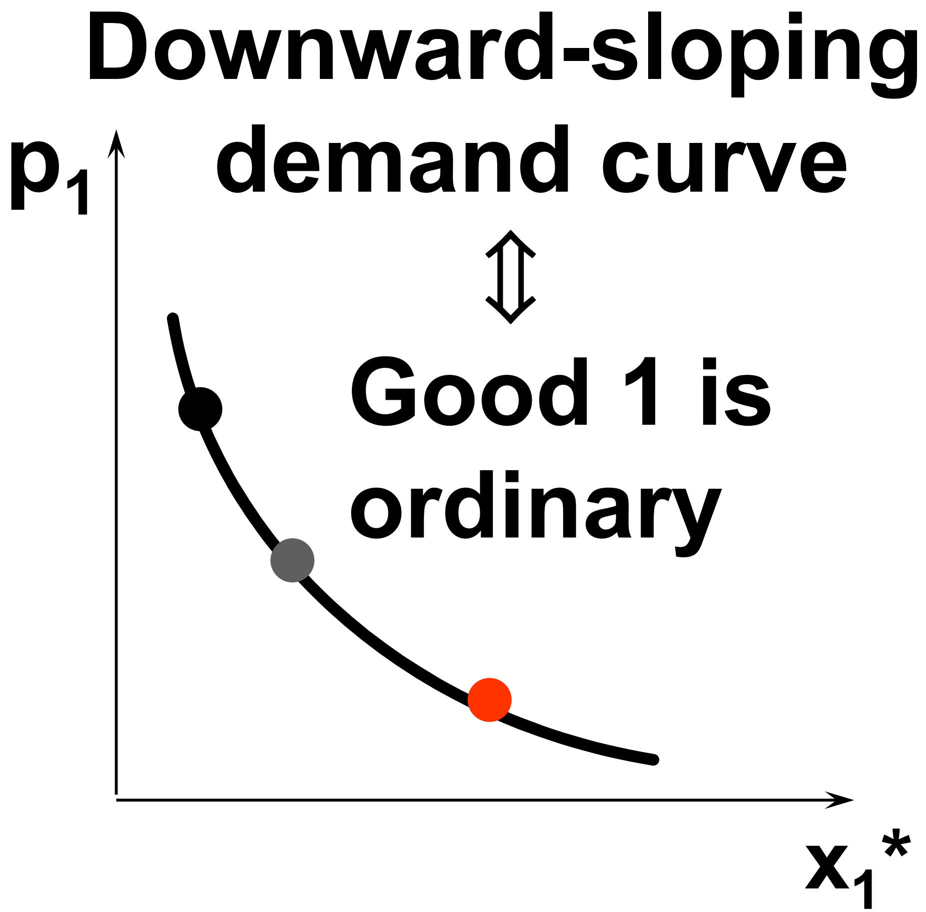
# Ordinary Goods

**Fixed $p_2$ and y.**



# Ordinary Goods



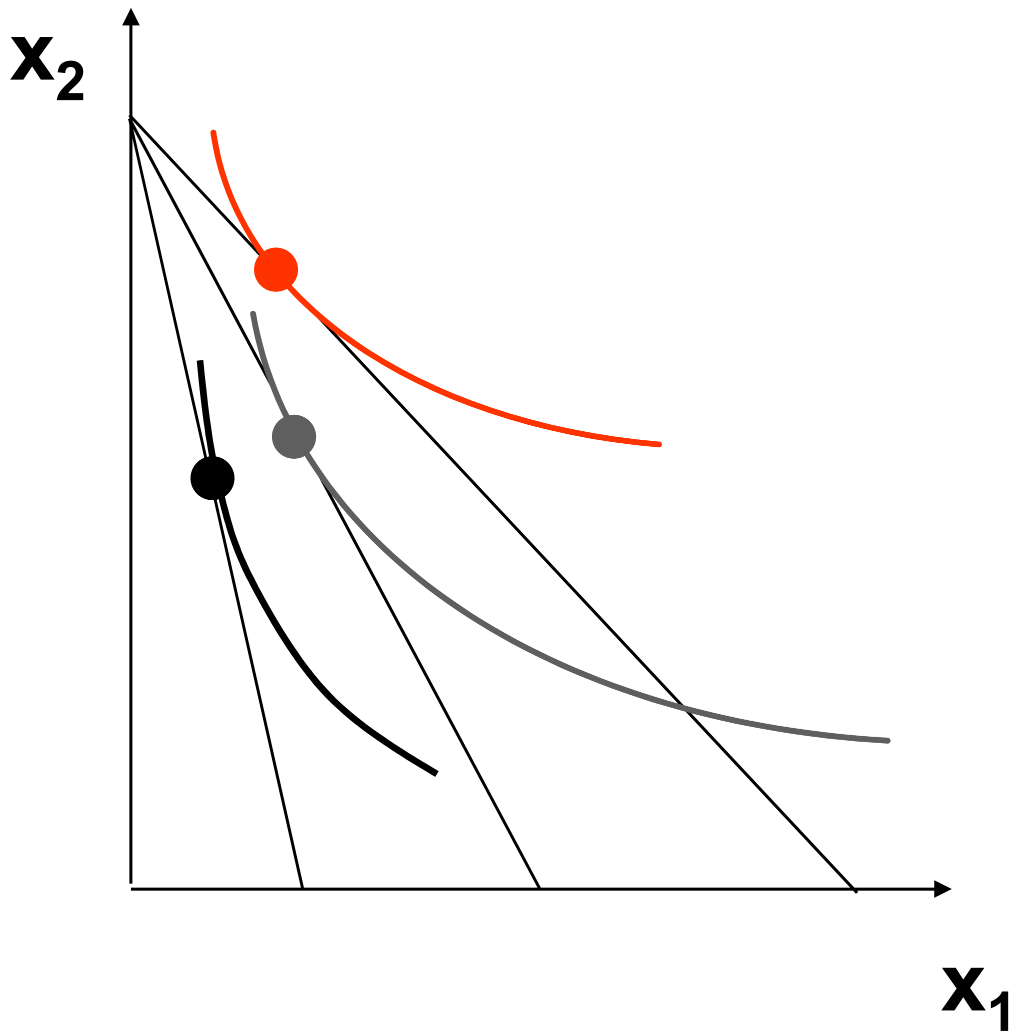



# Giffen Goods

- If, for some values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called Giffen.

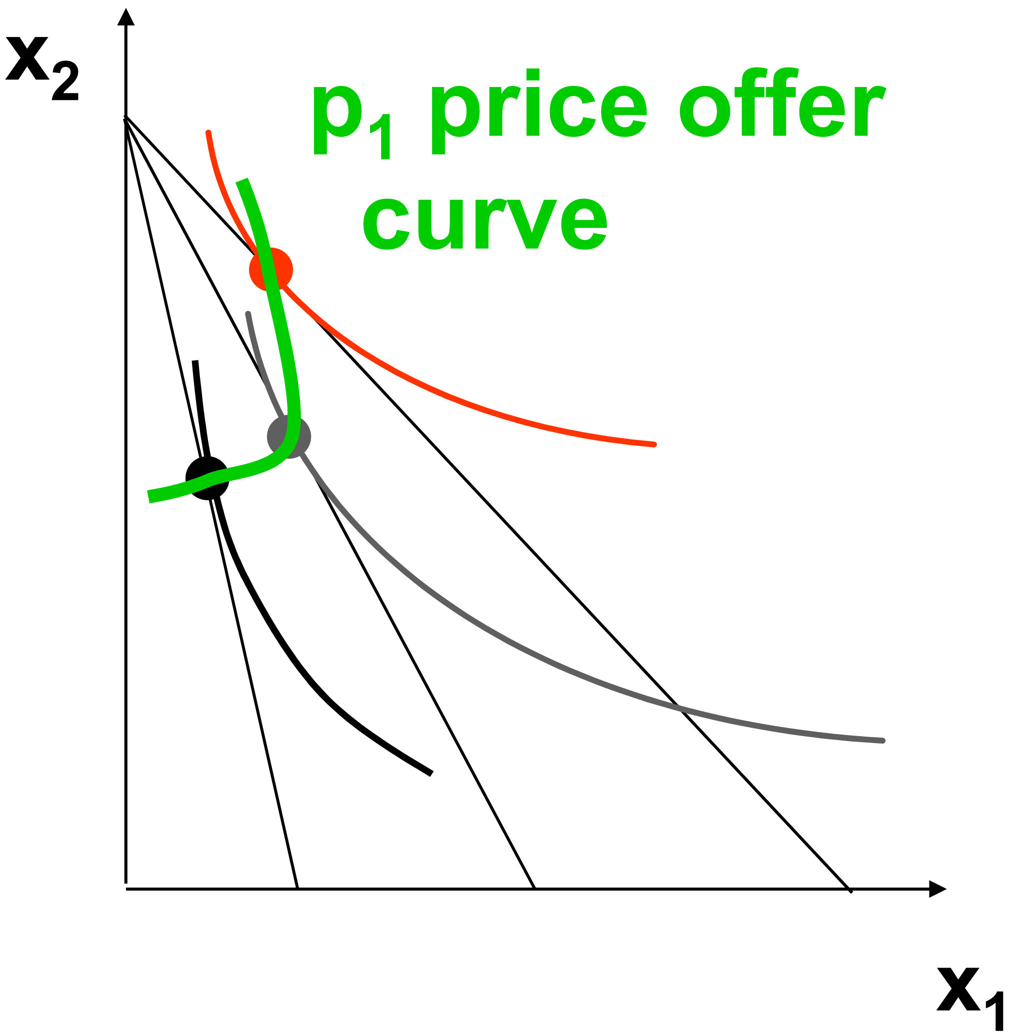
# Ordinary Goods

**Fixed $p_2$ and y.**



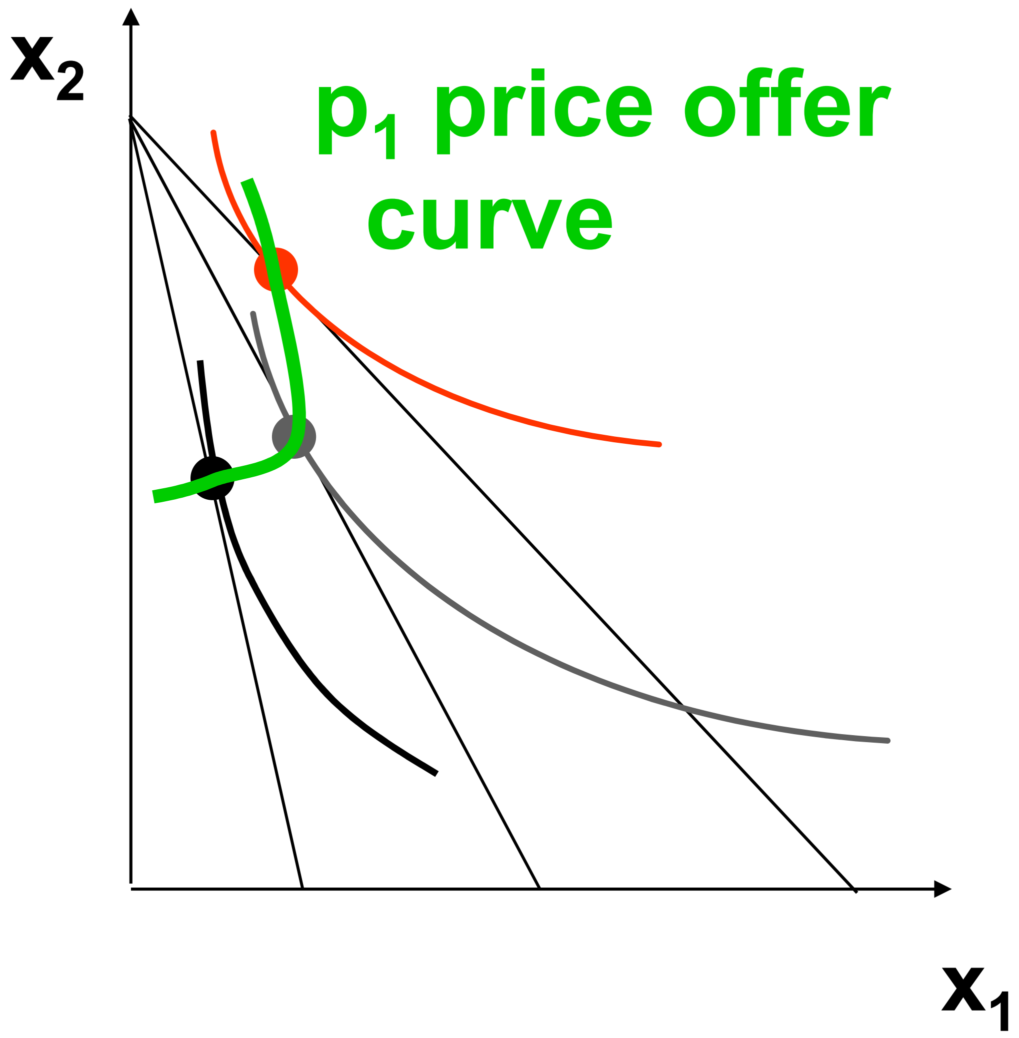
# Ordinary Goods

**Fixed $p_2$ and y.**



# Ordinary Goods

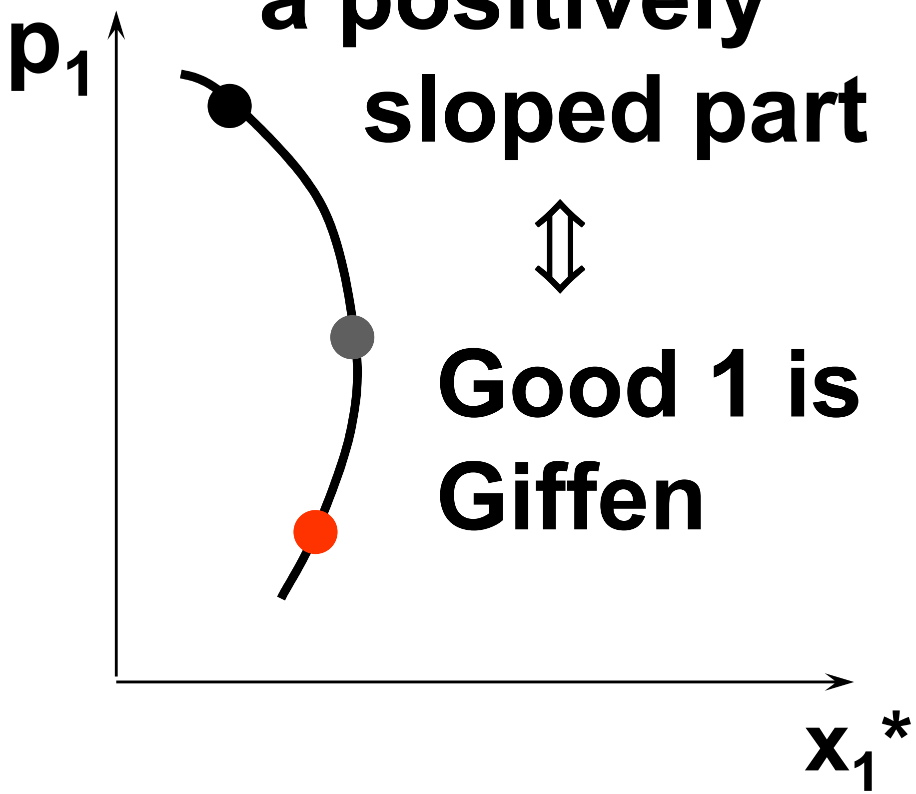
Fixed $p_2$ and y.




Demand curve has a positively sloped part

⇕

Good 1 is Giffen



# Cross-Price Effects

- **If an increase in $p_2$**
  - **increases demand for commodity 1 then commodity 1 is a gross substitute for commodity 2.**
  - **reduces demand for commodity 1 then commodity 1 is a gross complement for commodity 2.**

# Cross-Price Effects

**A perfect-complements example:**

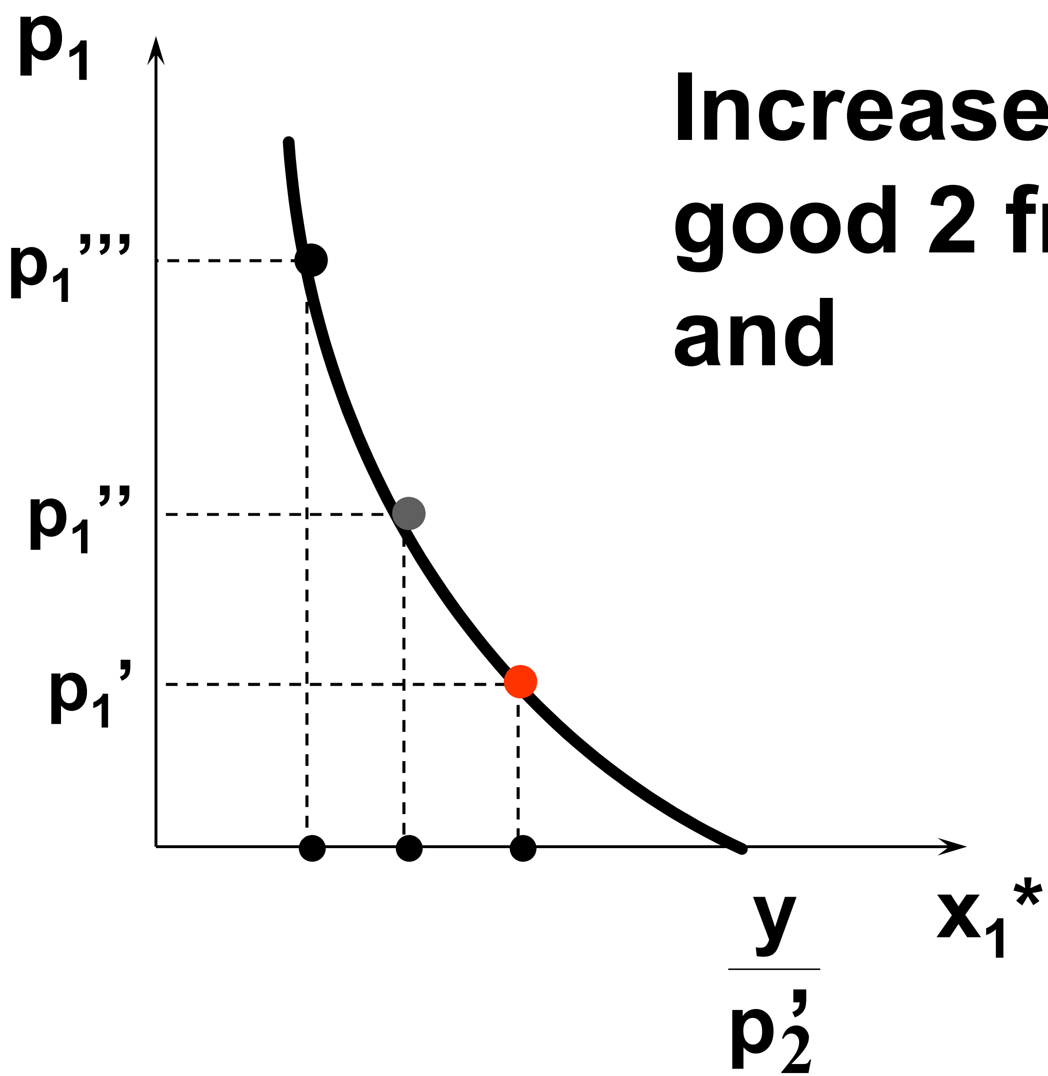
$$x_1^* = \frac{y}{p_1 + p_2}$$

**so**

$$\frac{\partial x_1^*}{\partial p_2} = -\frac{y}{(p_1 + p_2)^2} < 0.$$

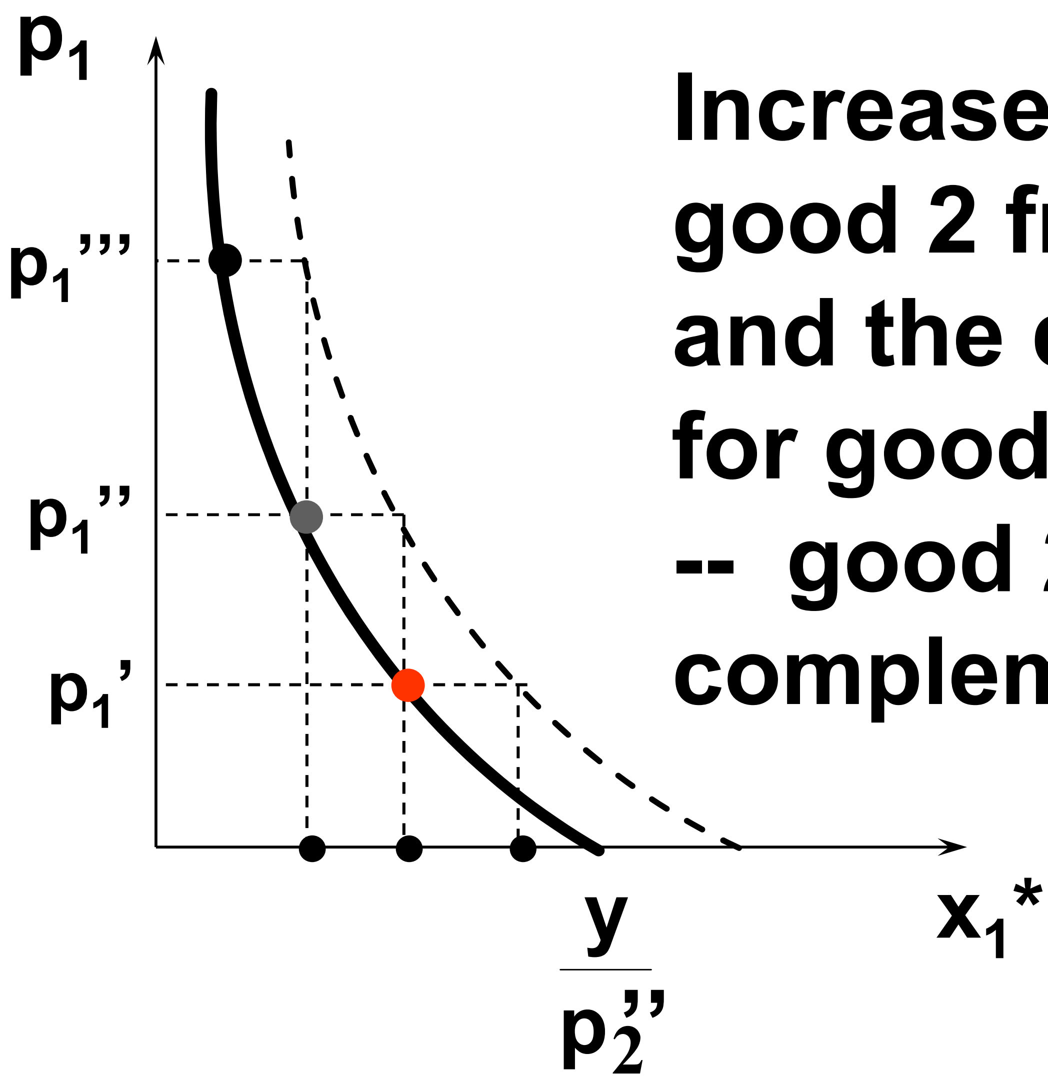
**Therefore commodity 2 is a gross complement for commodity 1.**

# Cross-Price Effects




**Increase the price of good 2 from $p_2$' to $p_2$'' and**

# Cross-Price Effects




**Increase the price of good 2 from $p_2'$ to $p_2''$ and the demand curve for good 1 shifts inwards -- good 2 is a complement for good 1.**

# Cross-Price Effects

**A Cobb- Douglas example:**

$$x_2^* = \frac{by}{(a+b)p_2}$$

**so**

# Cross-Price Effects

**A Cobb- Douglas example:**

$$x_2^* = \frac{by}{(a+b)p_2}$$

**so**

$$\frac{\partial x_2^*}{\partial p_1} = 0.$$

**Therefore commodity 1 is neither a gross complement nor a gross substitute for commodity 2.**