



INTERMEDIATE
MICROECONOMICS

NINTH EDITION

HAL R. VARIAN

Chapter 32

Exchange

Exchange

- ◆ **Two consumers, A and B.**
- ◆ **Their endowments of goods 1 and 2 are $\omega^A = (\omega_1^A, \omega_2^A)$ and $\omega^B = (\omega_1^B, \omega_2^B)$.**
- ◆ **E.g. $\omega^A = (6,4)$ and $\omega^B = (2,2)$.**
- ◆ **The total quantities available are $\omega_1^A + \omega_1^B = 6 + 2 = 8$ units of good 1 and $\omega_2^A + \omega_2^B = 4 + 2 = 6$ units of good 2.**

Exchange

- ◆ **Edgeworth and Bowley devised a diagram, called an Edgeworth box, to show all possible allocations of the available quantities of goods 1 and 2 between the two consumers.**

Starting an Edgeworth Box



Starting an Edgeworth Box



$$\text{Width} = \omega_1^{\text{A}} + \omega_1^{\text{B}} = 6 + 2 = 8$$

Starting an Edgeworth Box

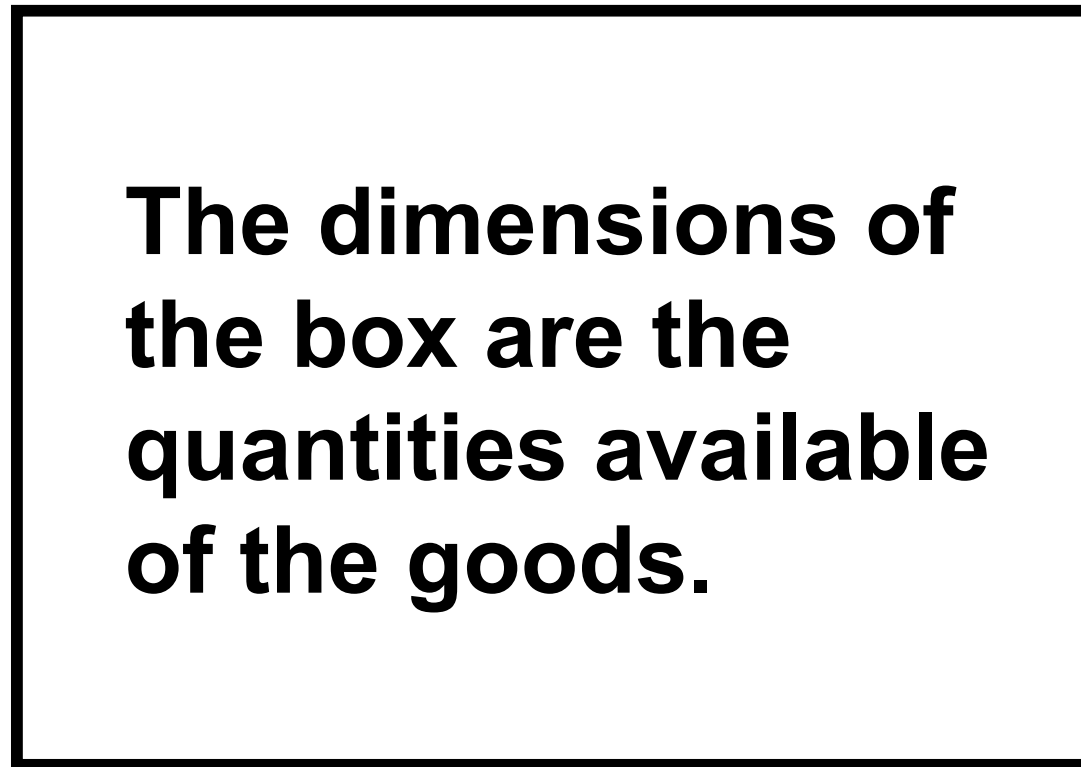
Height =
 $\omega_2^A + \omega_2^B$
 $= 4 + 2$
 $= 6$



Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

Starting an Edgeworth Box

$$\begin{aligned}\text{Height} &= \omega_2^A + \omega_2^B \\ &= 4 + 2 \\ &= 6\end{aligned}$$



$$\text{Width} = \omega_1^A + \omega_1^B = 6 + 2 = 8$$

Feasible Allocations

- ◆ **What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?**
- ◆ **How can all of the feasible allocations be depicted by the Edgeworth box diagram?**

Feasible Allocations

- ◆ **What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?**
- ◆ **How can all of the feasible allocations be depicted by the Edgeworth box diagram?**
- ◆ **One feasible allocation is the before-trade allocation; i.e. the endowment allocation.**

The Endowment Allocation

Height =

$$\omega_2^A + \omega_2^B$$

$$= 4 + 2$$

$$= 6$$

The endowment allocation is

$$\omega^A = (6, 4)$$

and

$$\omega^B = (2, 2).$$

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

The Endowment Allocation

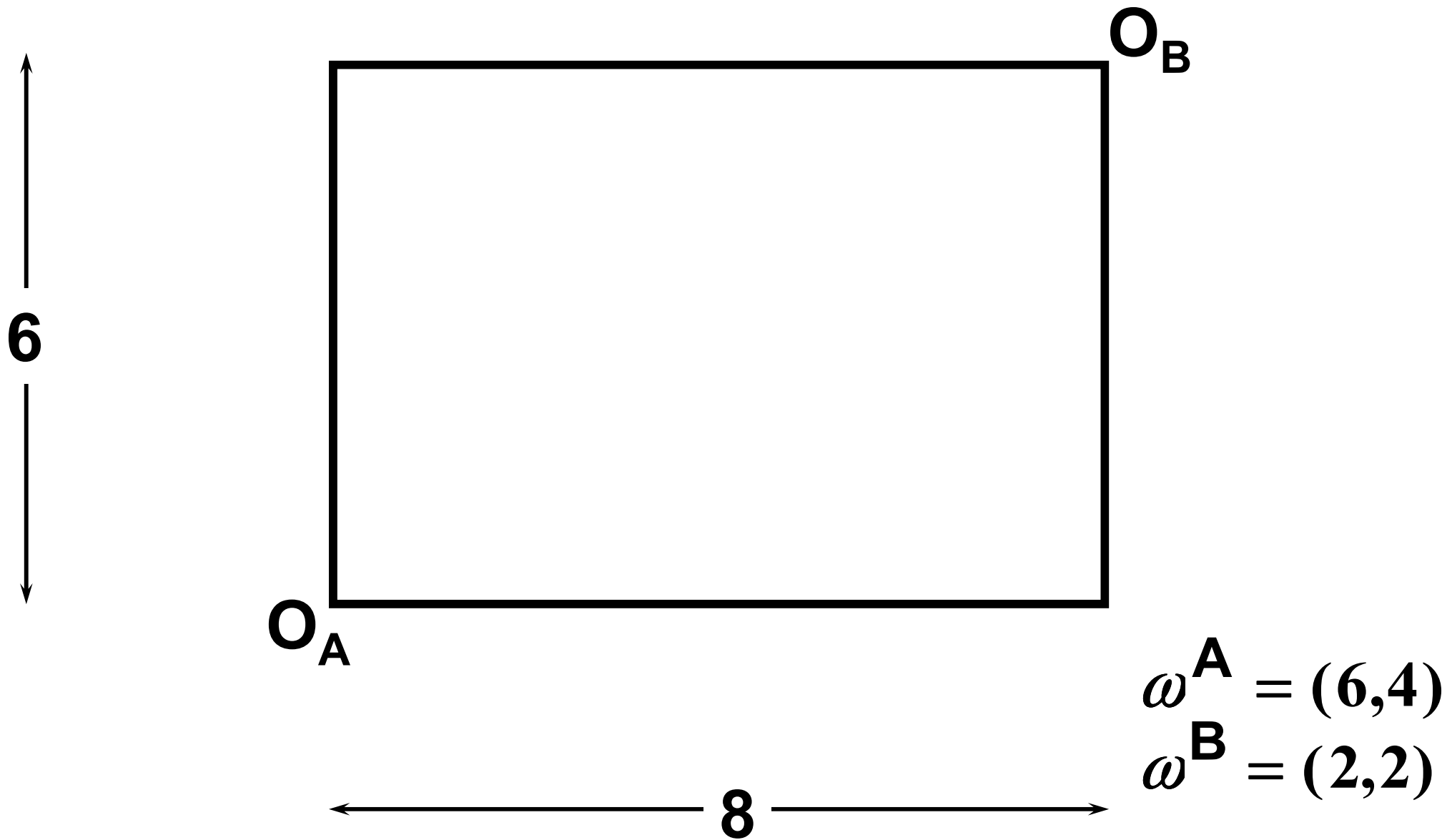
Height =
 $\omega_2^A + \omega_2^B$
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 $= 6$



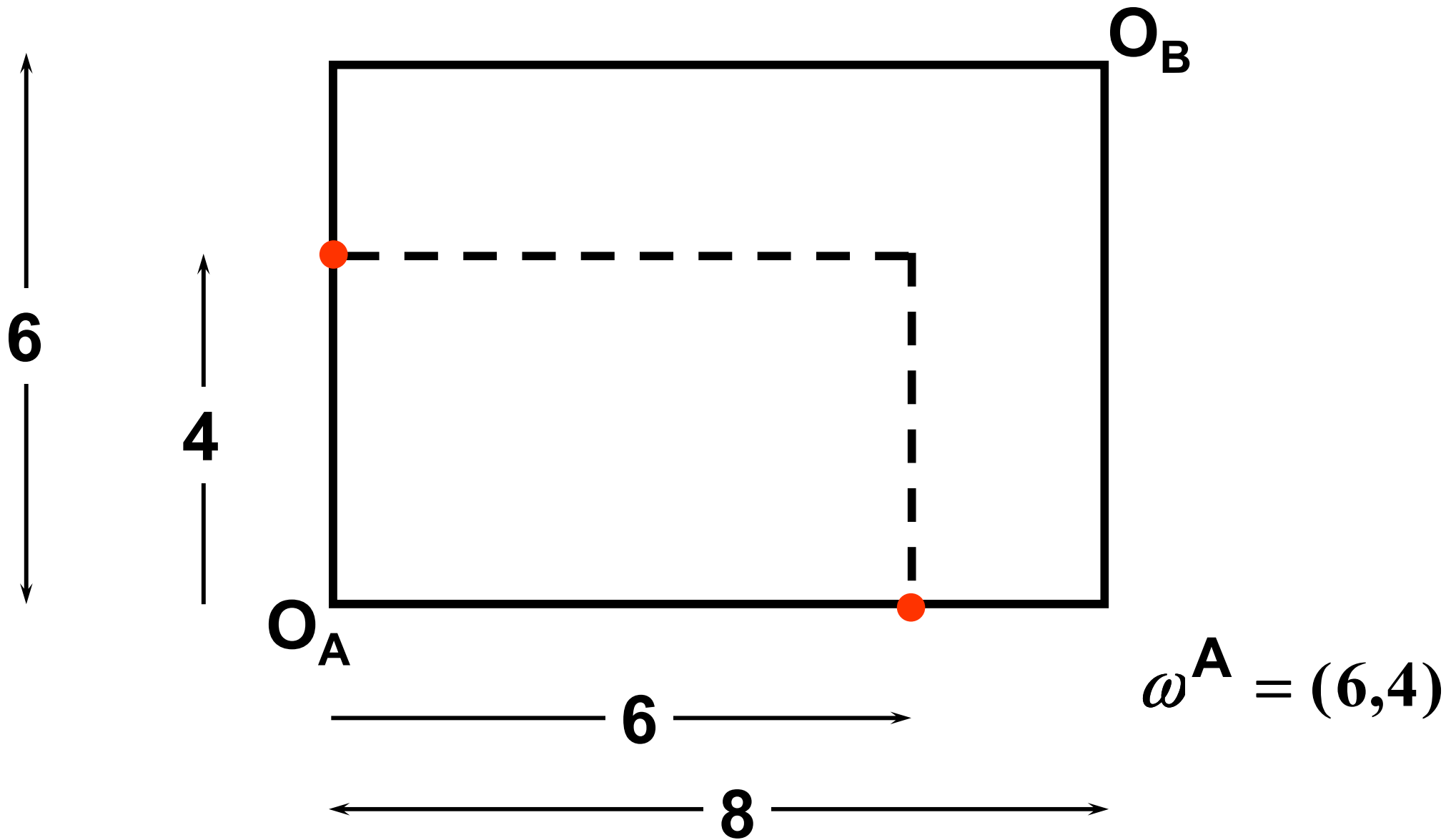
Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

$\omega^A = (6, 4)$
 $\omega^B = (2, 2)$

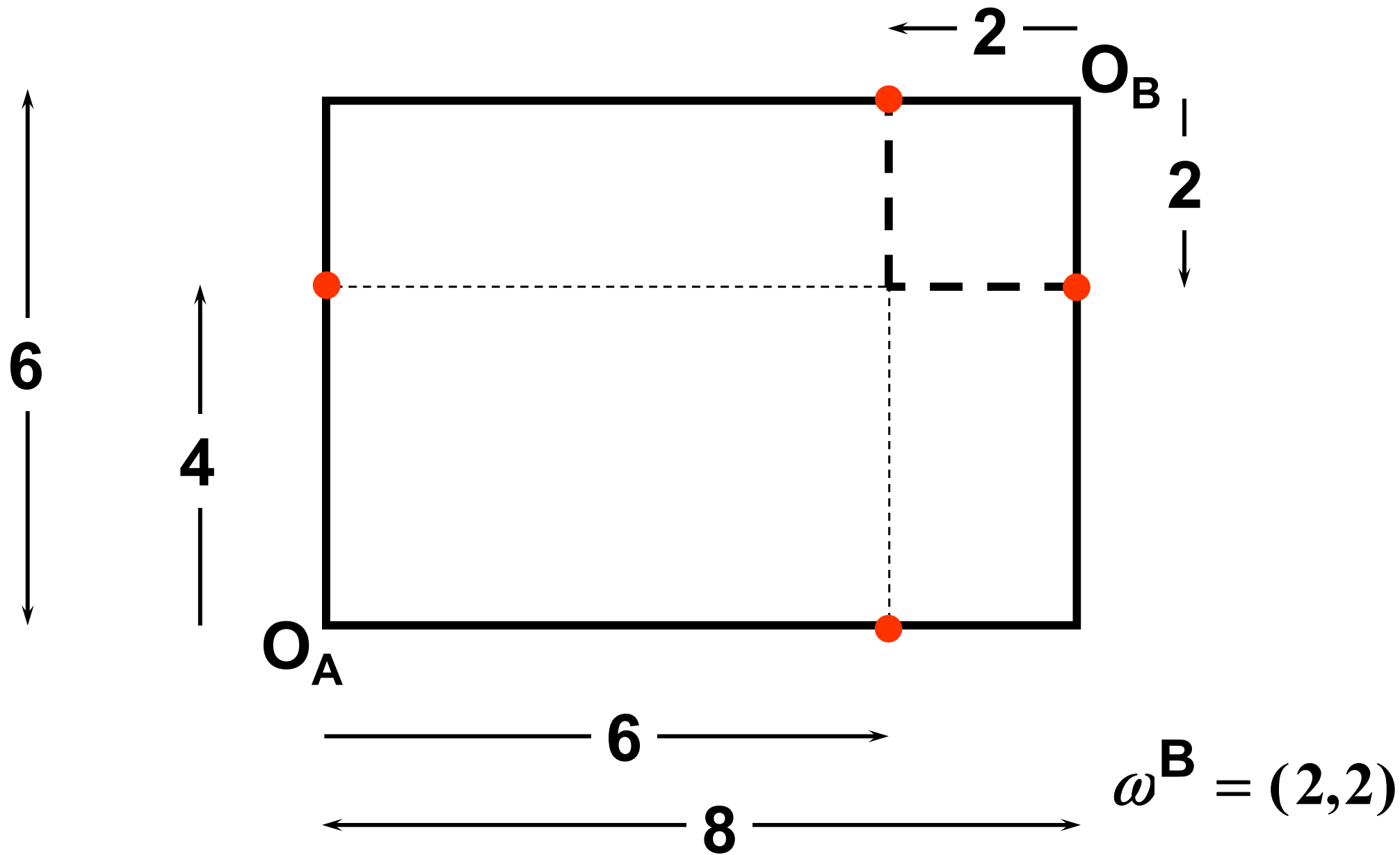
The Endowment Allocation



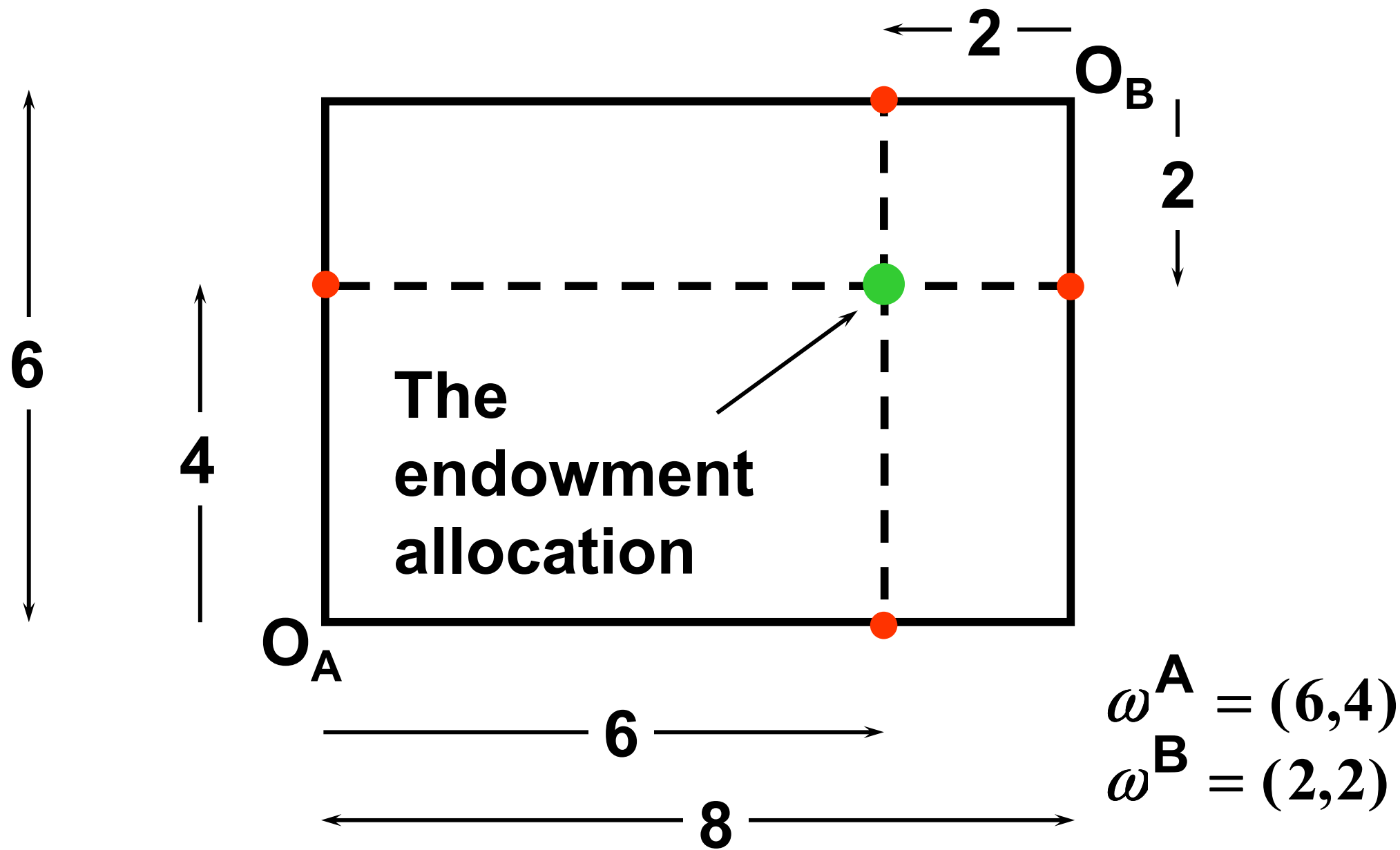
The Endowment Allocation



The Endowment Allocation



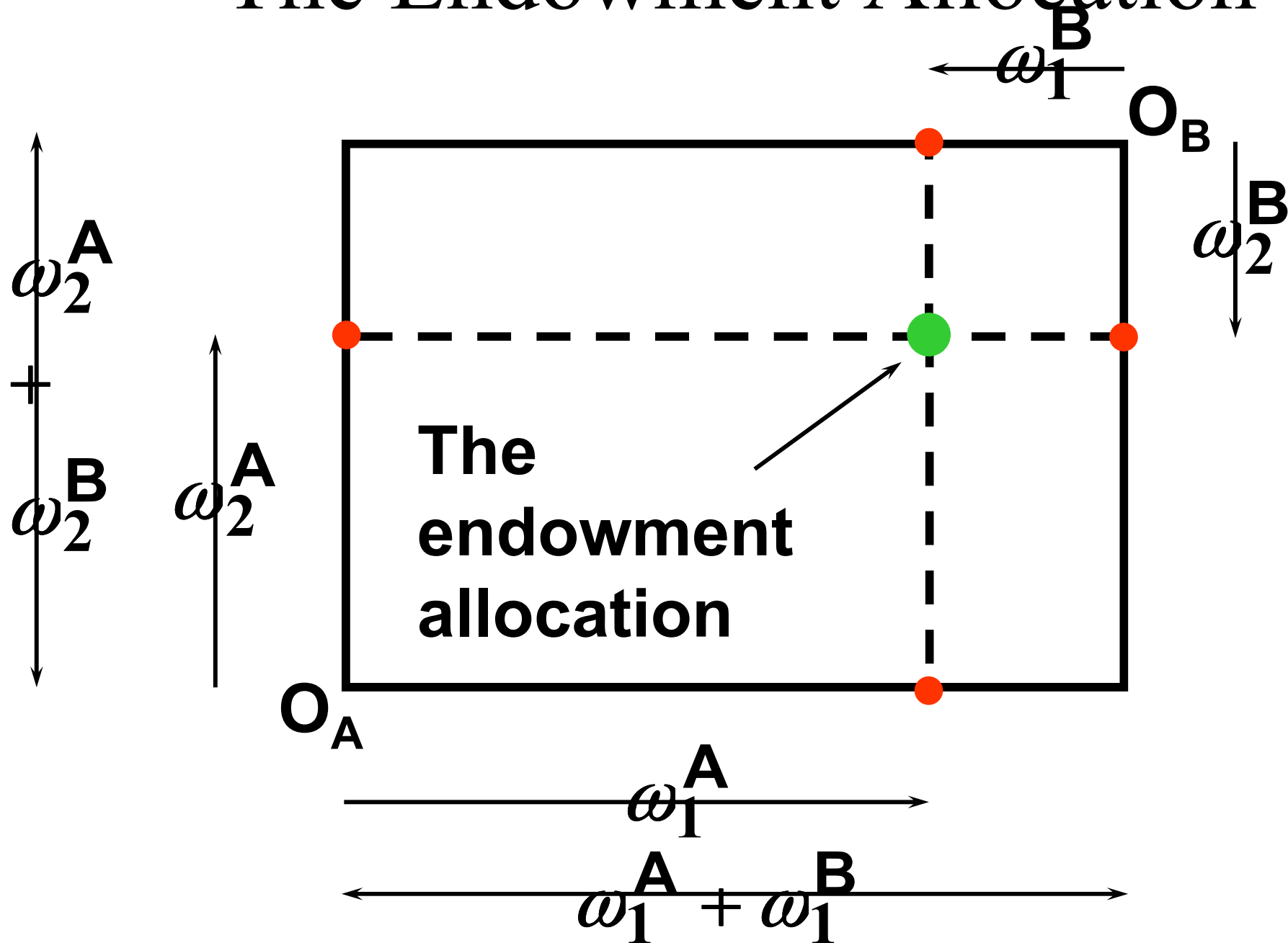
The Endowment Allocation



The Endowment Allocation

More generally, ...

The Endowment Allocation



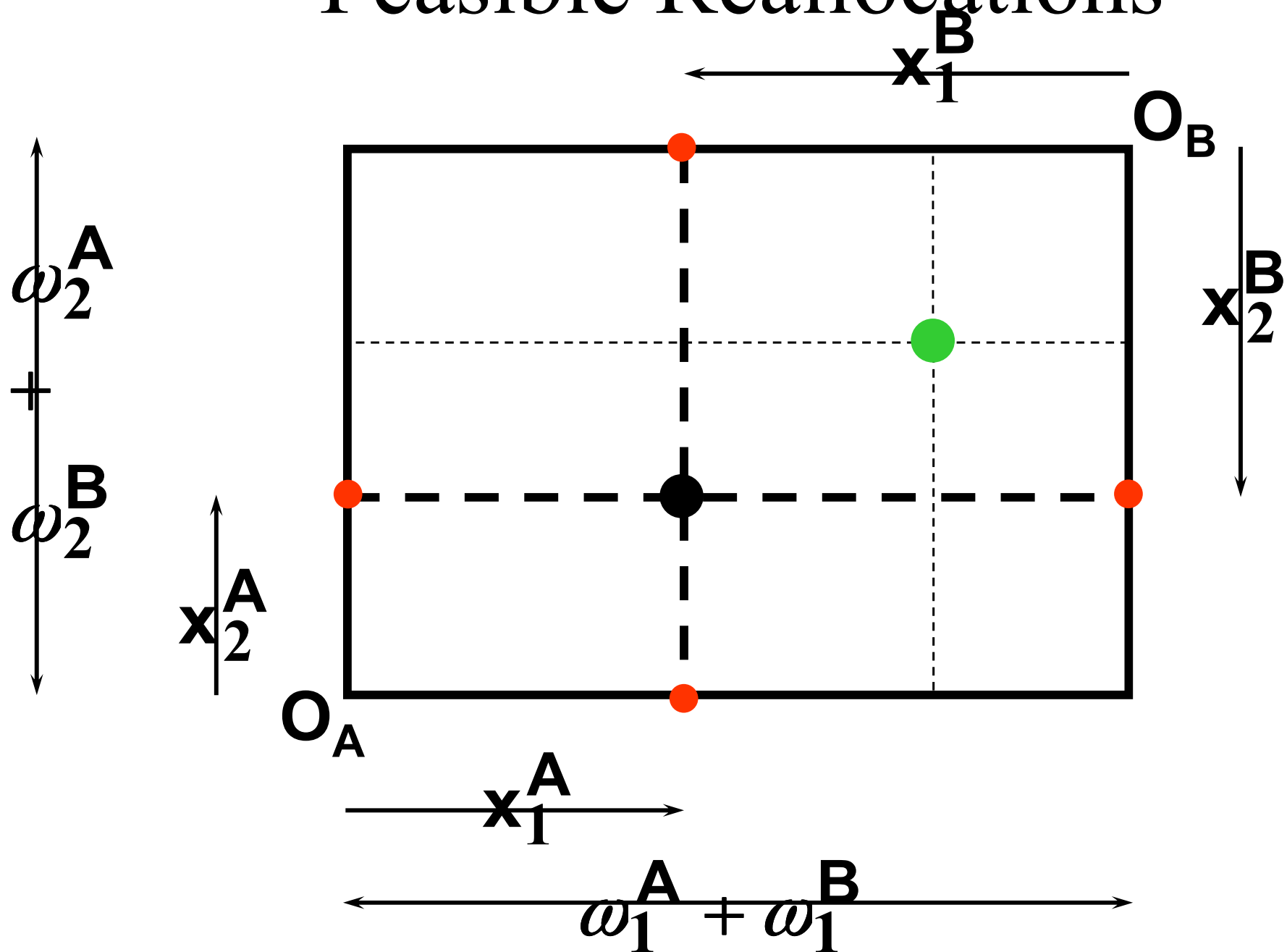
Other Feasible Allocations

- ◆ (x_1^A, x_2^A) denotes an allocation to consumer A.
- ◆ (x_1^B, x_2^B) denotes an allocation to consumer B.
- ◆ An allocation is feasible if and only if

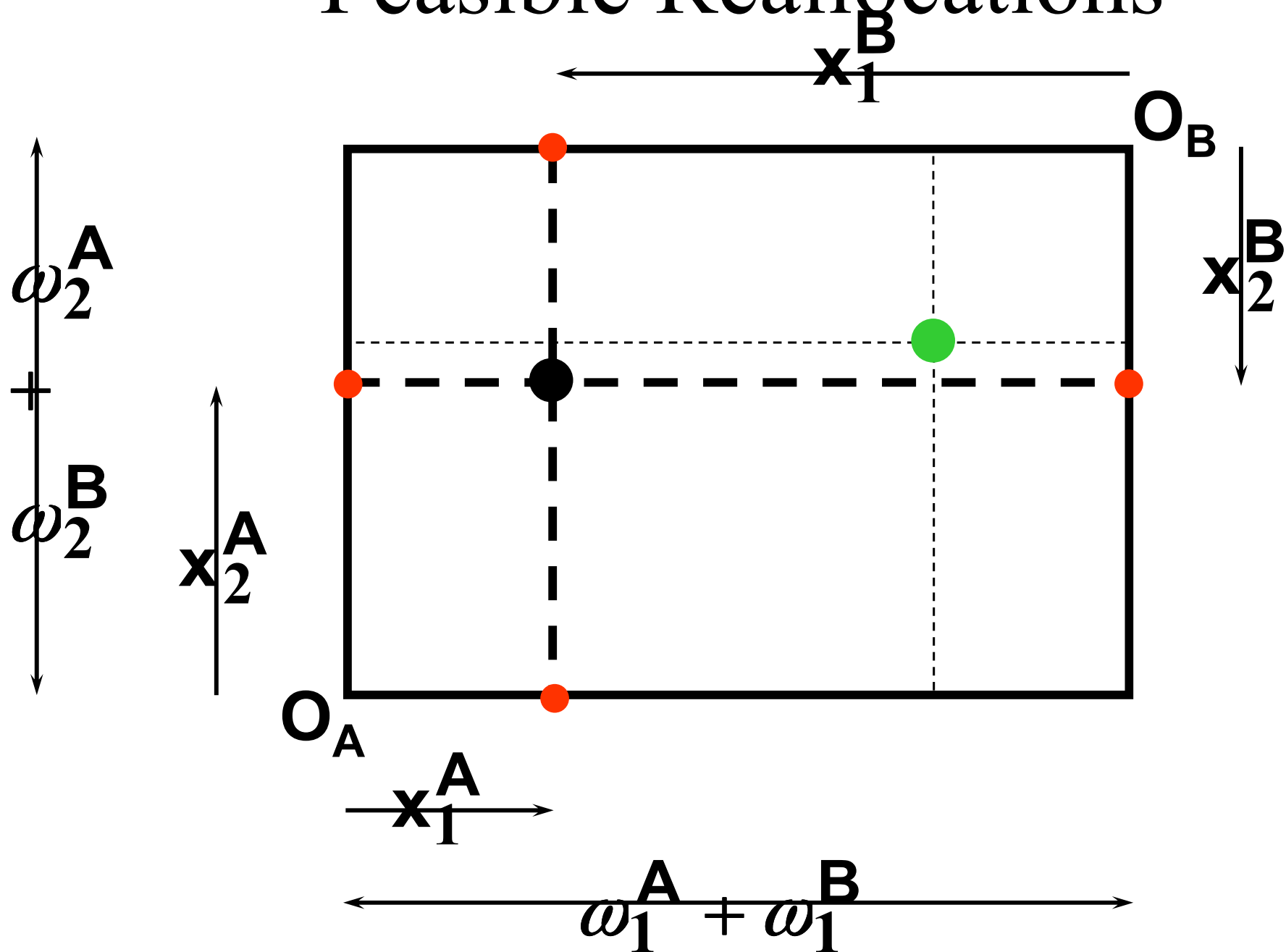
$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B$$

and $x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$.

Feasible Reallocations



Feasible Reallocations



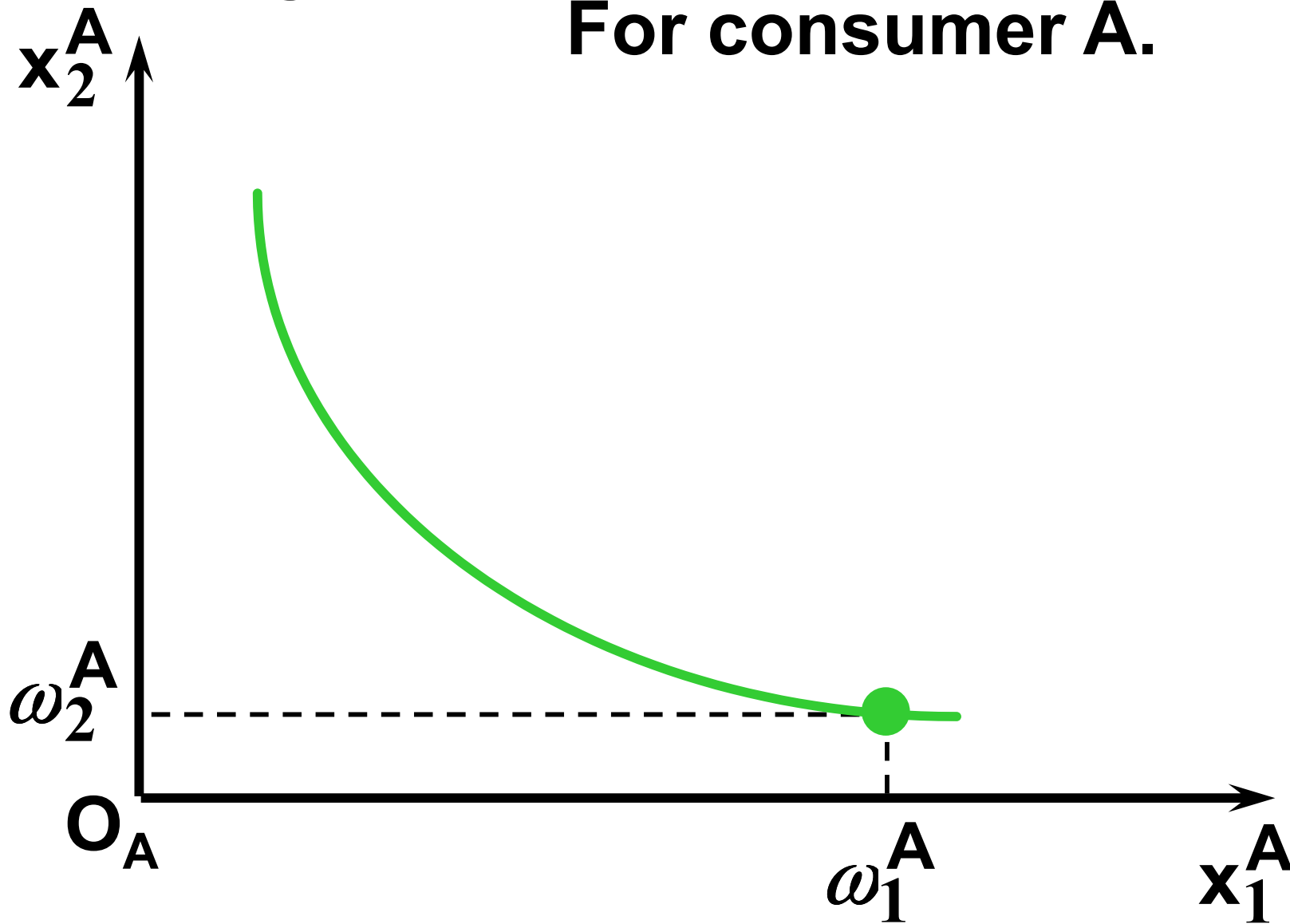
Feasible Reallocations

- ◆ **All points in the box, including the boundary, represent feasible allocations of the combined endowments.**

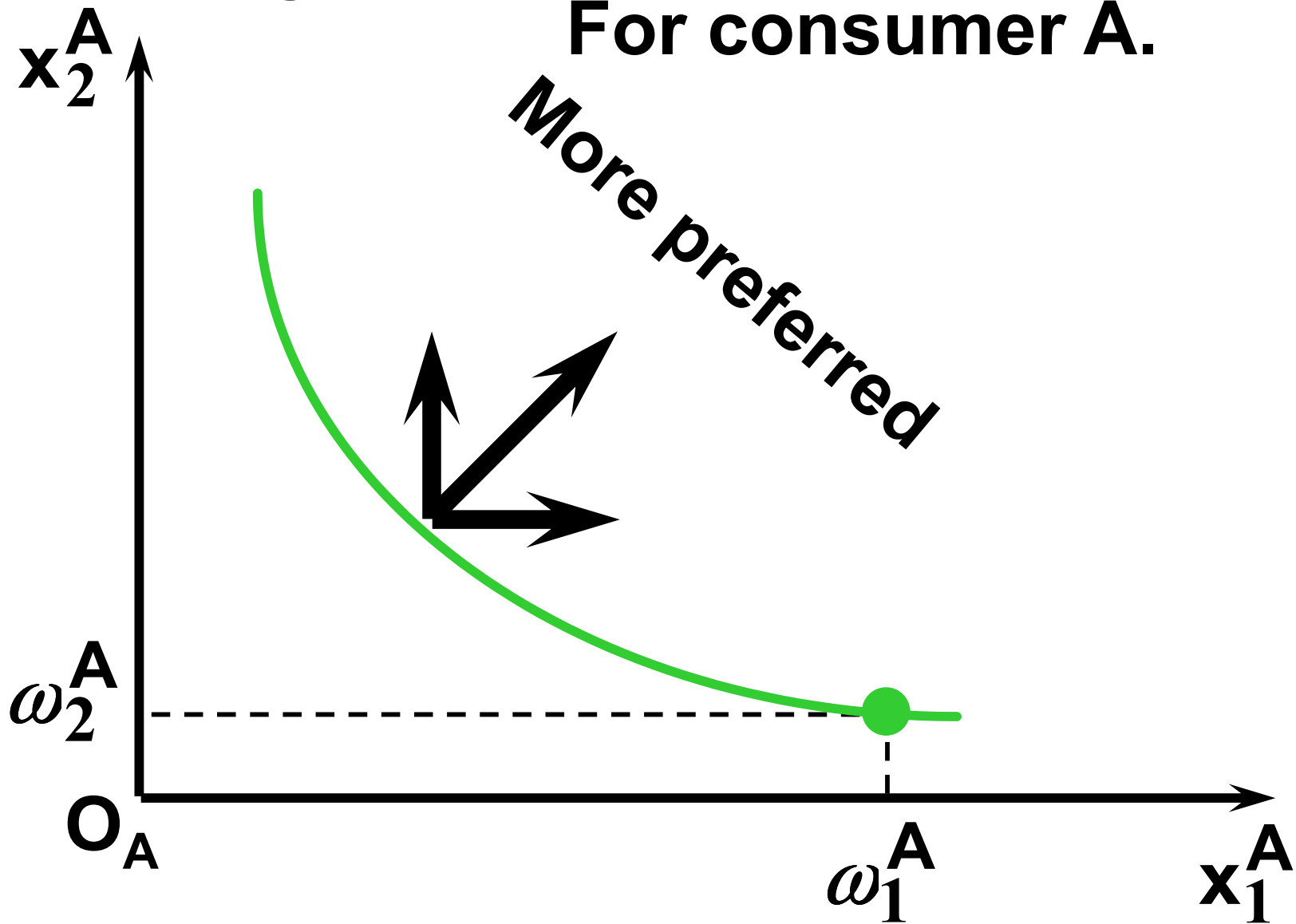
Feasible Reallocations

- ◆ **All points in the box, including the boundary, represent feasible allocations of the combined endowments.**
- ◆ **Which allocations will be blocked by one or both consumers?**
- ◆ **Which allocations make both consumers better off?**

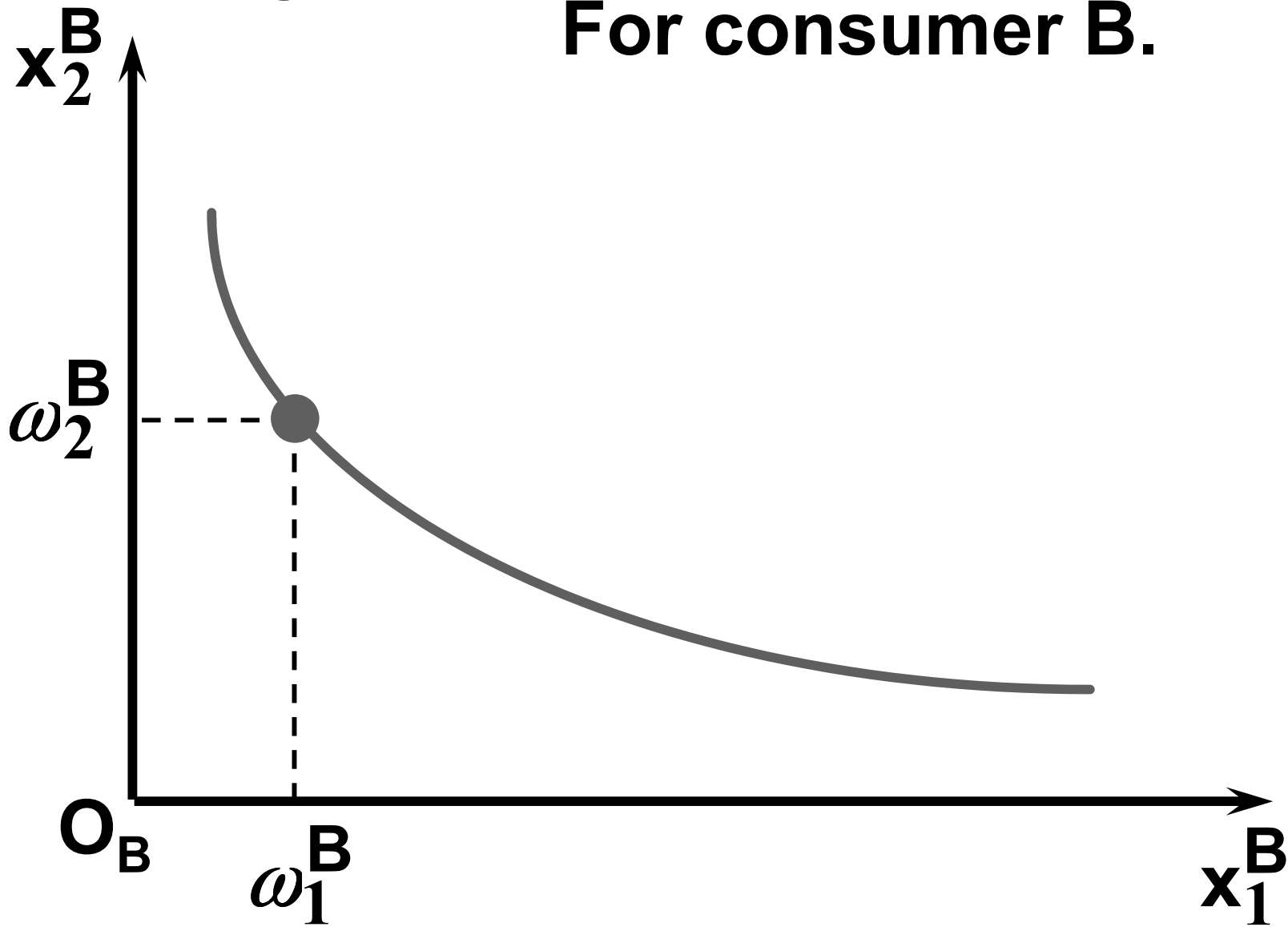
Adding Preferences to the Box For consumer A.



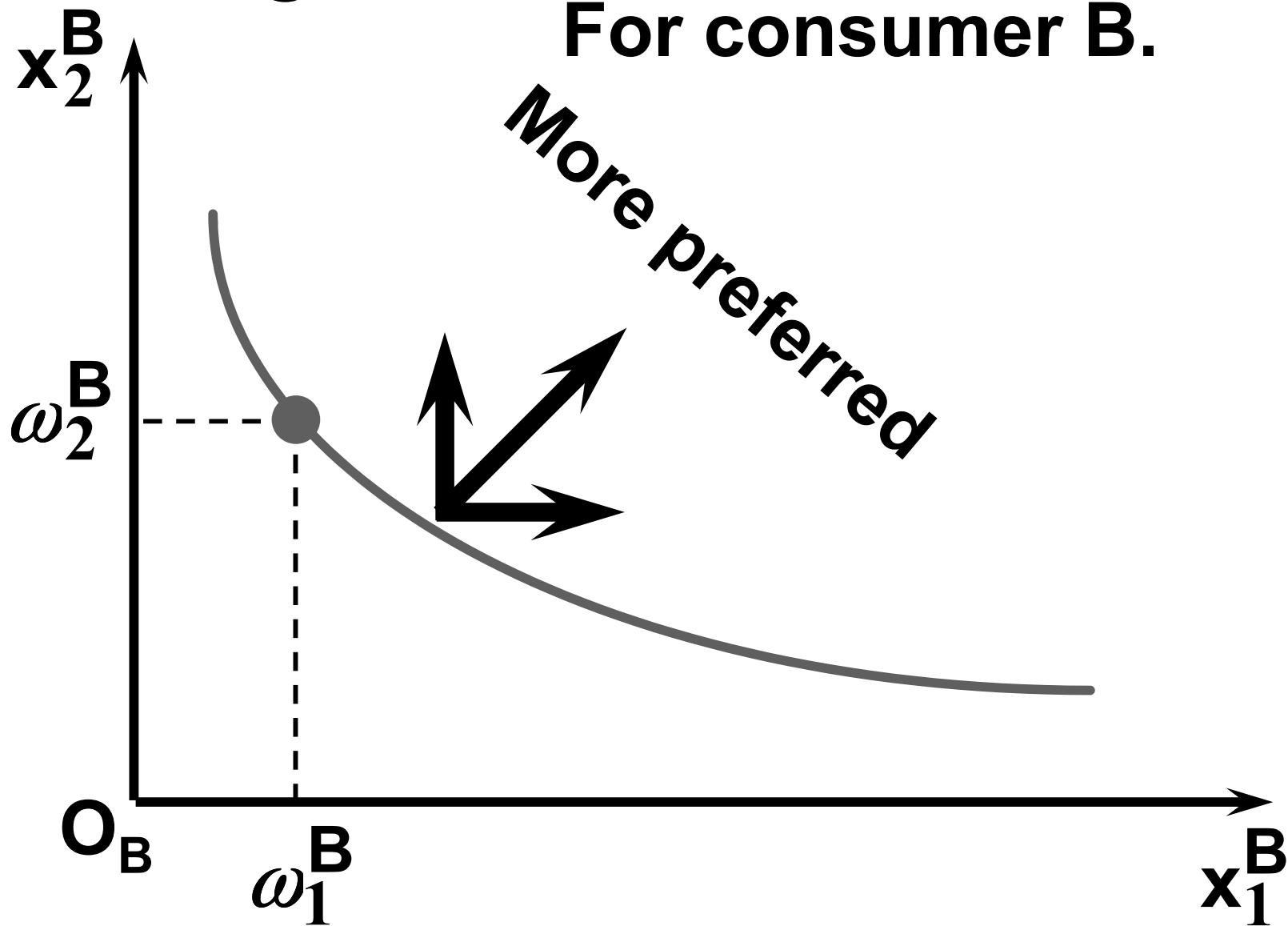
Adding Preferences to the Box For consumer A.



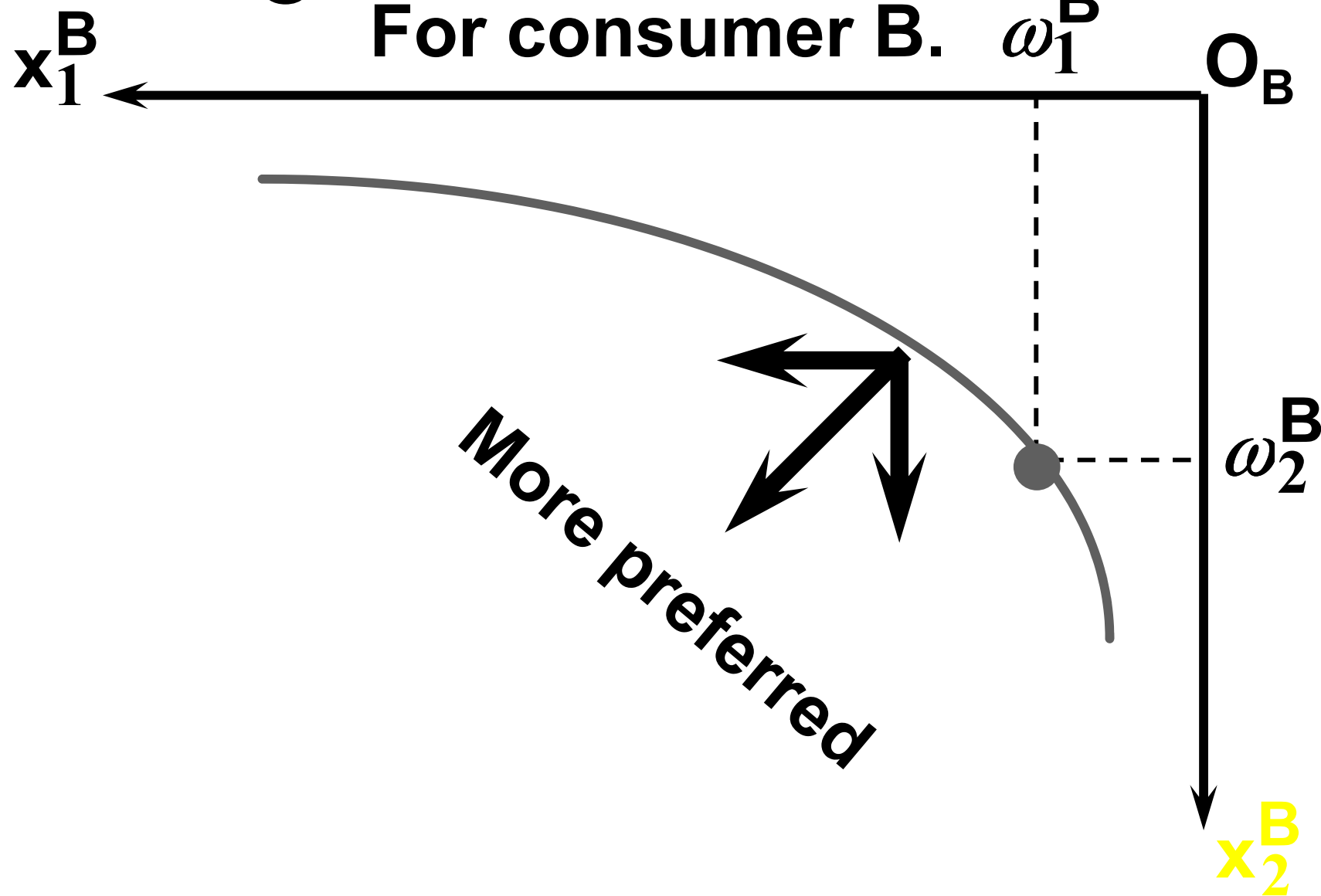
Adding Preferences to the Box For consumer B.



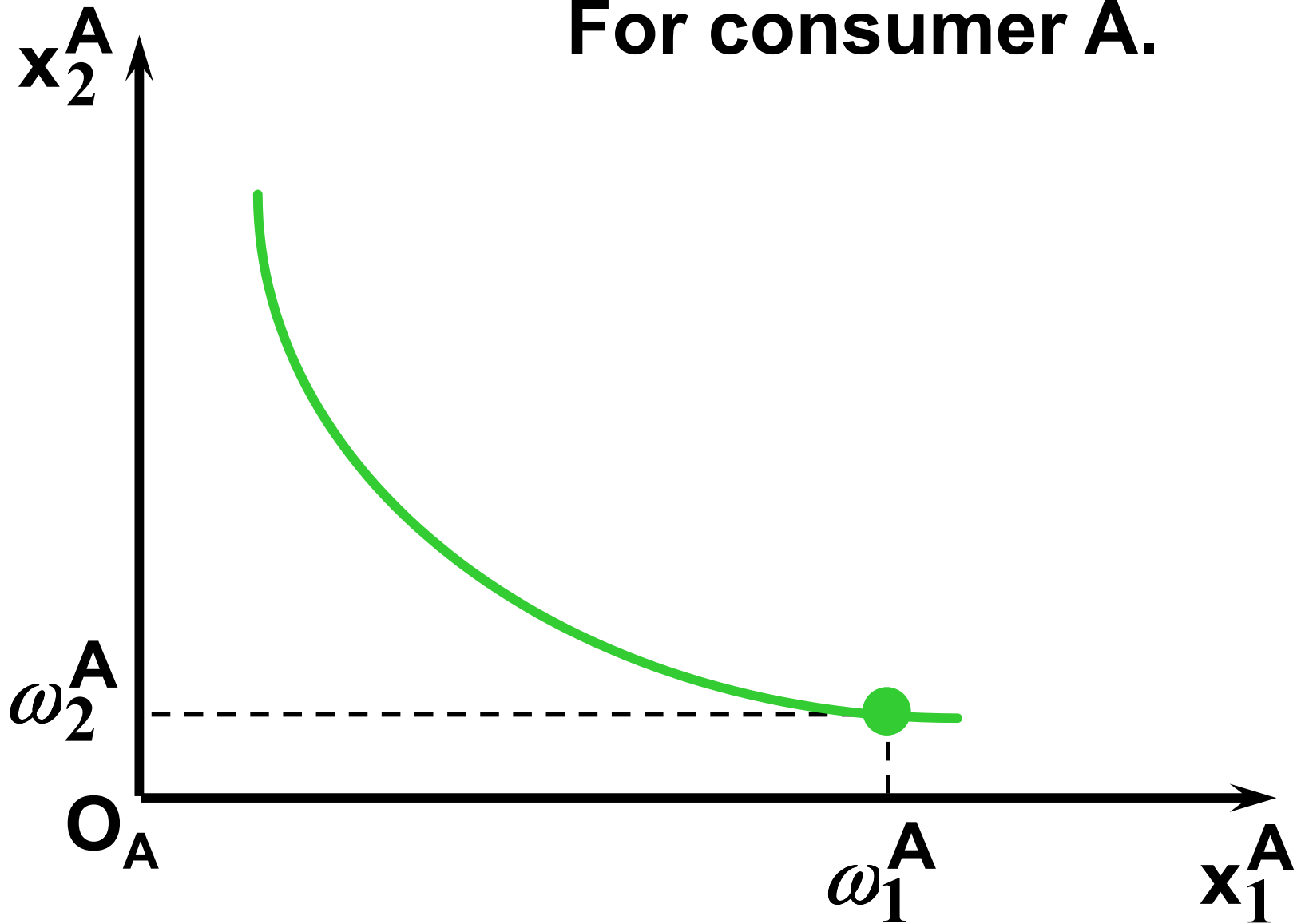
Adding Preferences to the Box For consumer B.



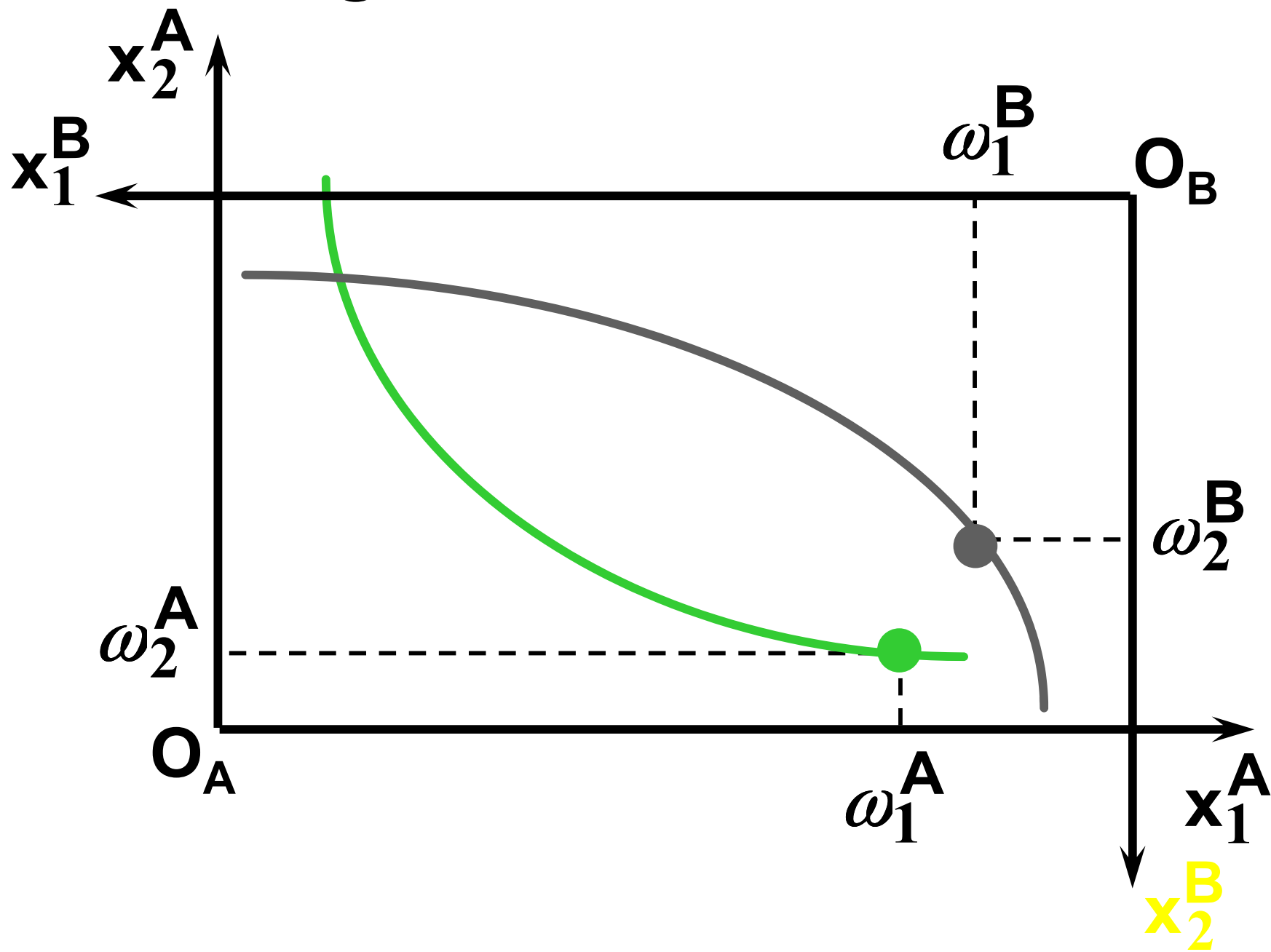
Adding Preferences to the Box



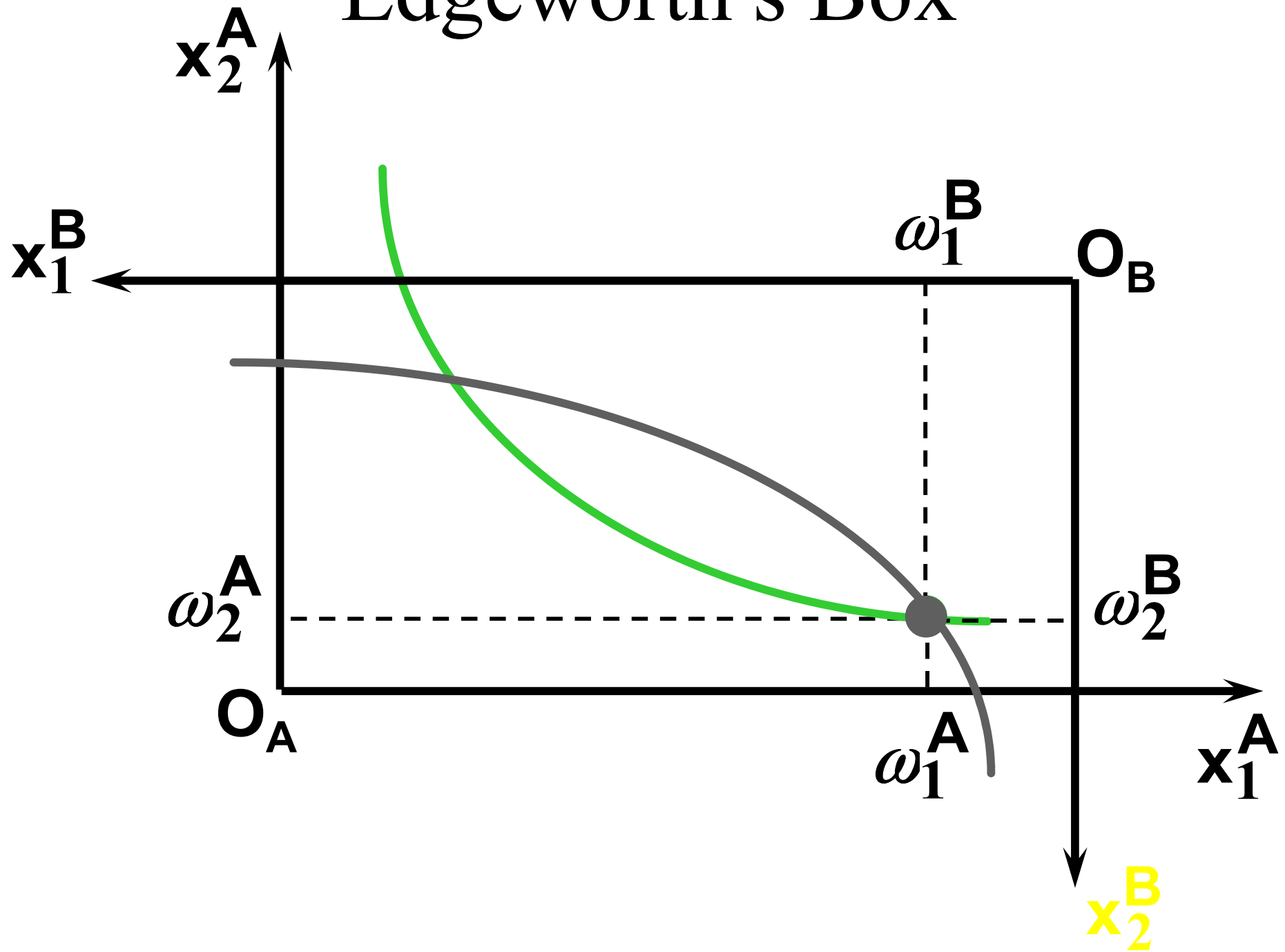
Adding Preferences to the Box For consumer A.



Adding Preferences to the Box



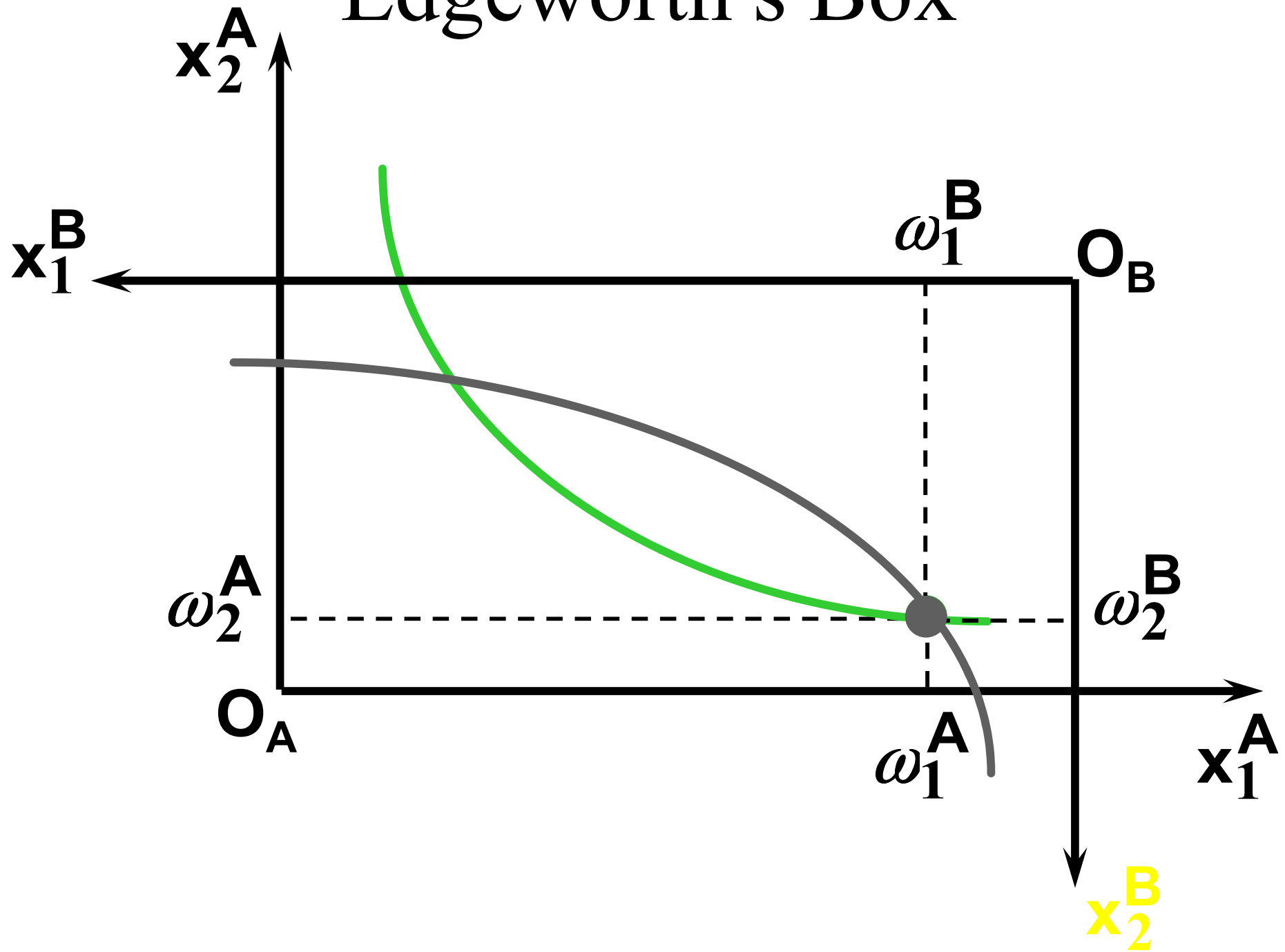
Edgeworth's Box



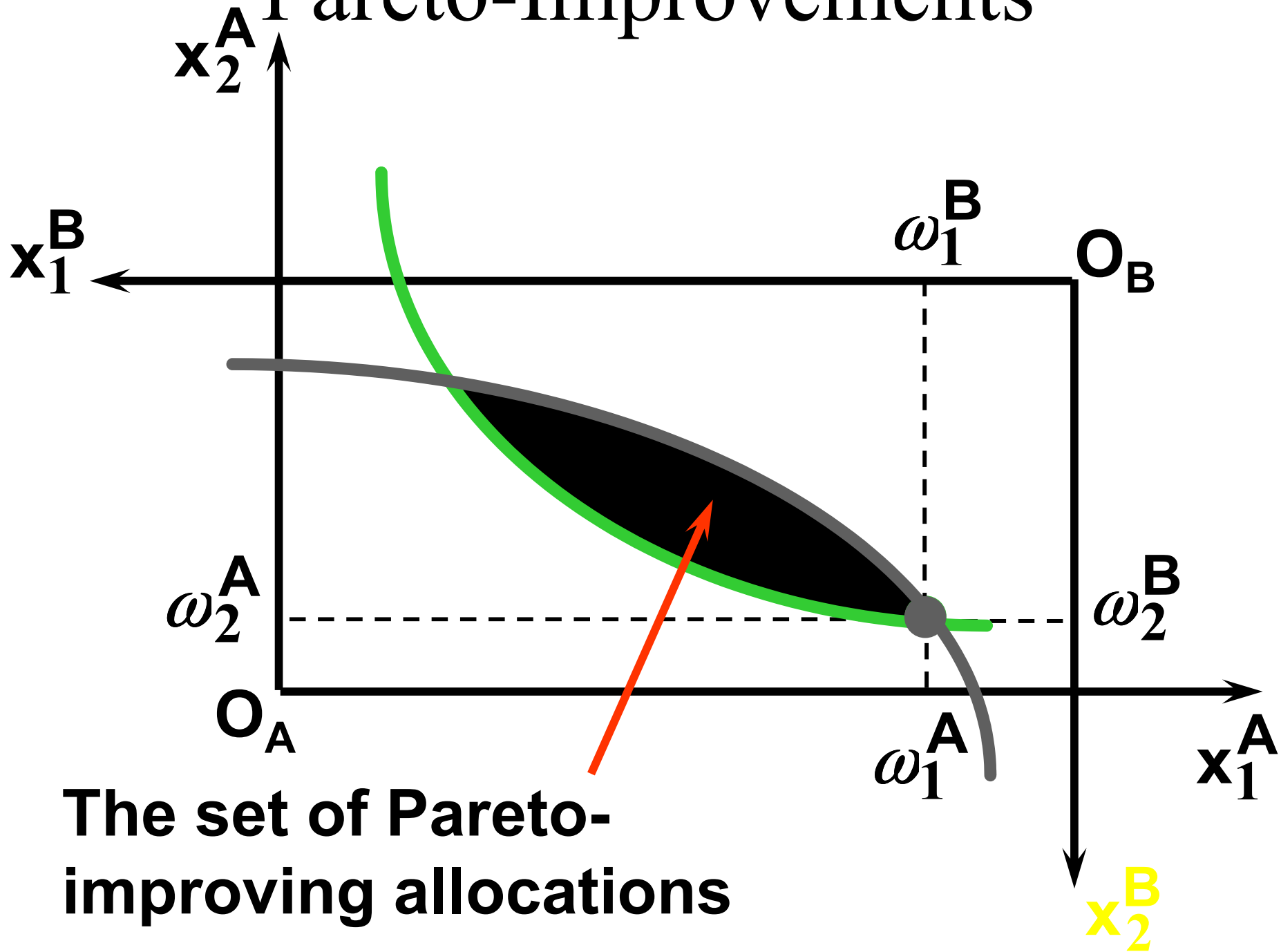
Pareto-Improvement

- ◆ **An allocation of the endowment that improves the welfare of a consumer without reducing the welfare of another is a Pareto-improving allocation.**
- ◆ **Where are the Pareto-improving allocations?**

Edgeworth's Box



Pareto-Improvements

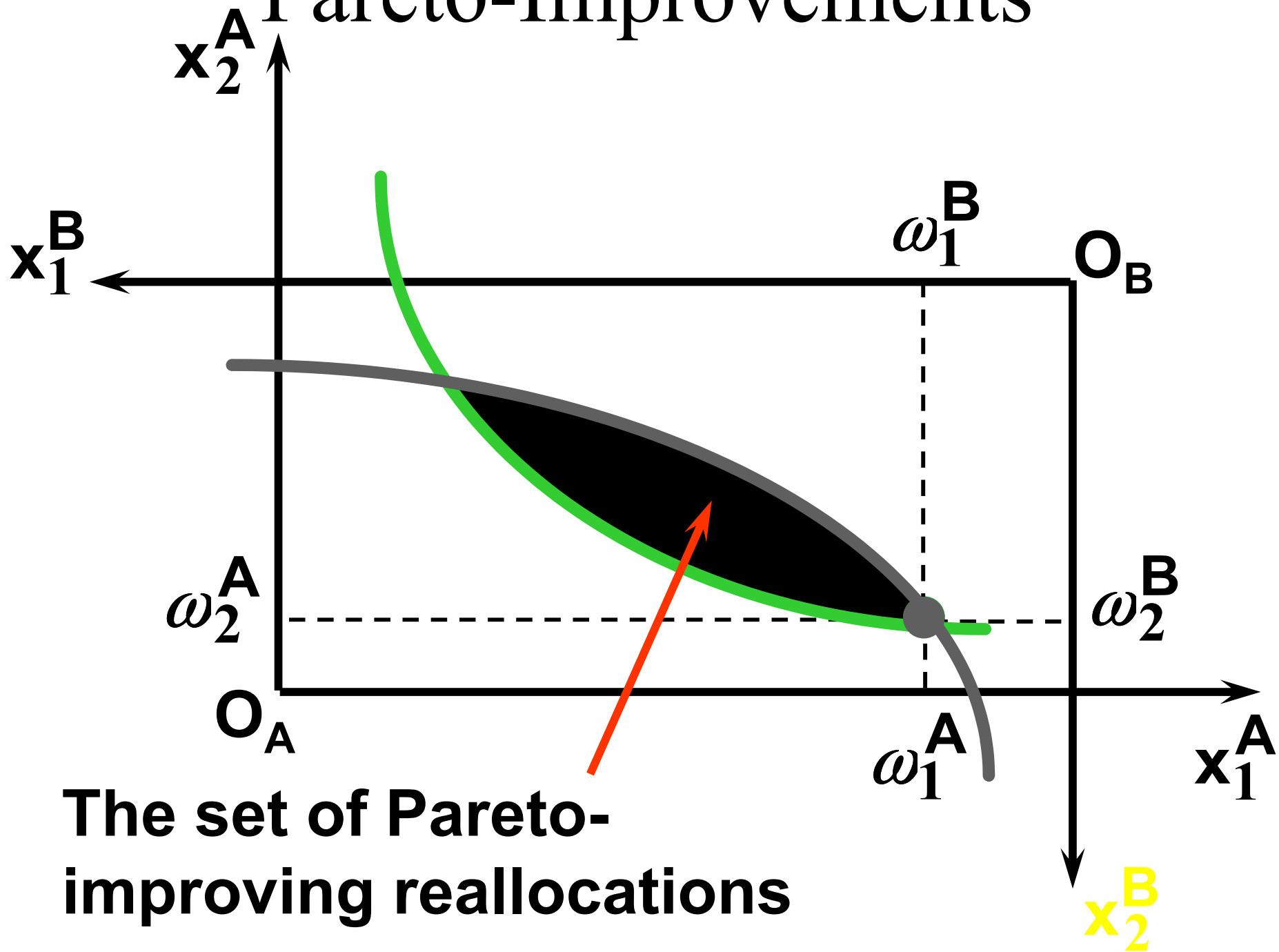


The set of Pareto-improving allocations

Pareto-Improvements

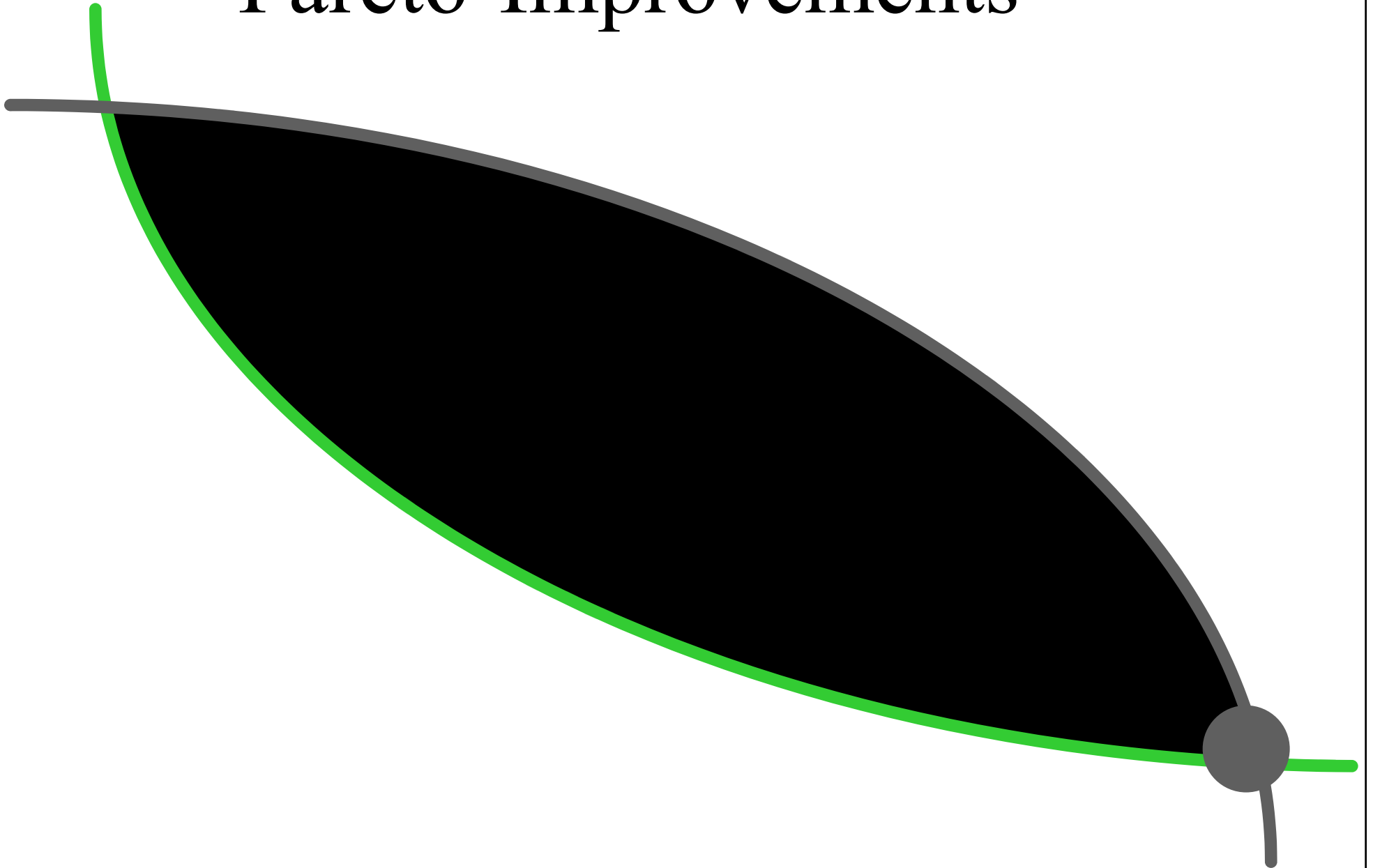
- ◆ **Since each consumer can refuse to trade, the only possible outcomes from exchange are Pareto-improving allocations.**
- ◆ **But which particular Pareto-improving allocation will be the outcome of trade?**

Pareto-Improvements

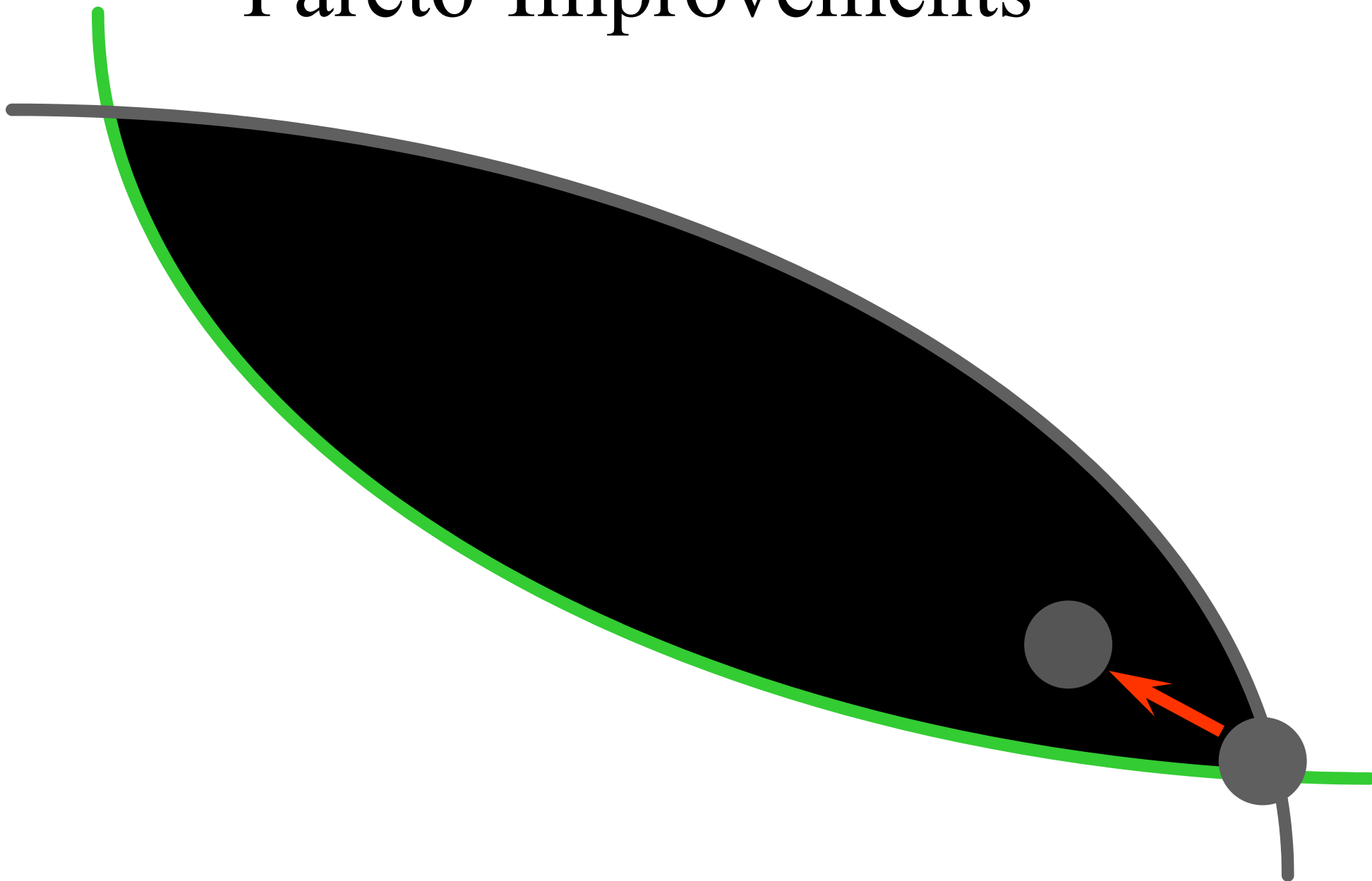


The set of Pareto-improving reallocations

Pareto-Improvements

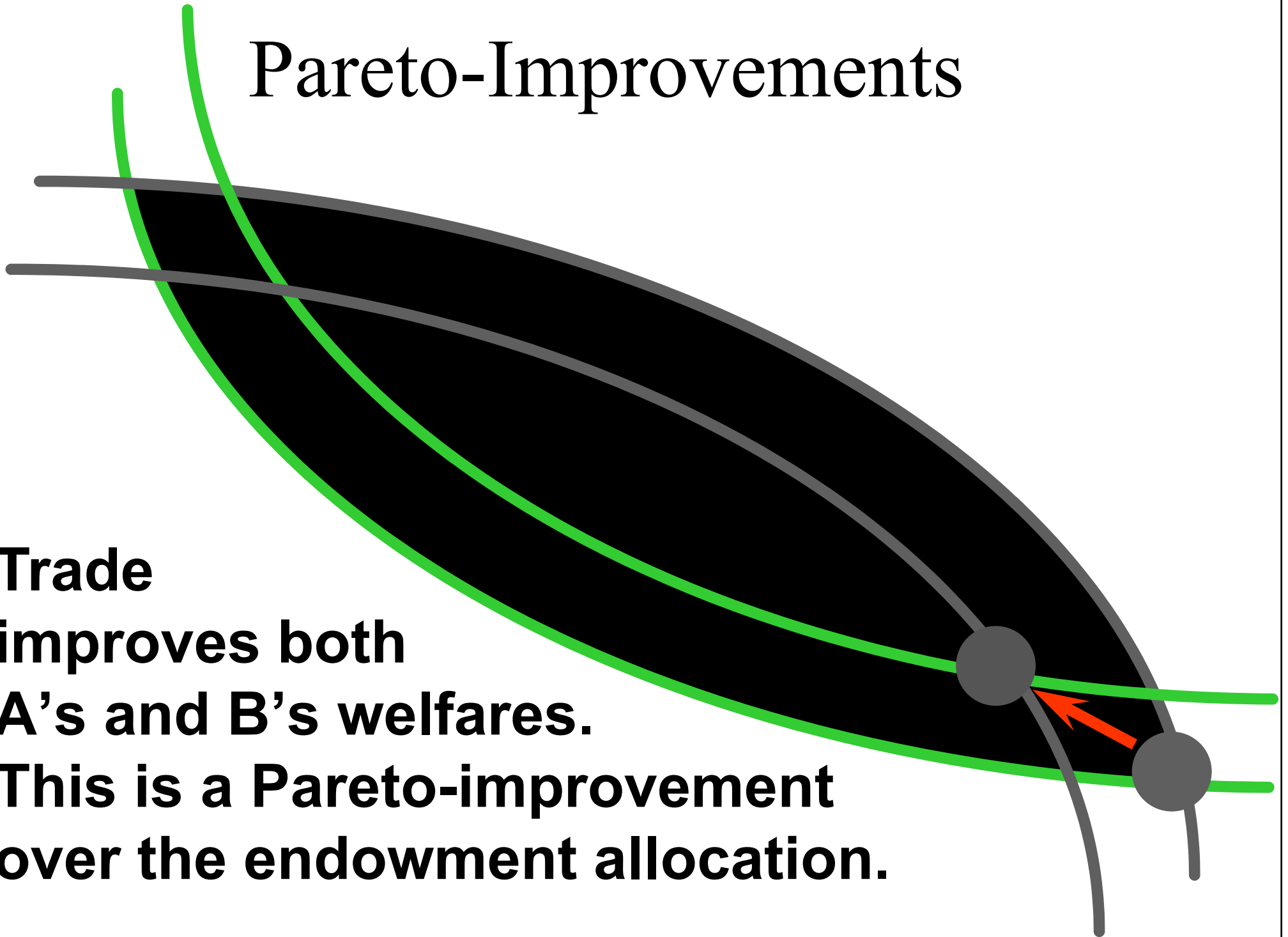


Pareto-Improvements



Pareto-Improvements

Trade improves both A's and B's welfares. This is a Pareto-improvement over the endowment allocation.



Pareto-Improvements
New mutual gains-to-trade region
is the set of all further Pareto-
improving
reallocations.

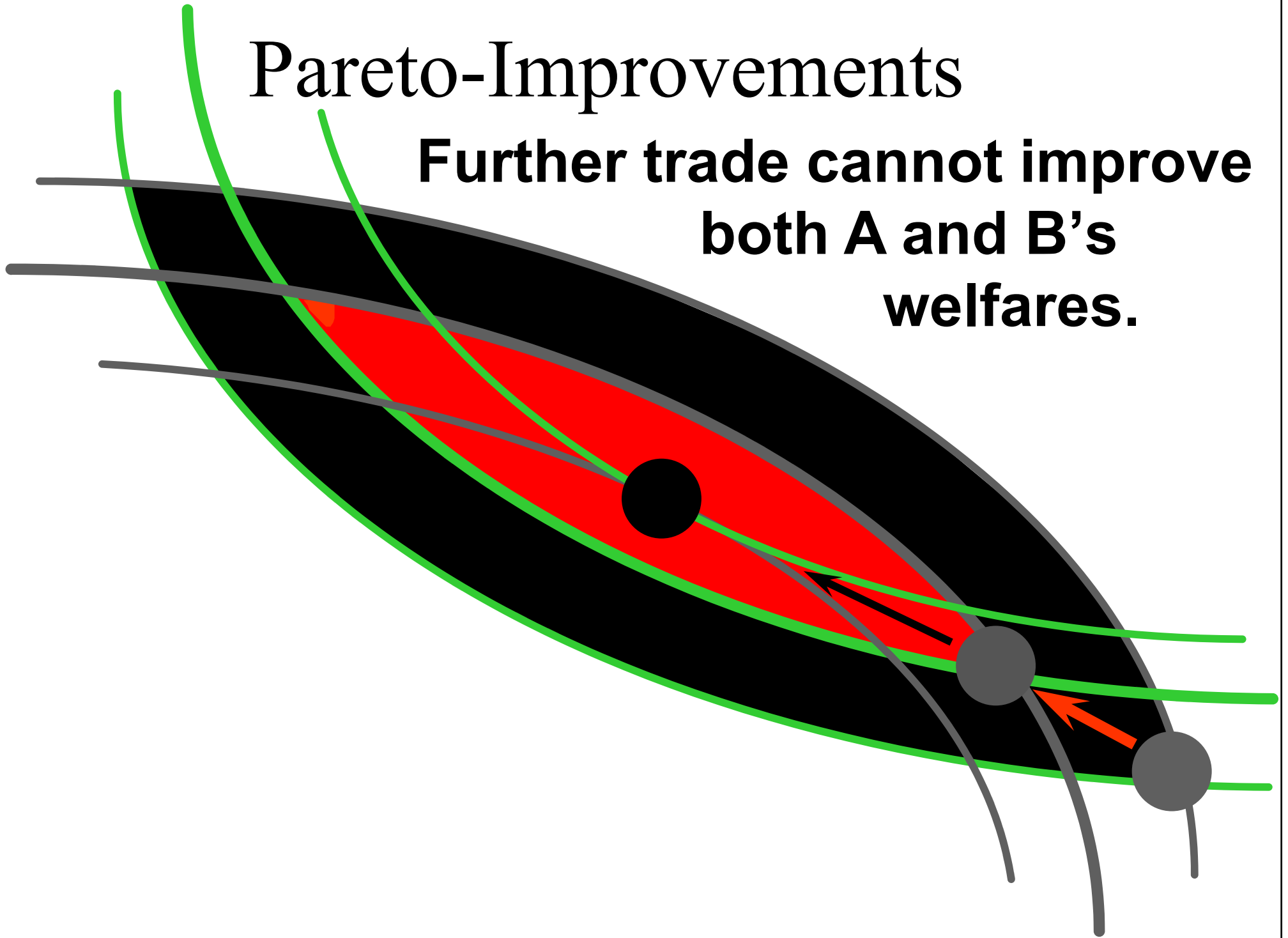


The diagram shows a concave production possibility frontier (PPF) in grey. Two indifference curves are shown: a green one on the left and a grey one on the right. The green curve is tangent to the PPF at a point marked with a grey dot. The grey curve is tangent to the PPF at a point further to the right, also marked with a grey dot. A red shaded region is bounded by the PPF, the green indifference curve, and a vertical line passing through the grey dot on the PPF. An arrow points from the text 'New mutual gains-to-trade region' to this red area. Another arrow points from the text 'improving reallocations' to the grey dot on the PPF. A red arrow points from the grey dot on the PPF towards the grey dot on the grey indifference curve, indicating a movement from the endowment allocation to a Pareto-improvement.

Trade
improves both
A's and B's welfares.
This is a Pareto-improvement
over the endowment allocation.

Pareto-Improvements

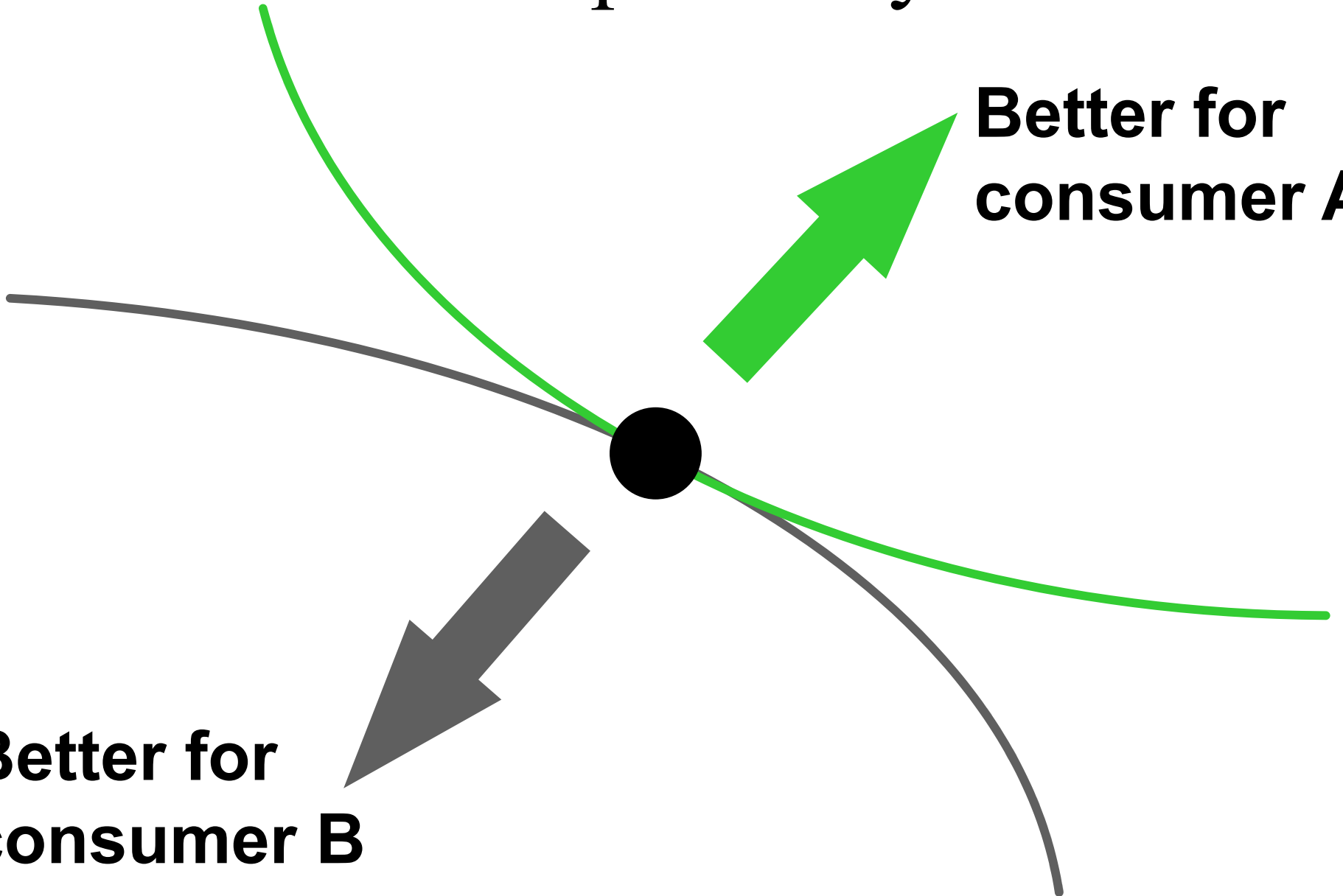
Further trade cannot improve both A and B's welfares.



Pareto-Optimality

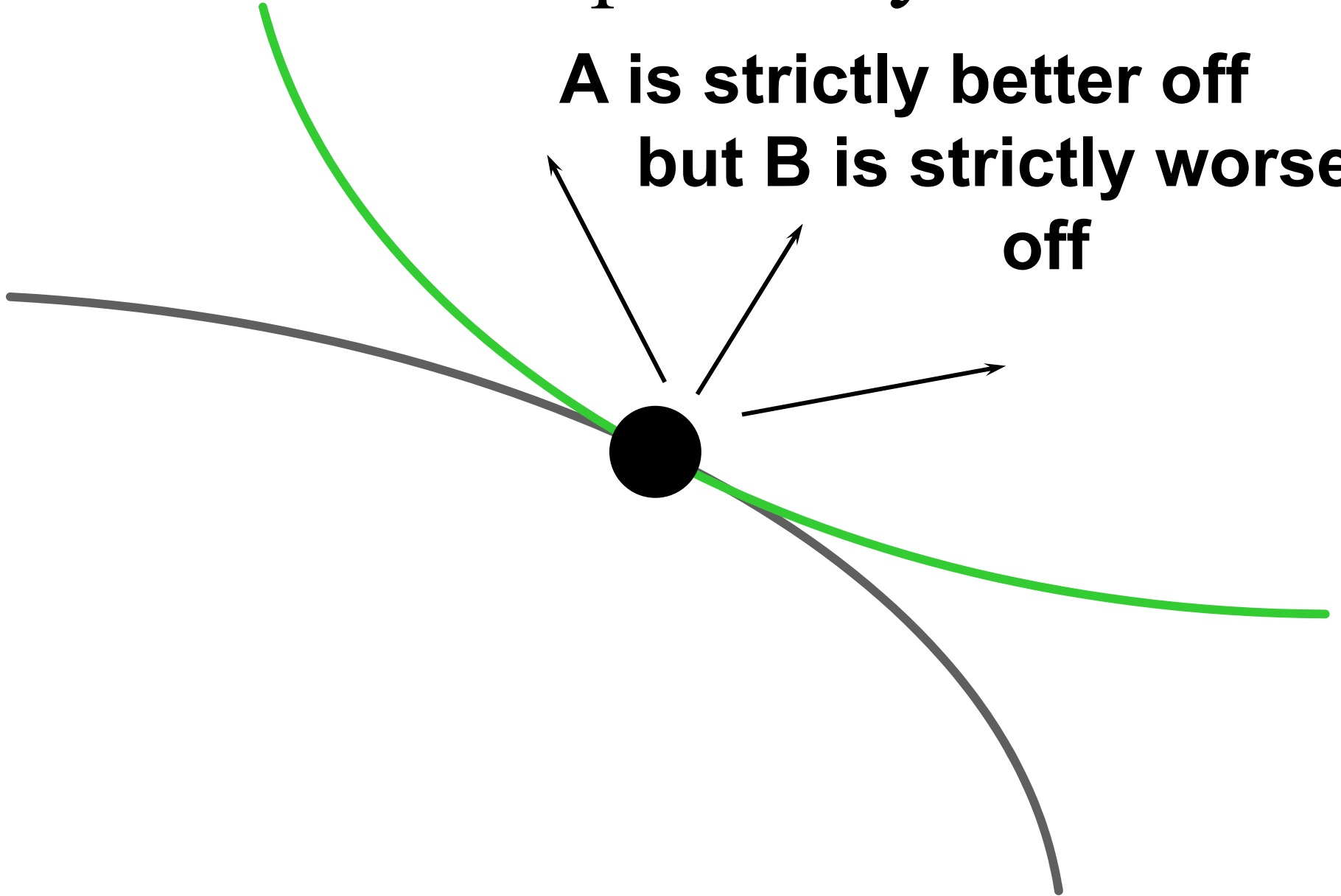
**Better for
consumer A**

**Better for
consumer B**



Pareto-Optimality

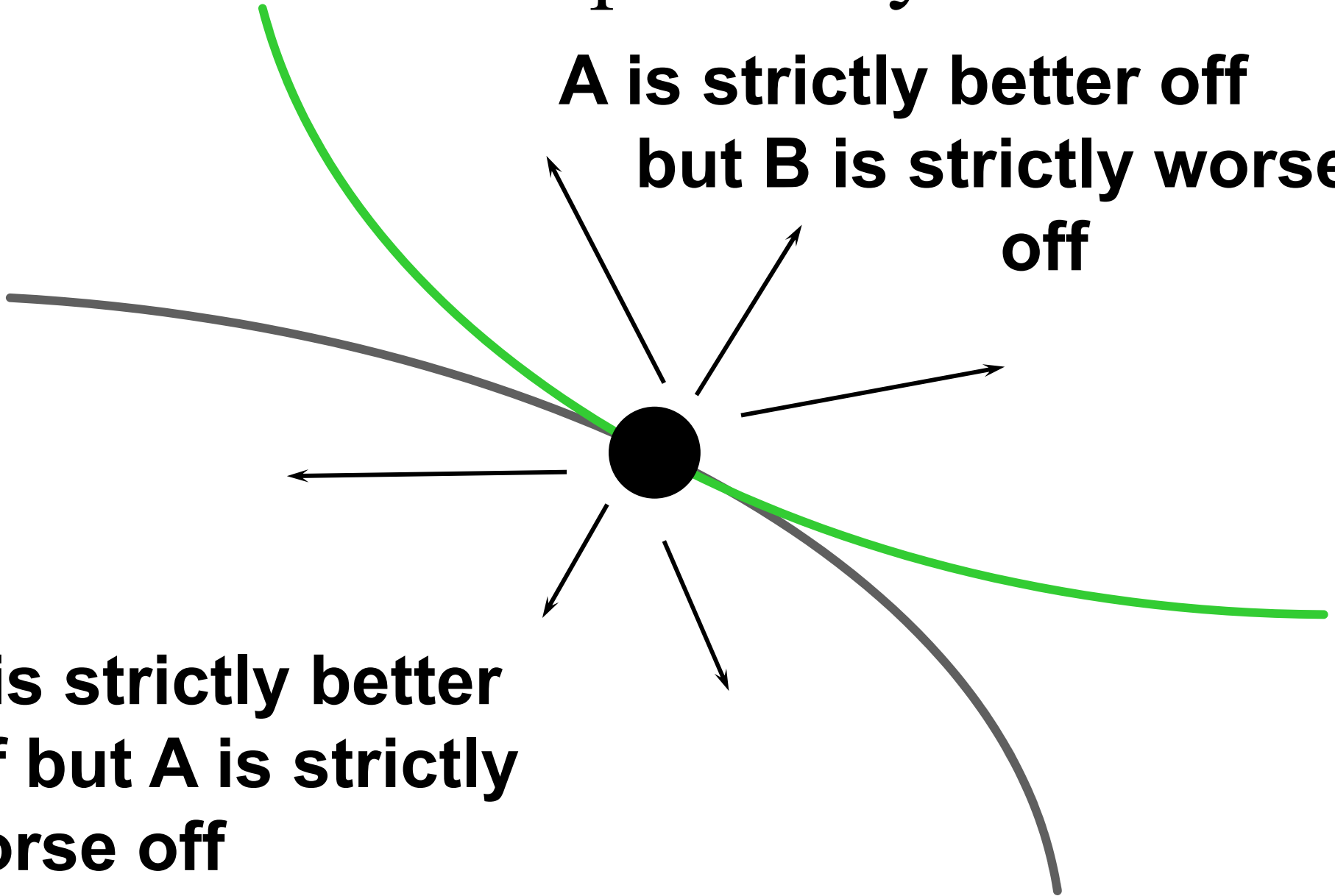
**A is strictly better off
but B is strictly worse
off**



Pareto-Optimality

**A is strictly better off
but B is strictly worse
off**

**B is strictly better
off but A is strictly
worse off**

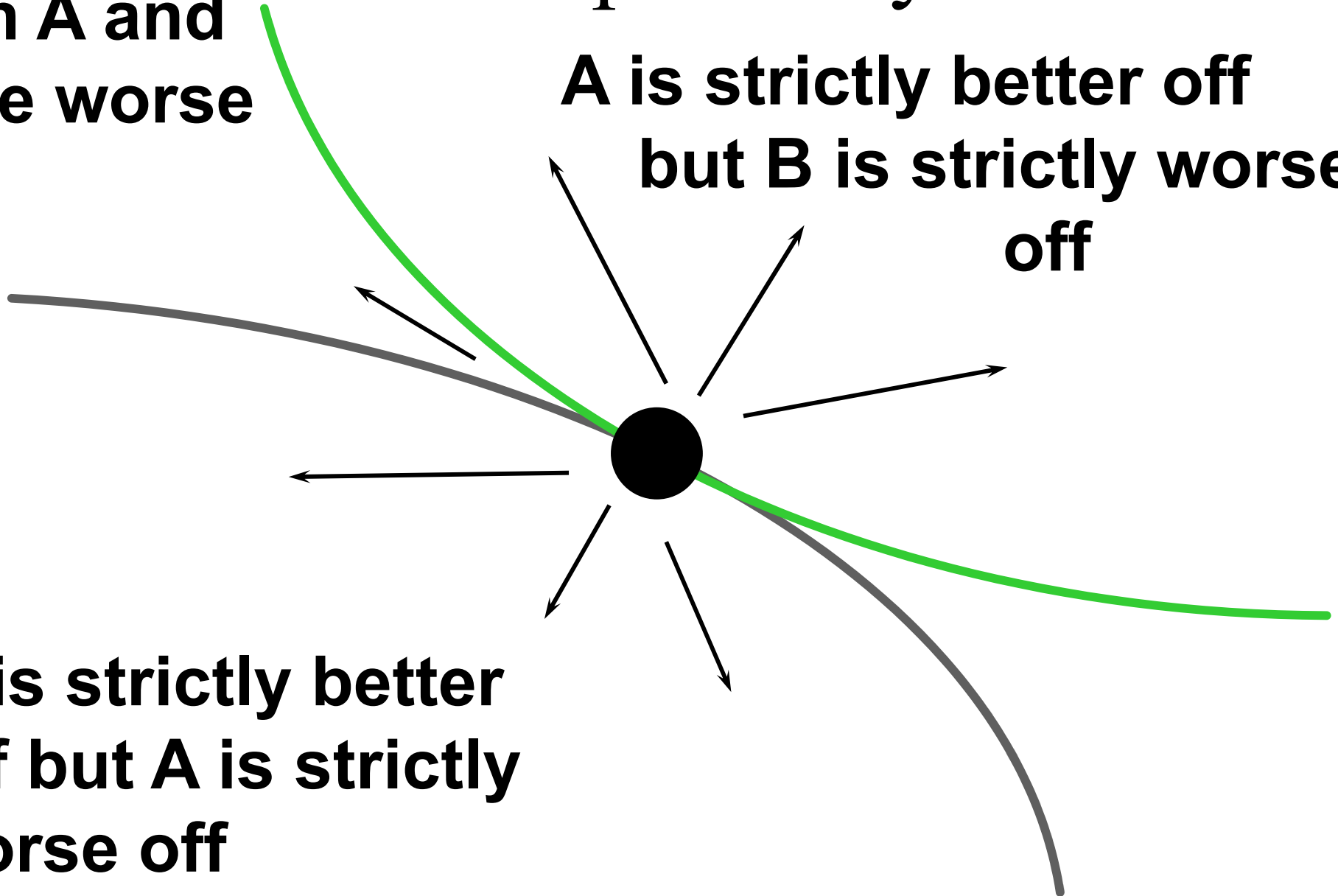


Pareto-Optimality

Both A and B are worse off

A is strictly better off but B is strictly worse off

B is strictly better off but A is strictly worse off



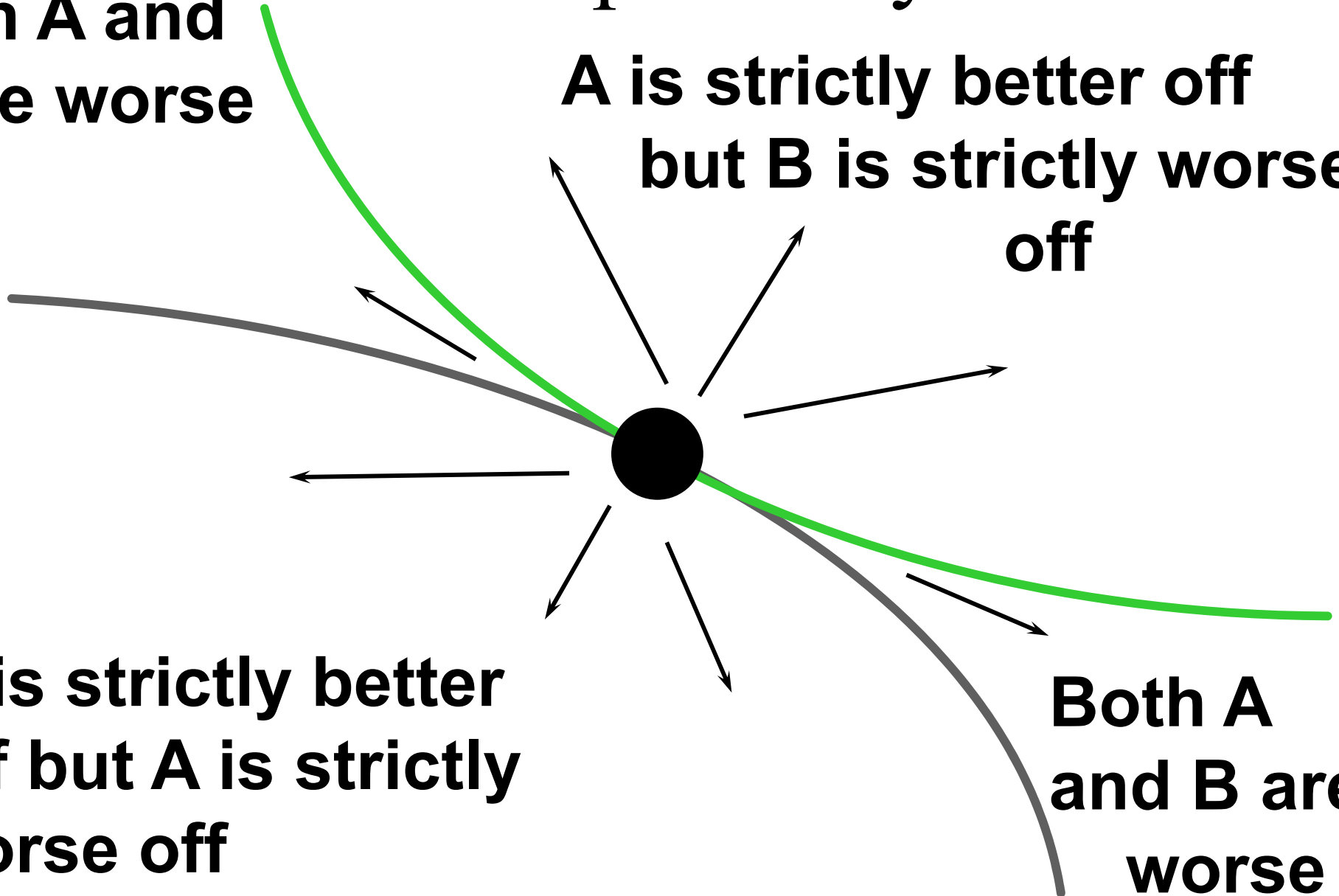
Pareto-Optimality

Both A and B are worse off

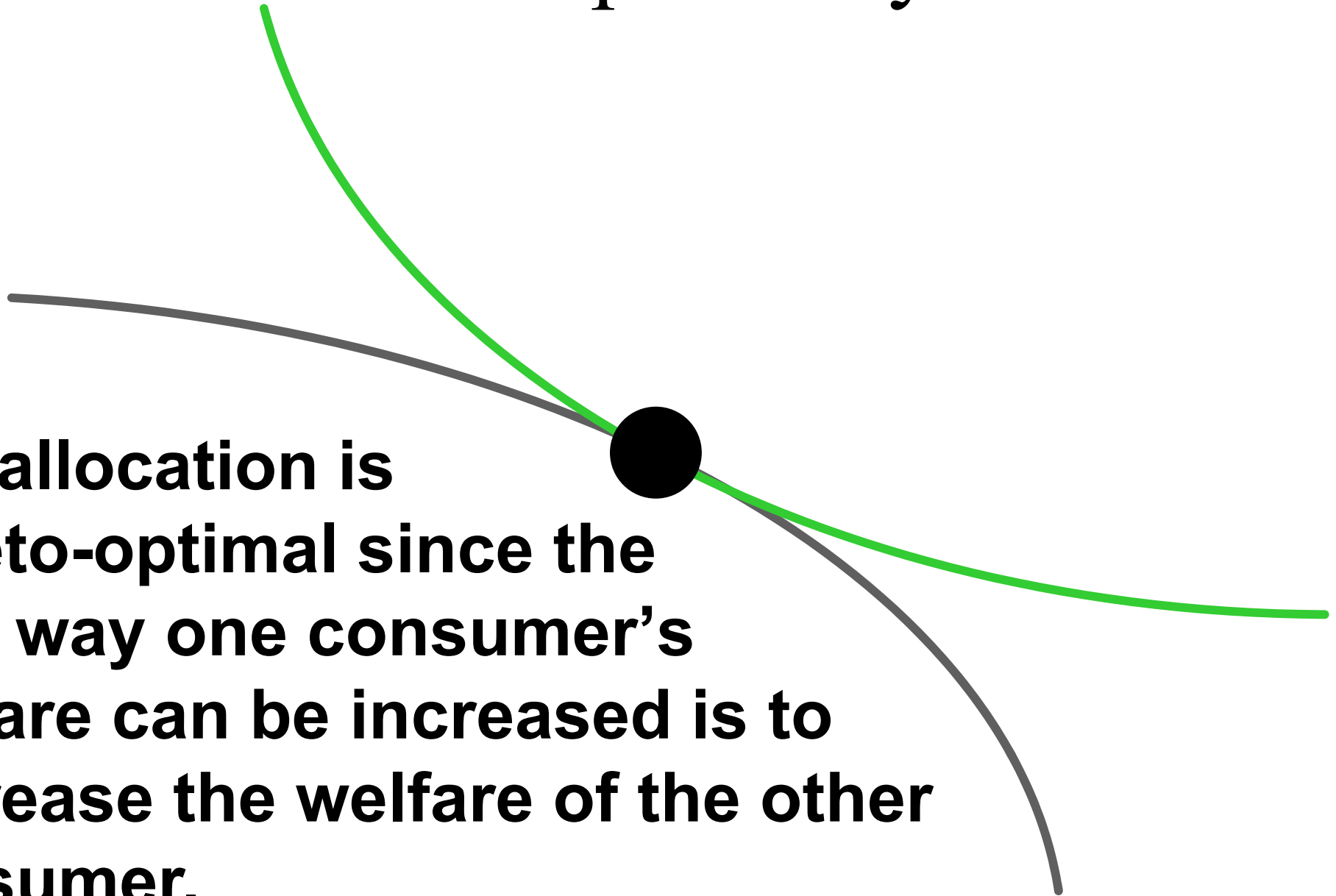
A is strictly better off but B is strictly worse off

B is strictly better off but A is strictly worse off

Both A and B are worse off



Pareto-Optimality

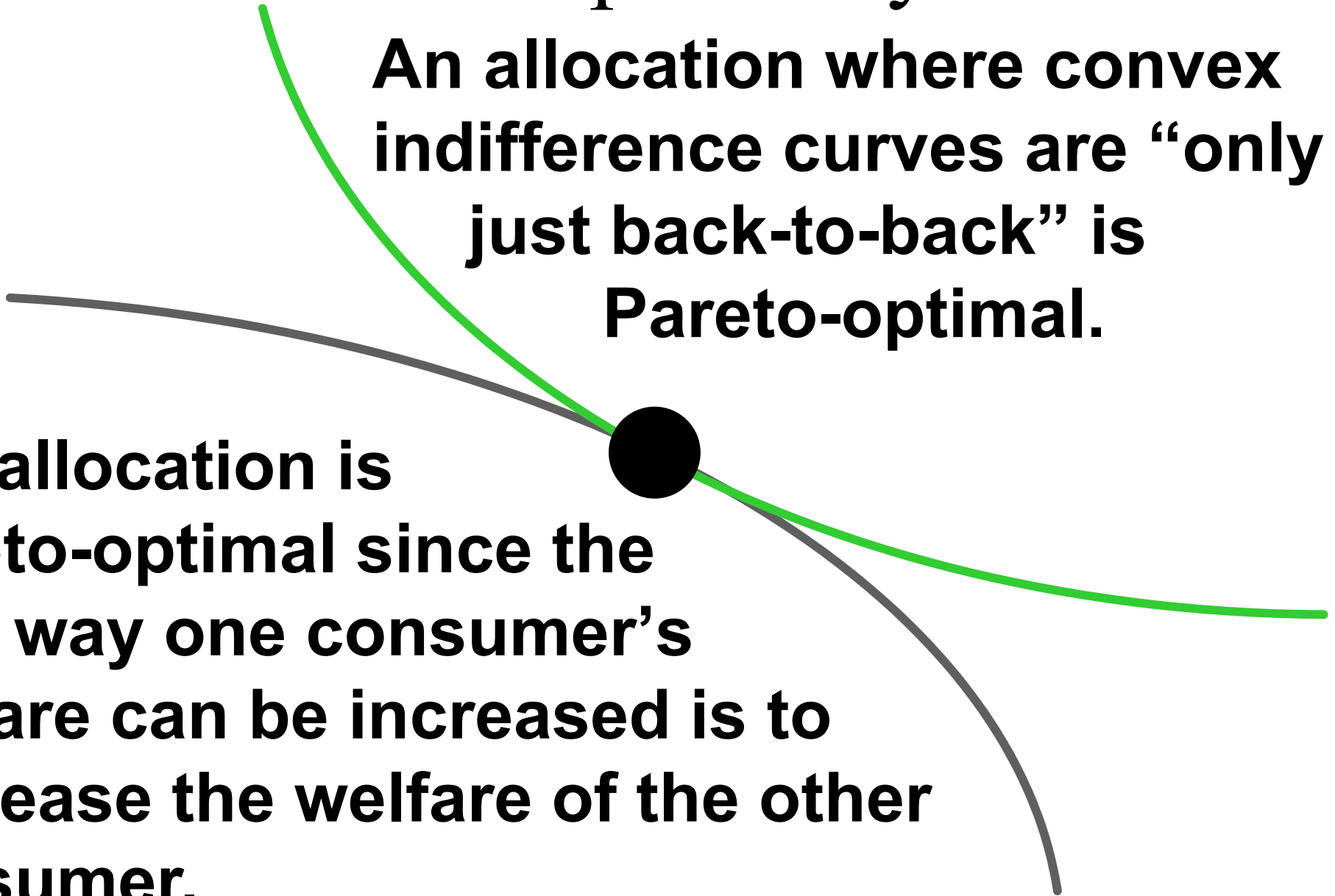


The allocation is Pareto-optimal since the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality

An allocation where convex indifference curves are “only just back-to-back” is Pareto-optimal.

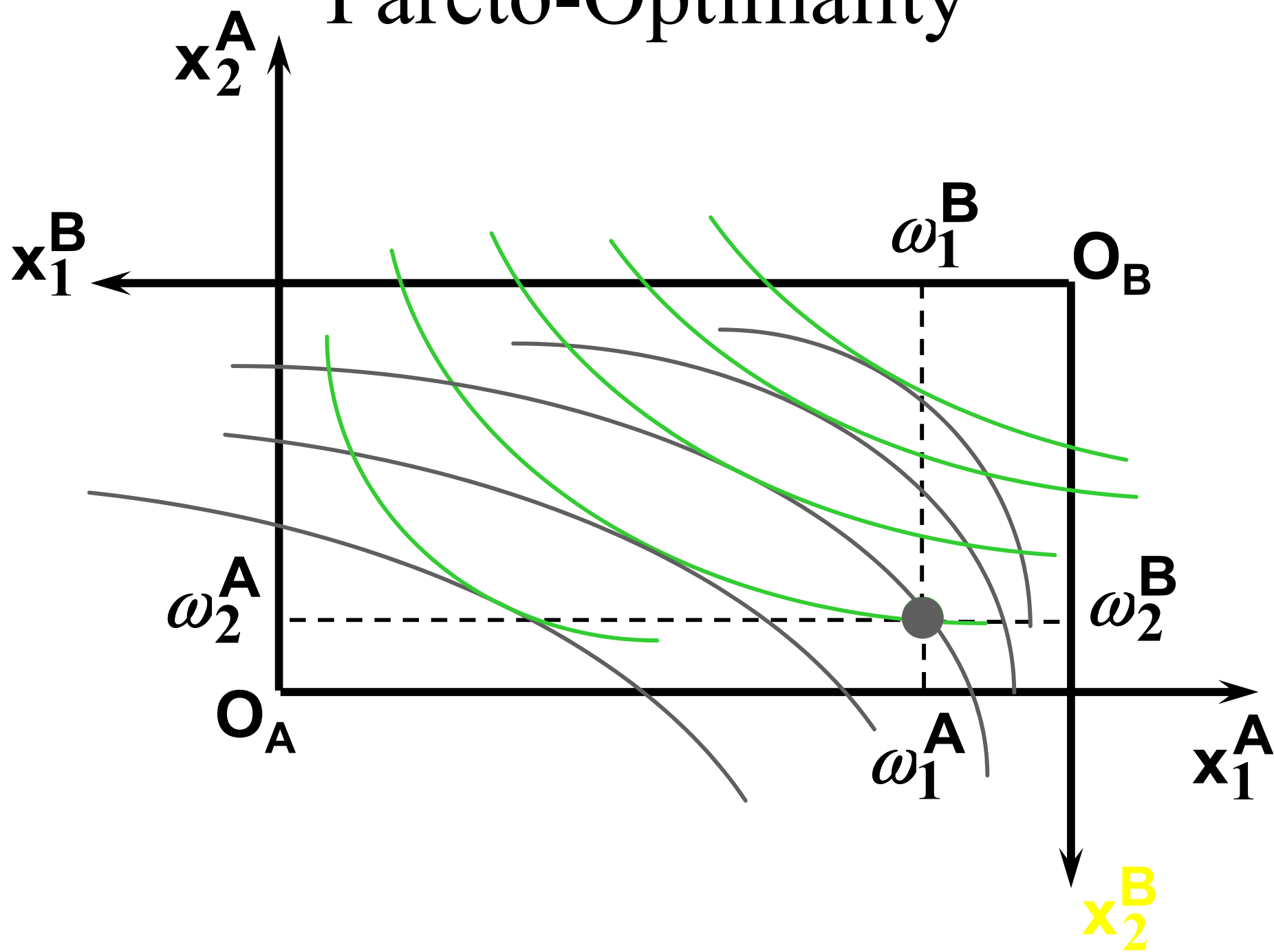
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Pareto-Optimality

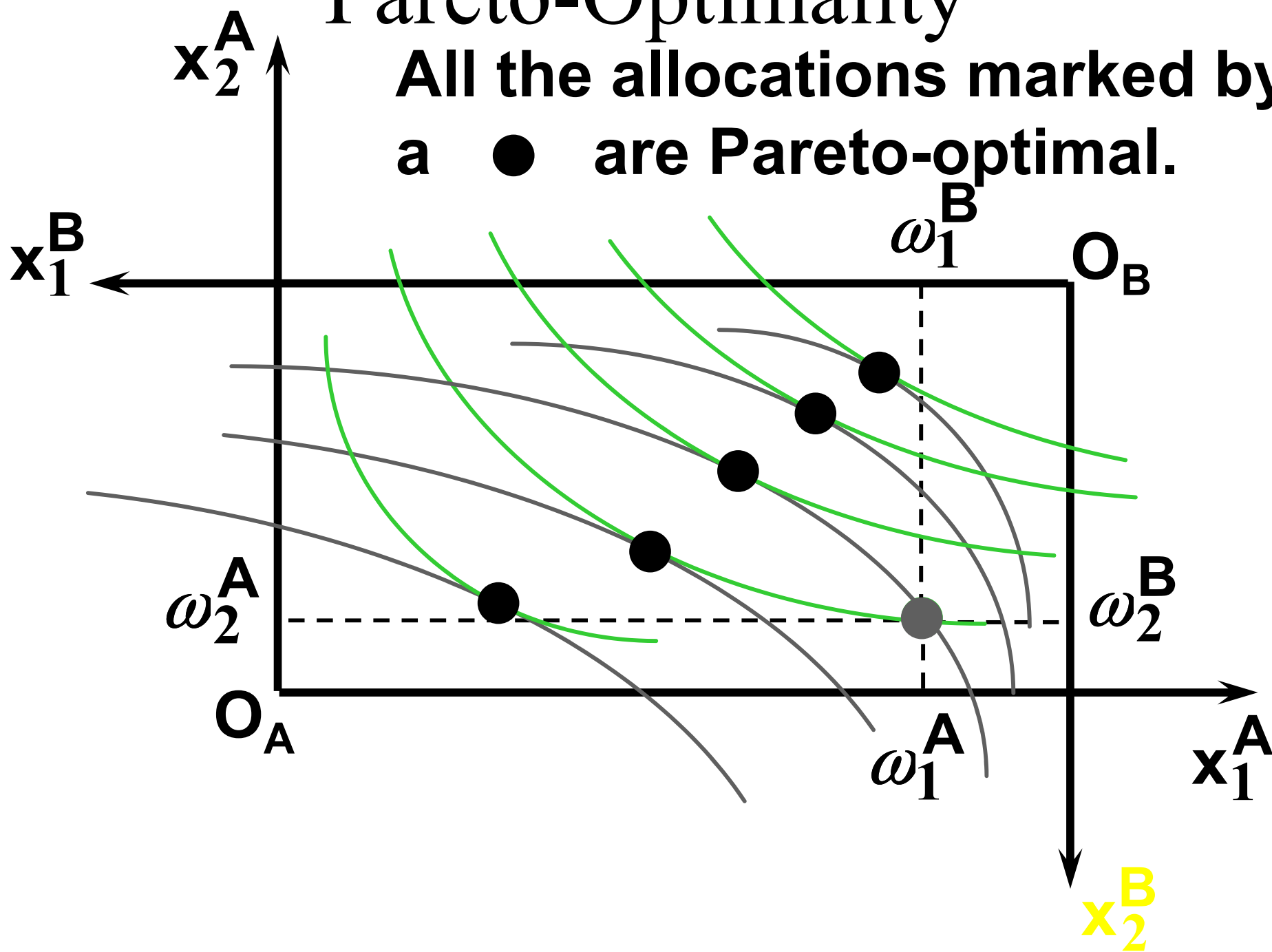
- ◆ **Where are all of the Pareto-optimal allocations of the endowment?**

Pareto-Optimality



Pareto-Optimality

All the allocations marked by a ● are Pareto-optimal.

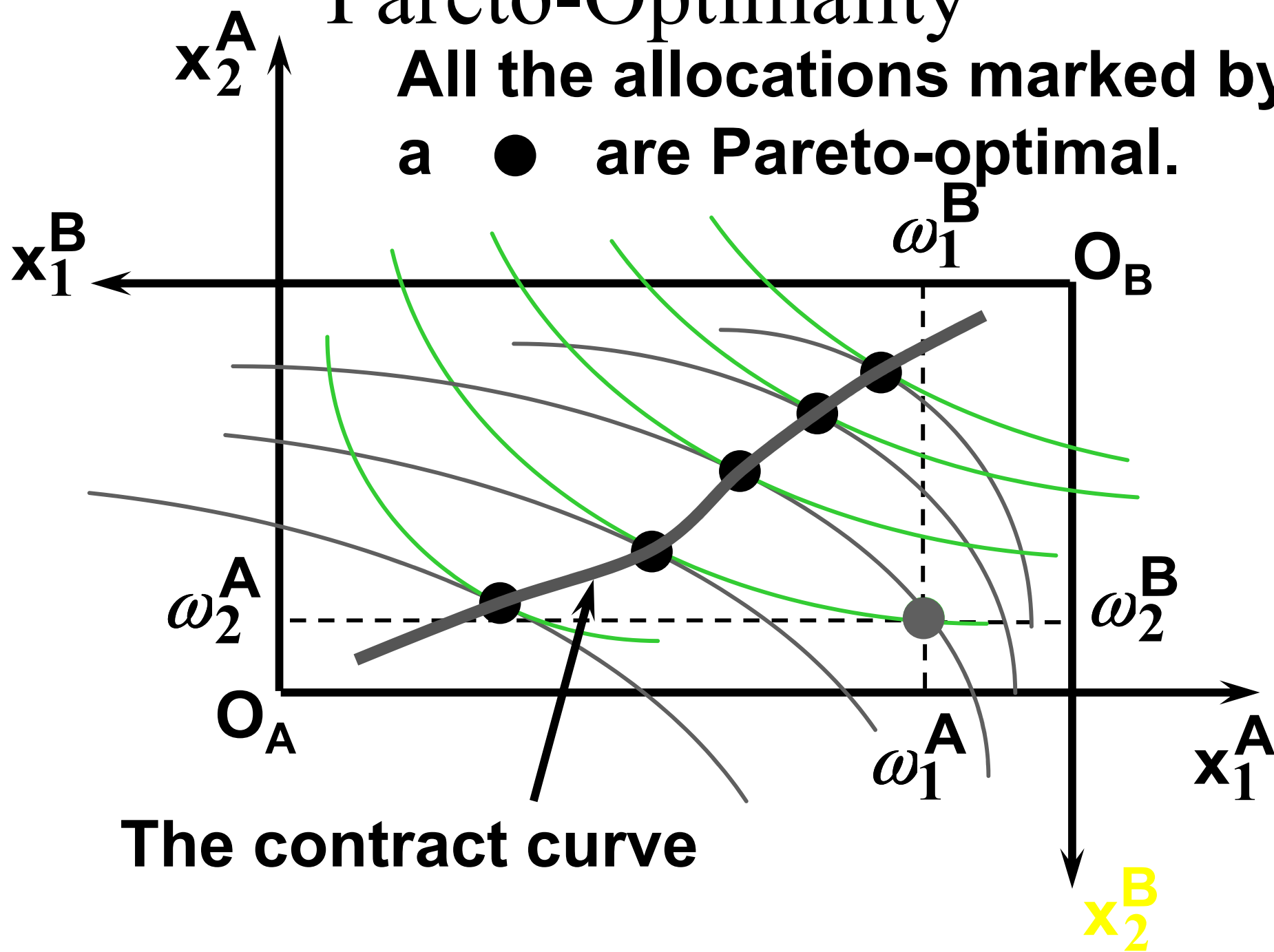


Pareto-Optimality

- ◆ **The contract curve is the set of all Pareto-optimal allocations.**

Pareto-Optimality

All the allocations marked by a ● are Pareto-optimal.

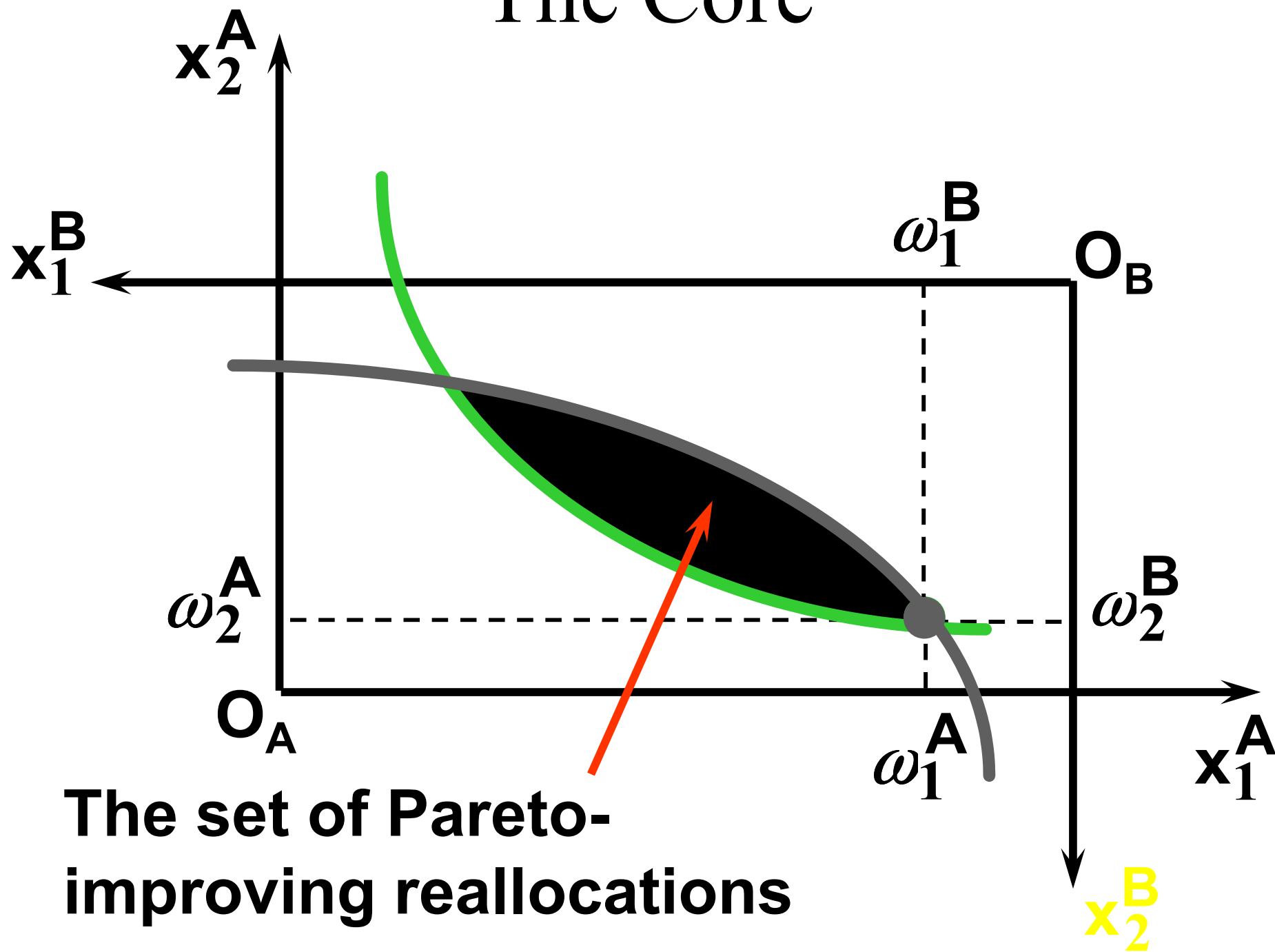


The contract curve

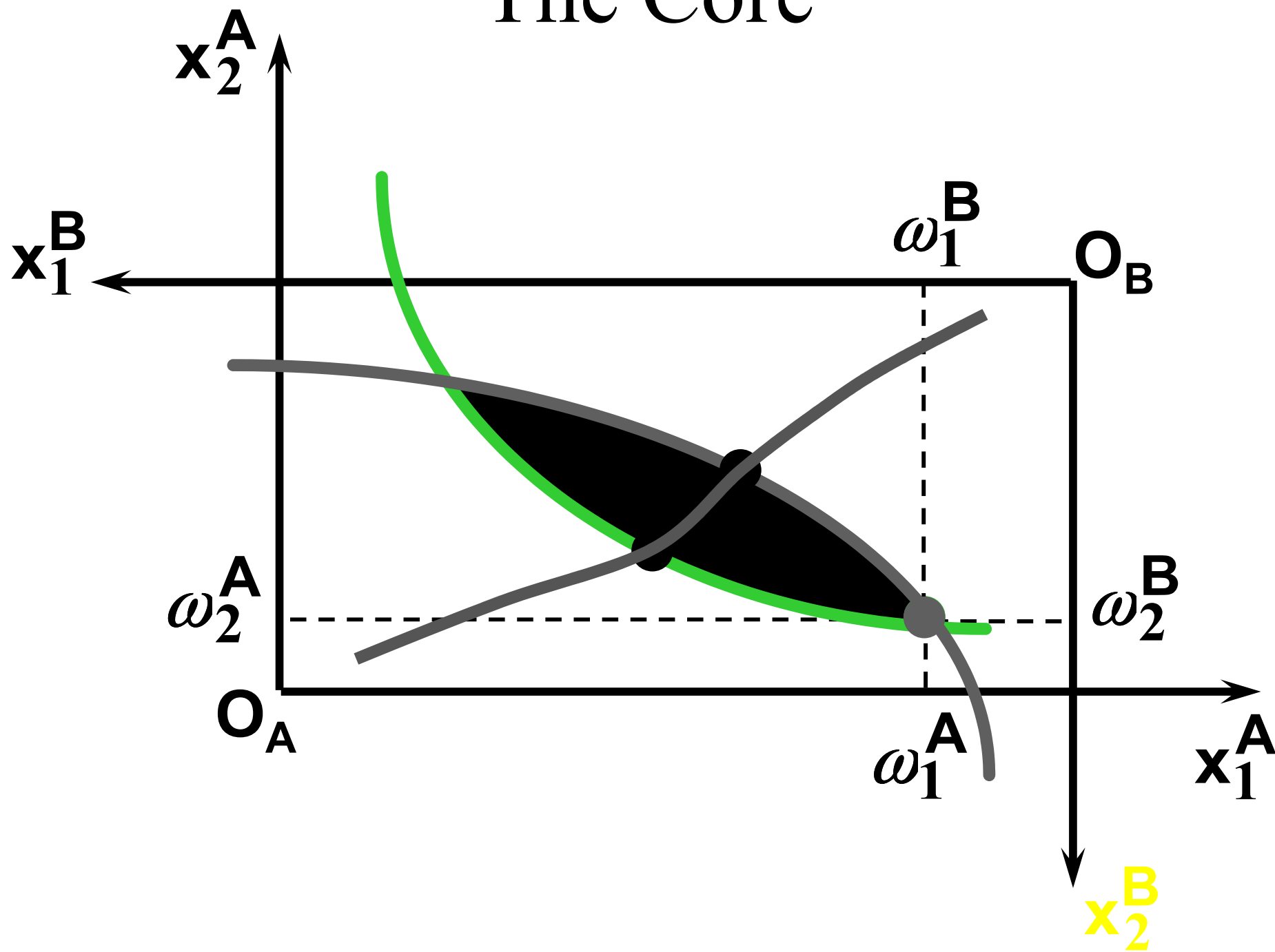
Pareto-Optimality

- ◆ **But to which of the many allocations on the contract curve will consumers trade?**
- ◆ **That depends upon how trade is conducted.**
- ◆ **In perfectly competitive markets? By one-on-one bargaining?**

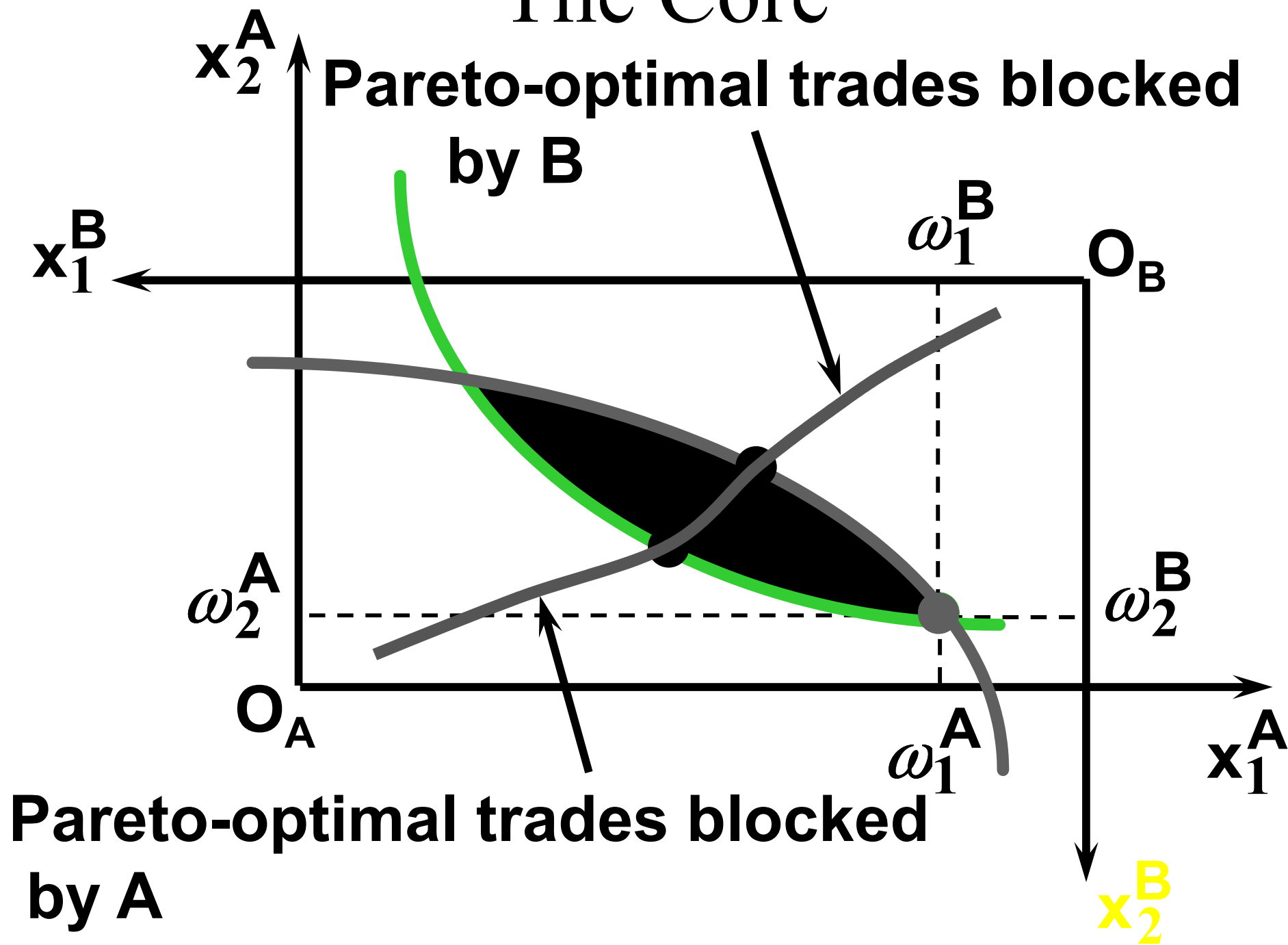
The Core



The Core

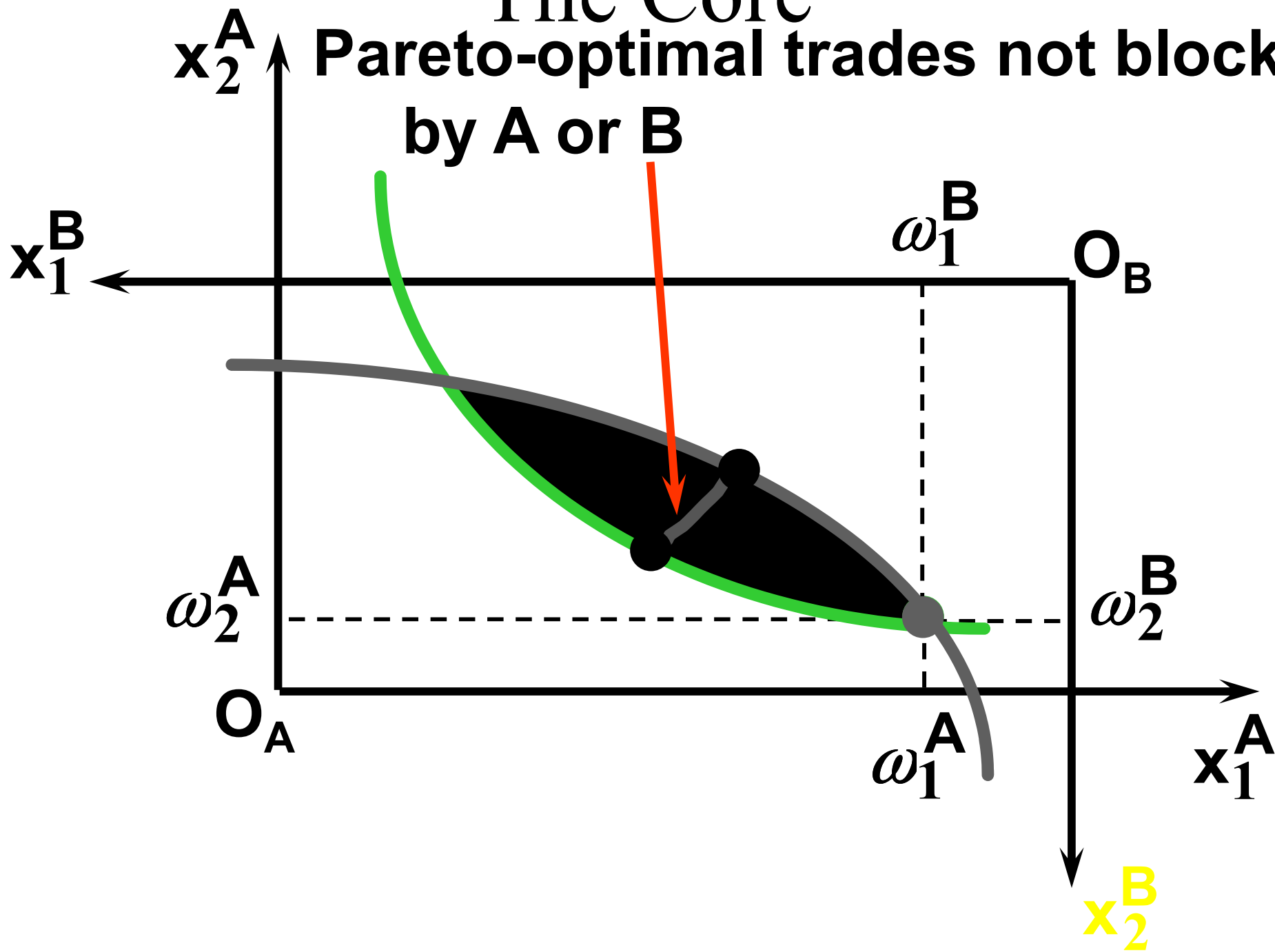


The Core



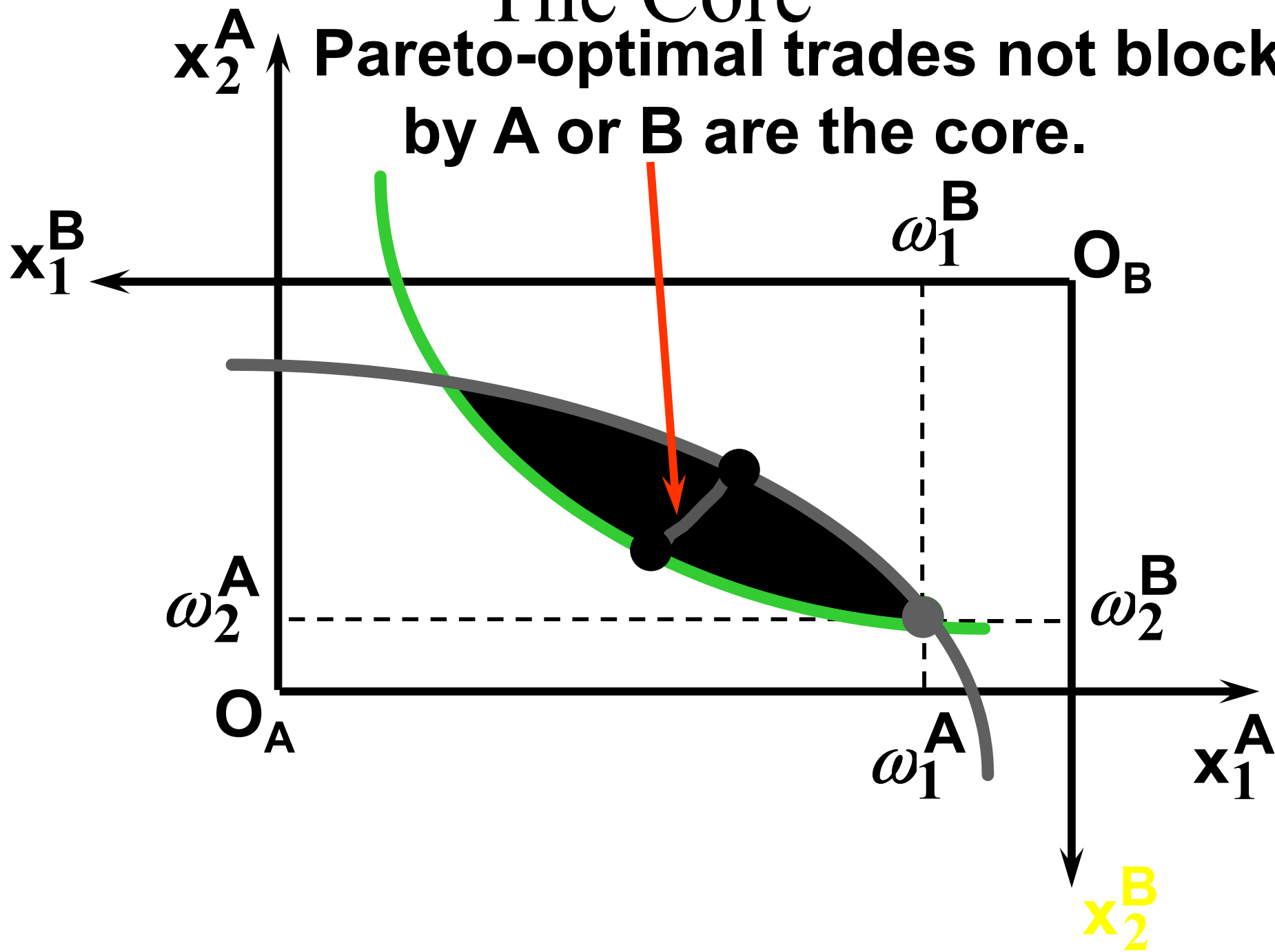
The Core

Pareto-optimal trades not blocked
by A or B



The Core

Pareto-optimal trades not blocked by A or B are the core.



The Core

- ◆ **The core is the set of all Pareto-optimal allocations that are welfare-improving for both consumers relative to their own endowments.**
- ◆ **Rational trade should achieve a core allocation.**

The Core

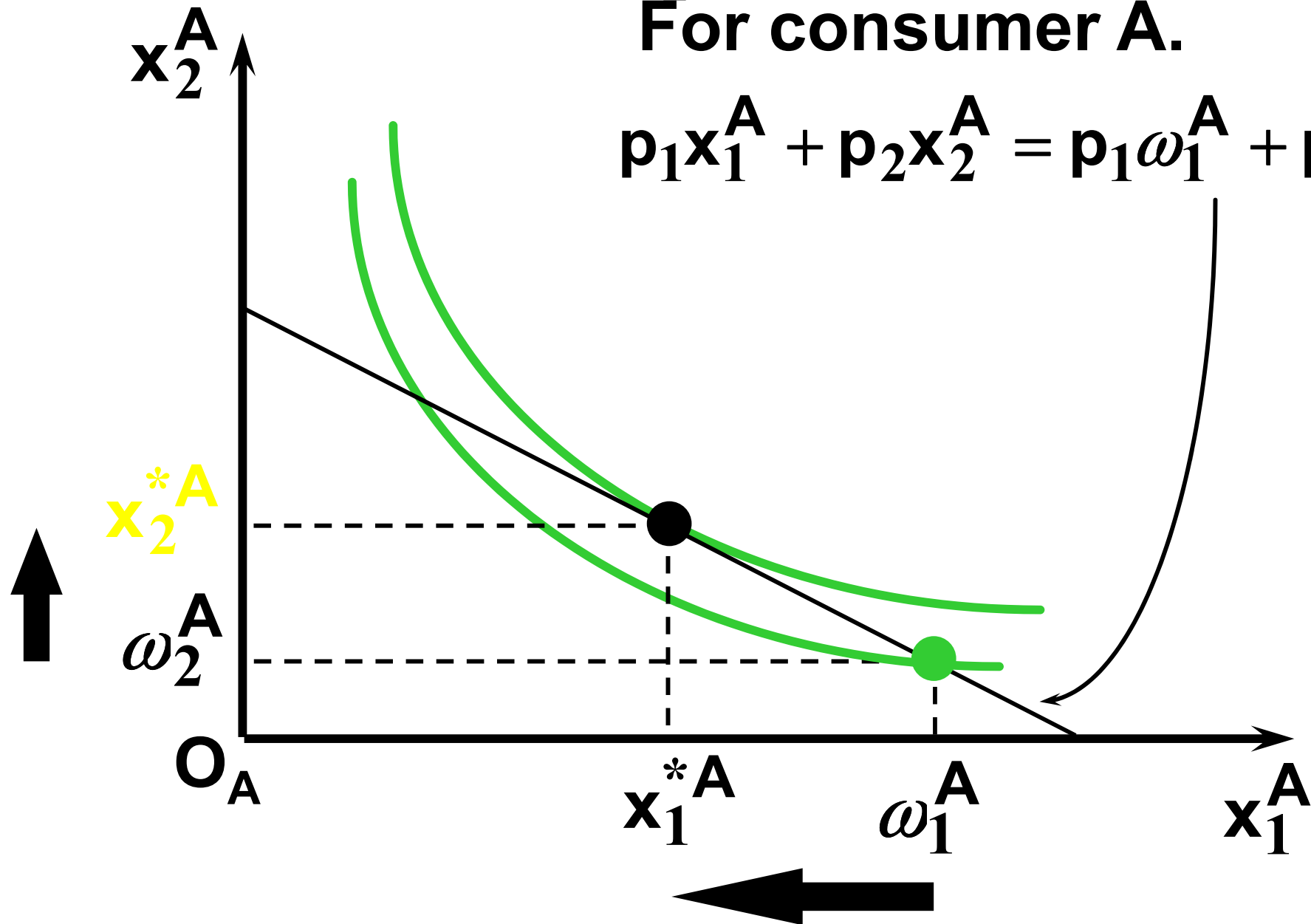
- ◆ **But which core allocation?**
- ◆ **Again, that depends upon the manner in which trade is conducted.**

Trade in Competitive Markets

- ◆ **Consider trade in perfectly competitive markets.**
- ◆ **Each consumer is a price-taker trying to maximize her own utility given p_1 , p_2 and her own endowment. That is, ...**

Trade in Competitive Markets For consumer A.

$$p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A$$



Trade in Competitive Markets

- ◆ So given p_1 and p_2 , consumer A's net demands for commodities 1 and 2 are

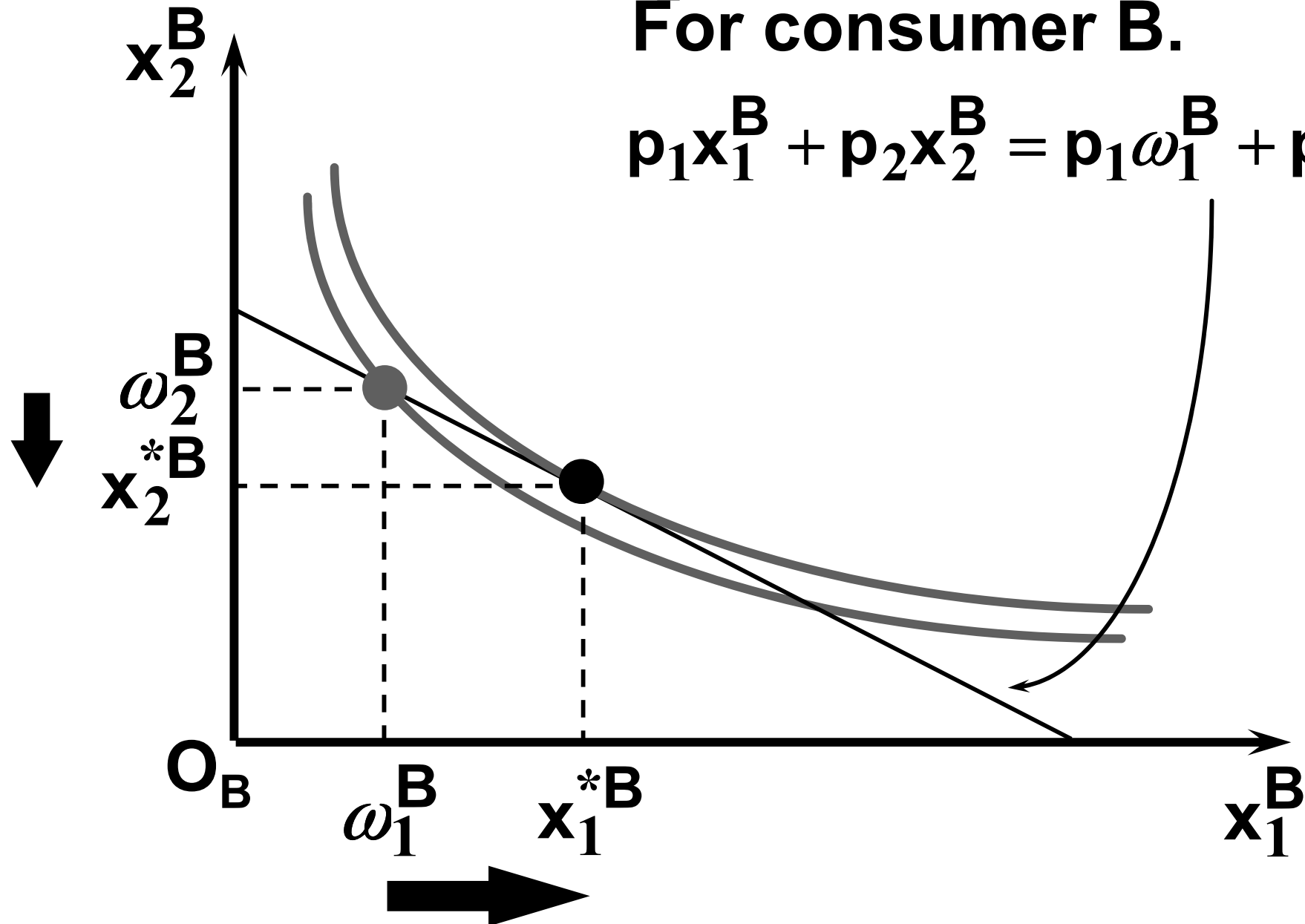
$$x_1^{*A} - \omega_1^A \quad \text{and} \quad x_2^{*A} - \omega_2^A .$$

Trade in Competitive Markets

◆ **And, similarly, for consumer B ...**

Trade in Competitive Markets For consumer B.

$$p_1 x_1^B + p_2 x_2^B = p_1 \omega_1^B + p_2 \omega_2^B$$



Trade in Competitive Markets

- ◆ So given p_1 and p_2 , consumer B's net demands for commodities 1 and 2 are

$$x_1^{*B} - \omega_1^B \quad \text{and} \quad x_2^{*B} - \omega_2^B .$$

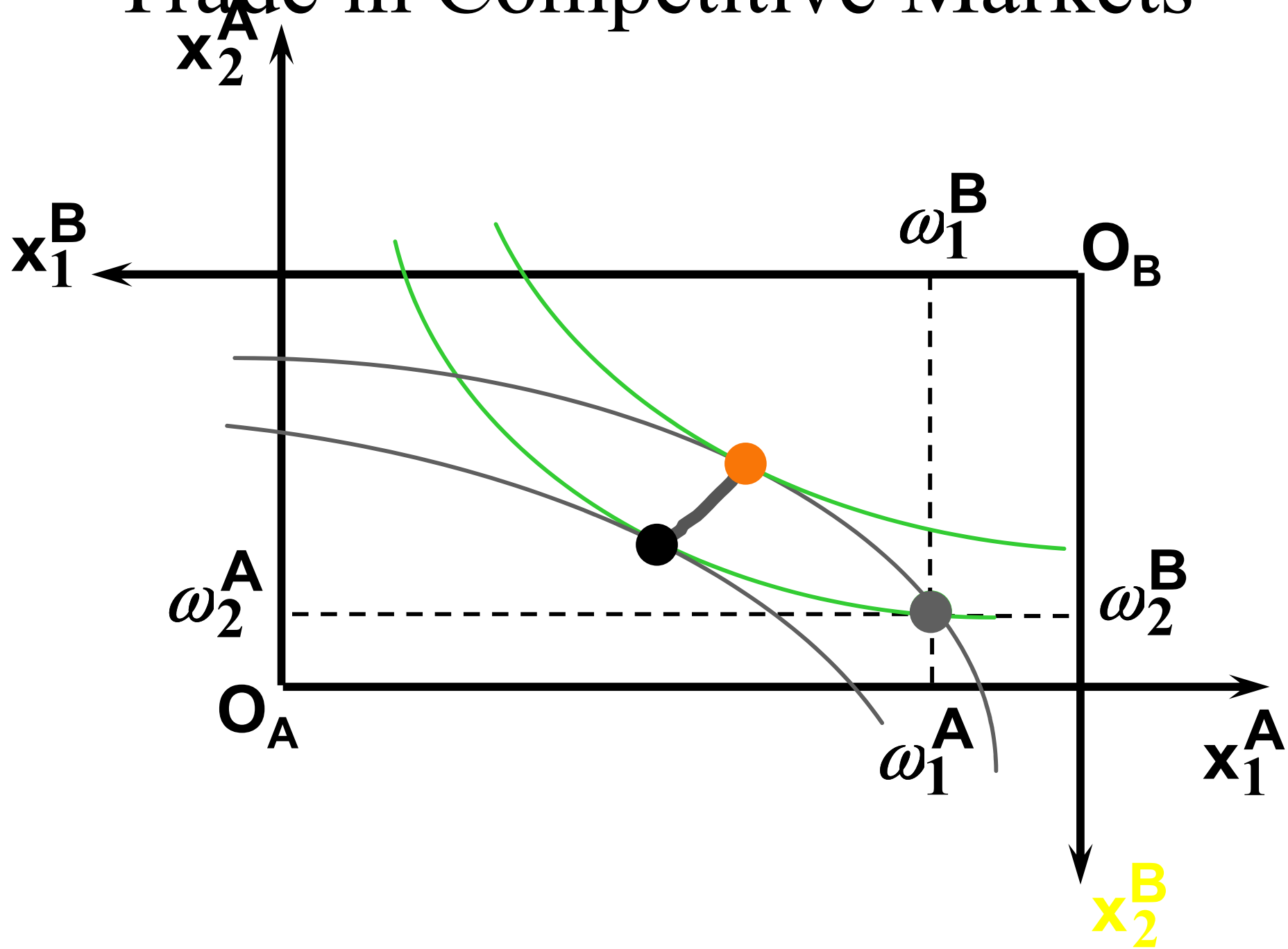
Trade in Competitive Markets

- ◆ A general equilibrium occurs when prices p_1 and p_2 cause both the markets for commodities 1 and 2 to clear; i.e.

$$\mathbf{x}_1^{*A} + \mathbf{x}_1^{*B} = \omega_1^A + \omega_1^B$$

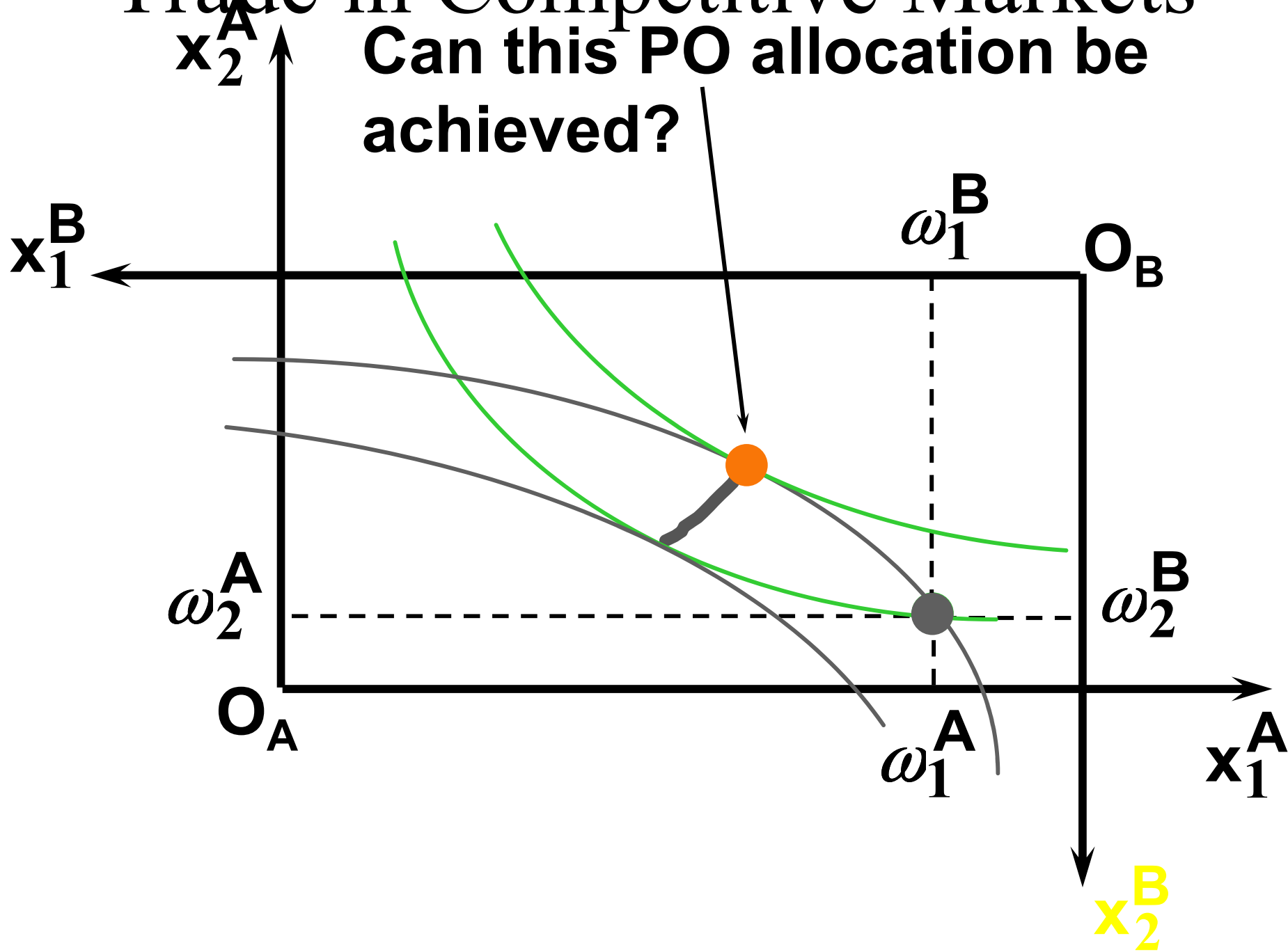
and $\mathbf{x}_2^{*A} + \mathbf{x}_2^{*B} = \omega_2^A + \omega_2^B.$

Trade in Competitive Markets



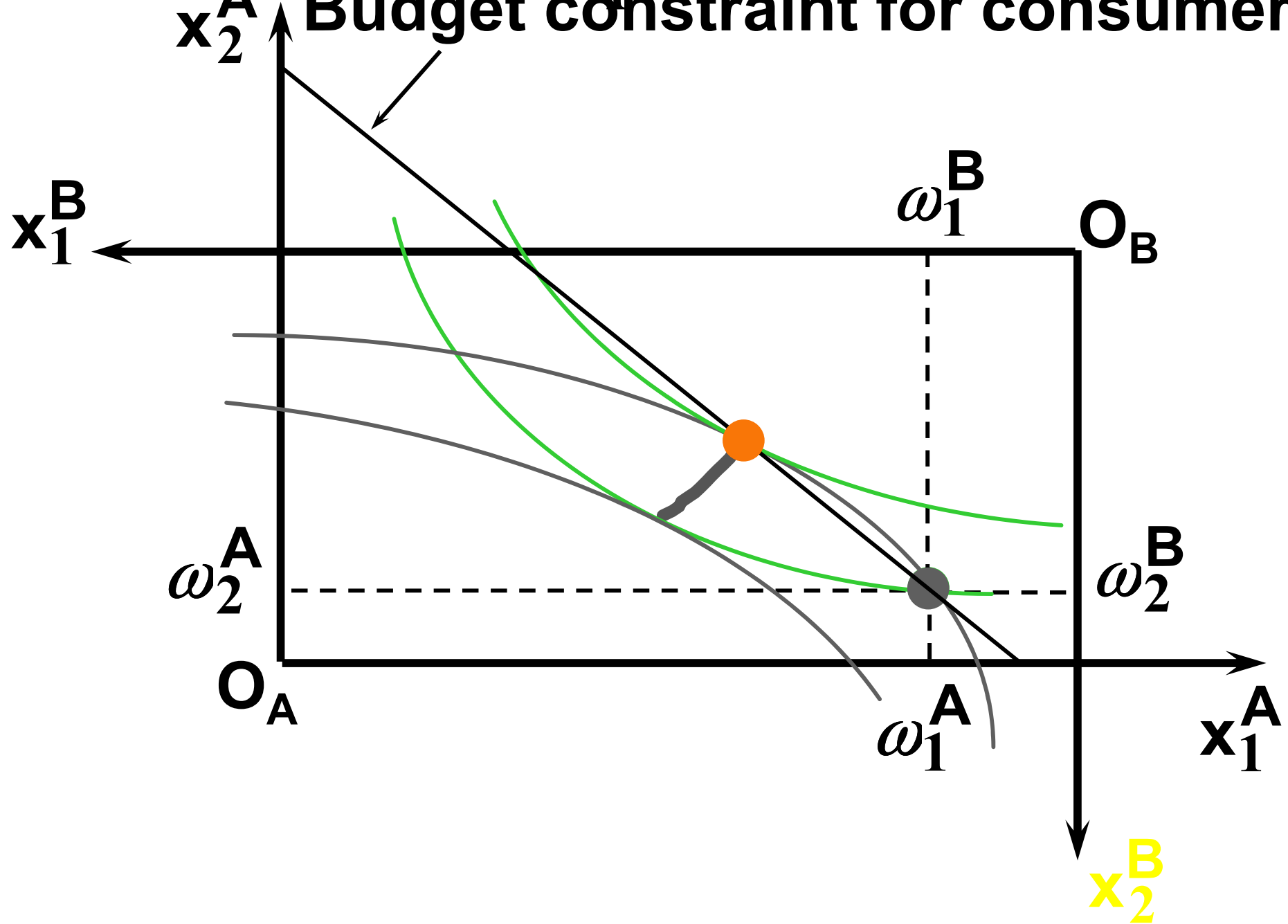
Trade in Competitive Markets

Can this PO allocation be achieved?



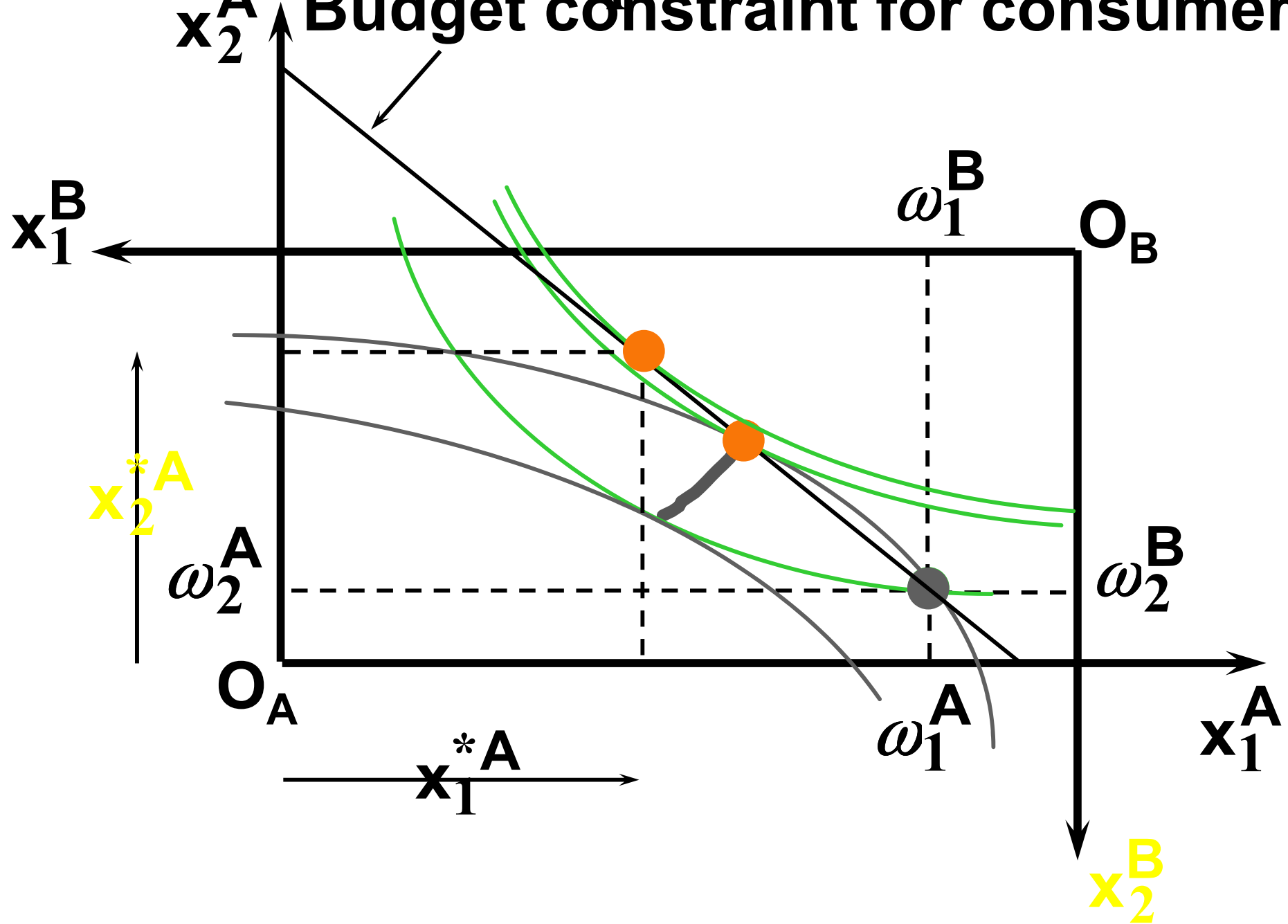
Trade in Competitive Markets

Budget constraint for consumer A

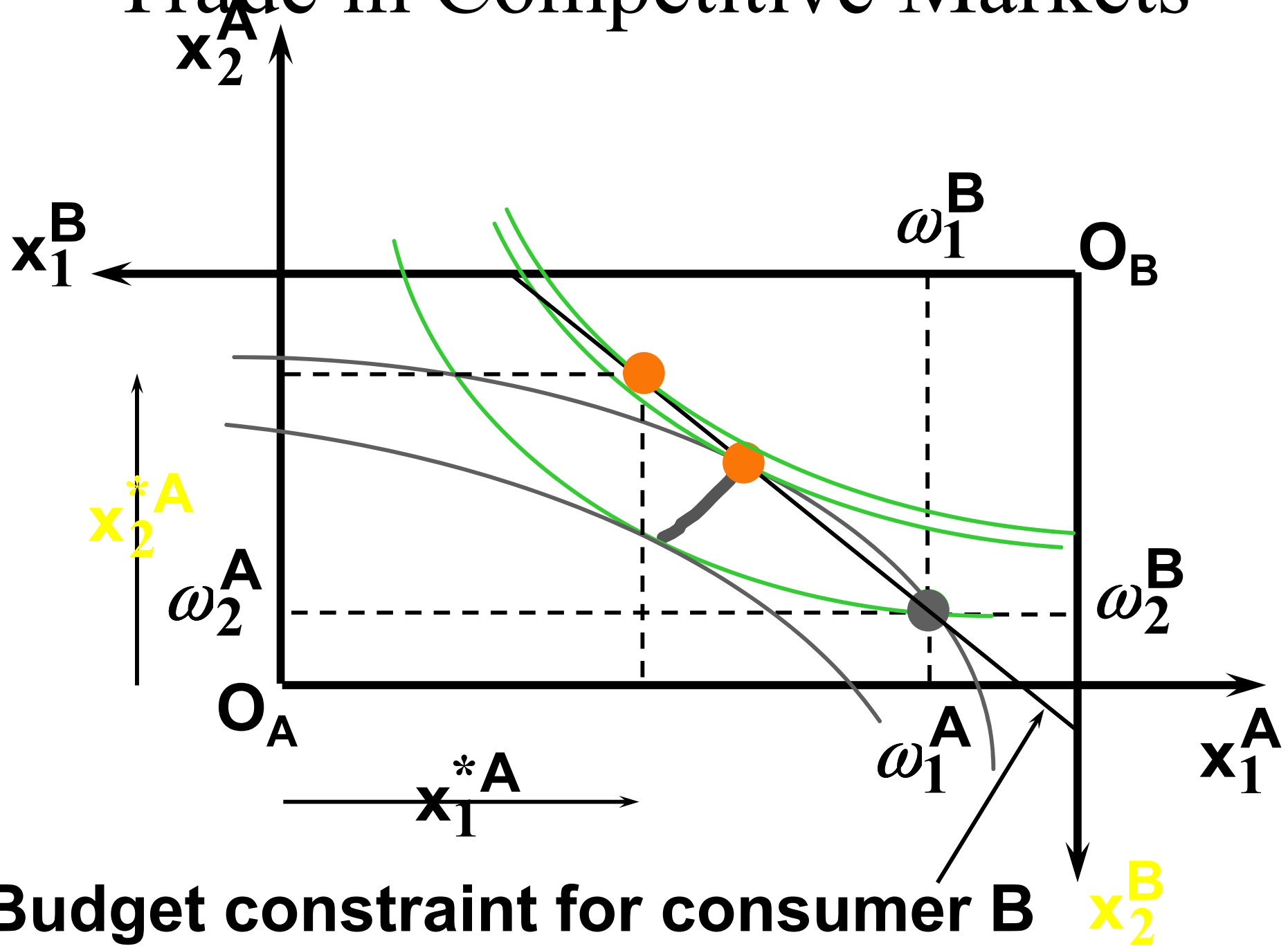


Trade in Competitive Markets

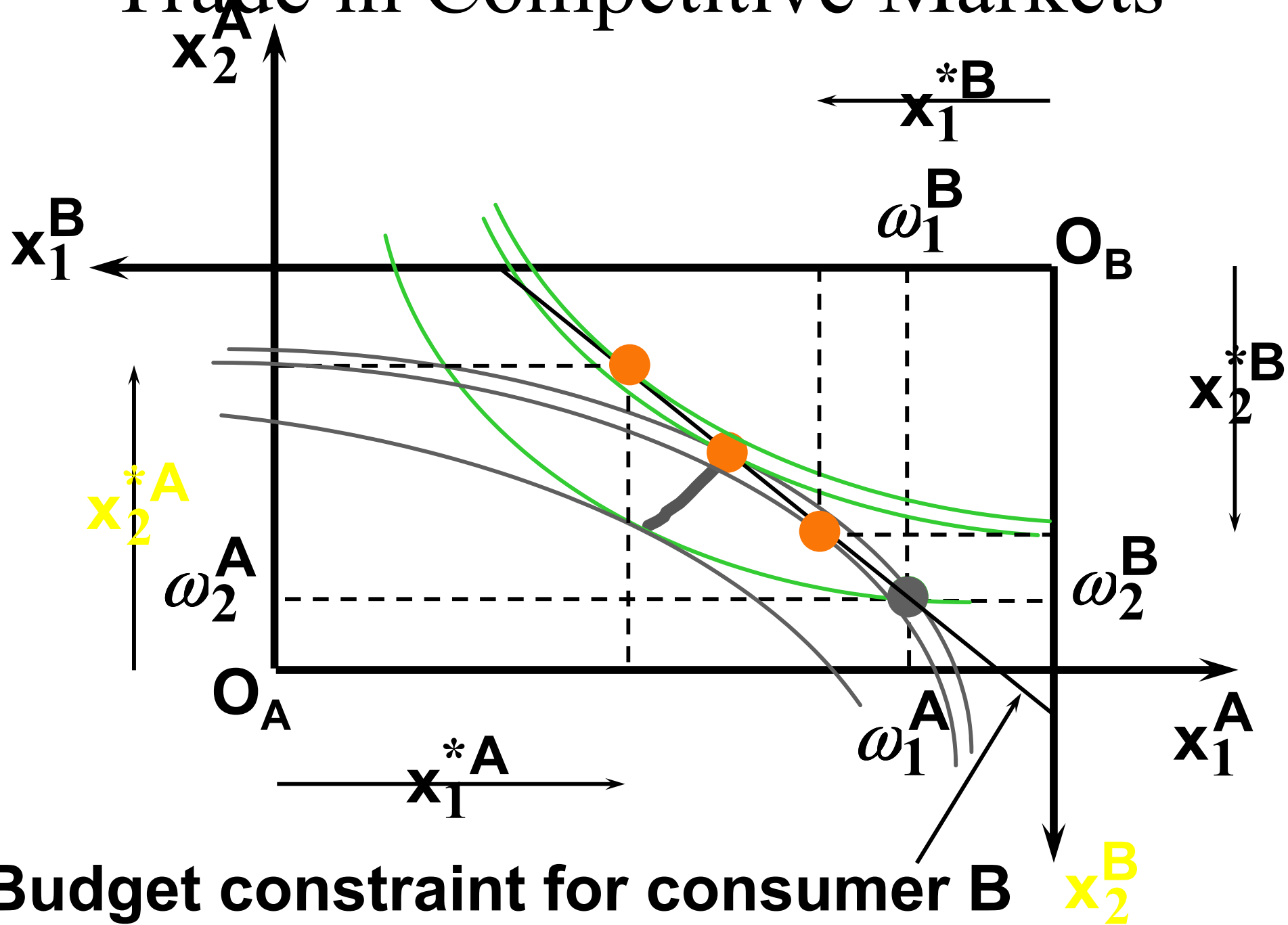
Budget constraint for consumer A



Trade in Competitive Markets

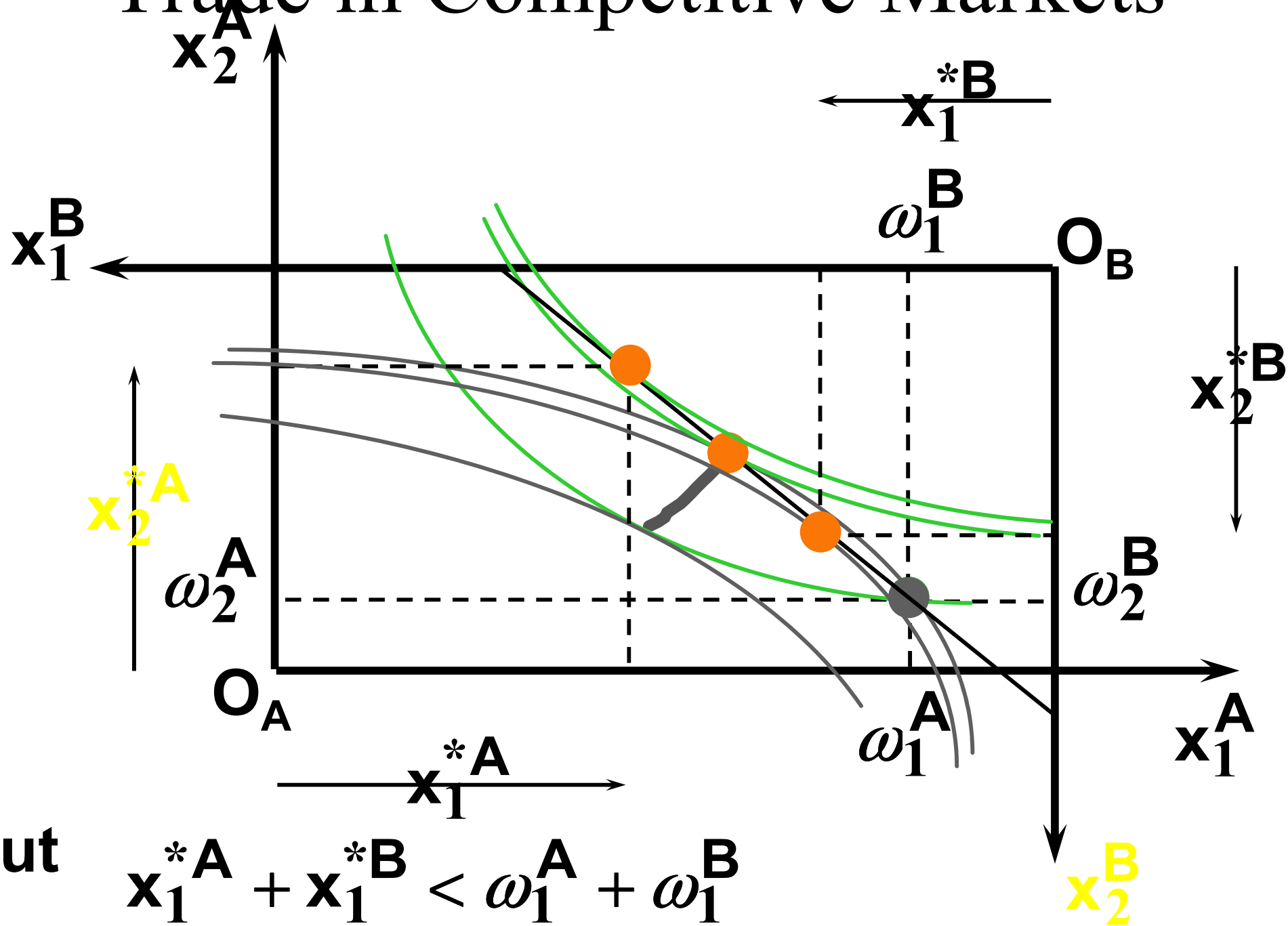


Trade in Competitive Markets

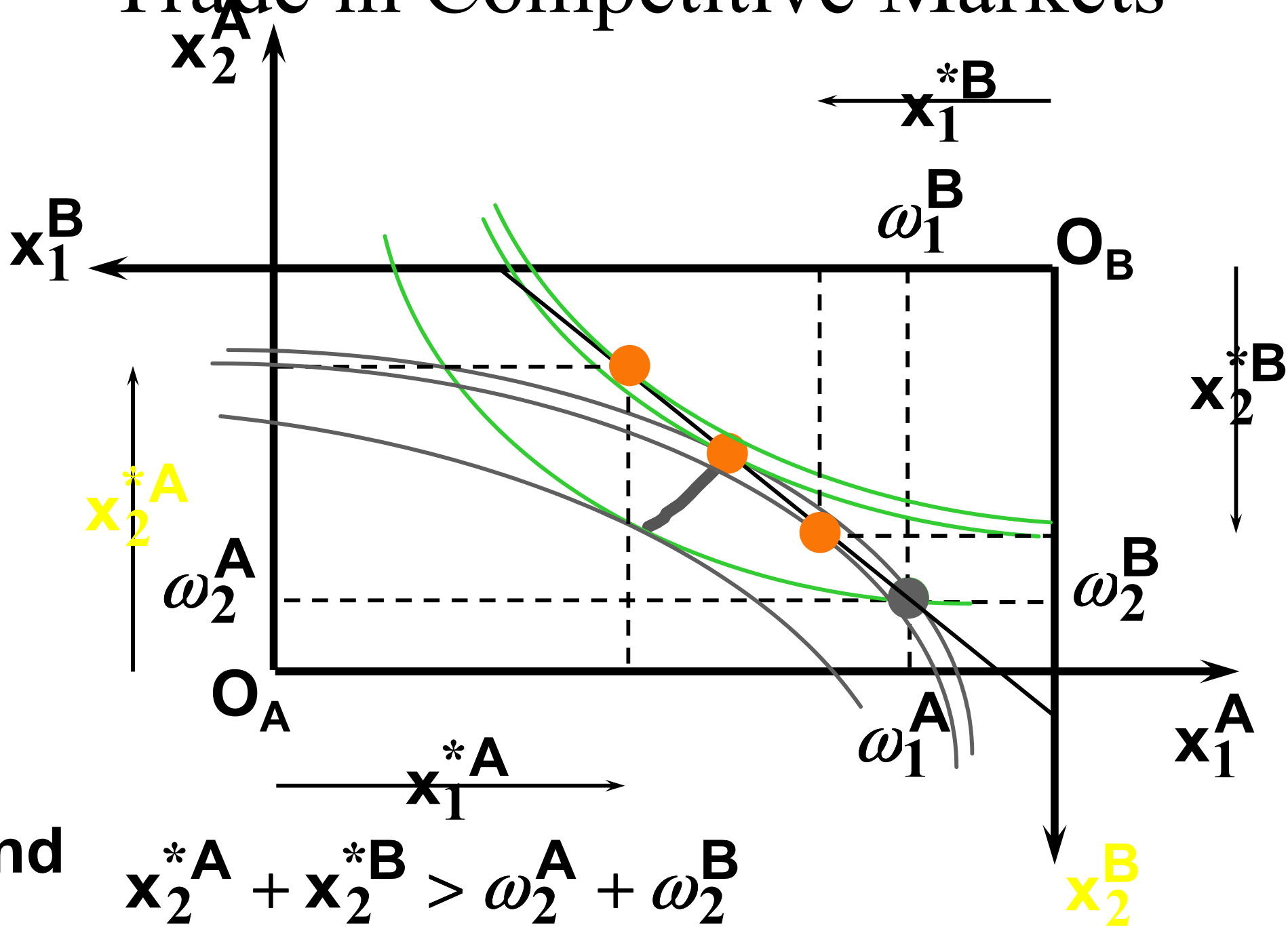


Budget constraint for consumer B

Trade in Competitive Markets



Trade in Competitive Markets

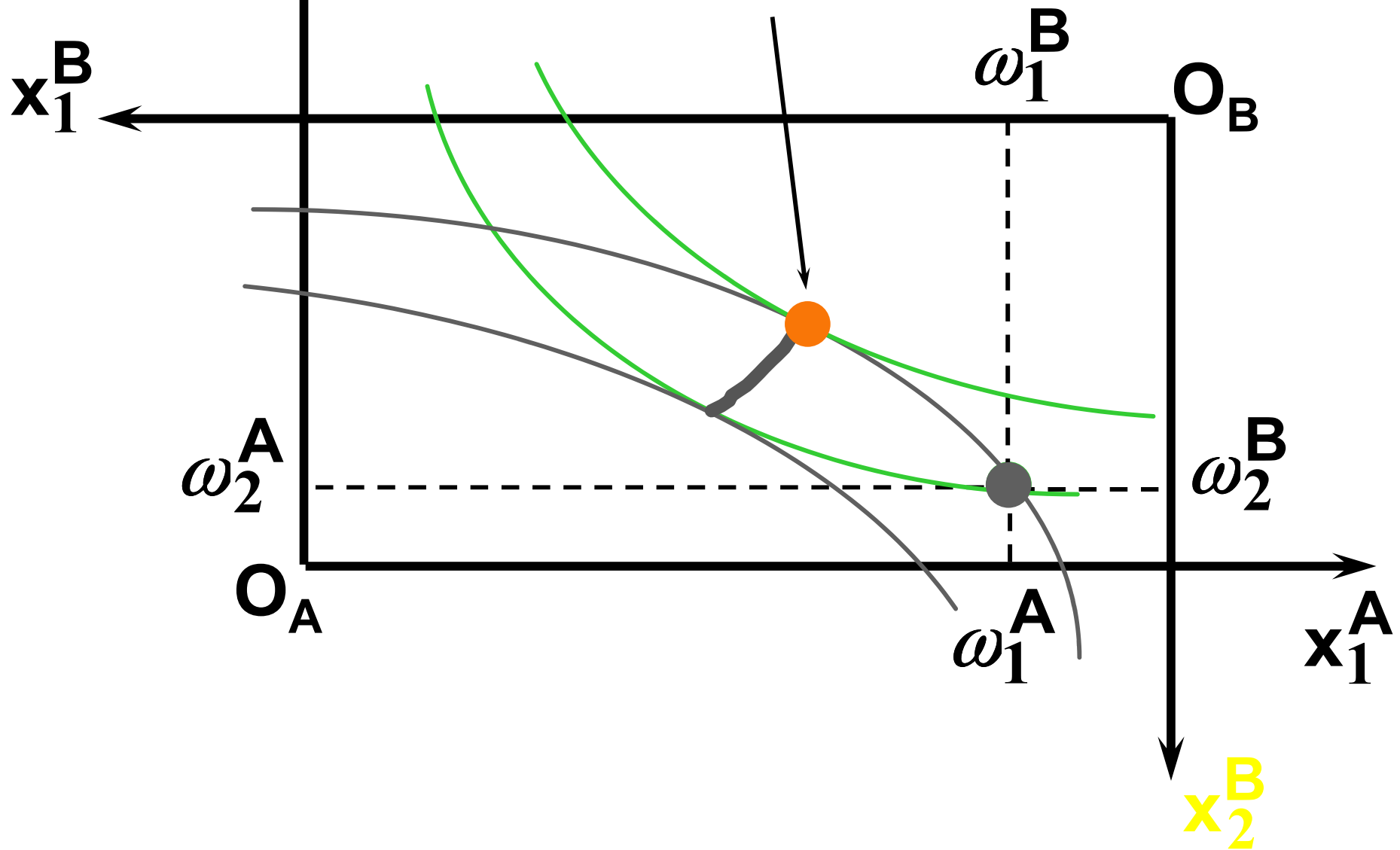


Trade in Competitive Markets

- ◆ **So at the given prices p_1 and p_2 there is an**
 - **excess supply of commodity 1**
 - **excess demand for commodity 2.**
- ◆ **Neither market clears so the prices p_1 and p_2 do not cause a general equilibrium.**

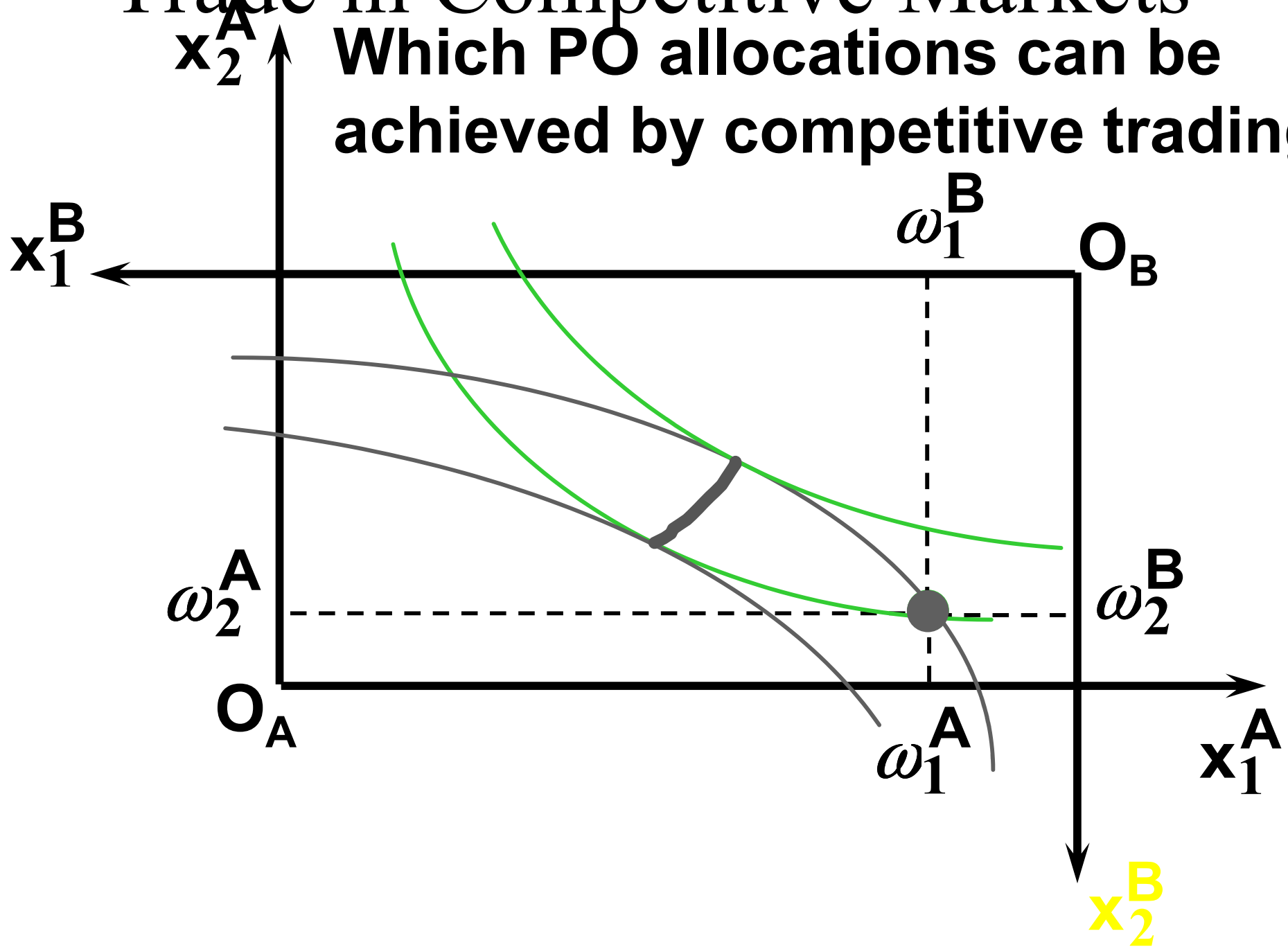
Trade in Competitive Markets

So this PO allocation cannot be achieved by competitive trading.



Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?

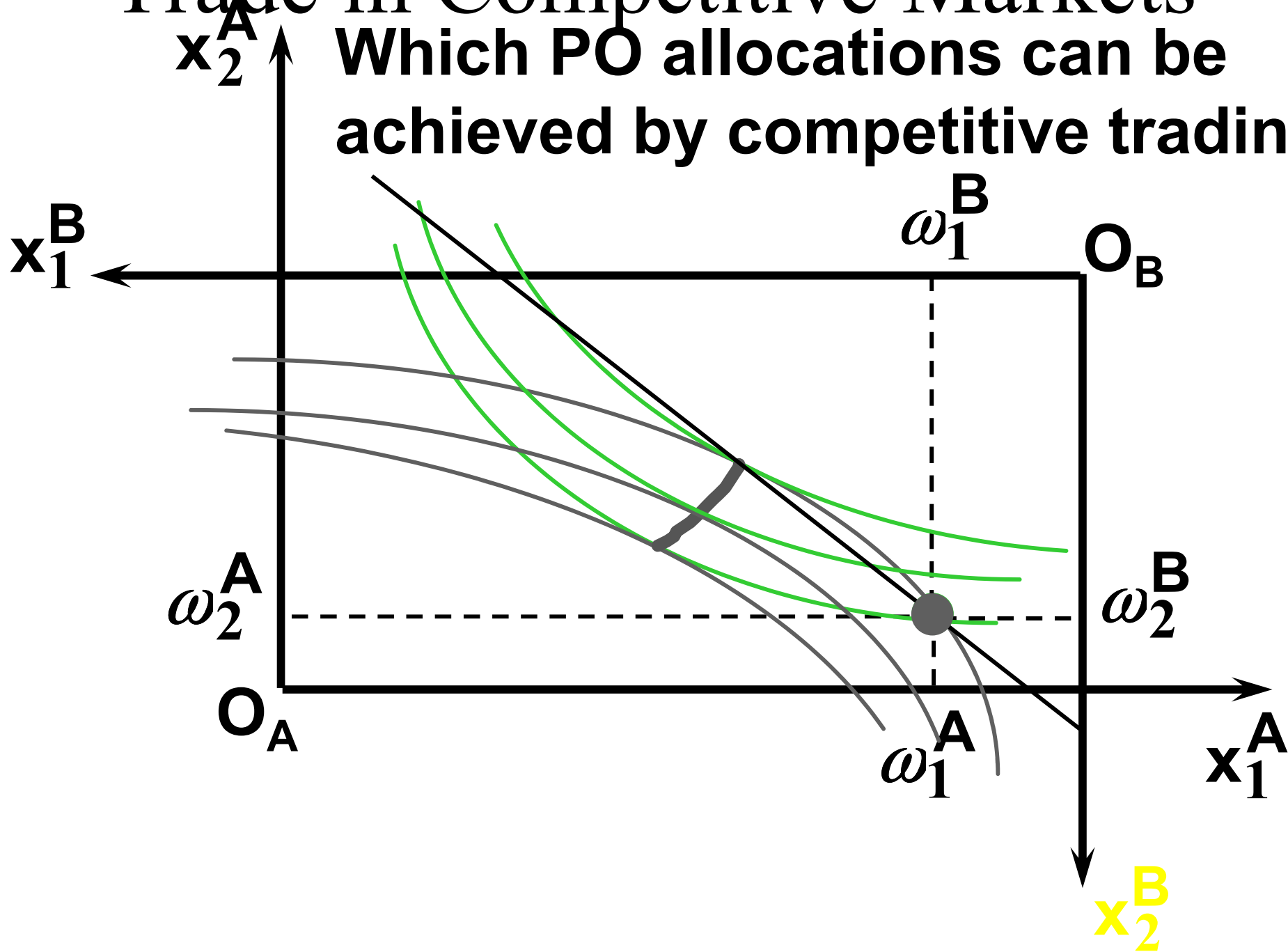


Trade in Competitive Markets

- ◆ **Since there is an excess demand for commodity 2, p_2 will rise.**
- ◆ **Since there is an excess supply of commodity 1, p_1 will fall.**
- ◆ **The slope of the budget constraints is $-p_1/p_2$ so the budget constraints will pivot about the endowment point and become less steep.**

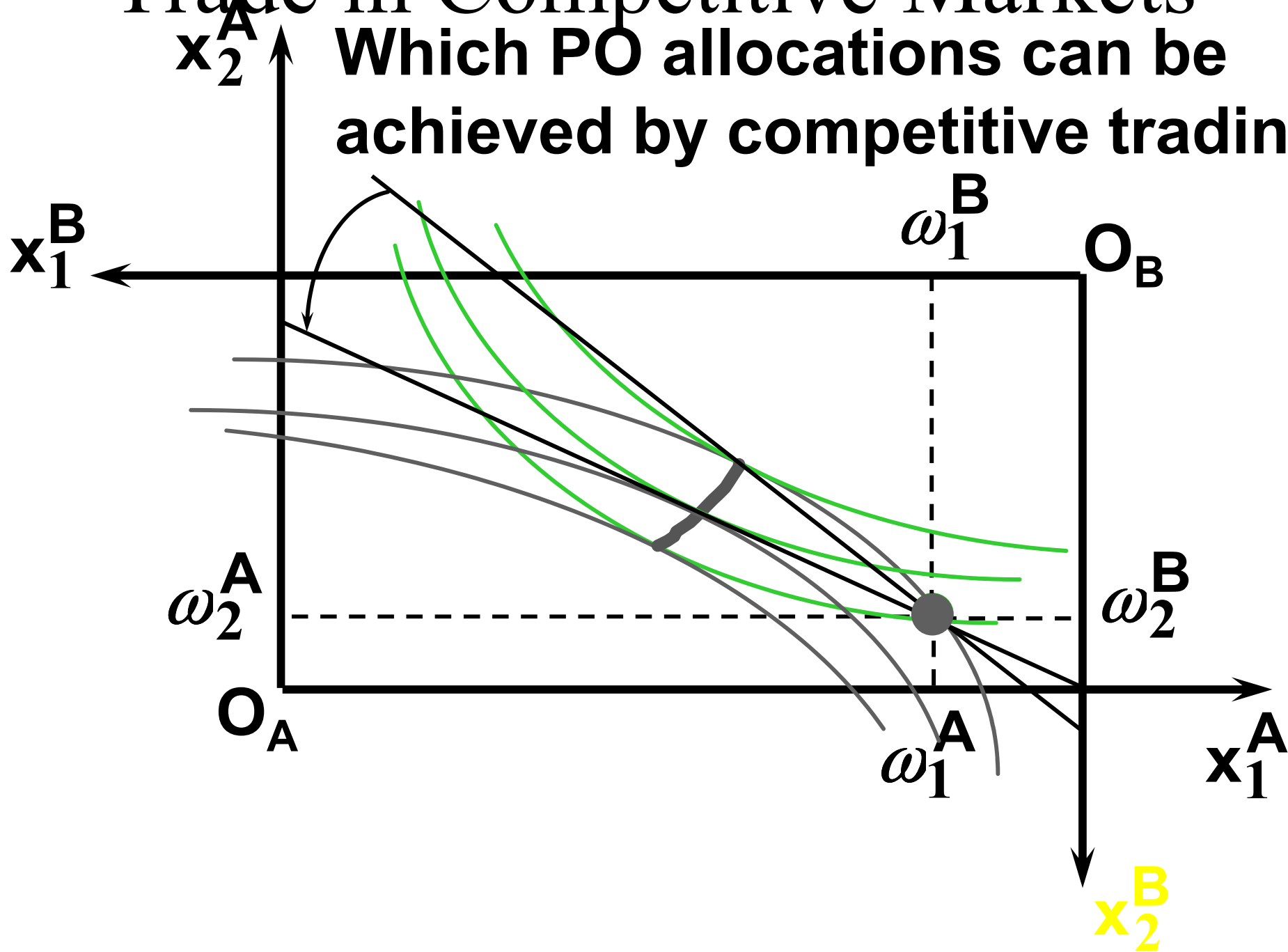
Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?



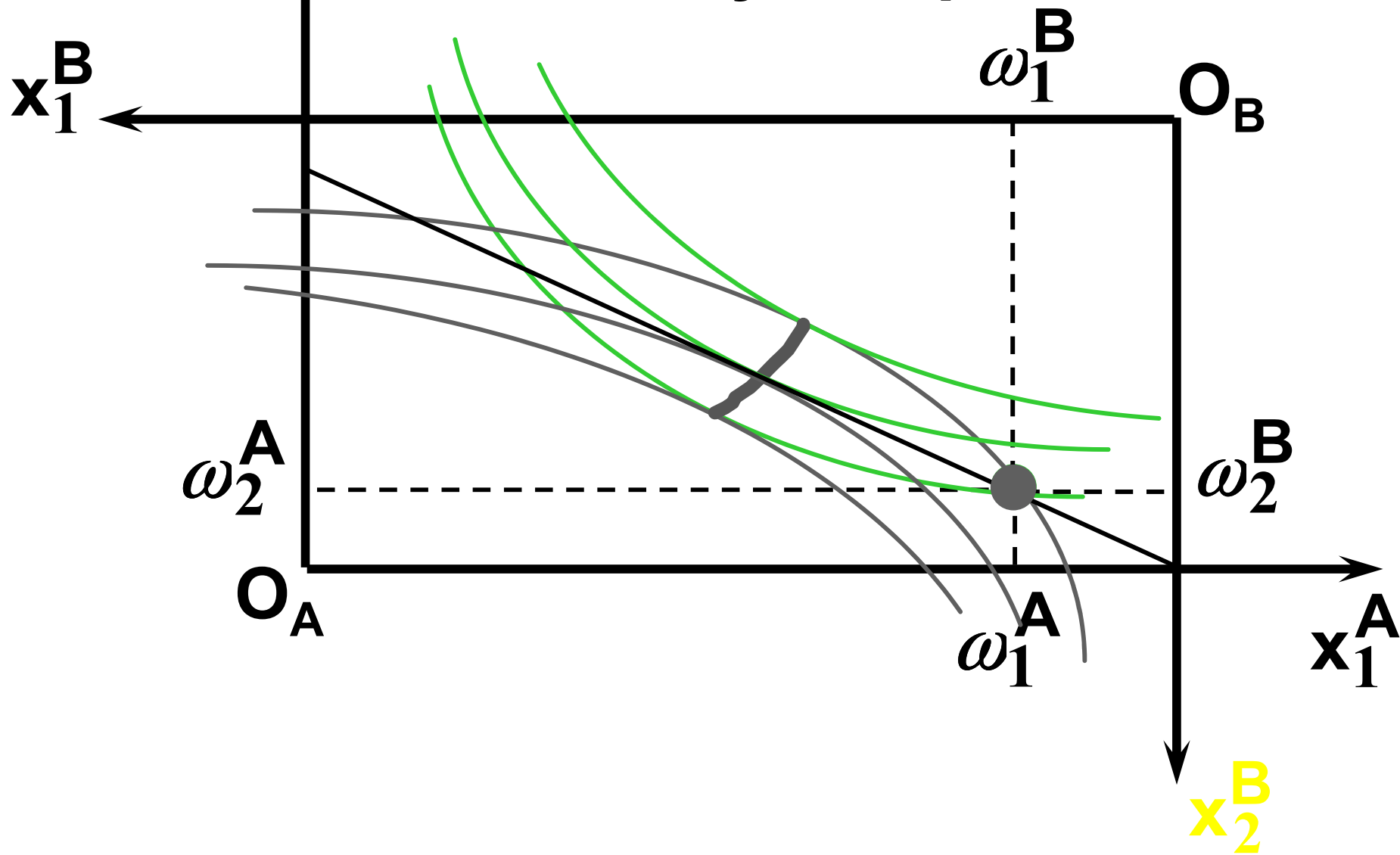
Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?



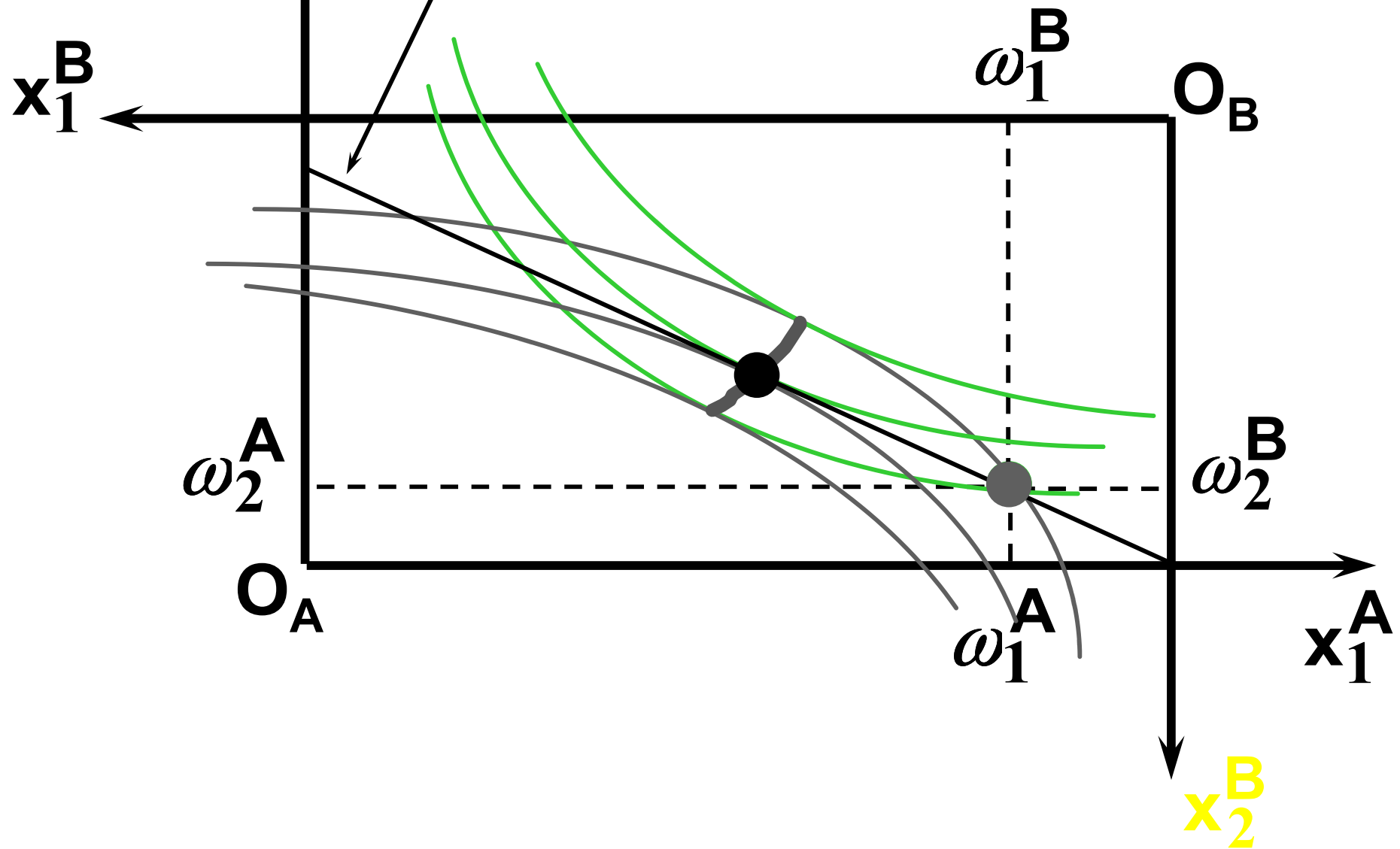
Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?



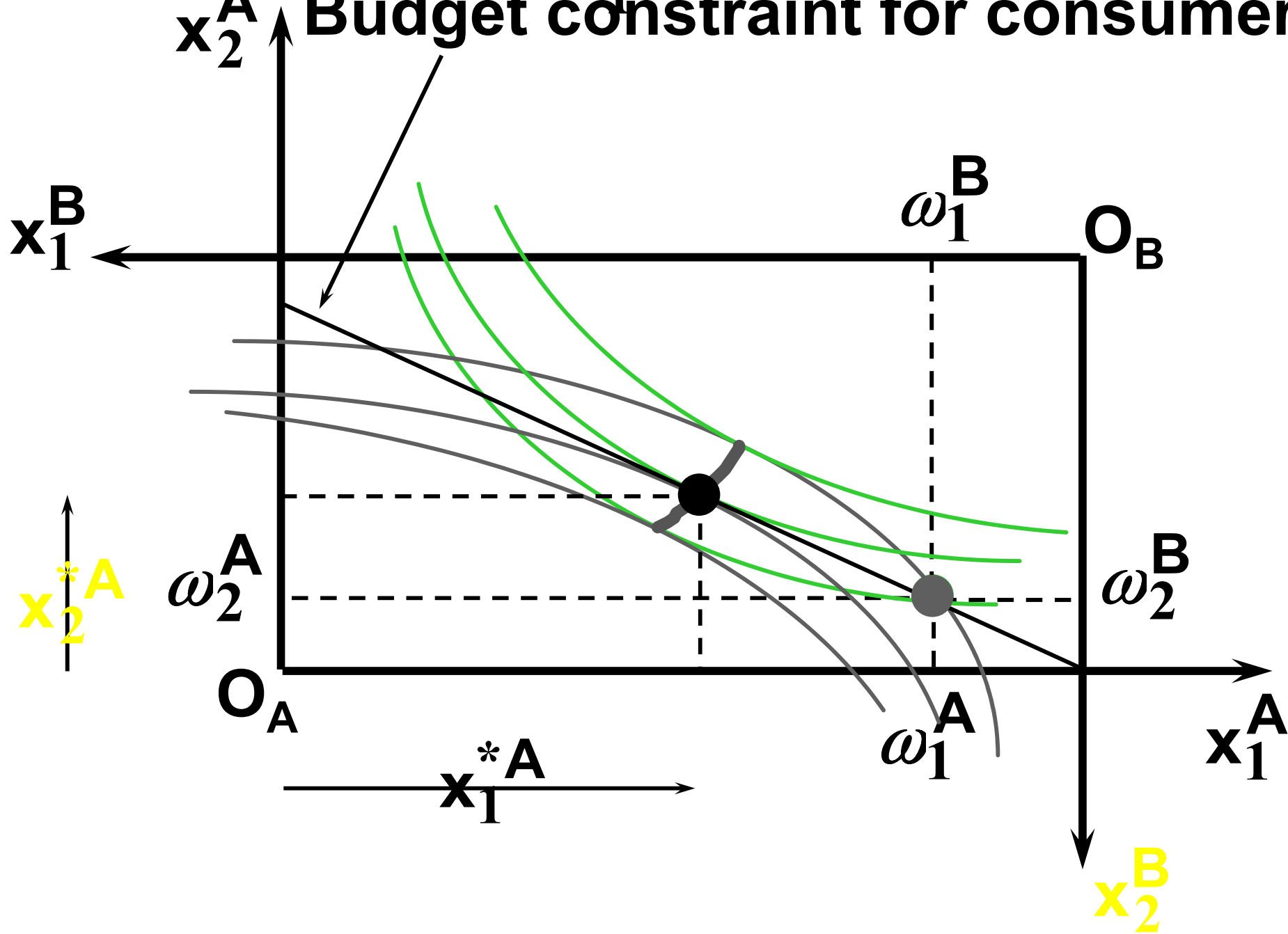
Trade in Competitive Markets

Budget constraint for consumer A

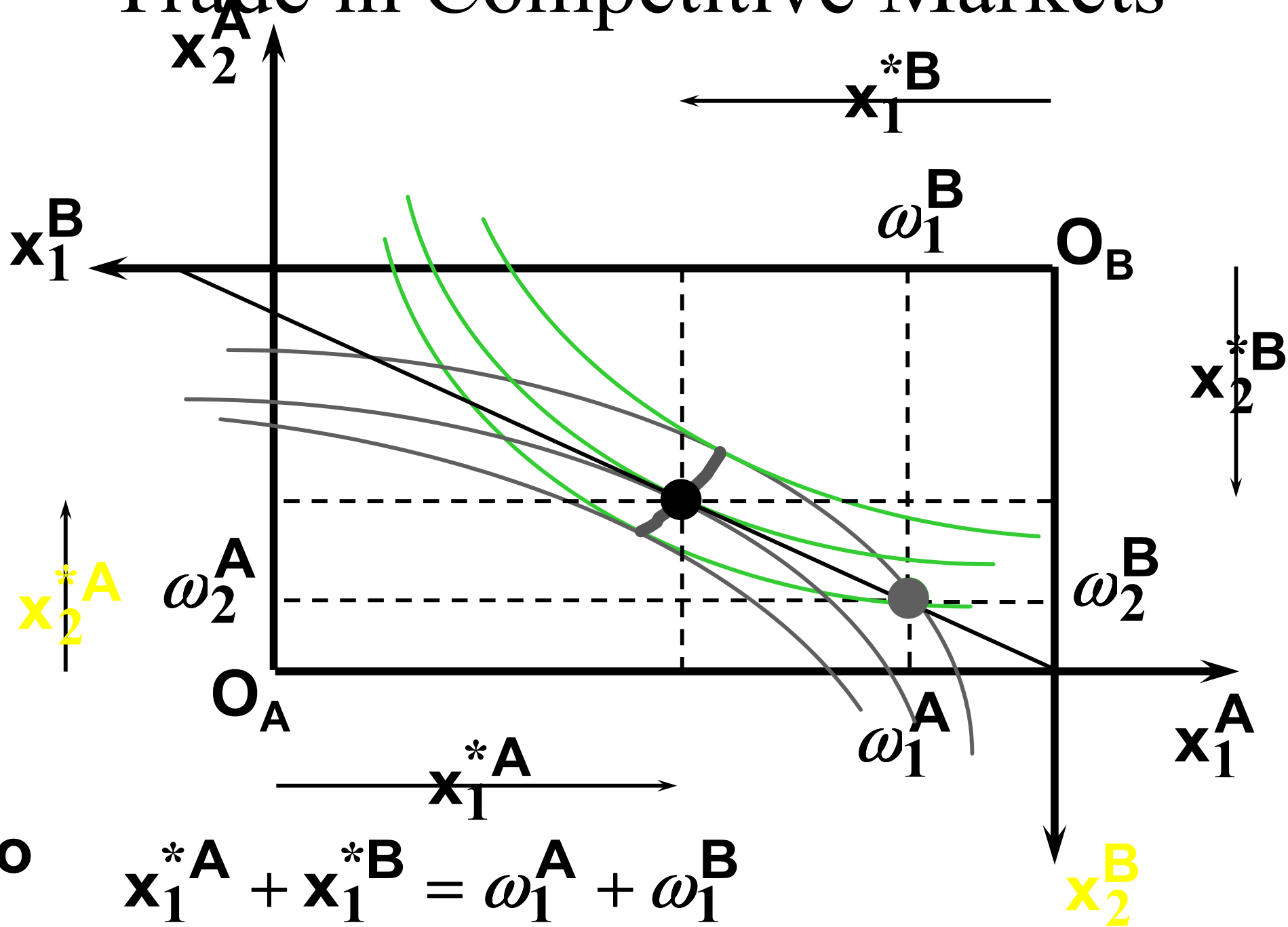


Trade in Competitive Markets

Budget constraint for consumer A



Trade in Competitive Markets

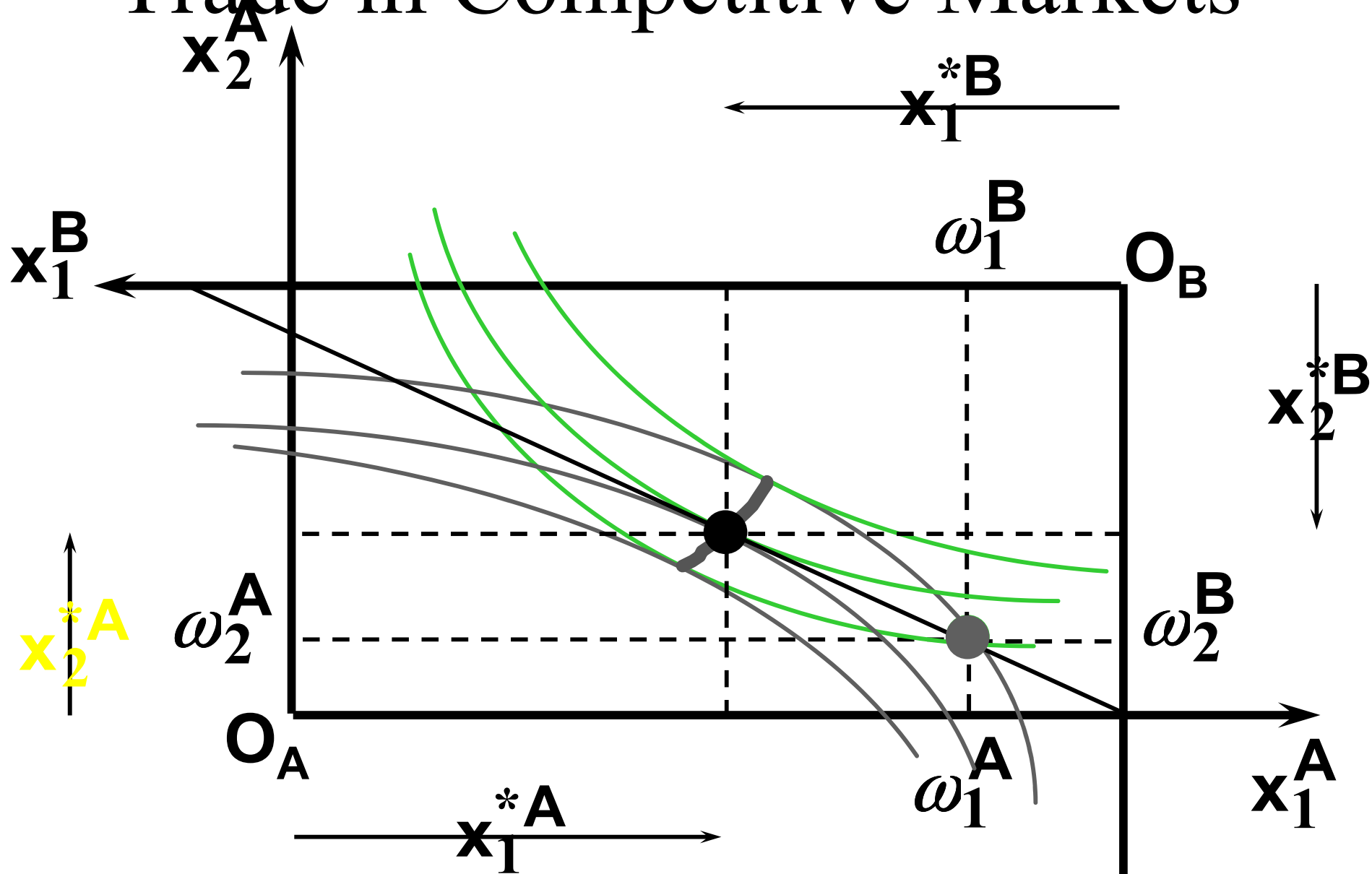


So

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

x_2^B

Trade in Competitive Markets



and

$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

x_2^B

Trade in Competitive Markets

- ◆ **At the new prices p_1 and p_2 both markets clear; there is a general equilibrium.**
- ◆ **Trading in competitive markets achieves a particular Pareto-optimal allocation of the endowments.**
- ◆ **This is an example of the First Fundamental Theorem of Welfare Economics.**

First Fundamental Theorem of Welfare Economics

- ◆ **Given that consumers' preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy's endowment.**

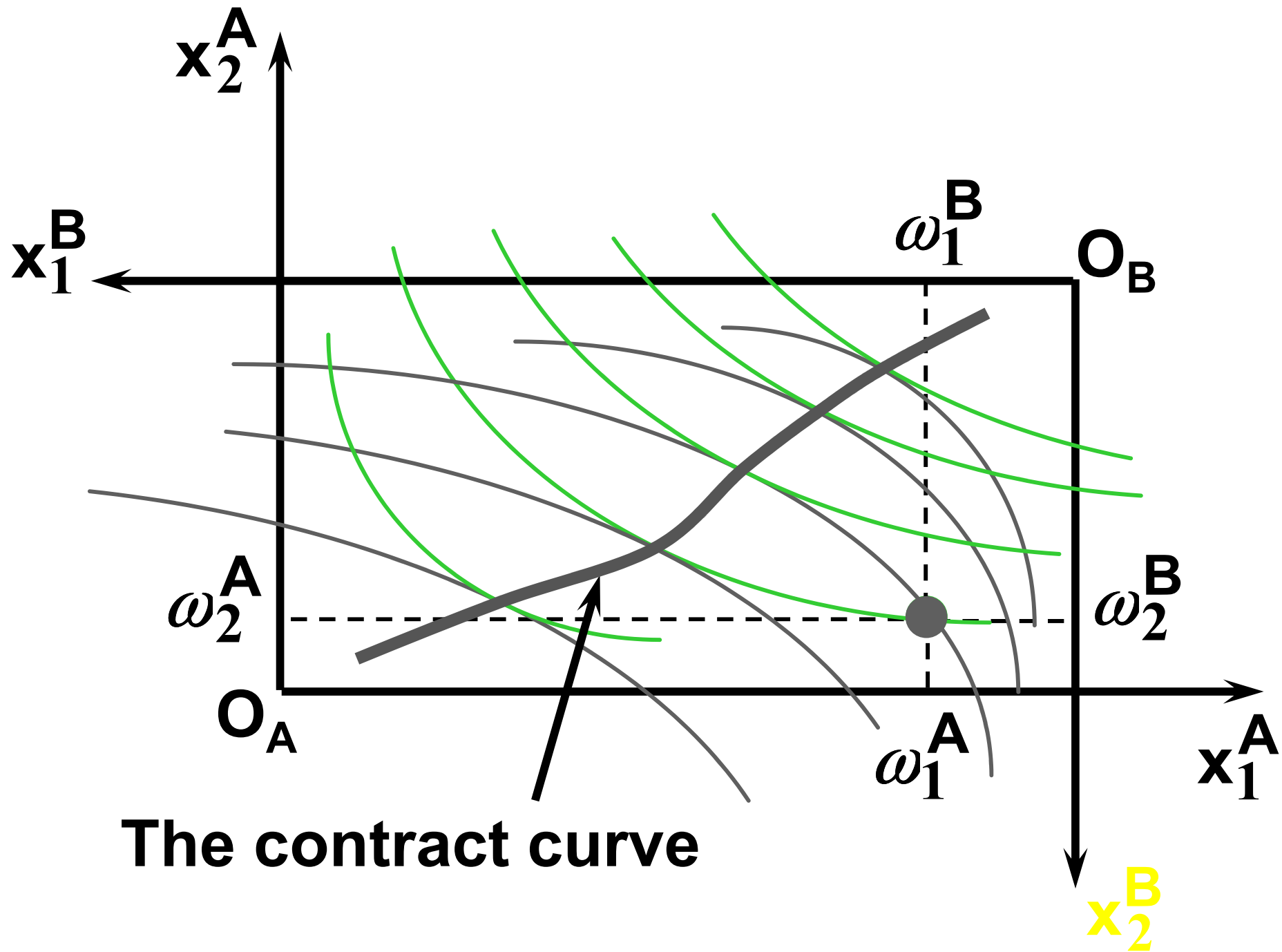
Second Fundamental Theorem of Welfare Economics

- ◆ **The First Theorem is followed by a second that states that any Pareto-optimal allocation (i.e. any point on the contract curve) can be achieved by trading in competitive markets provided that endowments are first appropriately rearranged amongst the consumers.**

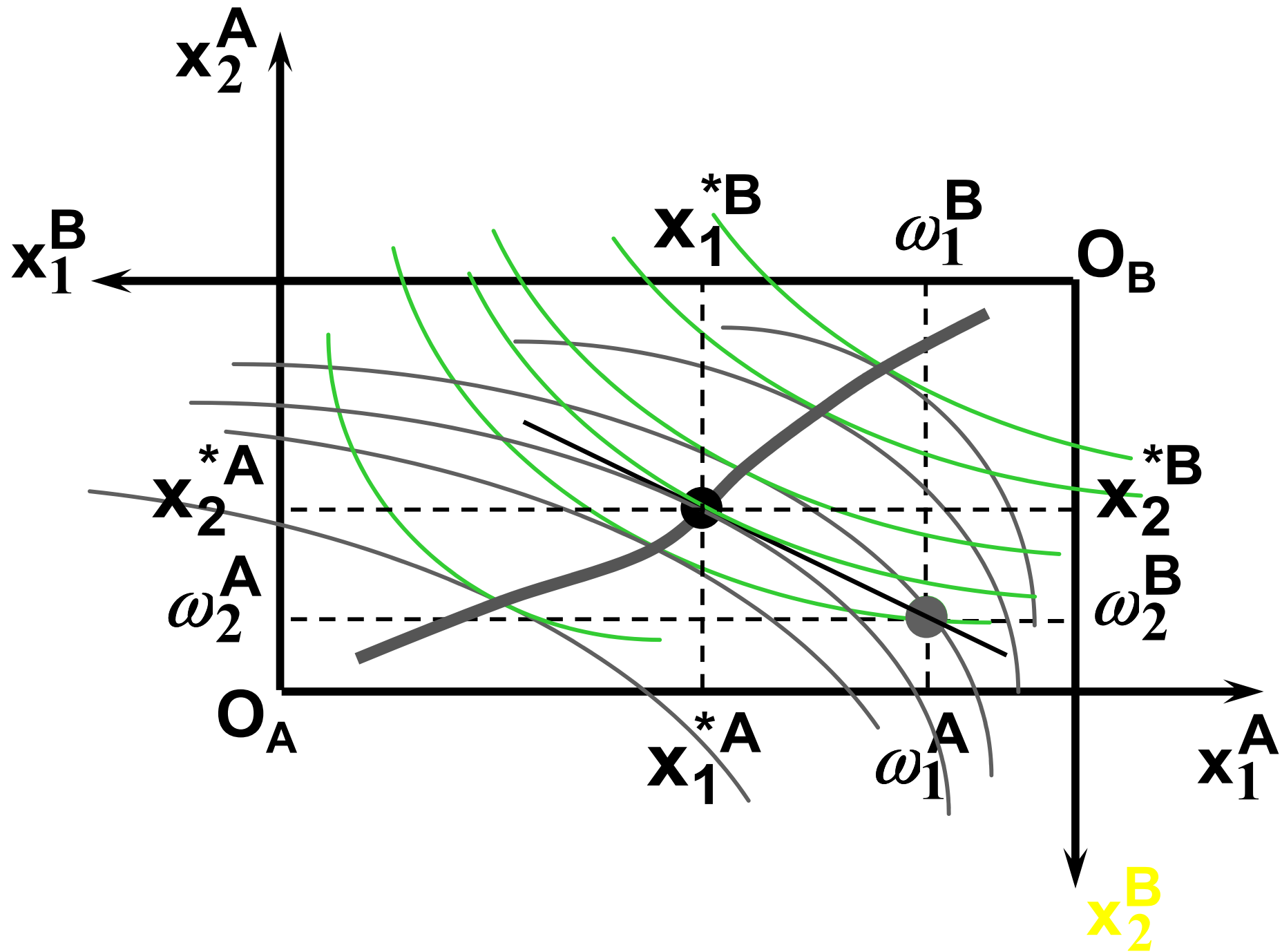
Second Fundamental Theorem of Welfare Economics

- ◆ **Given that consumers' preferences are well-behaved, for any Pareto-optimal allocation there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation implementable by trading in competitive markets.**

Second Fundamental Theorem

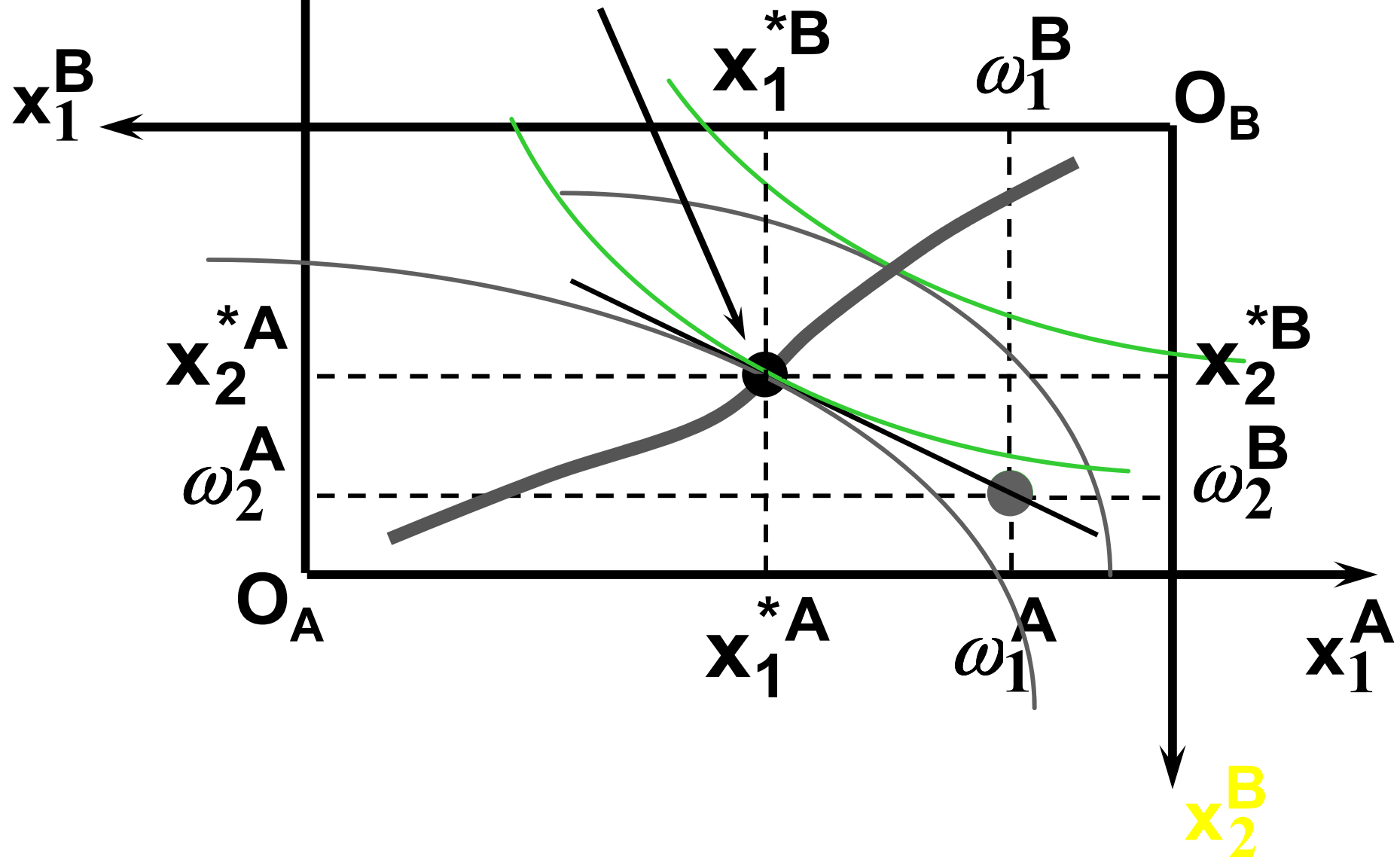


Second Fundamental Theorem



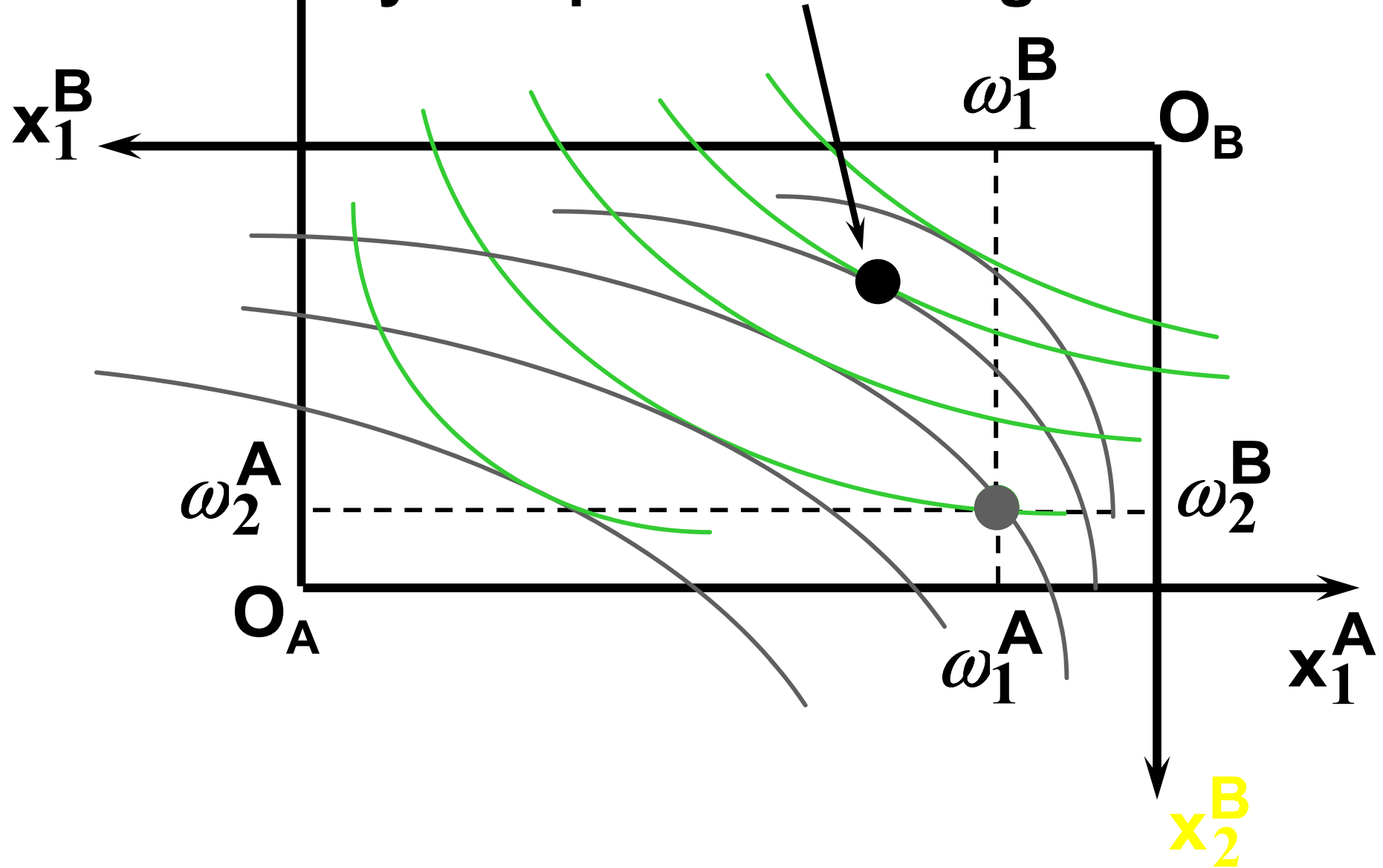
Second Fundamental Theorem

Implemented by competitive trading from the endowment ω .



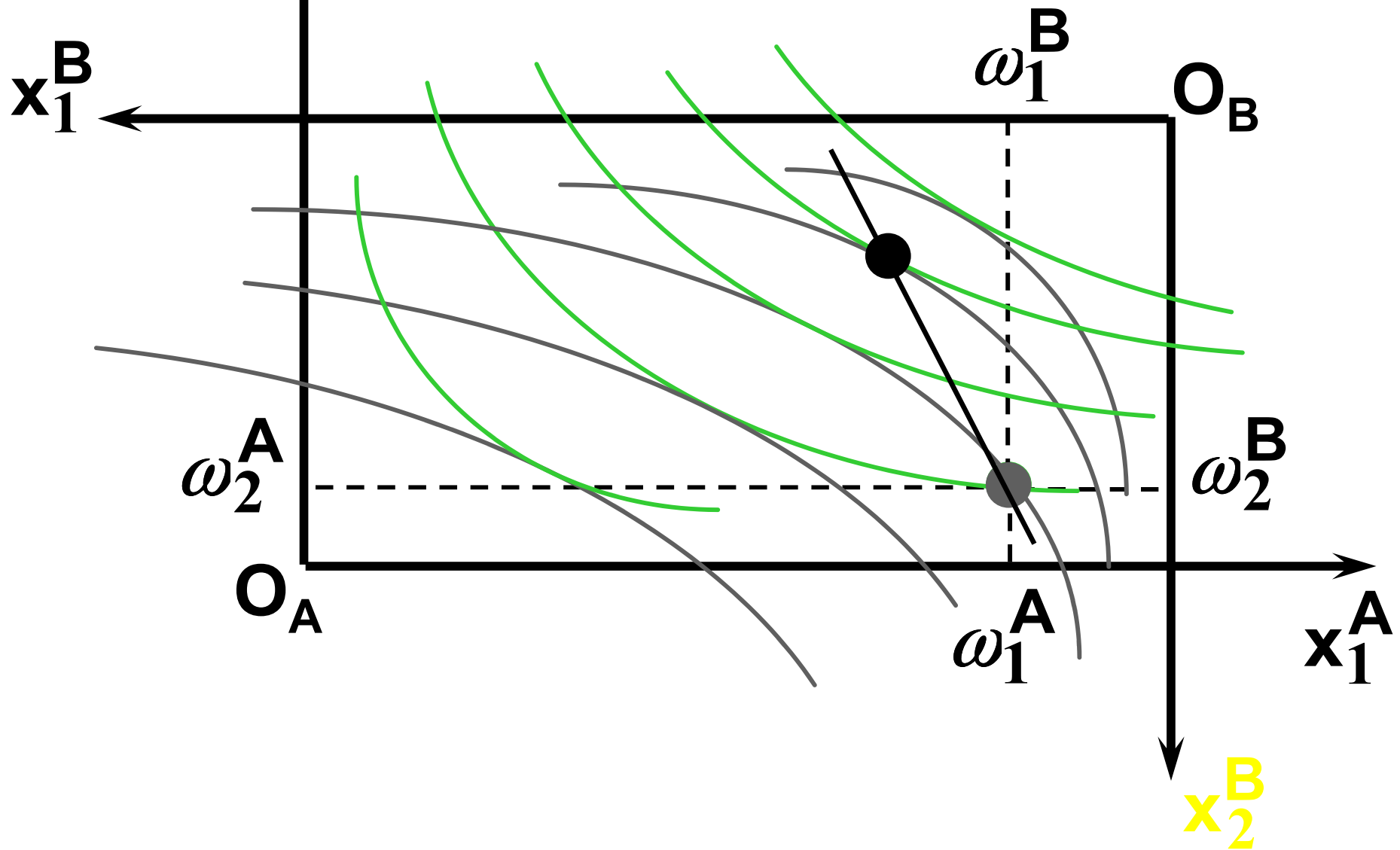
Second Fundamental Theorem

Can this allocation be implemented by competitive trading from ω ?



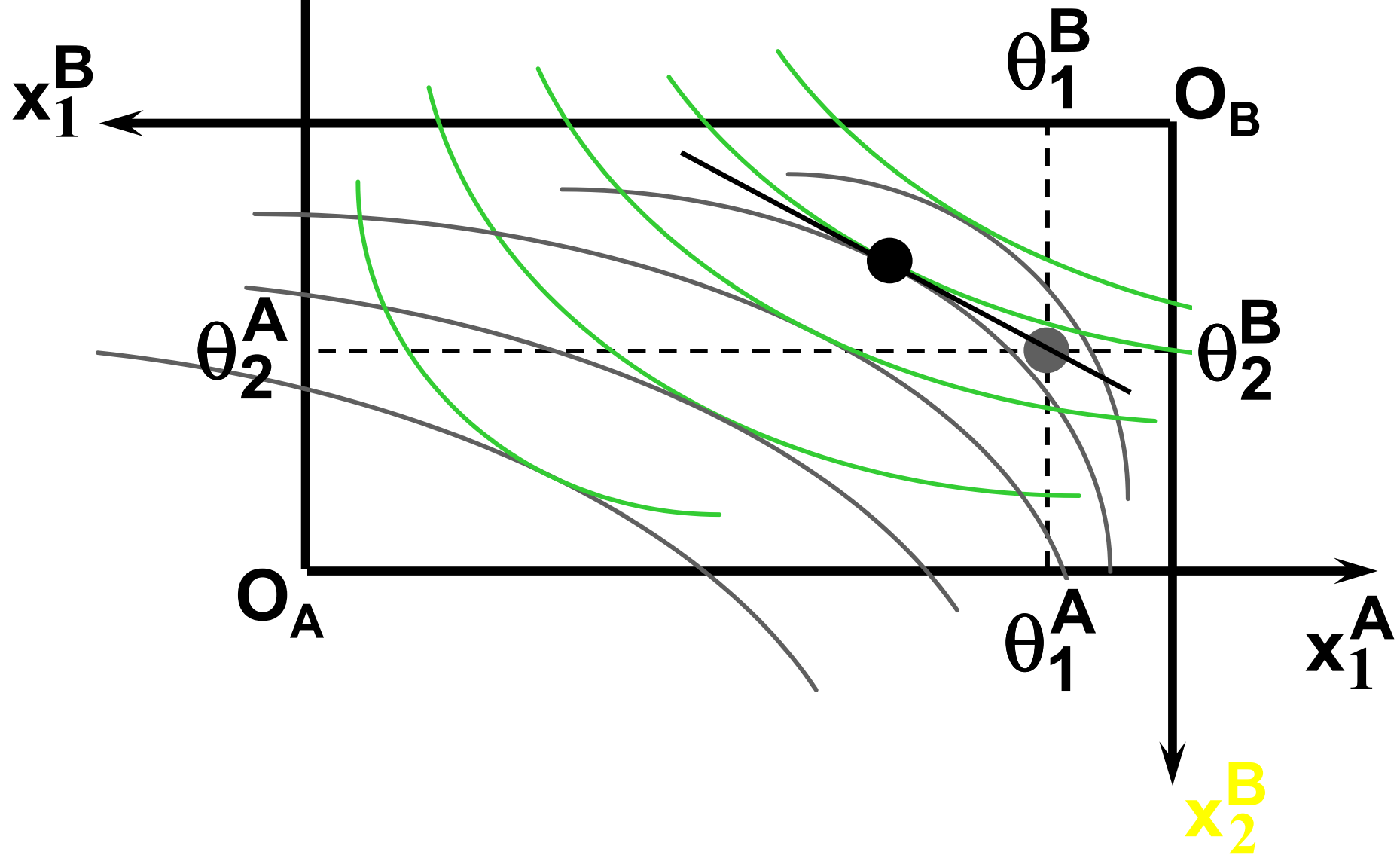
Second Fundamental Theorem

x_2^A **Can this allocation be implemented by competitive trading from ω ? No.**



Second Fundamental Theorem

But this allocation is implemented by competitive trading from θ .



Walras' Law

- ◆ **Walras' Law is an identity; i.e. a statement that is true for any positive prices (p_1, p_2) , whether these are equilibrium prices or not.**

Walras' Law

◆ Every consumer's preferences are well-behaved so, for any positive prices (p_1, p_2) , each consumer spends all of his budget.

◆ For consumer A:

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

For consumer B:

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Walras' Law

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Summing gives

$$\begin{aligned} & p_1 (x_1^{*A} + x_1^{*B}) + p_2 (x_2^{*A} + x_2^{*B}) \\ &= p_1 (\omega_1^A + \omega_1^B) + p_2 (\omega_2^A + \omega_2^B). \end{aligned}$$

Walras' Law

$$\begin{aligned} & \mathbf{p}_1(\mathbf{x}_1^{*A} + \mathbf{x}_1^{*B}) + \mathbf{p}_2(\mathbf{x}_2^{*A} + \mathbf{x}_2^{*B}) \\ &= \mathbf{p}_1(\omega_1^A + \omega_1^B) + \mathbf{p}_2(\omega_2^A + \omega_2^B). \end{aligned}$$

Rearranged,

$$\begin{aligned} & \mathbf{p}_1(\mathbf{x}_1^{*A} + \mathbf{x}_1^{*B} - \omega_1^A - \omega_1^B) + \\ & \mathbf{p}_2(\mathbf{x}_2^{*A} + \mathbf{x}_2^{*B} - \omega_2^A - \omega_2^B) = \mathbf{0}. \end{aligned}$$

That is, ...

Walras' Law

$$\begin{aligned} & p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) + \\ & p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) \\ & = 0. \end{aligned}$$

This says that the summed market value of excess demands is zero for **any positive prices p_1 and p_2 -- this is Walras' Law.**

Implications of Walras' Law

Suppose the market for commodity A is in equilibrium; that is,

$$\mathbf{x}_1^{*A} + \mathbf{x}_1^{*B} - \omega_1^A - \omega_1^B = \mathbf{0}.$$

Then

$$\mathbf{p}_1 (\mathbf{x}_1^{*A} + \mathbf{x}_1^{*B} - \omega_1^A - \omega_1^B) + \\ \mathbf{p}_2 (\mathbf{x}_2^{*A} + \mathbf{x}_2^{*B} - \omega_2^A - \omega_2^B) = \mathbf{0}$$

implies

$$\mathbf{x}_2^{*A} + \mathbf{x}_2^{*B} - \omega_2^A - \omega_2^B = \mathbf{0}.$$

Implications of Walras' Law

So one implication of Walras' Law for a two-commodity exchange economy is that if one market is in equilibrium then the other market must also be in equilibrium.

Implications of Walras' Law

What if, for some positive prices p_1 and p_2 , there is an excess quantity supplied of commodity 1? That is,

$$\mathbf{x}_1^{*A} + \mathbf{x}_1^{*B} - \omega_1^A - \omega_1^B < \mathbf{0}.$$

Then

$$\begin{aligned} & p_1 (\mathbf{x}_1^{*A} + \mathbf{x}_1^{*B} - \omega_1^A - \omega_1^B) + \\ & p_2 (\mathbf{x}_2^{*A} + \mathbf{x}_2^{*B} - \omega_2^A - \omega_2^B) = \mathbf{0} \end{aligned}$$

implies

$$\mathbf{x}_2^{*A} + \mathbf{x}_2^{*B} - \omega_2^A - \omega_2^B > \mathbf{0}.$$

Implications of Walras' Law

So a second implication of Walras' Law for a two-commodity exchange economy is that an excess supply in one market implies an excess demand in the other market.