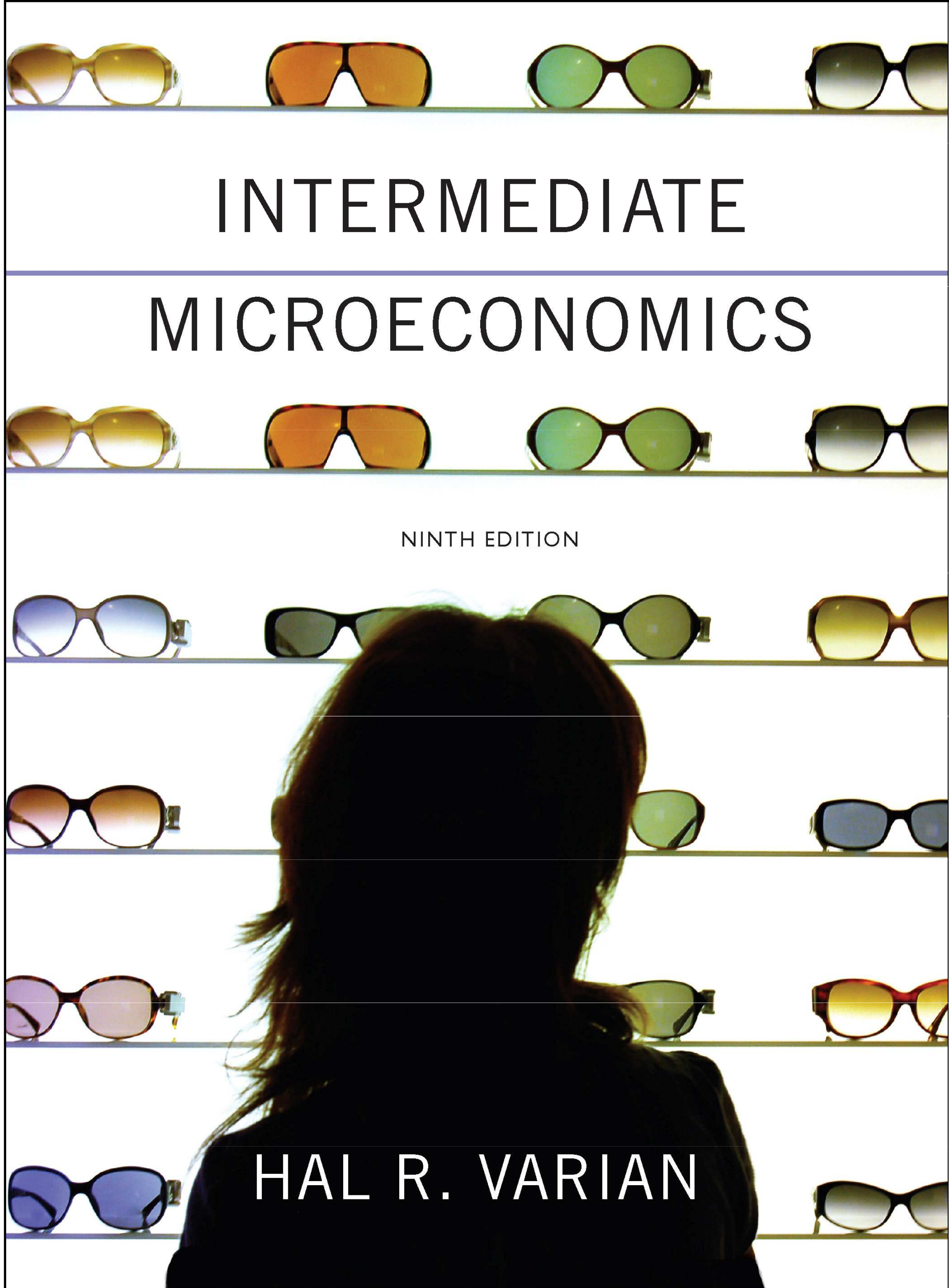



# Chapter 15

# Market Demand

# From Individual to Market Demand Functions

- **Think of an economy containing n consumers, denoted by i = 1, … ,n.**
- **Consumer i's ordinary demand function for commodity j is**

$$x_j^{*i}(p_1, p_2, m^i)$$

# From Individual to Market Demand Functions

- **When all consumers are price-takers, the market demand function for commodity j is**

$$X_j(p_1, p_2, m^1, \cdots, m^n) = \sum_{i=1}^{n} x_j^{*i}(p_1, p_2, m^i).$$

- **If all consumers are identical then**

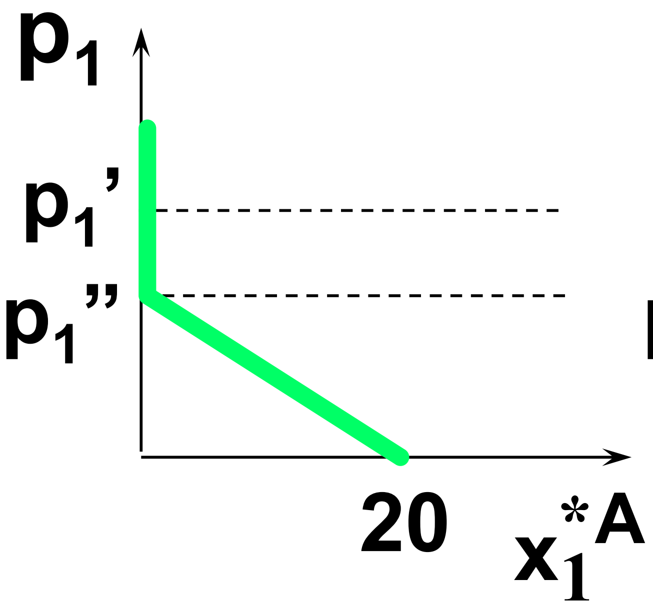
$$X_j(p_1, p_2, M) = n \times x_j^*(p_1, p_2, m)$$

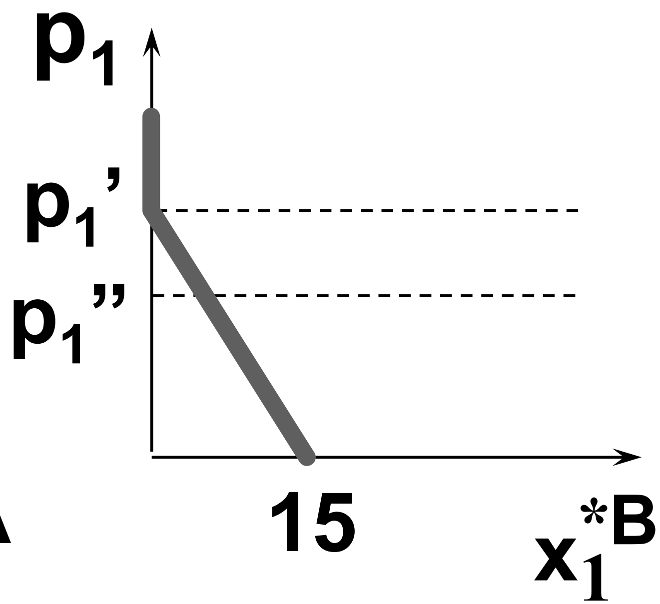
**where M = nm.**

# From Individual to Market Demand Functions

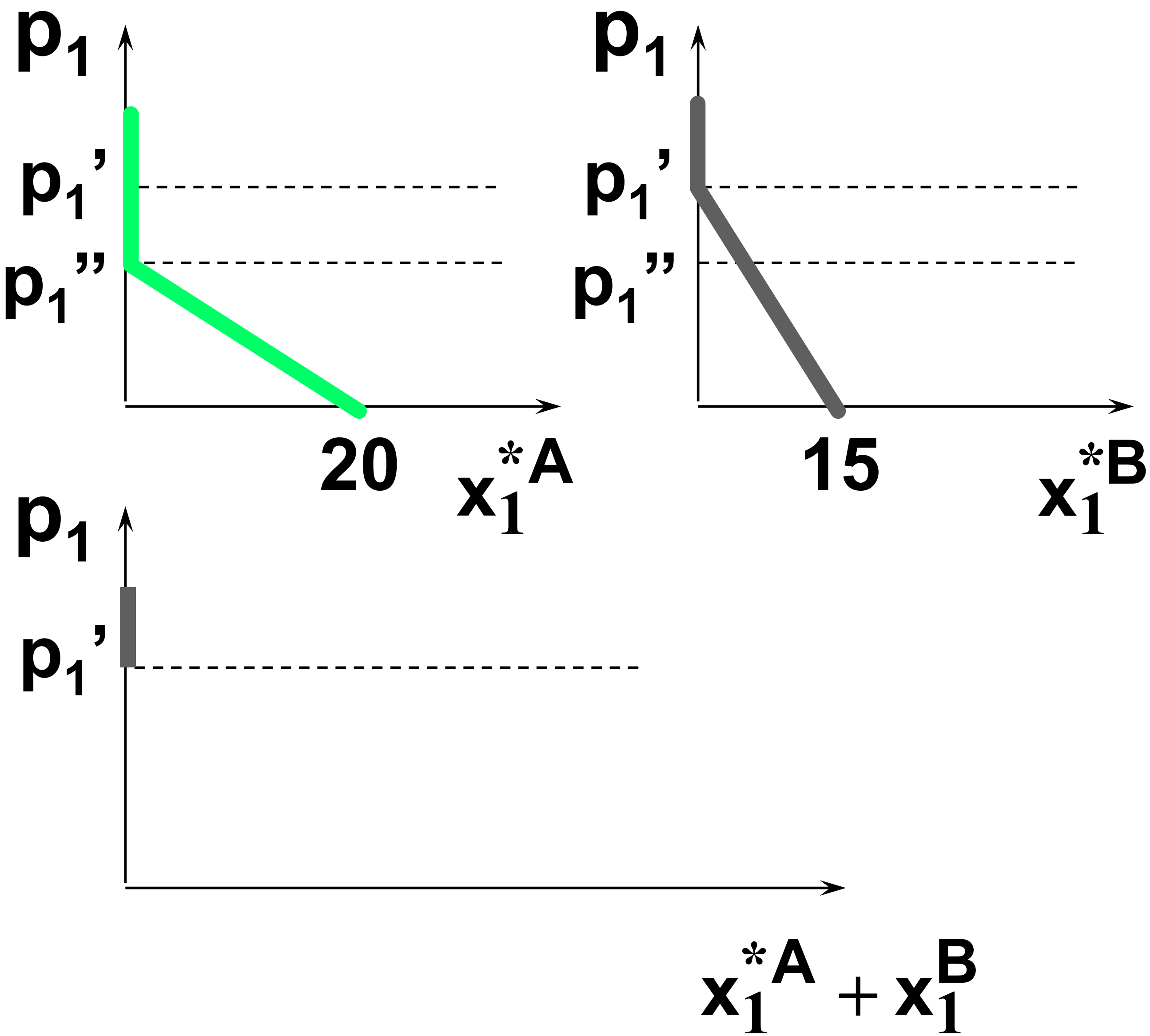
- **The market demand curve is the “horizontal sum” of the individual consumers’ demand curves.**
- **E.g. suppose there are only two consumers; i = A,B.**

# From Individual to Market Demand Functions

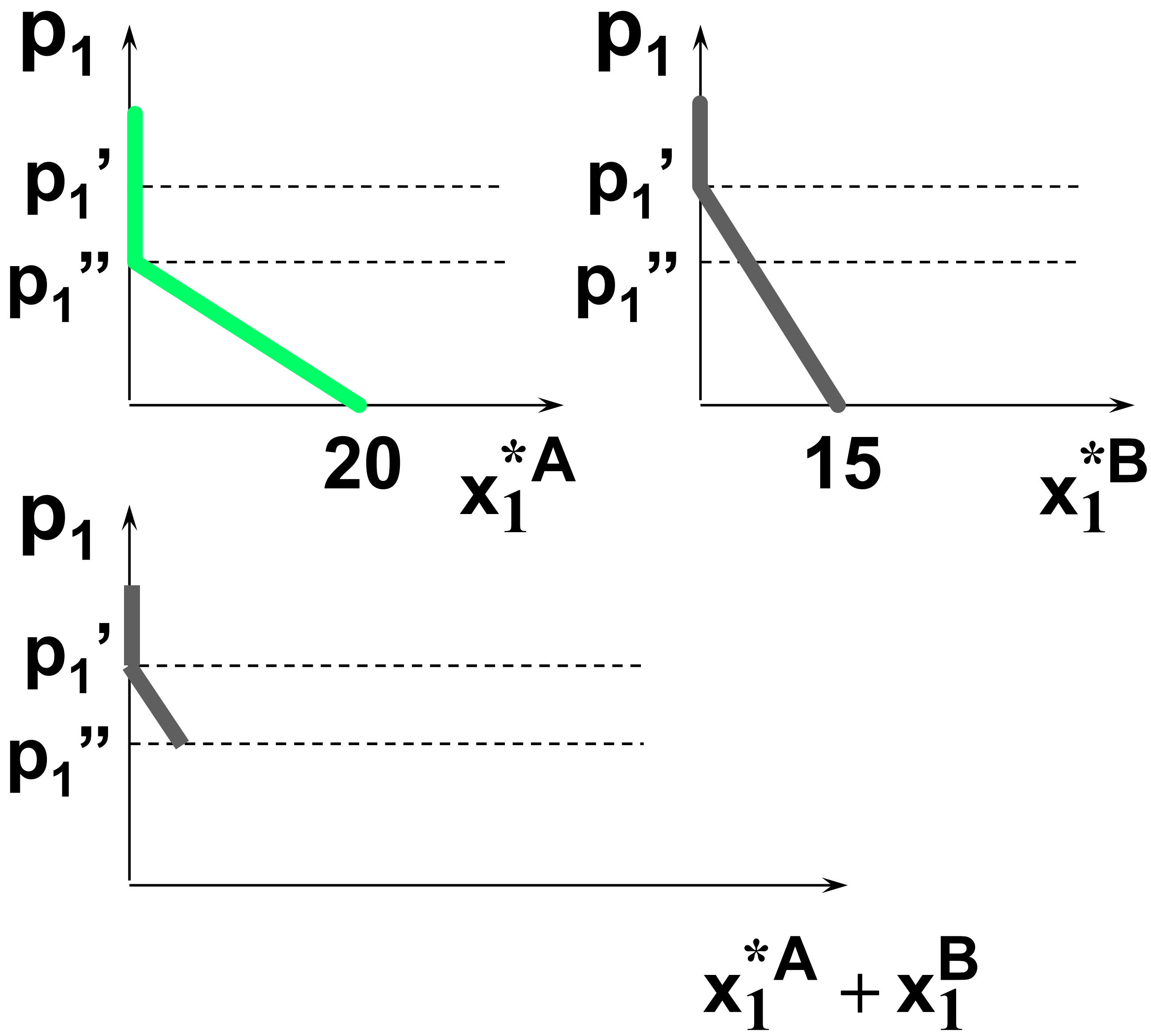


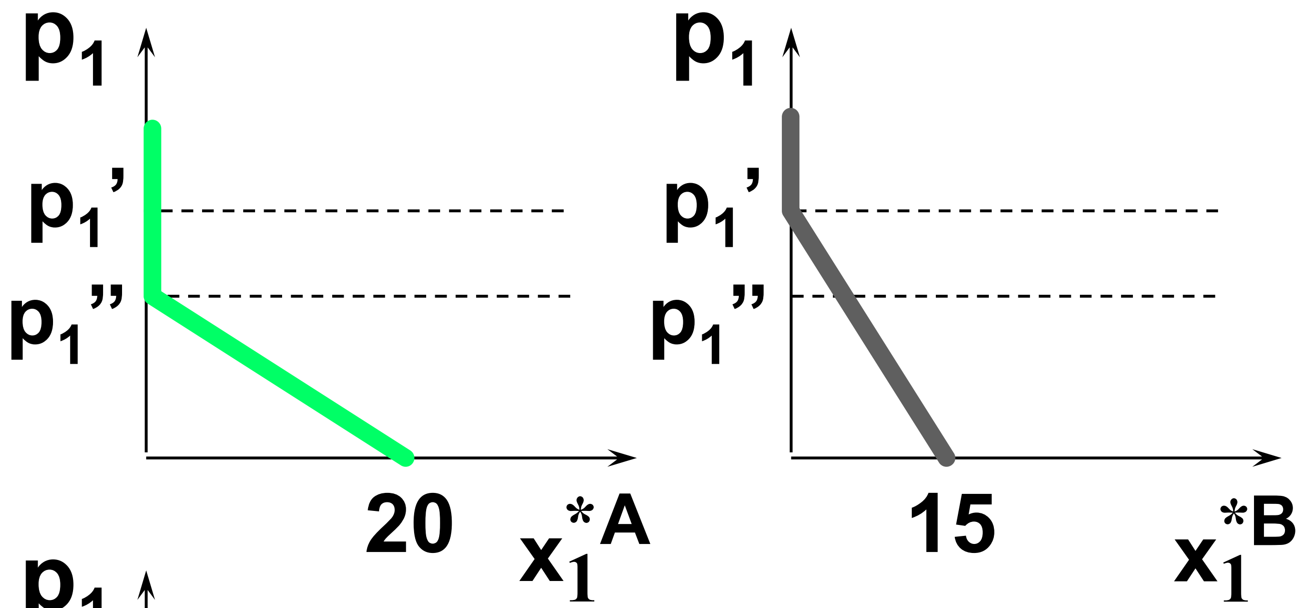
# From Individual to Market Demand Functions

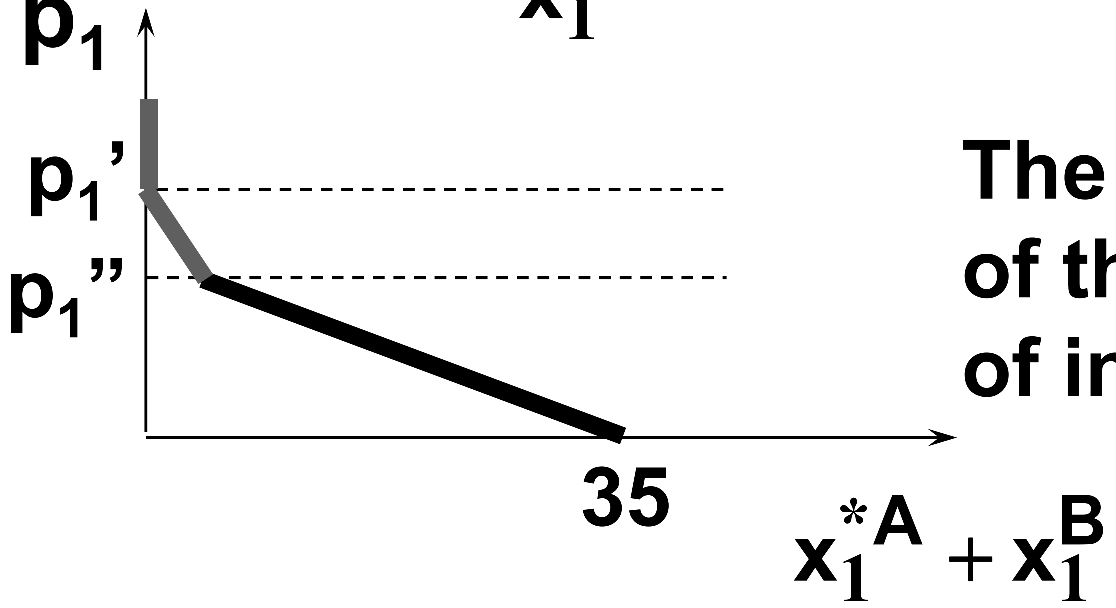


# From Individual to Market Demand Functions



# From Individual to Market Demand Functions




**The "horizontal sum" of the demand curves of individuals A and B.**

# Elasticities

- **Elasticity measures the "sensitivity" of one variable with respect to another.**
- **The elasticity of variable X with respect to variable Y is**

$$\varepsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y}.$$

# Economic Applications of Elasticity

- **Economists use elasticities to measure the sensitivity of**
  - **quantity demanded of commodity i with respect to the price of commodity i (own-price elasticity of demand)**
  - **demand for commodity i with respect to the price of commodity j (cross-price elasticity of demand).**

# Economic Applications of Elasticity

- **demand for commodity i with respect to income (income elasticity of demand)**
- **quantity supplied of commodity i with respect to the price of commodity i (own-price elasticity of supply)**

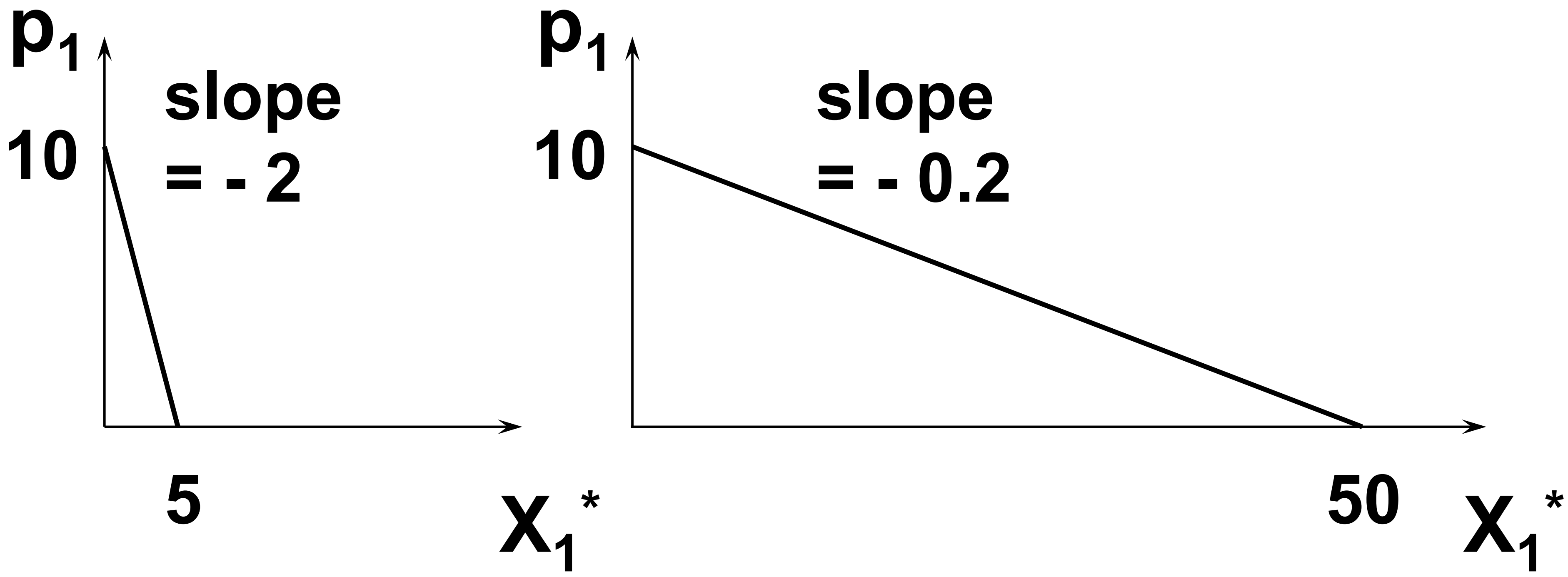
# Economic Applications of Elasticity

- **quantity supplied of commodity i with respect to the wage rate (elasticity of supply with respect to the price of labor)**
- **and many, many others.**

# Own-Price Elasticity of Demand

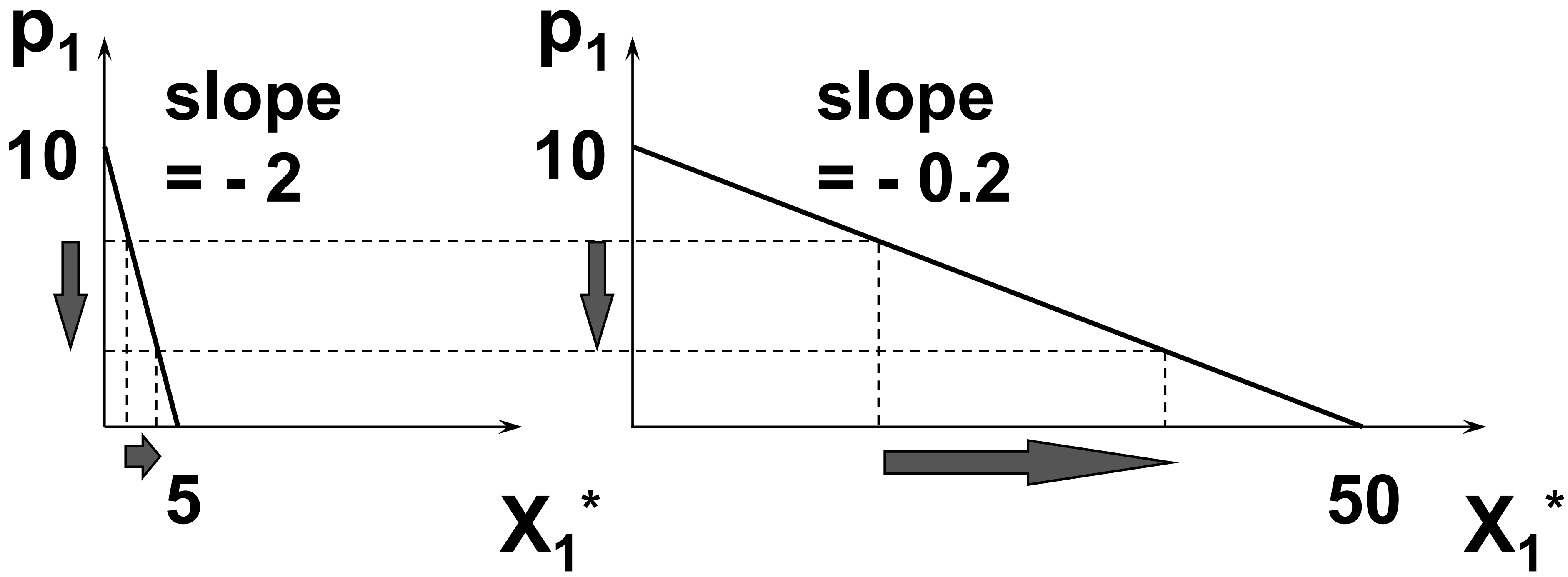
- **Q: Why not use a demand curve's slope to measure the sensitivity of quantity demanded to a change in a commodity's own price?**

# Own-Price Elasticity of Demand



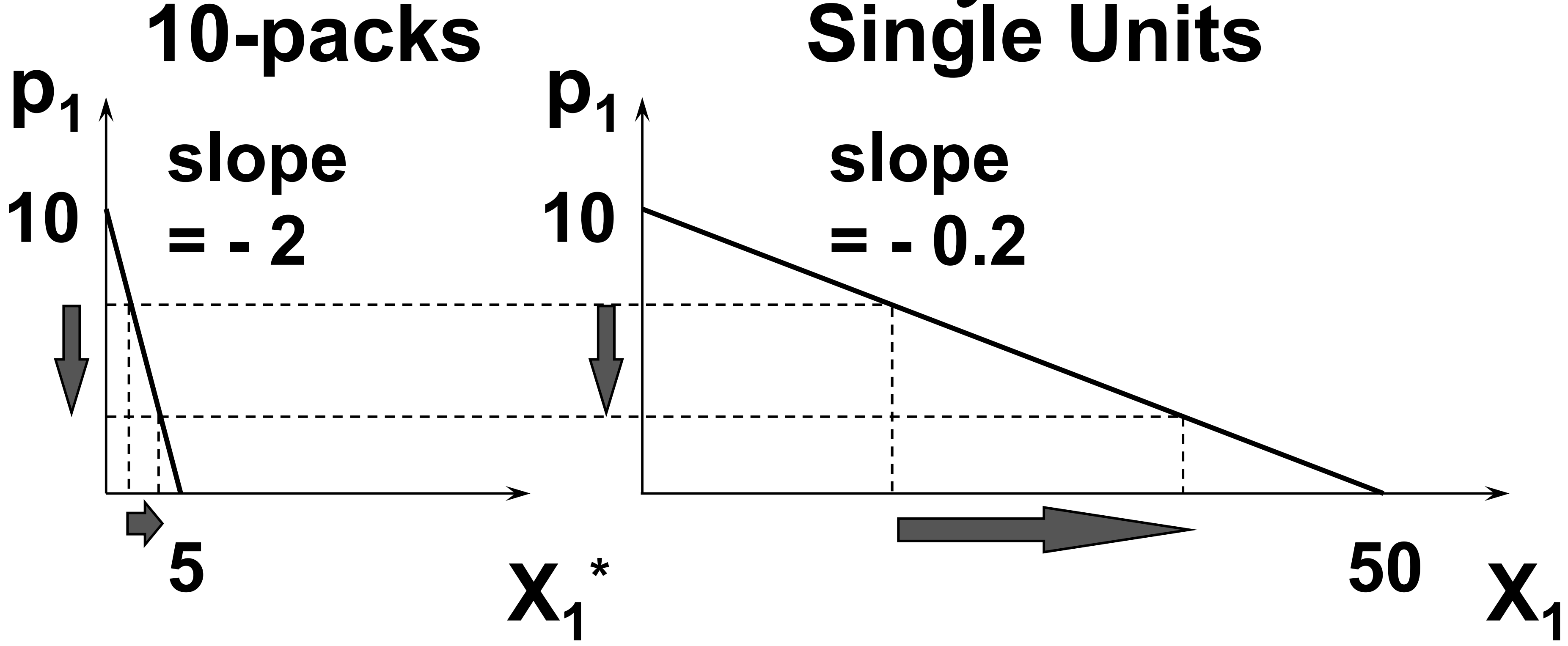

**In which case is the quantity demanded $X_1^*$ more sensitive to changes to $p_1$?**

# Own-Price Elasticity of Demand




**In which case is the quantity demanded $X_1$* more sensitive to changes to $p_1$?**

# Own-Price Elasticity of Demand

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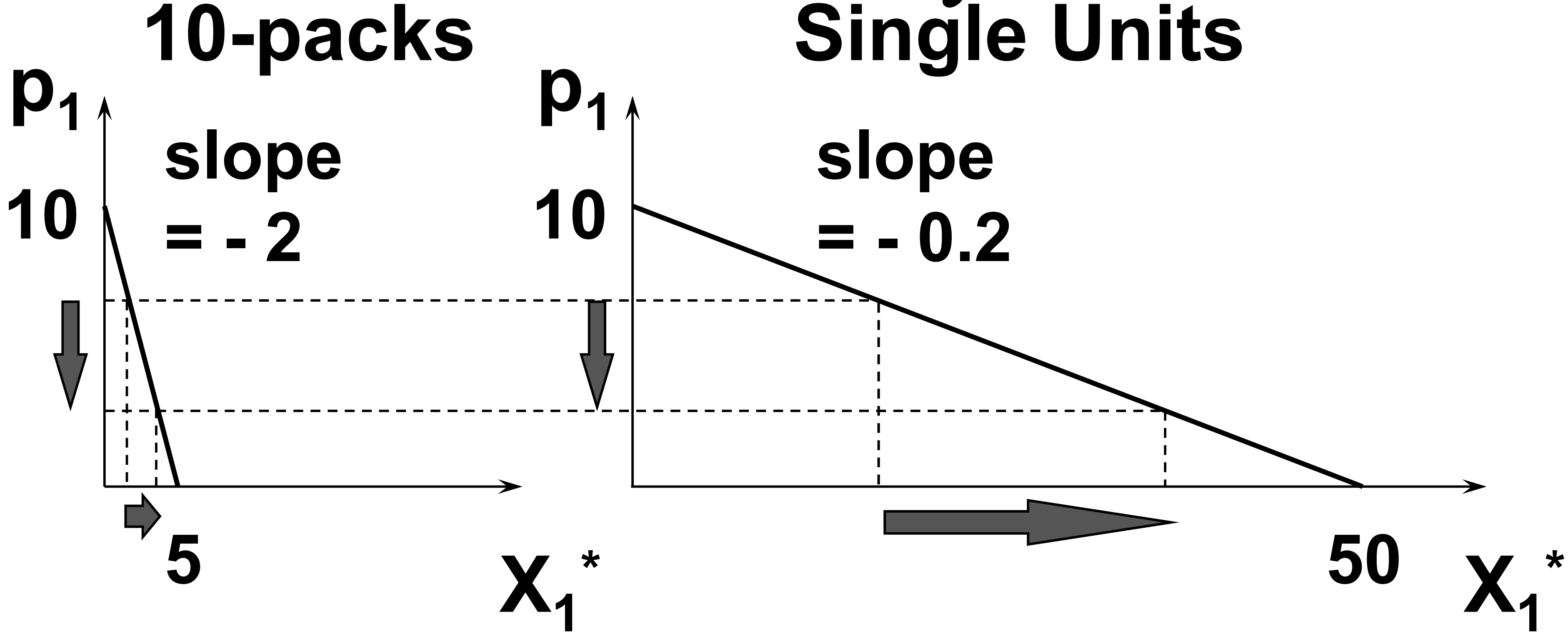
# Own-Price Elasticity of Demand




**In which case is the quantity demanded $X_1^*$ more sensitive to changes to $p_1$?**
**It is the same in both cases.**

# Own-Price Elasticity of Demand

- **Q: Why not just use the slope of a demand curve to measure the sensitivity of quantity demanded to a change in a commodity's own price?**
- **A: Because the value of sensitivity then depends upon the (arbitrary) units of measurement used for quantity demanded.**

# Own-Price Elasticity of Demand

$$\varepsilon_{x_1^*, p_1} = \frac{\%\Delta x_1^*}{\%\Delta p_1}$$

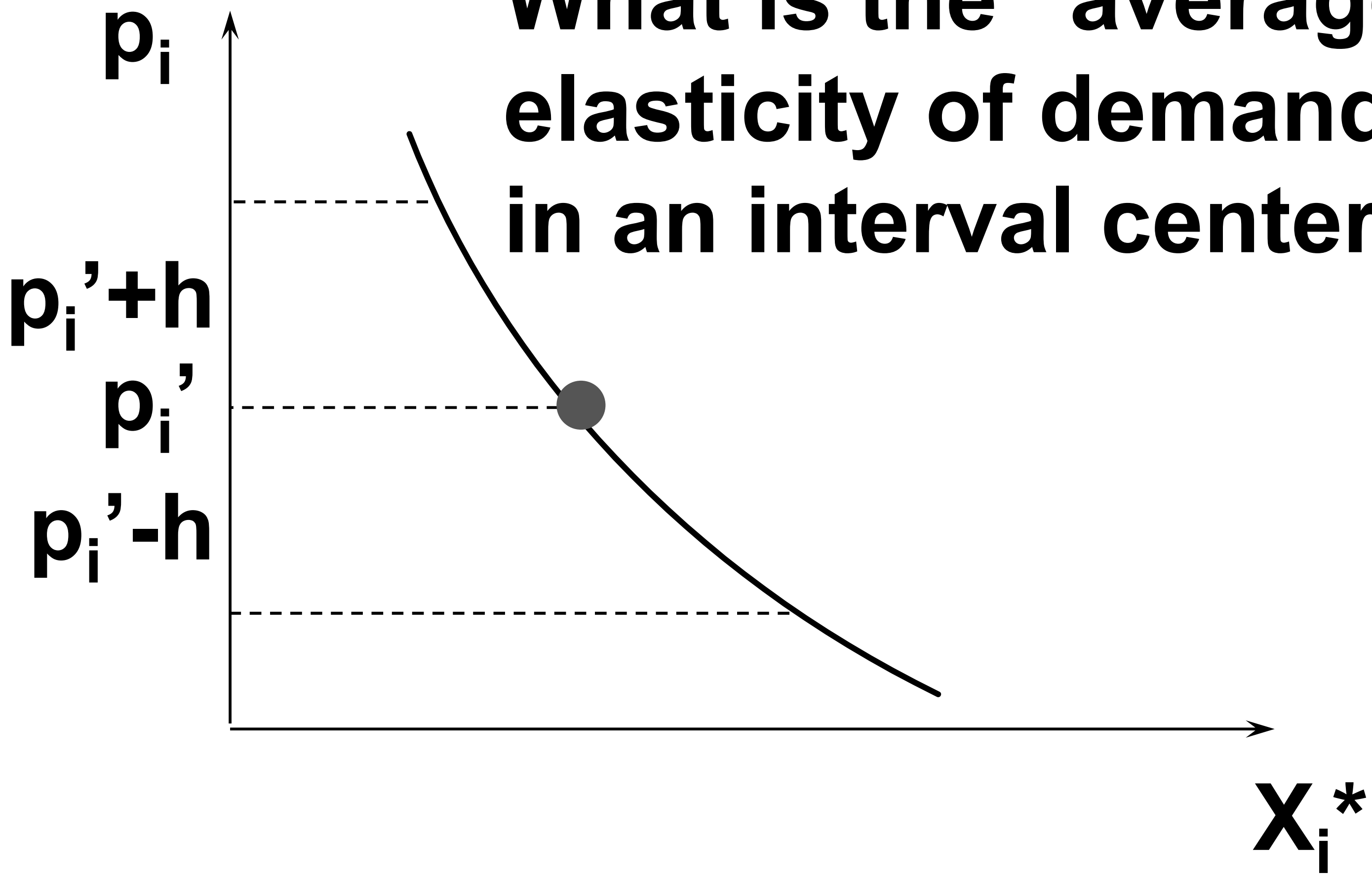
**is a ratio of percentages and so has no units of measurement.**
**Hence own-price elasticity of demand is a sensitivity measure that is independent of units of measurement.**

# Arc and Point Elasticities

- An "average" own-price elasticity of demand for commodity i over an interval of values for $p_i$ is an arc-elasticity, usually computed by a mid-point formula.
- Elasticity computed for a single value of $p_i$ is a point elasticity.

# Arc Own-Price Elasticity

**What is the "average" own-price elasticity of demand for prices in an interval centered on $p_i$'?**



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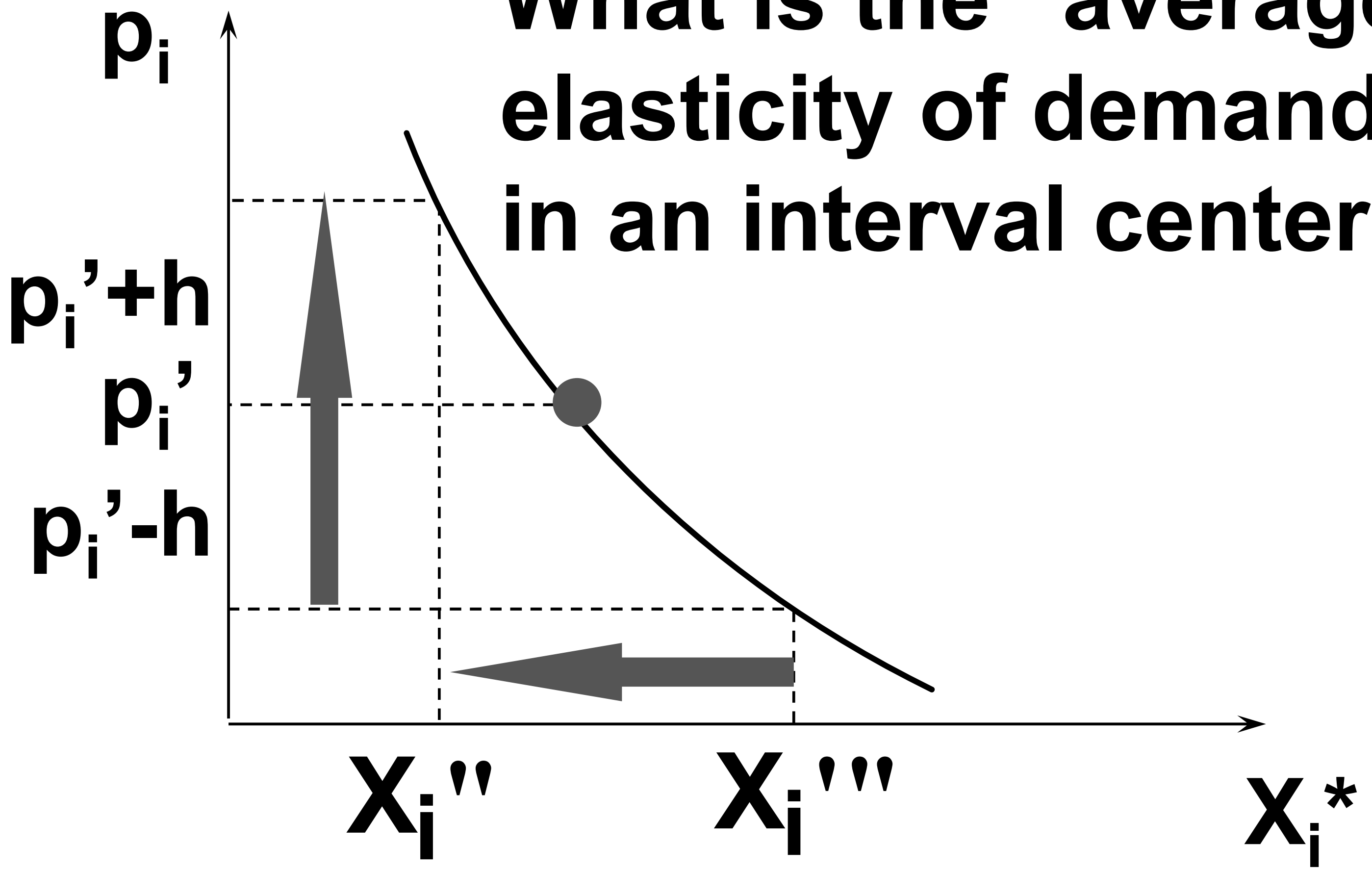

# Arc Own-Price Elasticity

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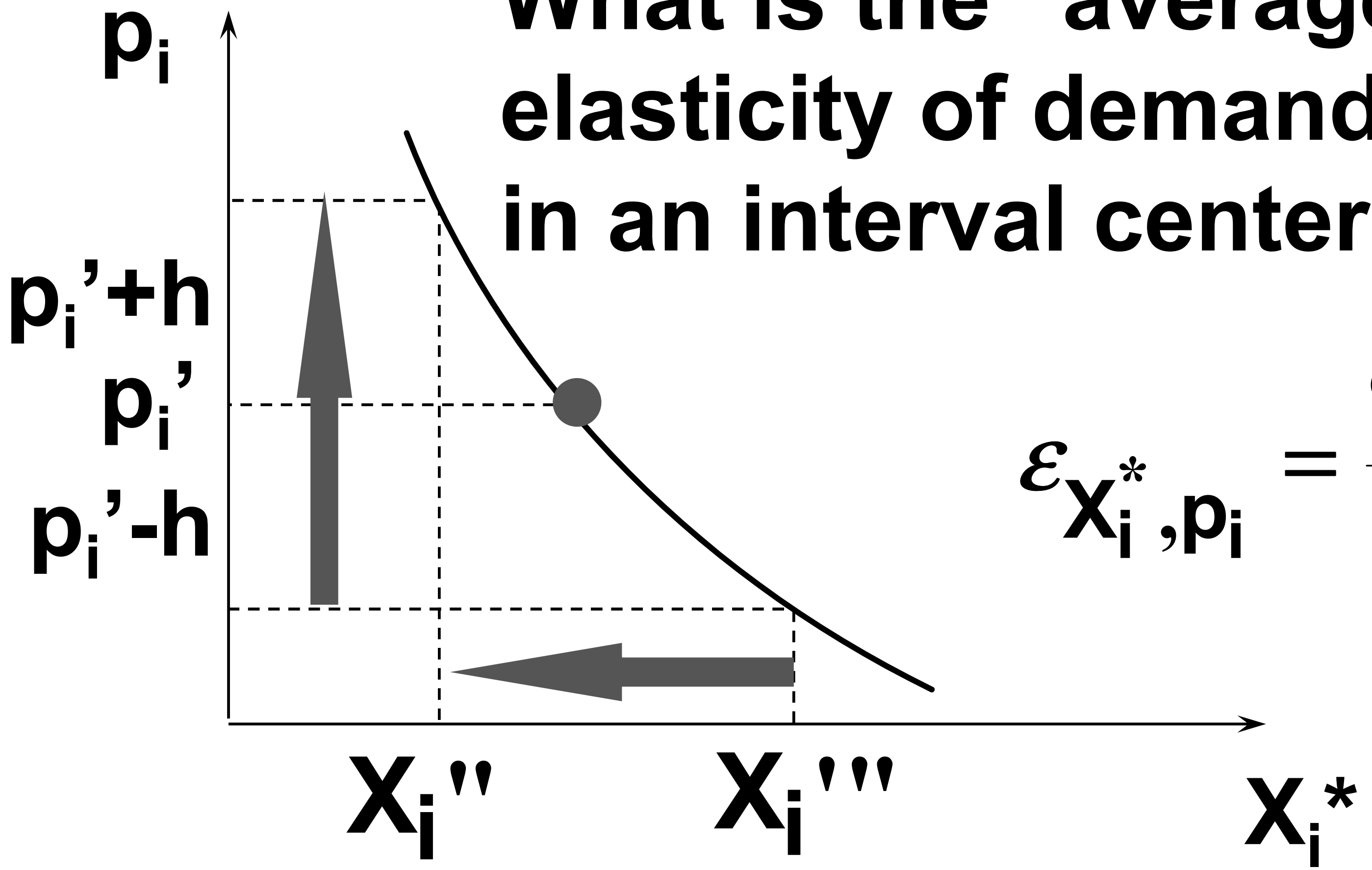
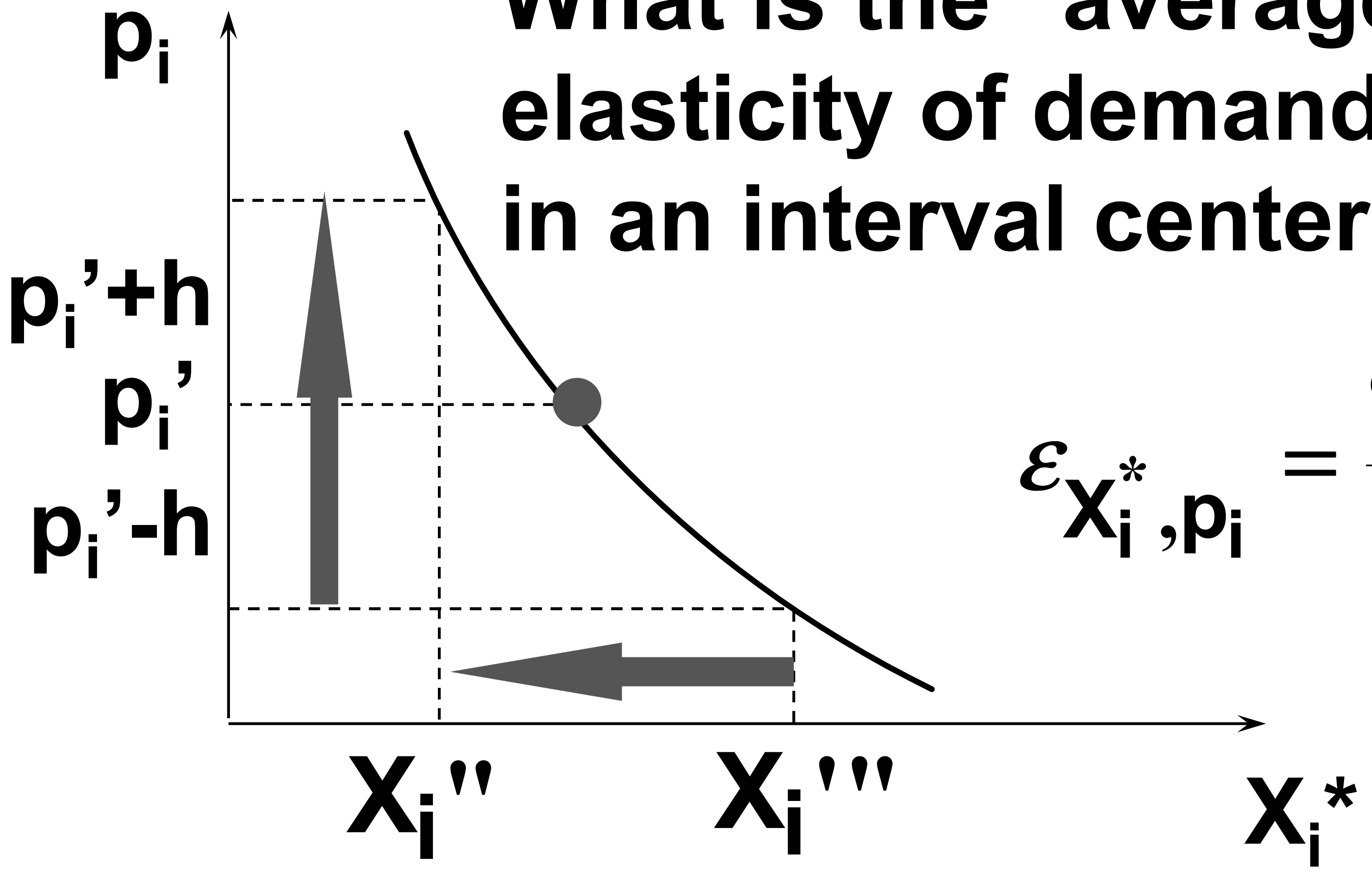



$$\varepsilon_{X_i^*, p_i} = \frac{\% \Delta X_i^*}{\% \Delta p_i}$$

# Arc Own-Price Elasticity

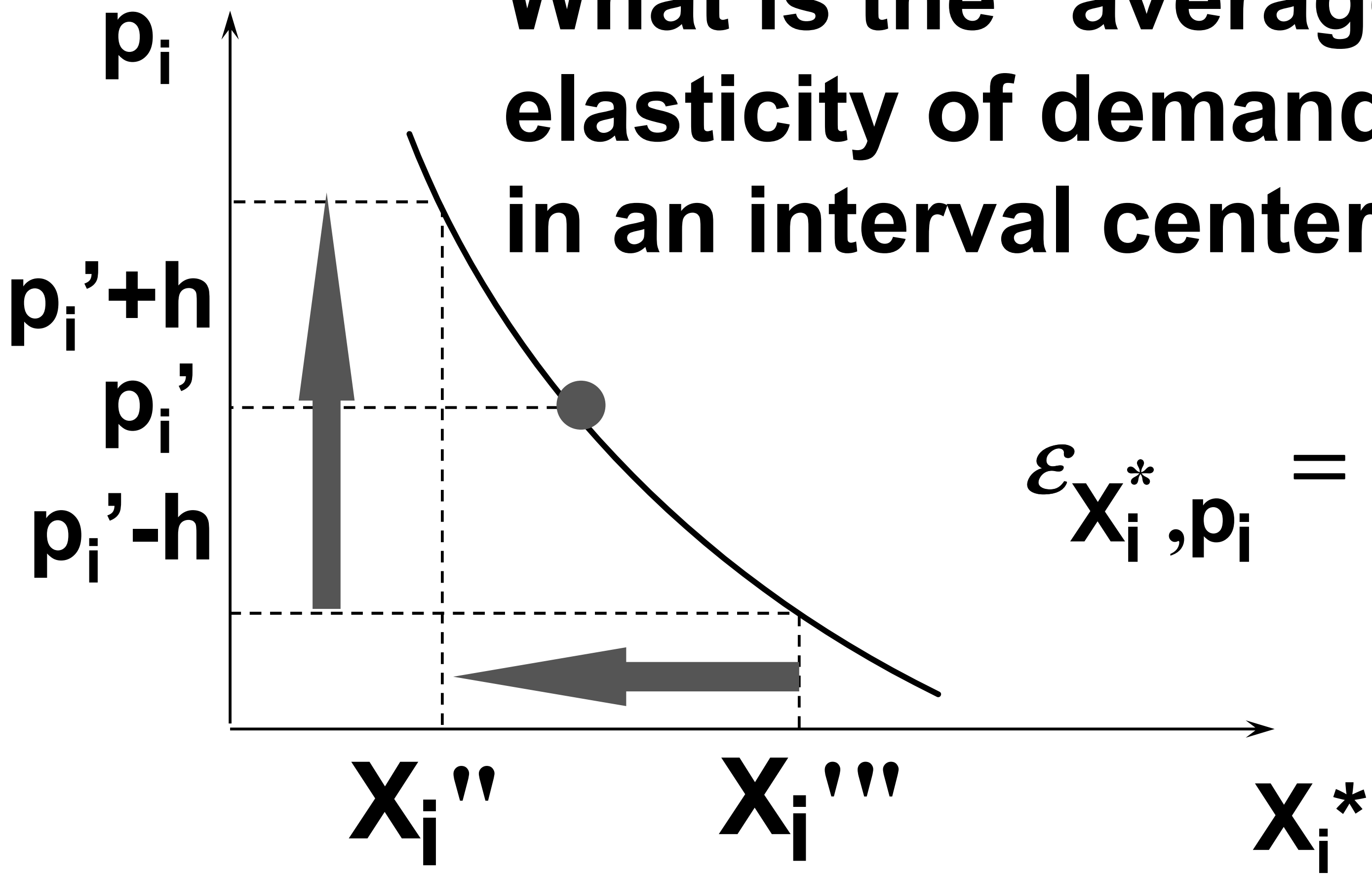
**What is the "average" own-price elasticity of demand for prices in an interval centered on $p_i$'?**




$$\varepsilon_{X_i^*, p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'}$$

# Arc Own-Price Elasticity




**What is the "average" own-price elasticity of demand for prices in an interval centered on $p_i'$?**

$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'} \qquad \%\Delta X_i^* = 100 \times \frac{(X_i'' - X_i''')}{(X_i'' + X_i''')/2}$$

# Arc Own-Price Elasticity

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'}$$

$$\varepsilon_{x_i^*, p_i} = \frac{\%\Delta x_i^*}{\%\Delta p_i}$$

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$$\% \Delta X_i^* = 100 \times \frac{(X_i'' - X_i''')}{(X_i'' + X_i''')/2}$$

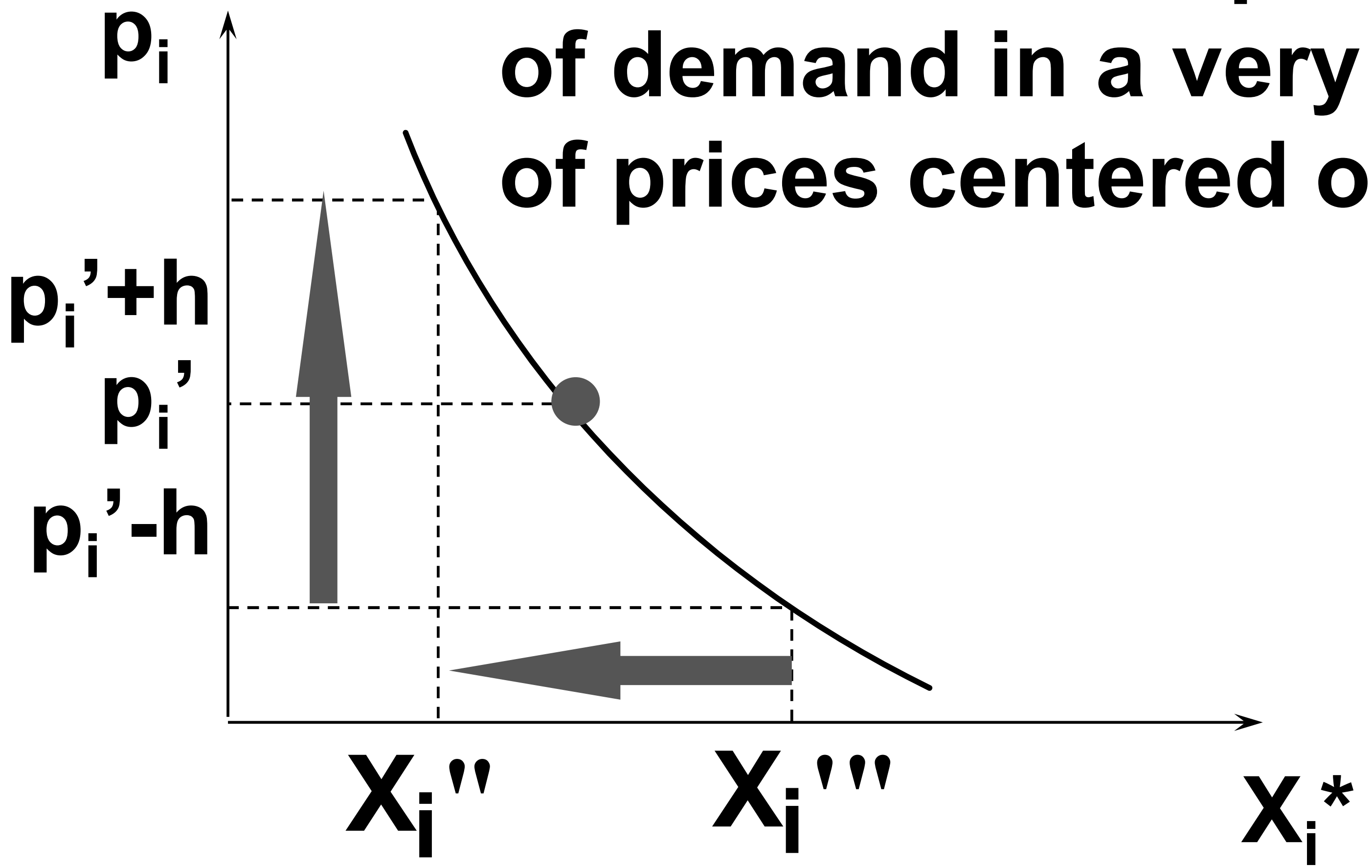
**So**

$$\varepsilon_{X_i^*, p_i} = \frac{\% \Delta X_i^*}{\% \Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

**is the arc own-price elasticity of demand.**

# Point Own-Price Elasticity

**What is the own-price elasticity of demand in a very small interval of prices centered on $p_i$'?**

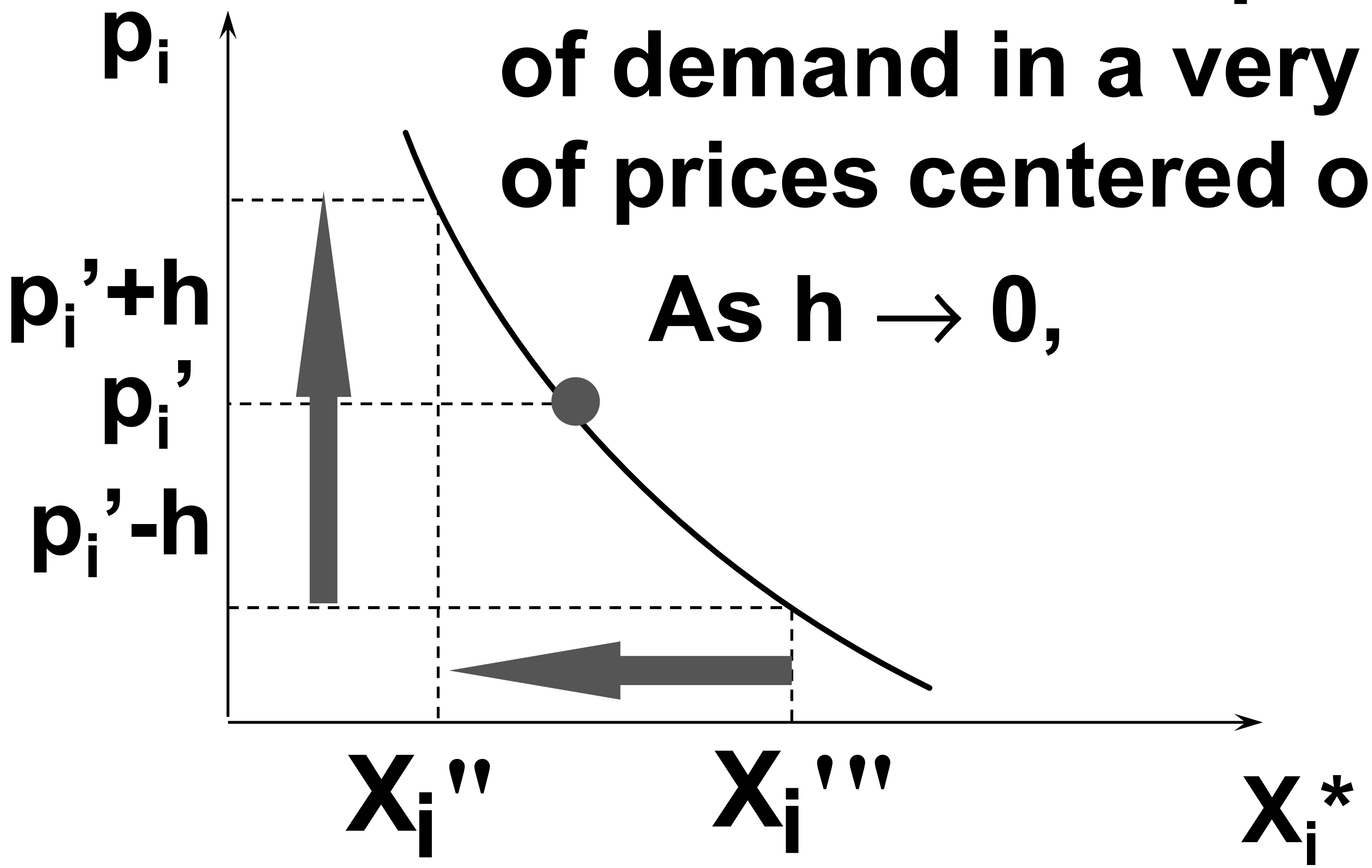



$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

# Point Own-Price Elasticity

**What is the own-price elasticity of demand in a very small interval of prices centered on $p_i$'?**

**As $h \to 0$,**




$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i''+X_i''')/2} \times \frac{(X_i''-X_i''')}{2h}.$$

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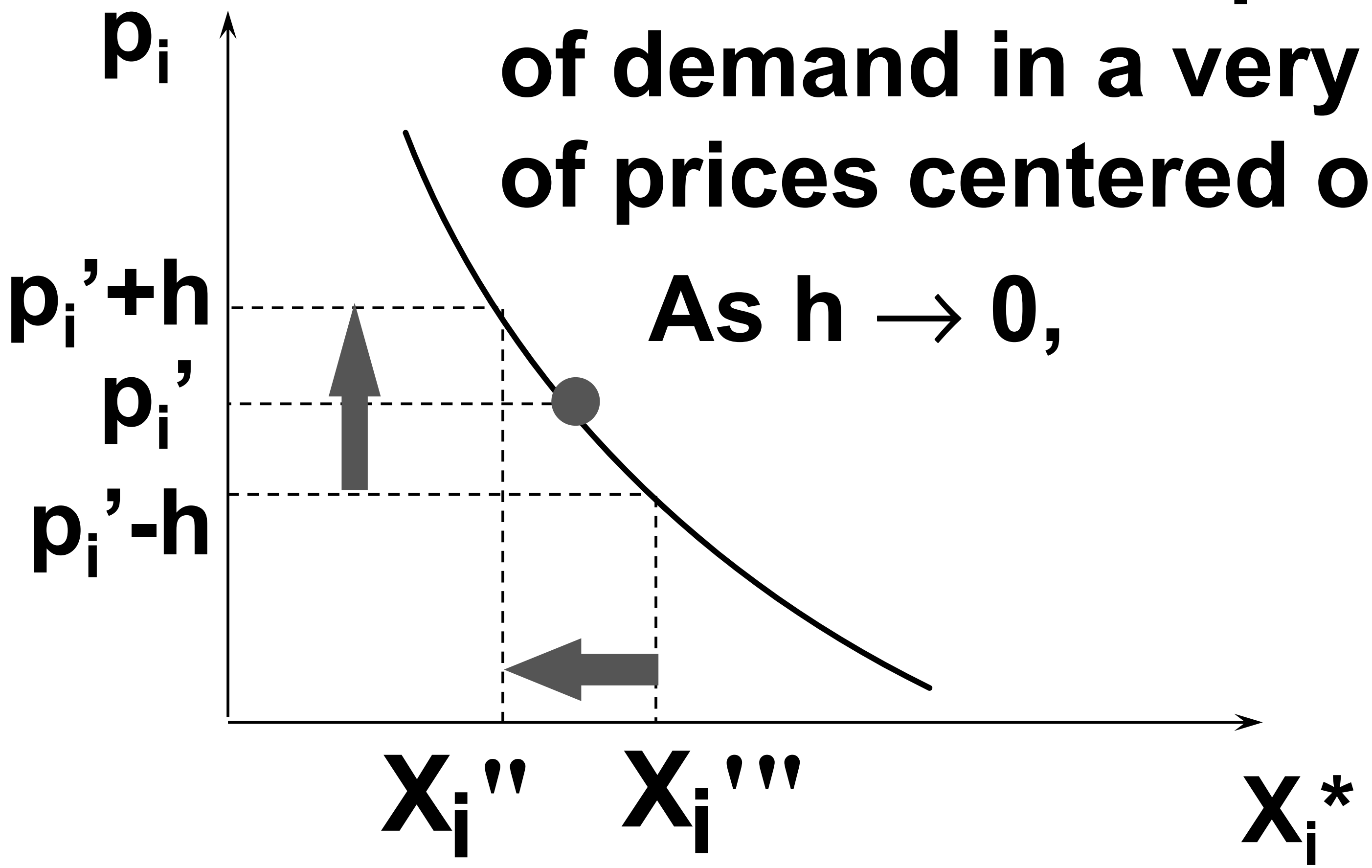


**As $h \to 0$,**

$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i''+X_i''')/2} \times \frac{(X_i''-X_i''')}{2h}.$$

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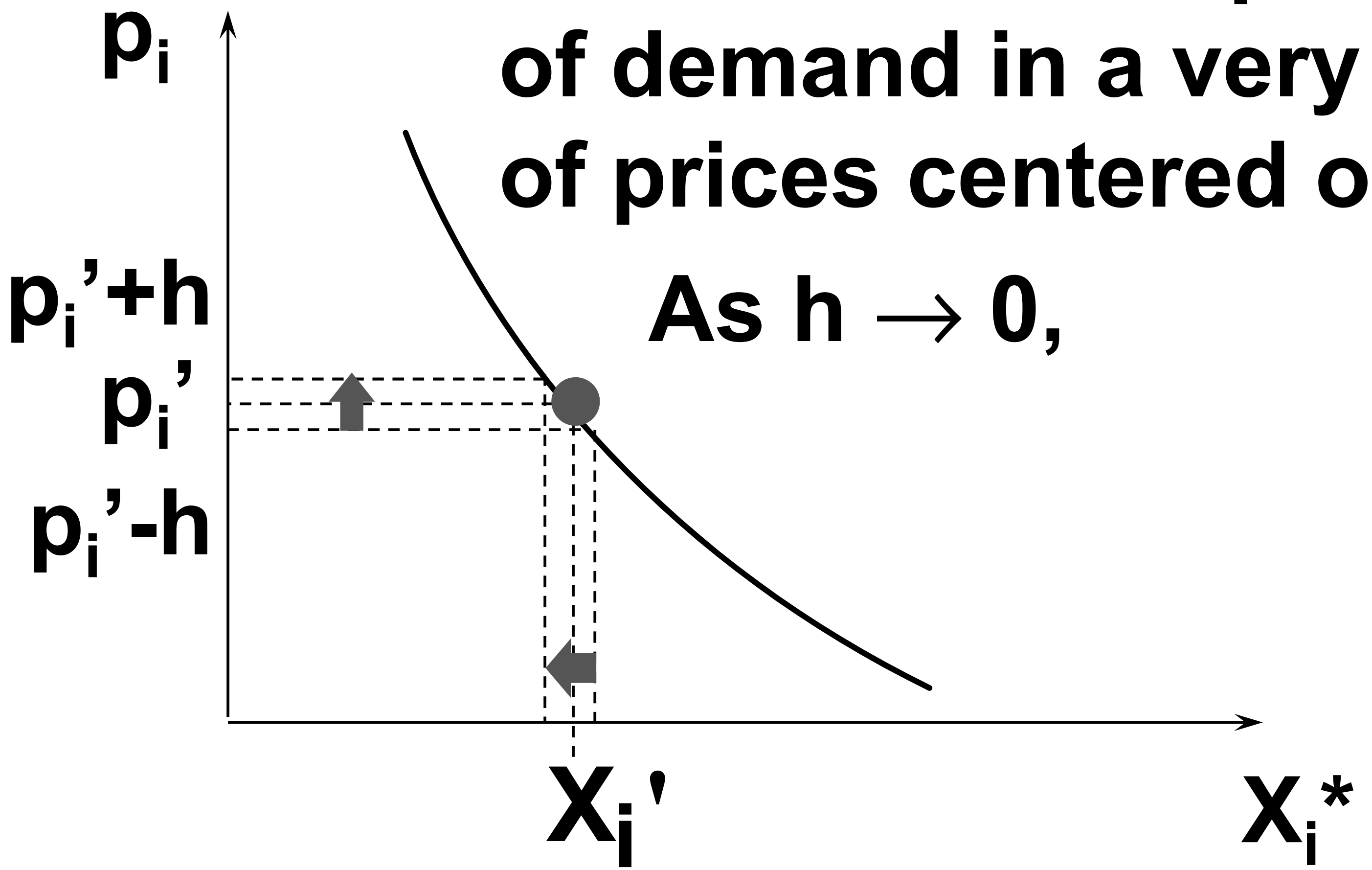


$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i''+X_i''')/2} \times \frac{(X_i''-X_i''')}{2h}.$$

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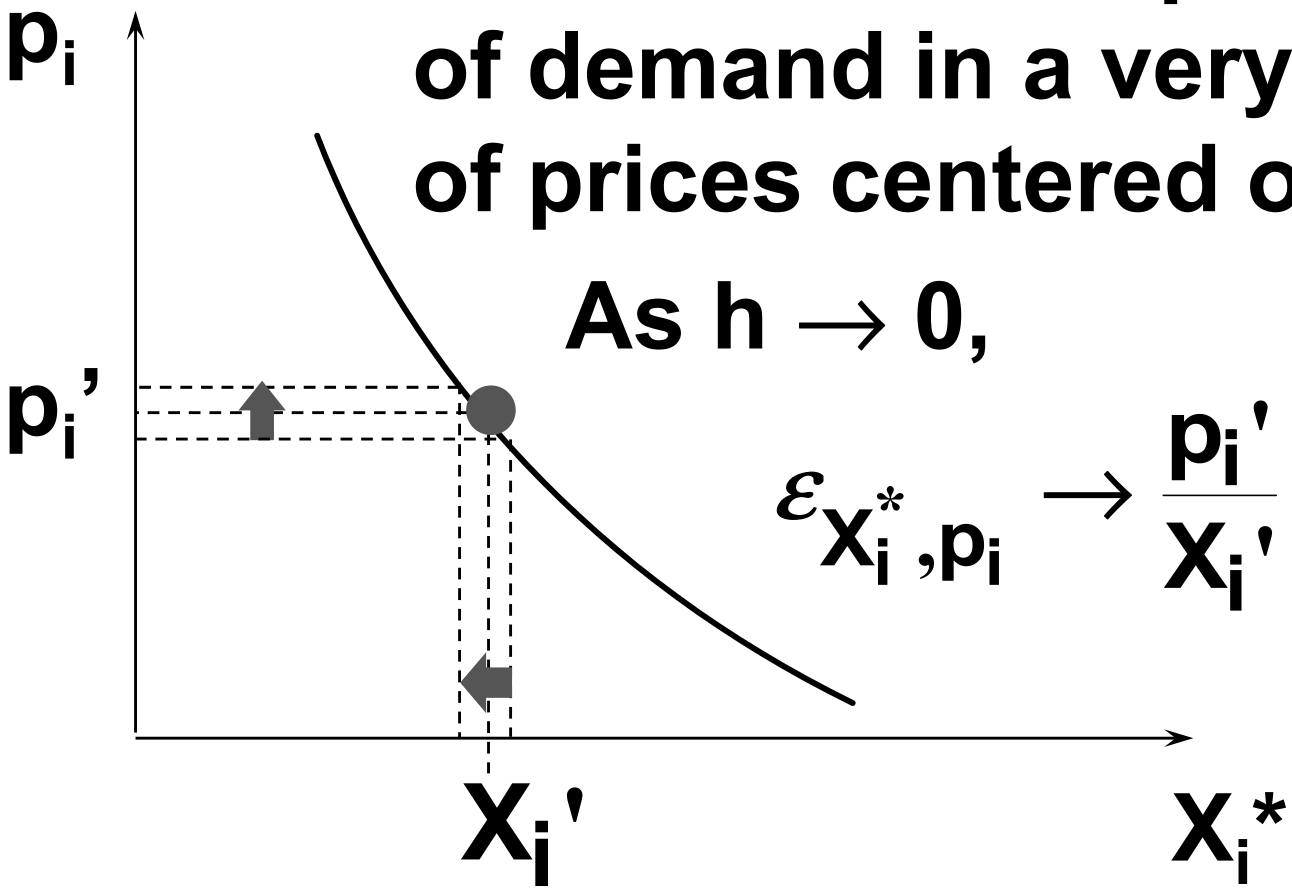


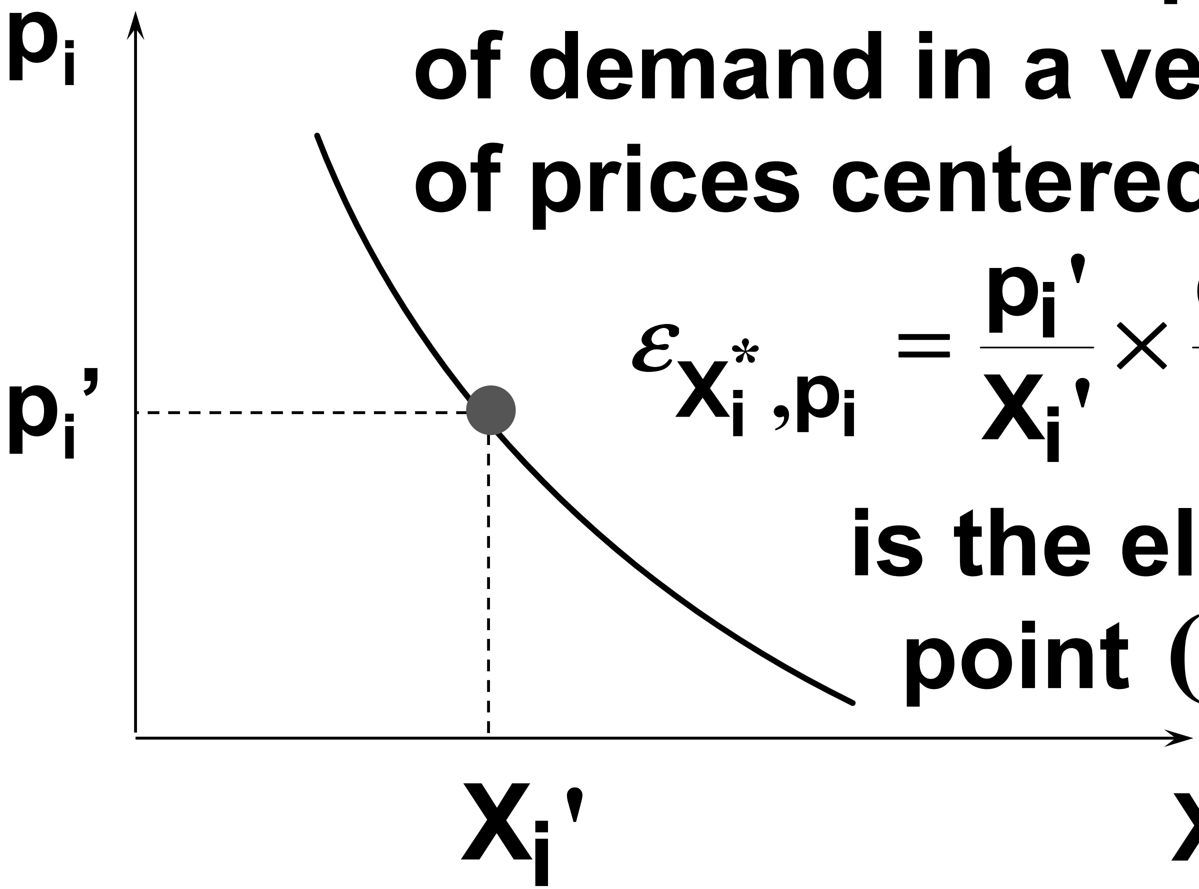

**As $h \to 0$,**

$$\varepsilon_{X_i^*,p_i} \to \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

# Point Own-Price Elasticity

**What is the own-price elasticity of demand in a very small interval of prices centered on $p_i$'?**




$$\varepsilon_{X_i^*, p_i} = \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

**is the elasticity at the point $(X_i', p_i')$.**

# Point Own-Price Elasticity

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{X_i^*} \times \frac{dX_i^*}{dp_i}$$

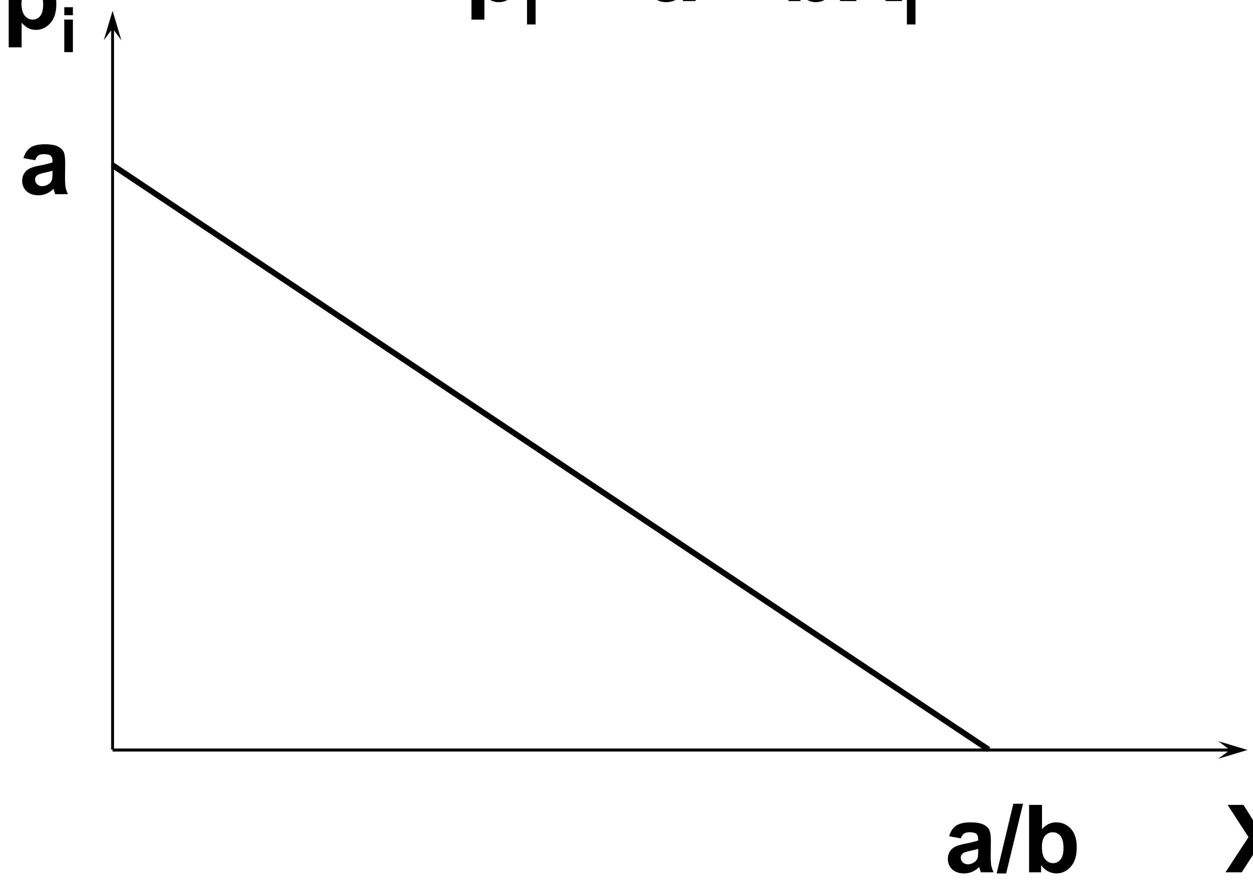
**E.g. Suppose $p_i = a - bX_i$.**
**Then $X_i = (a-p_i)/b$ and**

$$\frac{dX_i^*}{dp_i} = -\frac{1}{b}.$$ **Therefore,**

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{(a - p_i)/b} \times \left(-\frac{1}{b}\right) = -\frac{p_i}{a - p_i}.$$

# Point Own-Price Elasticity

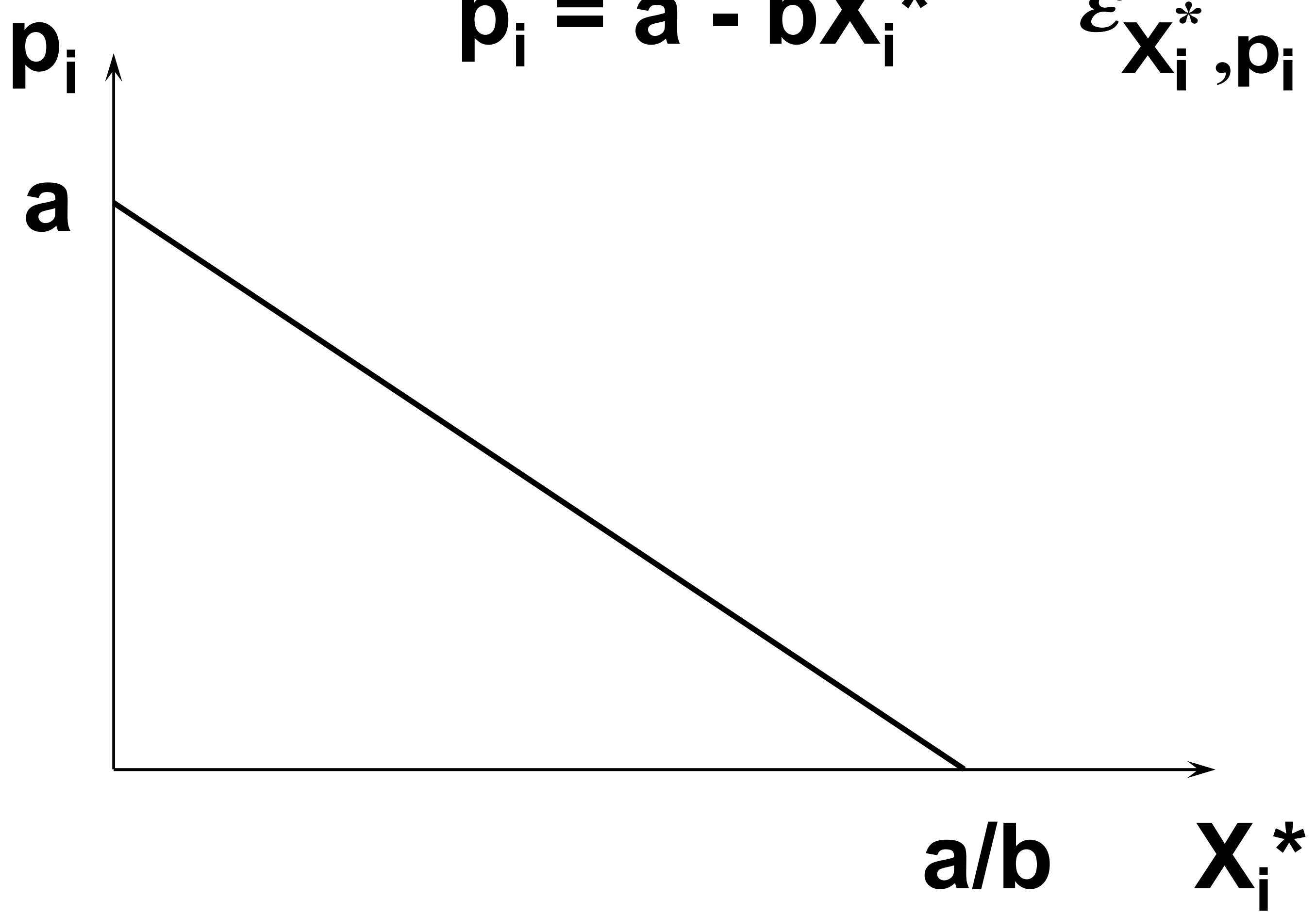
$p_i = a - bX_i^*$



# Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

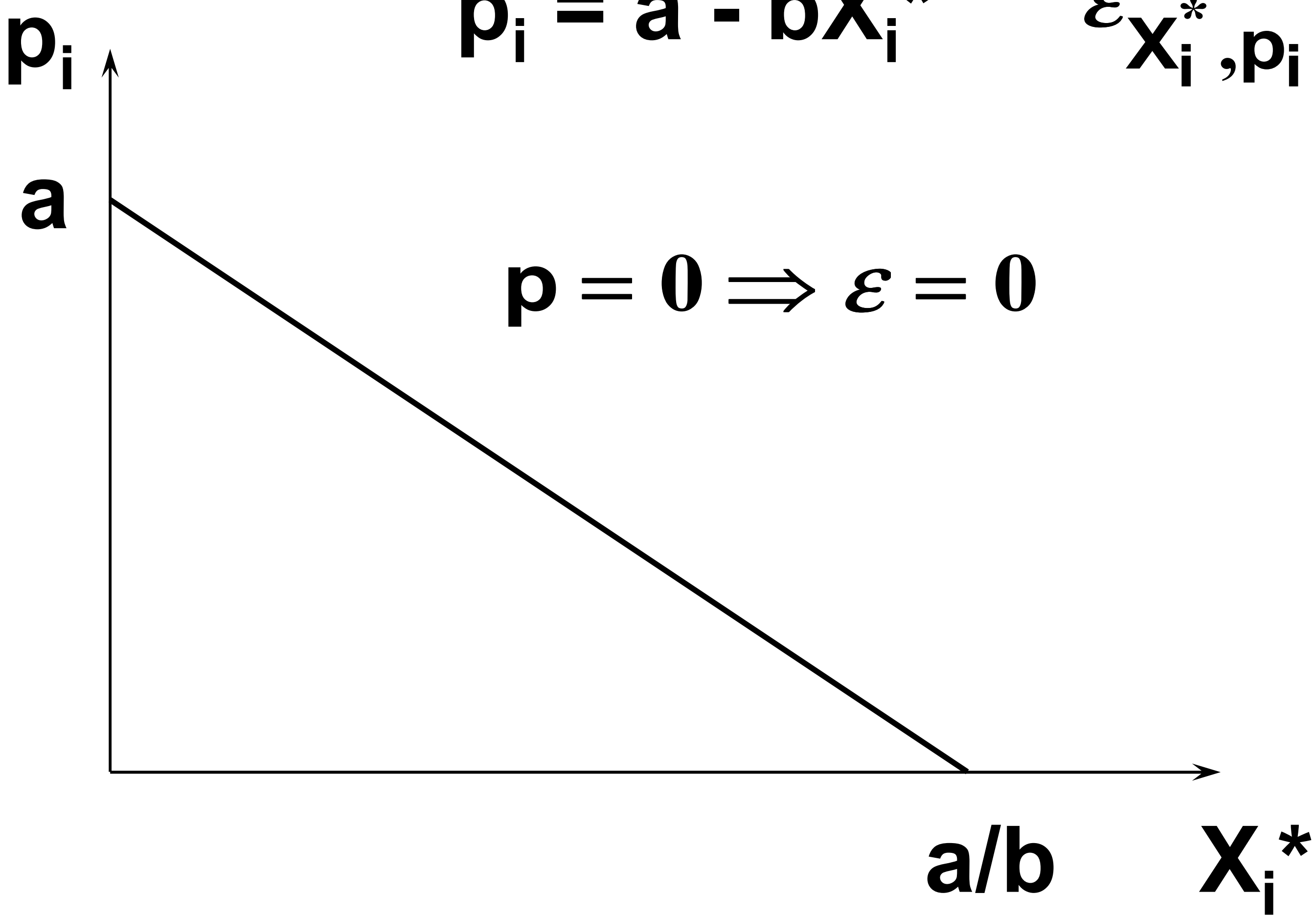


# Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

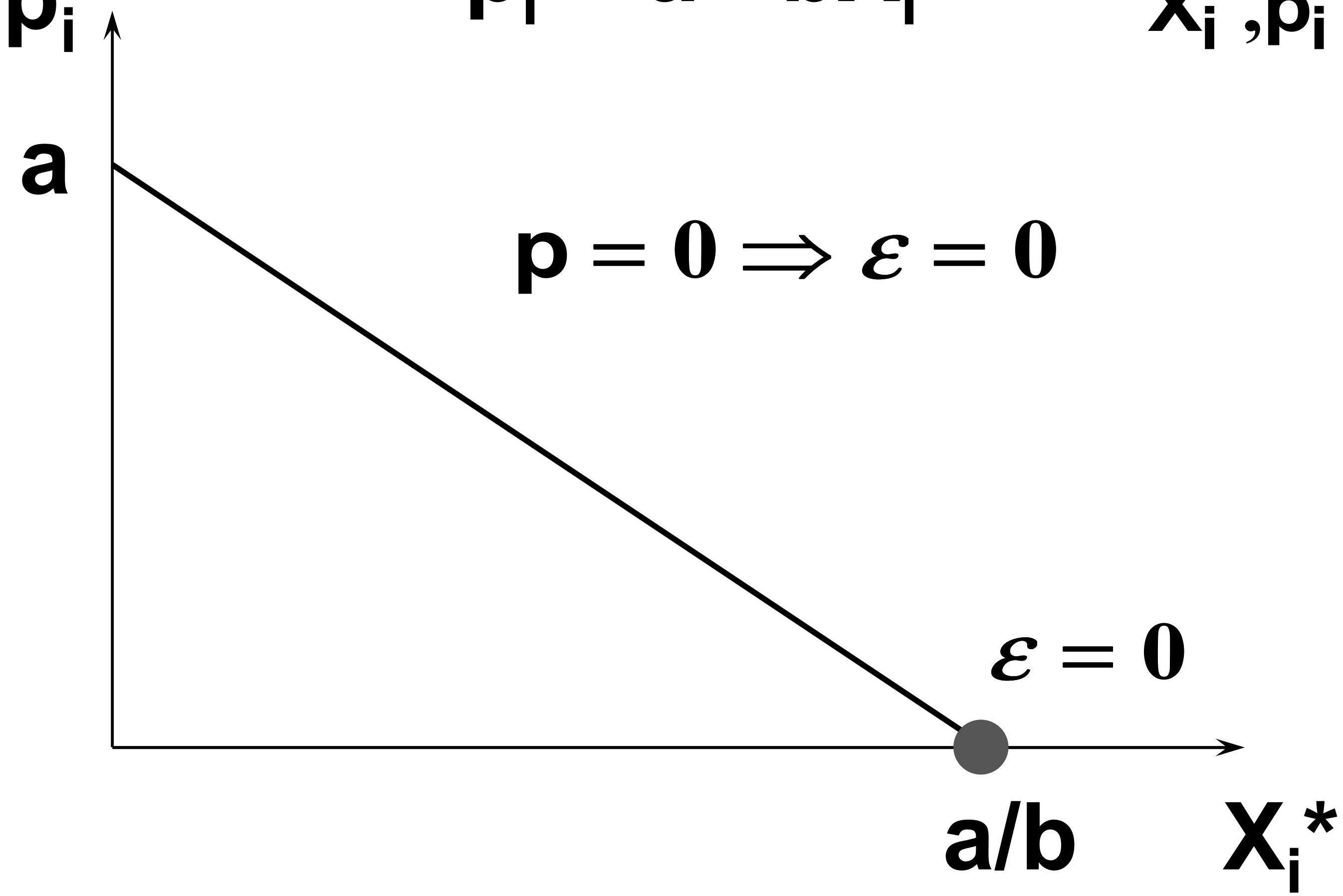
$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = 0 \Rightarrow \varepsilon = 0$$



# Point Own-Price Elasticity

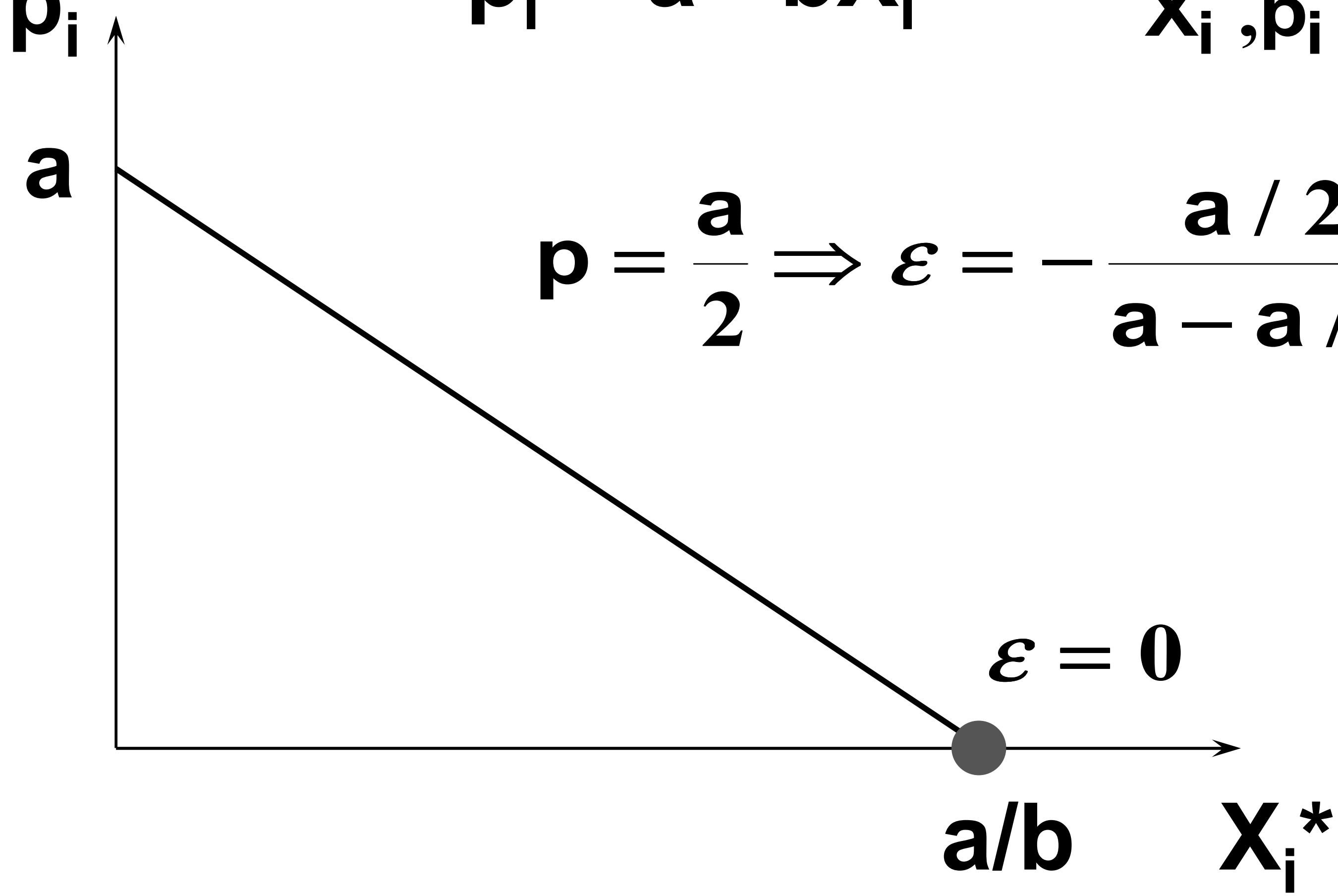
$$\mathbf{p_i = a - bX_i^*} \qquad \varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$



# Point Own-Price Elasticity

$$p_i = a - bX_i^* \qquad \varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = \frac{a}{2} \Rightarrow \varepsilon = -\frac{a/2}{a - a/2} = -1$$



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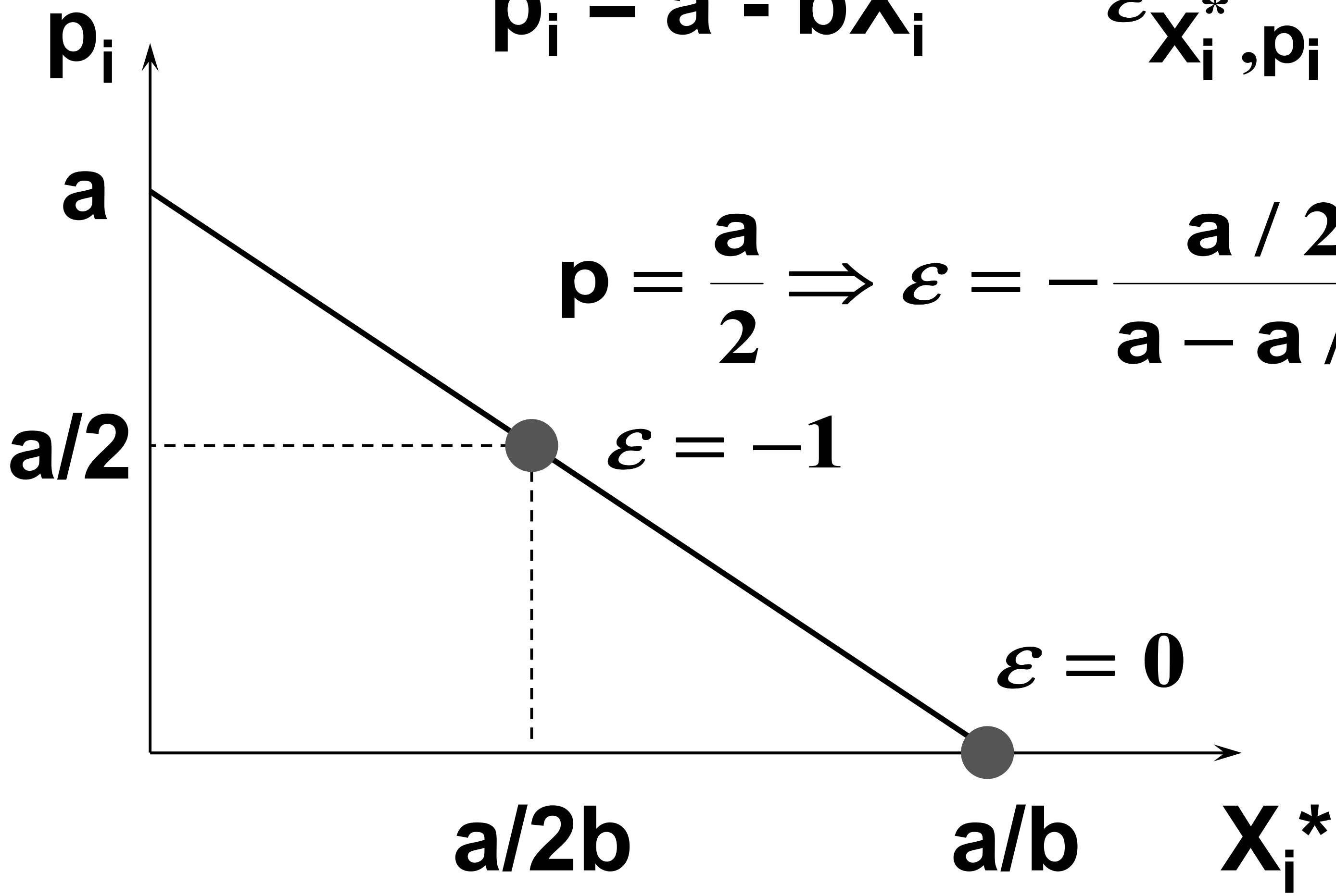
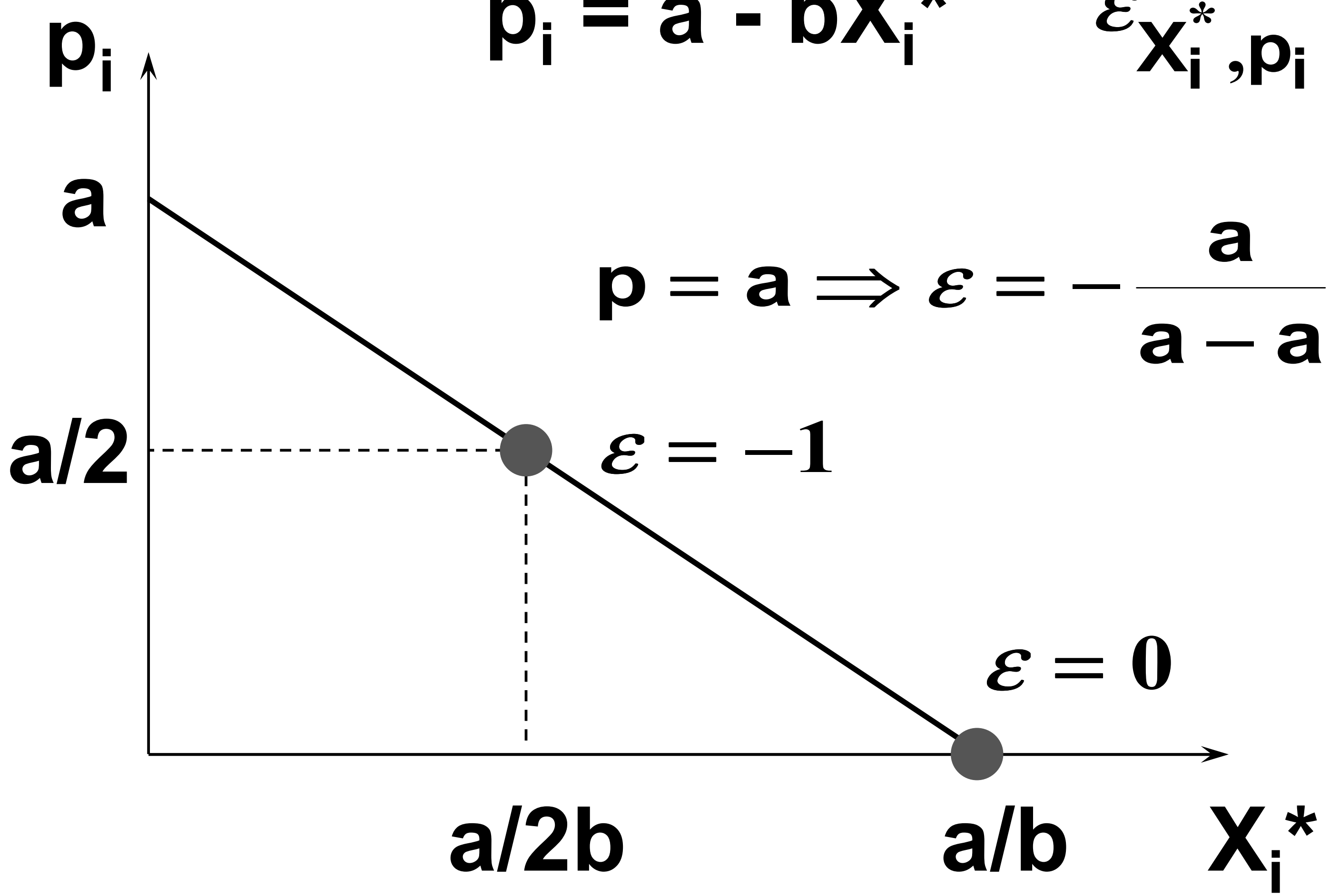


# Point Own-Price Elasticity

$$p_i = a - bX_i^* \qquad \varepsilon_{X_i^*,p_i} = -\frac{p_i}{a - p_i}$$

$$p = a \Rightarrow \varepsilon = -\frac{a}{a-a} = -\infty$$



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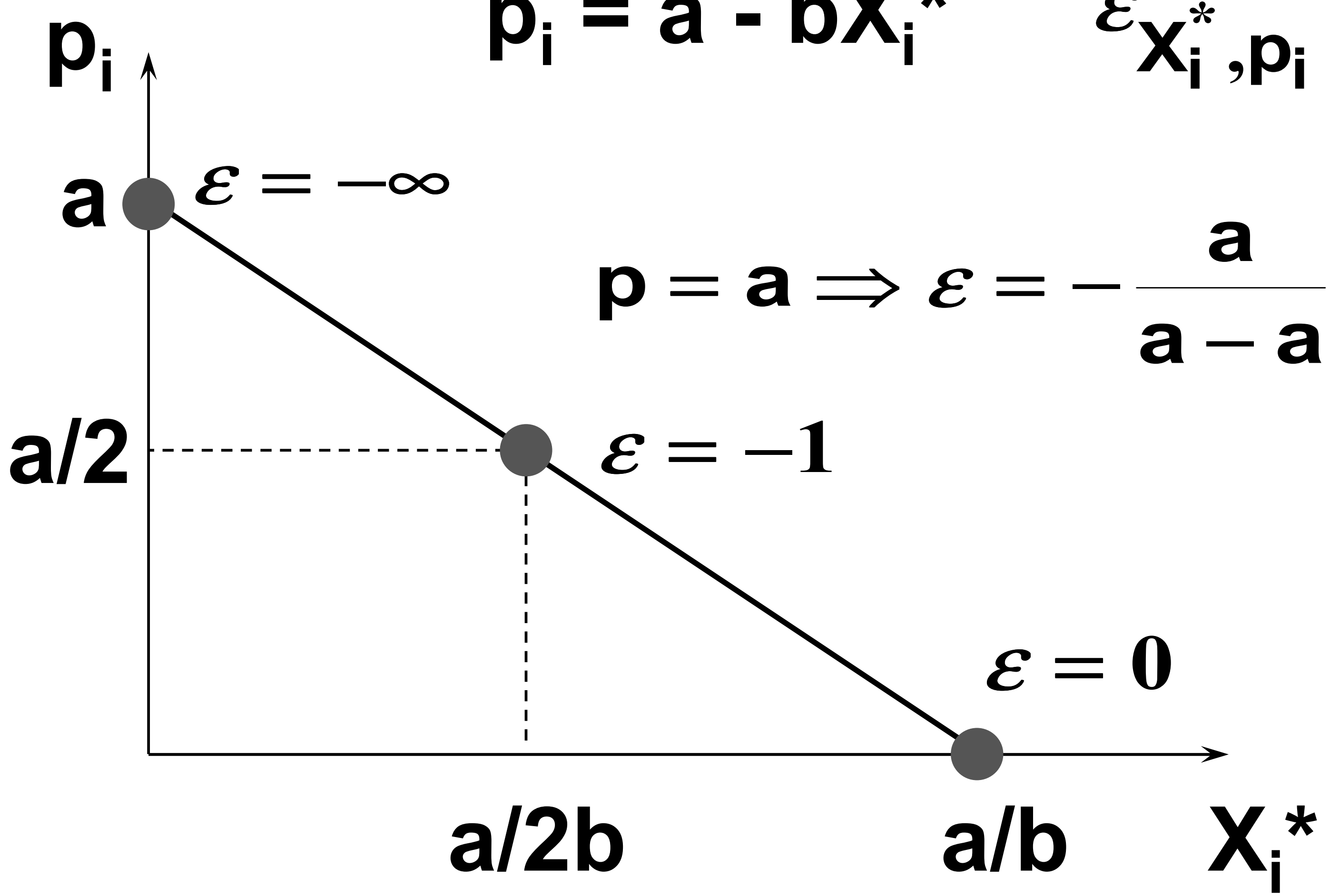


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# Point Own-Price Elasticity

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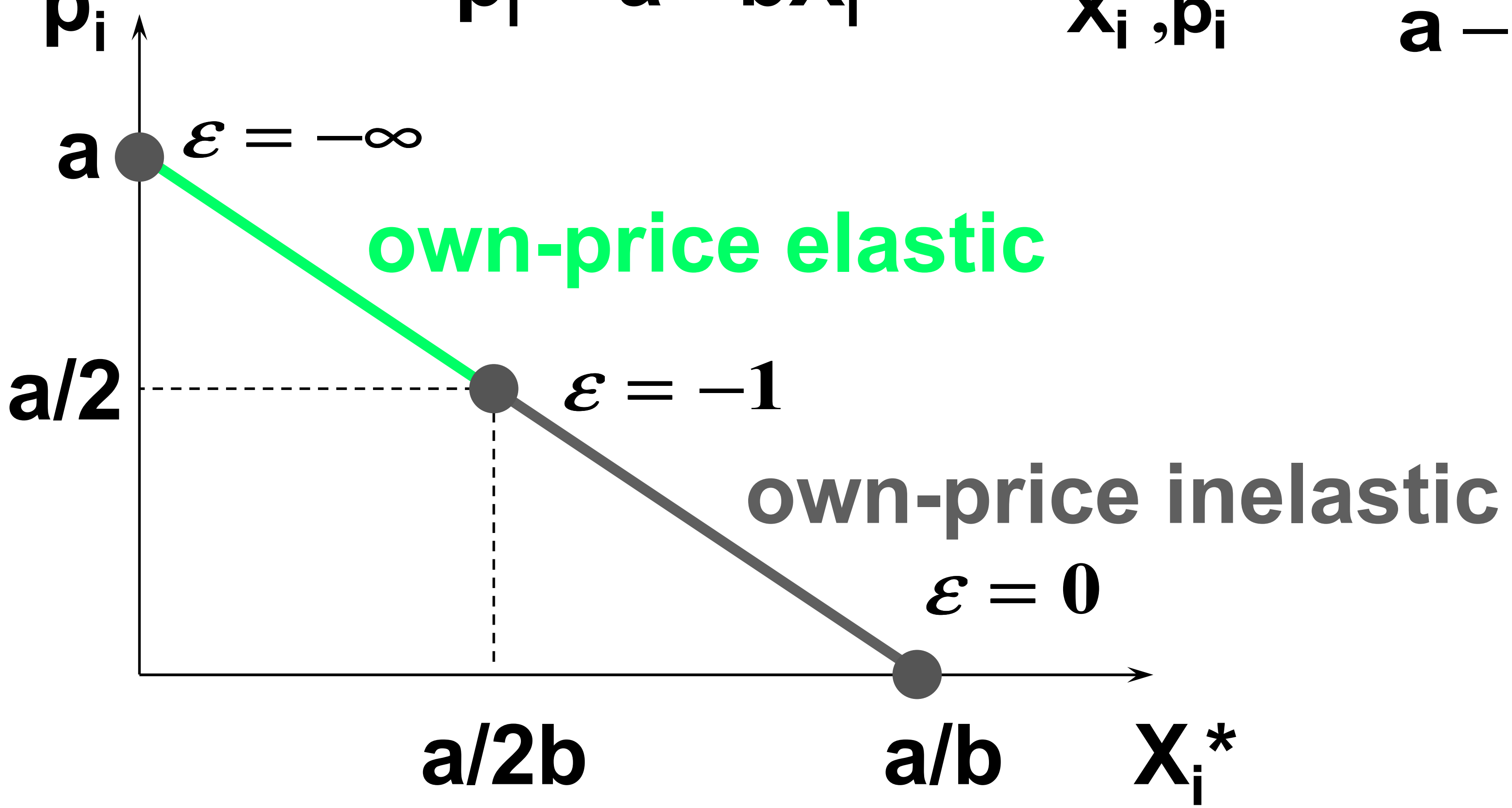

# Point Own-Price Elasticity

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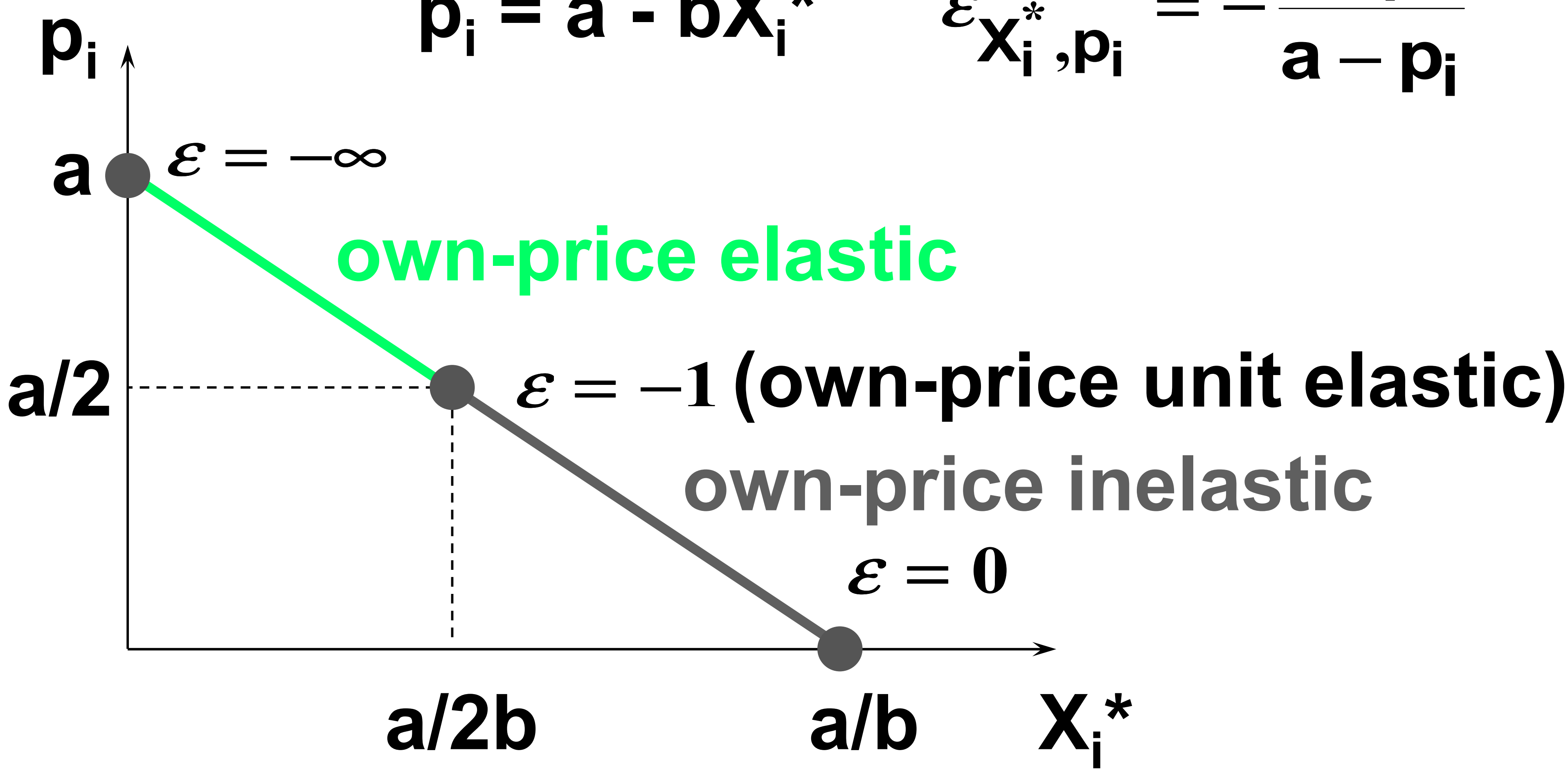


# Point Own-Price Elasticity

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{X_i^*} \times \frac{dX_i^*}{dp_i}$$

E.g. $X_i^* = kp_i^a$. Then $\dfrac{dX_i^*}{dp_i} = a p_i^{a-1}$

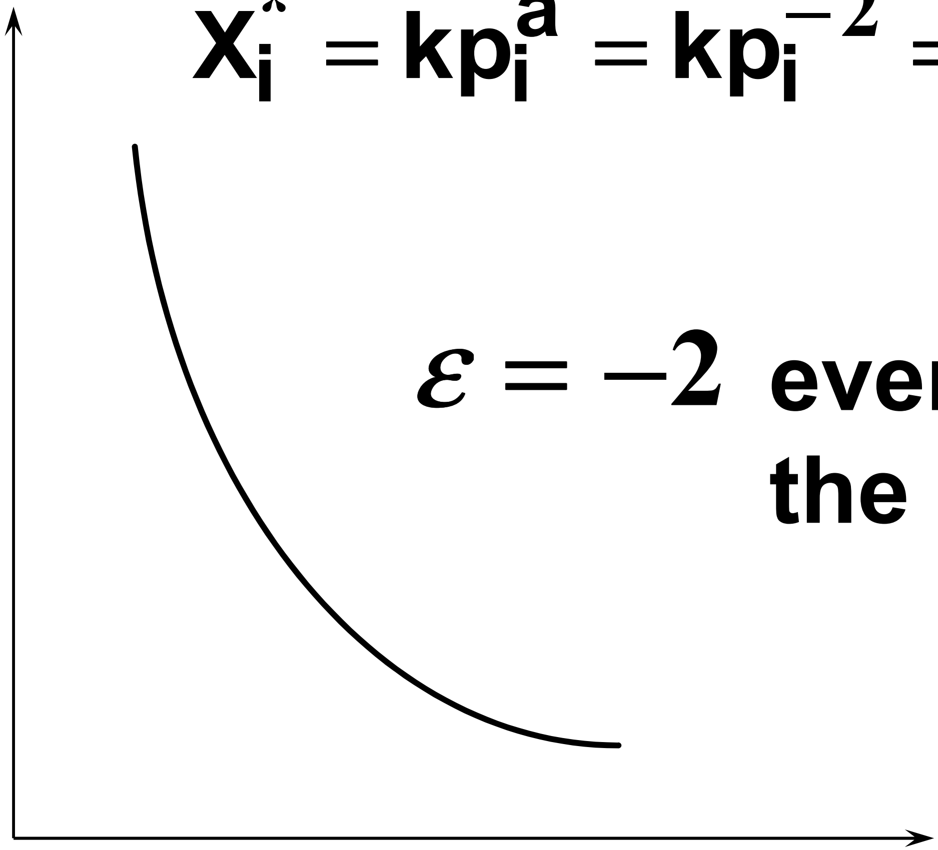
so

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{kp_i^a} \times ka p_i^{a-1} = a \frac{p_i^a}{p_i^a} = a.$$

# Point Own-Price Elasticity

$$X_i^* = kp_i^a = kp_i^{-2} = \frac{k}{p_i^2}$$

$p_i$



$\varepsilon = -2$ **everywhere along the demand curve.**

$X_i^*$

# Revenue and Own-Price Elasticity of Demand

- **If raising a commodity's price causes little decrease in quantity demanded, then sellers' revenues rise.**
- **Hence own-price inelastic demand causes sellers' revenues to rise as price rises.**

# Revenue and Own-Price Elasticity of Demand

- **If raising a commodity's price causes a large decrease in quantity demanded, then sellers' revenues fall.**
- **Hence own-price elastic demand causes sellers' revenues to fall as price rises.**

# Revenue and Own-Price Elasticity of Demand

**Sellers' revenue is** $\mathbf{R(p) = p \times X^*(p)}$.

# Revenue and Own-Price Elasticity of Demand

**Sellers' revenue is** $R(p) = p \times X^*(p).$

**So** $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

# Revenue and Own-Price Elasticity of Demand

**Sellers' revenue is** $\mathbf{R(p) = p \times X^*(p).}$

**So** $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

# Revenue and Own-Price Elasticity of Demand

**Sellers' revenue is** $R(p) = p \times X^*(p).$

**So** $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

$$= X^*(p)[1 + \varepsilon].$$

# Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)\left[1 + \varepsilon\right]$$

# Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

**so if** $\varepsilon = -1$ **then** $\frac{dR}{dp} = 0$

**and a change to price does not alter sellers' revenue.**

# Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1 + \varepsilon]$$

**but if** $-1 < \varepsilon \leq 0$ **then** $\frac{dR}{dp} > 0$

**and a price increase raises sellers' revenue.**

# Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

**And if** $\varepsilon < -1$ **then** $\frac{dR}{dp} < 0$

**and a price increase reduces sellers' revenue.**

# Revenue and Own-Price Elasticity of Demand

**In summary:**

**Own-price inelastic demand;** $-1 < \varepsilon \leq 0$
**price rise causes rise in sellers' revenue.**

**Own-price unit elastic demand;** $\varepsilon = -1$
**price rise causes no change in sellers' revenue.**

**Own-price elastic demand;** $\varepsilon < -1$
**price rise causes fall in sellers' revenue.**

# Marginal Revenue and Own-Price Elasticity of Demand

- **A seller's marginal revenue is the rate at which revenue changes with the number of units sold by the seller.**

$$\mathbf{MR(q) = \frac{dR(q)}{dq}.}$$

# Marginal Revenue and Own-Price Elasticity of Demand

**p(q) denotes the seller's inverse demand function; i.e. the price at which the seller can sell q units. Then**

$$R(q) = p(q) \times q$$

**so**

$$MR(q) = \frac{dR(q)}{dq} = \frac{dp(q)}{dq} q + p(q)$$

$$= p(q)\left[1 + \frac{q}{p(q)} \frac{dp(q)}{dq}\right].$$

# Marginal Revenue and Own-Price Elasticity of Demand

$$\mathrm{MR}(q) = p(q)\left[1 + \frac{q}{p(q)}\frac{dp(q)}{dq}\right].$$

and $$\varepsilon = \frac{dq}{dp} \times \frac{p}{q}$$

so $$\mathrm{MR}(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right].$$

# Marginal Revenue and Own-Price Elasticity of Demand

$$\mathbf{MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]}$$ **says that the rate at which a seller's revenue changes with the number of units it sells depends on the sensitivity of quantity demanded to price; *i.e.*, upon the of the own-price elasticity of demand.**

# Marginal Revenue and Own-Price Elasticity of Demand

$$\mathbf{MR(q)} = \mathbf{p(q)}\left[\mathbf{1} + \frac{\mathbf{1}}{\varepsilon}\right]$$

**If** $\varepsilon = -1$ **then** $\mathbf{MR(q) = 0.}$

**If** $-1 < \varepsilon \le 0$ **then** $\mathbf{MR(q) < 0.}$

**If** $\varepsilon < -1$ **then** $\mathbf{MR(q) > 0.}$

# Marginal Revenue and Own-Price Elasticity of Demand

**If $\varepsilon = -1$ then $\mathrm{MR}(q) = 0$. Selling one more unit does not change the seller's revenue.**

**If $-1 < \varepsilon \leq 0$ then $\mathrm{MR}(q) < 0$. Selling one more unit reduces the seller's revenue.**

**If $\varepsilon < -1$ then $\mathrm{MR}(q) > 0$. Selling one more unit raises the seller's revenue.**

# Marginal Revenue and Own-Price Elasticity of Demand

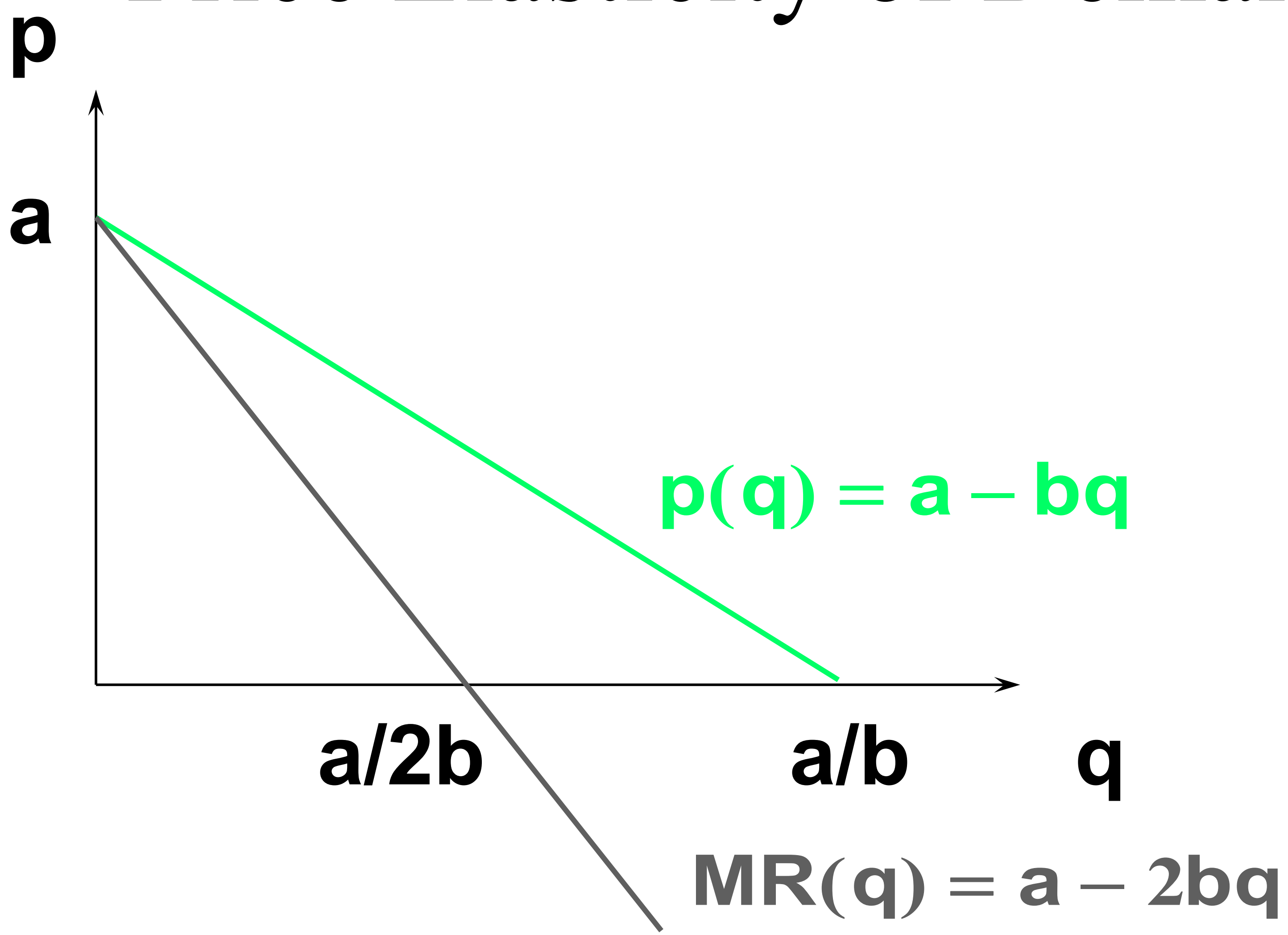
**An example with linear inverse demand.**

$$p(q) = a - bq.$$

**Then** $R(q) = p(q)q = (a - bq)q$

**and** $MR(q) = a - 2bq.$

# Marginal Revenue and Own-Price Elasticity of Demand



# Marginal Revenue and Own-Price Elasticity of Demand