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Applied Financial Econometrics

Class 8: GARCH and Value at Risk (VaR)

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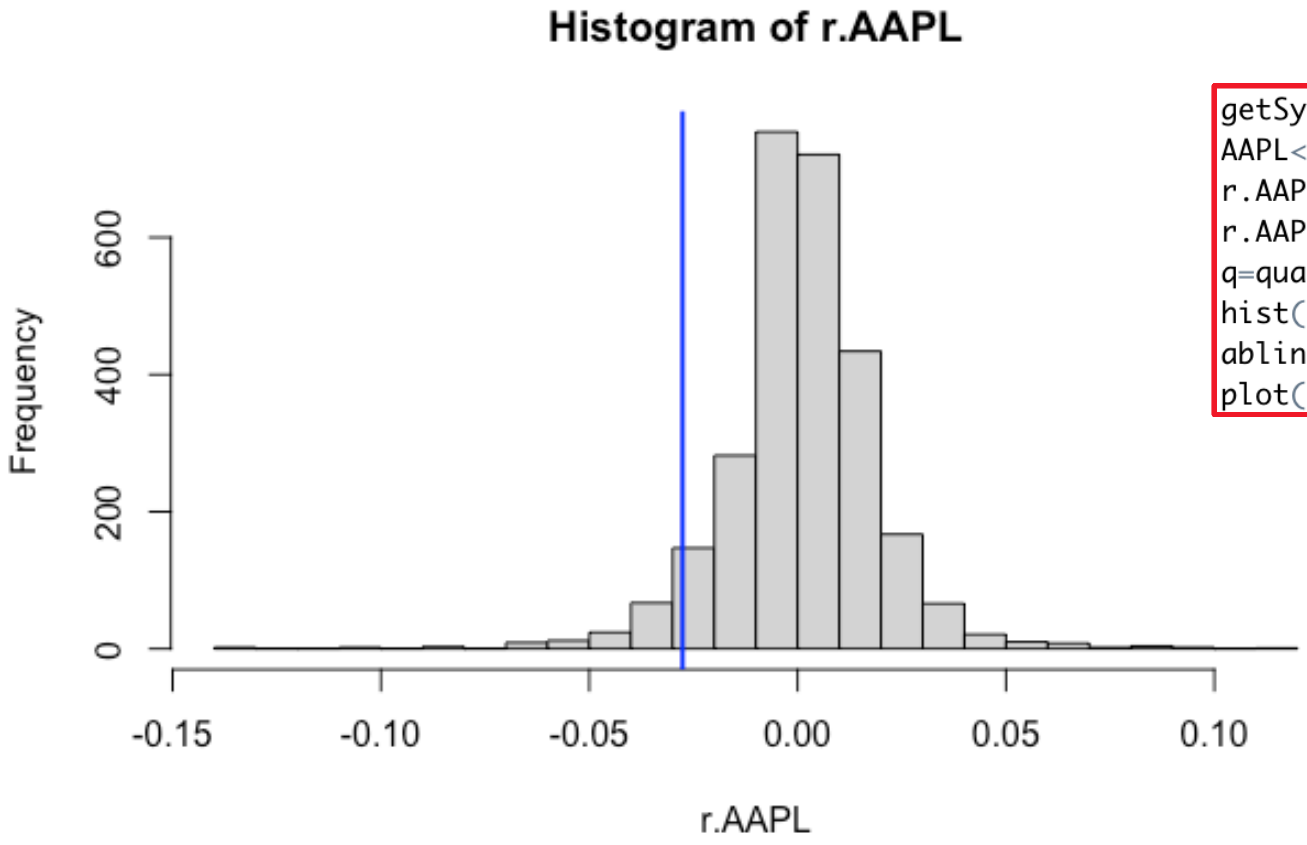
Risk measures: Value at Risk (VaR)

- Probability of (potential) losses given some probability.
- Downside risk based on current levels and normal market conditions.
- For a computed *T*-horizon *p*-VaR, a loss (*L*) will not exceed p-VaR with *p* probability *T*-periods ahead. $Prob(L > \text{VaR}) \leq 1-p$.
- Example: One-day 95% VaR of 100.
    - 5% of losses falls over 100.
    - Expected losses greater than 100 for 1 day over 20 days.
    - 95% of confidence of don't have losses greater than 100.

Computing VaR




```r
getSymbols('AAPL',src='yahoo', from="2012-01-01",periodicity = 'daily')
AAPL<-AAPL[,6]
r.AAPL<-diff(log(AAPL))
r.AAPL<-r.AAPL[2:length(AAPL)]
q=quantile(r.AAPL,0.05)
hist(r.AAPL,breaks = 20)
abline(v=q,col="blue",lwd=2)
plot(density(r.AAPL))
```

```
> quantile(r.AAPL,0.05)
         5%
-0.02761346
```

– 1-day 95% VaR: 2.76 %

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Estimating VaR (in-sample)

- Assuming normality:

$$VaR(a) = \bar{x} + \sigma * N^{-1}(a)$$

$$VaR(95) = \bar{x} - \sigma * 1.96$$

Assignment 6

1. For the S&P500 and another asset of your choice, estimate the in-sample 1-day 95%VaR assuming normality and:
   a) Constant volatility and ARIMA(0,0,0) for the returns
   b) Constant volatility (finding the best ARIMA model possible for the returns, if any)
   c) GARCH(1,1) volatility.

2. Compare the results.

Review of the assignment 6 (1.a)

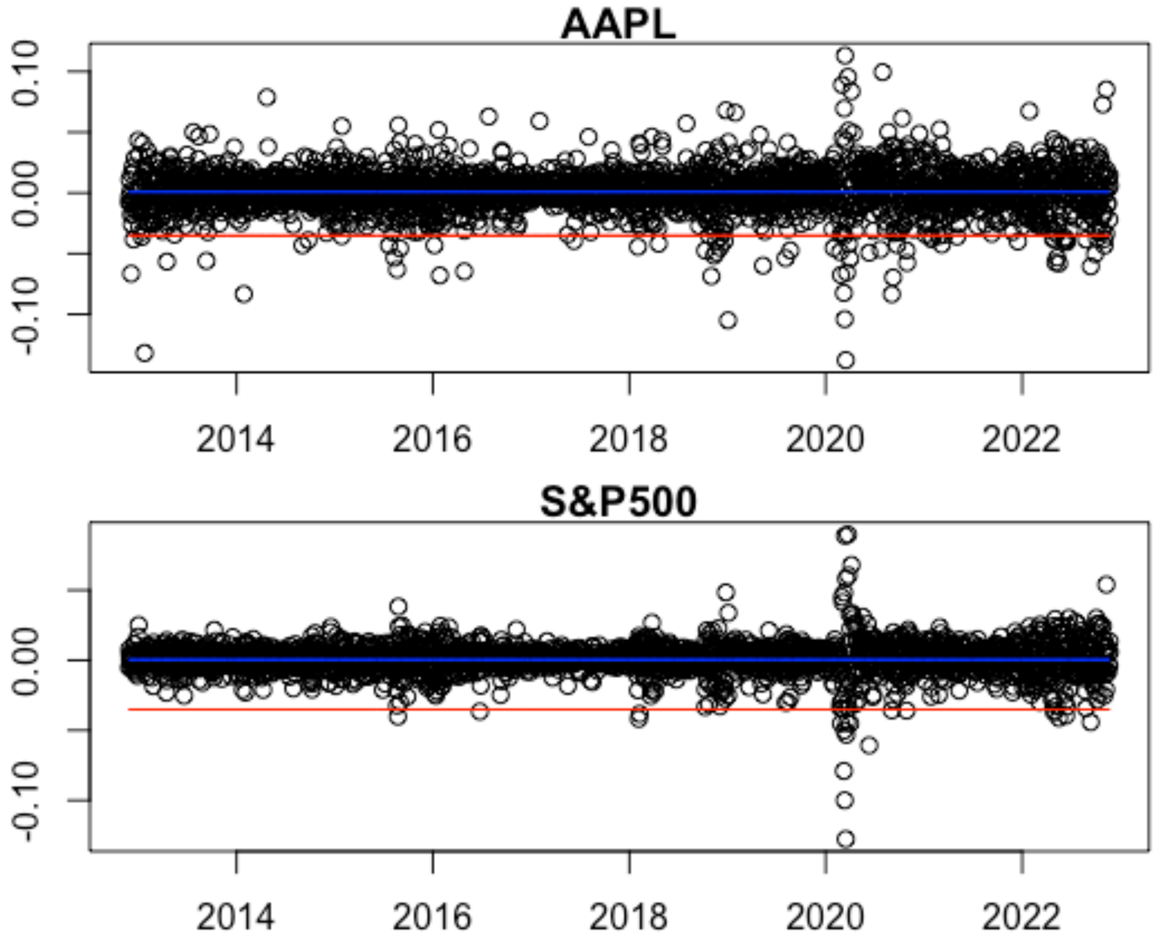
```r
library('quantmod')
getSymbols(c('AAPL','^GSPC'),src='yahoo',
           from="2012-11-25",periodicity = 'daily')
r.AAPL<-diff(log(AAPL[,6]))
r.SP500<-diff(log(GSPC[,6]))
r.AAPL<-r.AAPL[2:length(r.AAPL)]
r.SP500<-na.omit(r.SP500)
```

```r
# 95-VAR under normality
VAR_AAPL<-arima0$coef-1.96*vol0AAPL
VAR_SP500<-mean(r.SP500)-1.96*vol0SP500
```

```r
arima0<-arima(r.AAPL,order=c(0,0,0))
res0AAPL<-arima0$residuals
arima0SP500<-mean(r.SP500)
res0SP500<r.SP500-mean(r.SP500)
vol0AAPL<-sqrt(arima0$sigma2)
vol0SP500<-sqrt(var(r.SP500))
```

Review of the assignment 6 (1.a)

```r
par(mfrow=c(2,1),mar=c(2.5,2.5,1,1))
plot(time(r.AAPL),r.AAPL,title('AAPL'))
lines(time(r.AAPL),rep(arima0$coef,
      length(r.AAPL)),col="blue")
lines(time(r.AAPL),rep(VAR_AAPL,
      length(r.AAPL)),col="red")
plot(time(r.SP500),r.SP500,title('S&P500'))
lines(time(r.SP500),rep(mean(r.SP500),
      length(r.SP500)),col="blue")
lines(time(r.SP500),rep(VAR_AAPL,
      length(r.AAPL)),col="red")
```



Review of the assignment 6 (1.b)

```r
library(forecast)
arimaAAPL<-auto.arima(r.AAPL)
arimaSP500<-auto.arima(r.SP500)
arimaAAPL; arimaSP500
```

```
Series: r.AAPL
ARIMA(0,0,1) with non-zero mean

Coefficients:
          ma1    mean
      -0.0637   8e-04
s.e.   0.0200   3e-04
```

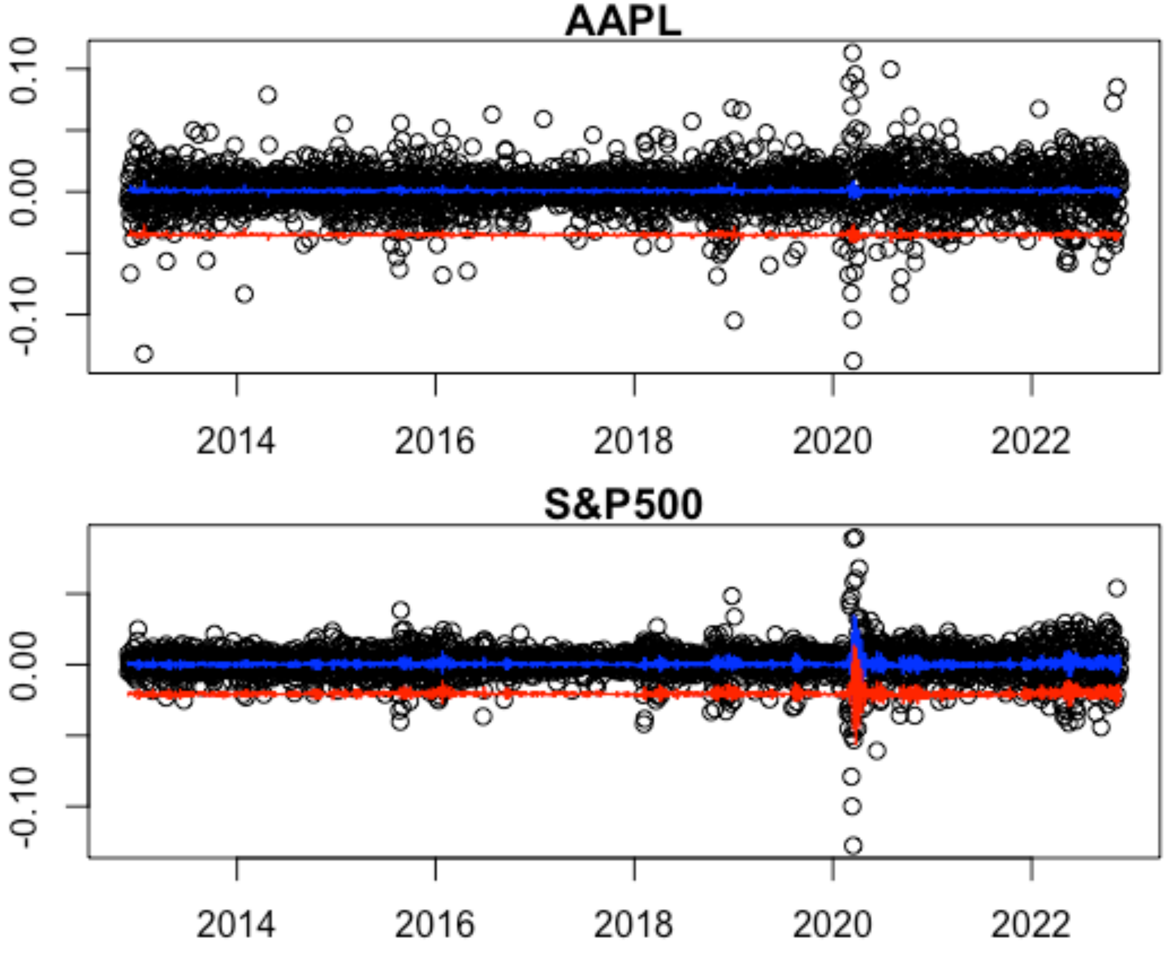
```
Series: r.SP500
ARIMA(3,0,3) with non-zero mean

Coefficients:
          ar1     ar2     ar3     ma1      ma2      ma3   mean
      -0.7649  0.8182  0.8577  0.6572  -0.8582  -0.7461  4e-04
s.e.   0.0317  0.0344  0.0276  0.0373   0.0273   0.0327  1e-04
```

```r
vol2AAPL<-sqrt(arimaAAPL$sigma2)
vol2SP500<-sqrt(arimaSP500$sigma2)
model2_AAPL<-arimaAAPL$fitted
model2_SP500<-arimaSP500$fitted
```

```r
# 95-VAR under normality
VAR_AAPL_2<-model2_AAPL-1.96*vol2AAPL
VAR_SP500_2<-model2_SP500-1.96*vol2SP500
```

Review of the assignment 6 (1.b)

```
plot(time(r.AAPL),r.AAPL,title('AAPL'))
lines(time(r.AAPL),model2_AAPL,col="blue")
lines(time(r.AAPL),VAR_AAPL_2,col="red")
plot(time(r.SP500),r.SP500,title('S&P500'))
lines(time(r.SP500),model2_SP500,col="blue")
lines(time(r.SP500),VAR_SP500_2,col="red")
```

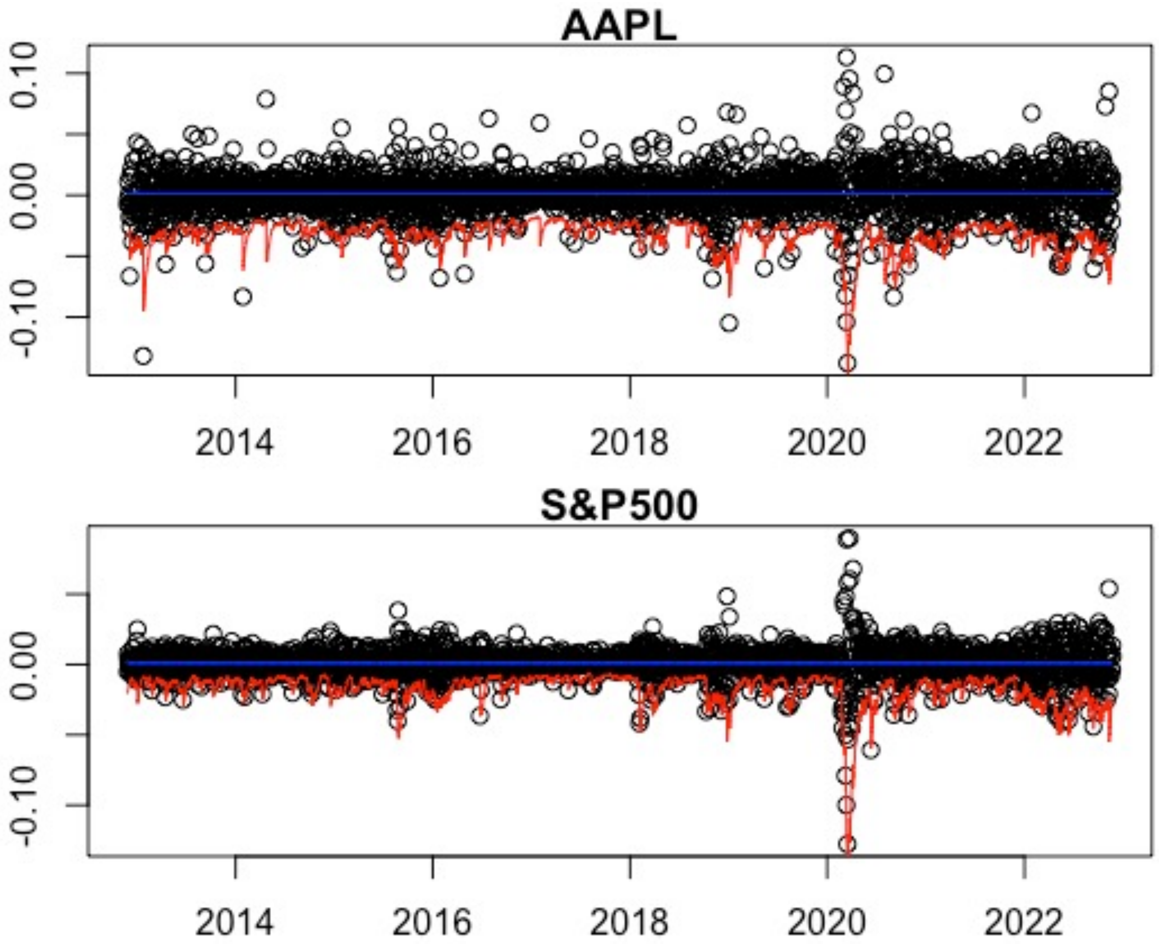


Review of the assignment 6 (1.c)

```
library('rugarch')
garchSpec <- ugarchspec(
  variance.model=list(model="sGARCH",
                      garchOrder=c(1,1)),
  mean.model=list(armaOrder=c(0,0)),
  distribution.model="norm")
garchFit <- ugarchfit(spec=garchSpec, data=r.AAPL)
AAPL_hat <- garchFit@fit$fitted.values
AAPL_vol_hat <- ts(garchFit@fit$sigma)
garchFit2 <- ugarchfit(spec=garchSpec, data=r.SP500)
SP_hat <- garchFit2@fit$fitted.values
SP_vol_hat <- ts(garchFit2@fit$sigma)
```

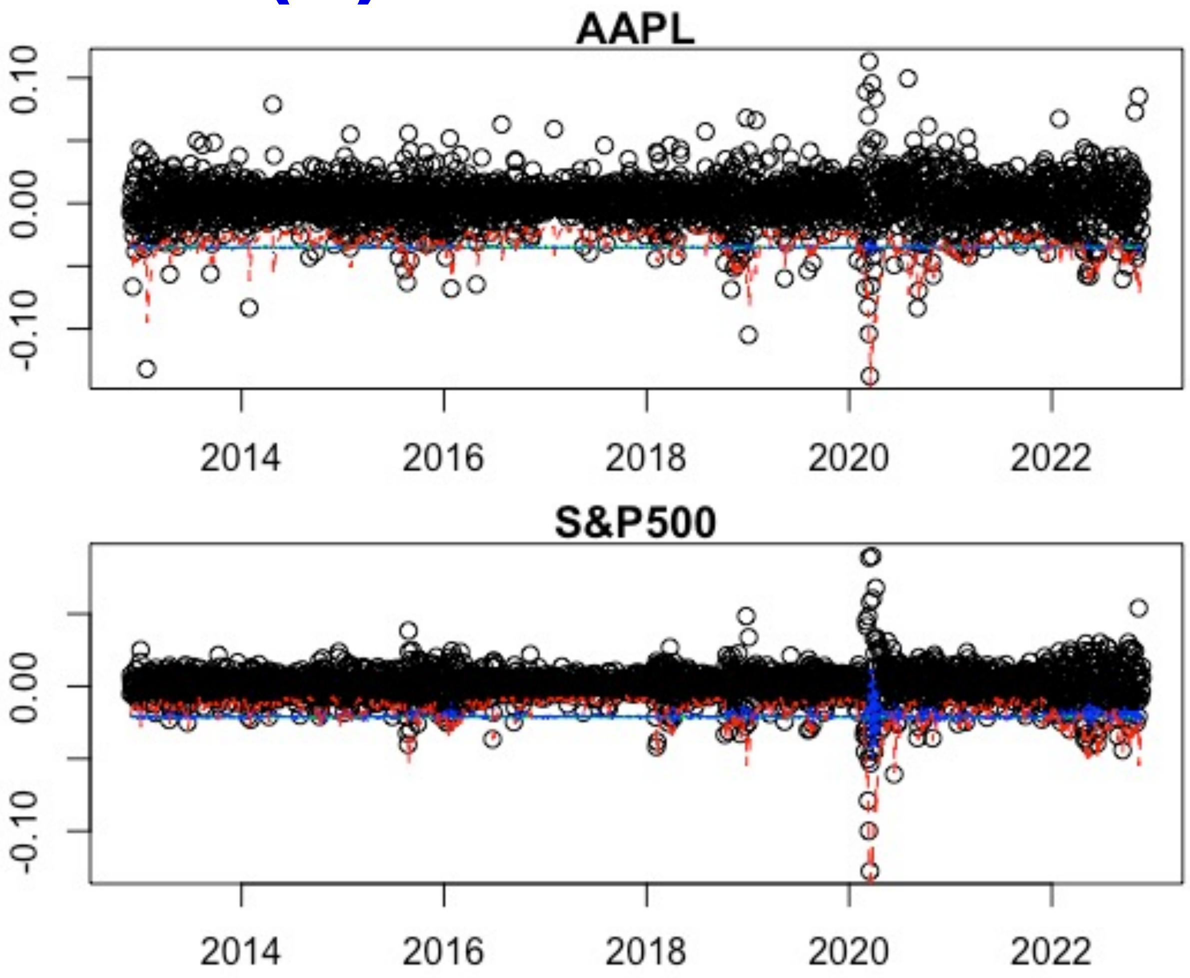
```
# 95-VAR under normality
VAR_AAPL_3<-AAPL_hat-1.96*AAPL_vol_hat
VAR_SP500_3<-SP_hat-1.96*SP_vol_hat
```

```
plot(time(r.AAPL),r.AAPL,title('AAPL'))
lines(time(r.AAPL),AAPL_hat,col="blue")
lines(time(r.AAPL),VAR_AAPL_3,col="red")
plot(time(r.SP500),r.SP500,title('S&P500'))
lines(time(r.SP500),SP_hat,col="blue")
lines(time(r.SP500),VAR_SP500_3,col="red")
```



Review of the assignment 6 (2)

```
#Comparison
plot(time(r.AAPL),r.AAPL,title('AAPL'))
lines(time(r.AAPL),rep(VAR_AAPL,
      length(r.AAPL)),col="green")
lines(time(r.AAPL),VAR_AAPL_2,col="blue",lty='dotted')
lines(time(r.AAPL),VAR_AAPL_3,col="red",lty='dashed')
plot(time(r.SP500),r.SP500,title('S&P500'))
lines(time(r.SP500),rep(VAR_SP500,
      length(r.AAPL)),col="green")
lines(time(r.SP500),VAR_SP500_2,col="blue",lty='dotted')
lines(time(r.SP500),VAR_SP500_3,col="red",lty='dashed')
```



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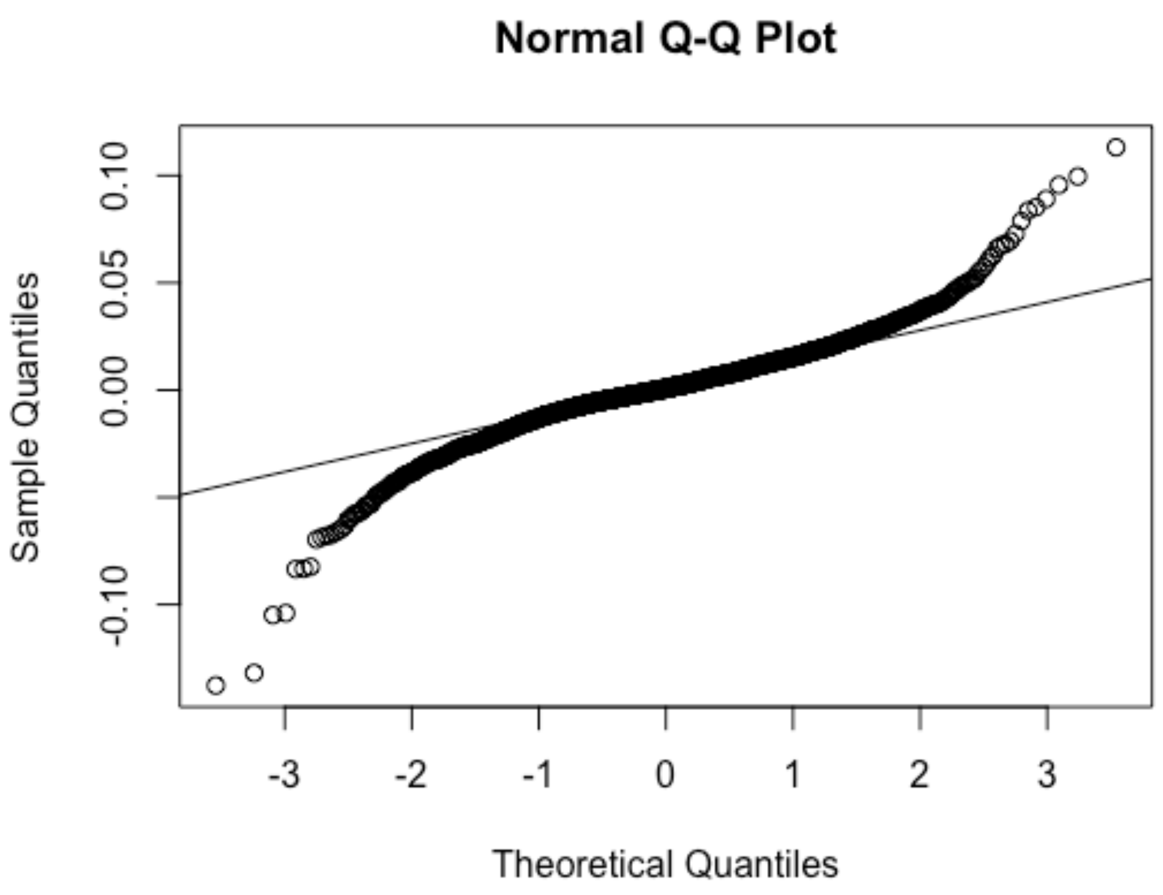
VaR computation

- Returns exhibits heavy tails than normal distribution.

- t-student distribution: Similar shape than normal distribution with heavy tails.



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VaR computation

```
pars<-fitdist(distribution = 'std' , x = r.AAPL)$pars
```

```
> pars
         mu       sigma       shape
0.001187087 0.019318014 3.284576519
```

```
pars<-fitdist(distribution = 'std' , x = r.AAPL)$pars
quantile(r.AAPL,0.05)
qnorm(p = 0.05,mean=mean(r.AAPL), sd=sqrt(var(r.AAPL)))
qdist(distribution = 'std' ,mu=pars[1], sigma=pars[2],
      shape = pars[3] , p = 0.05)
```

```
quantile(r.AAPL,0.01)
qnorm(p = 0.01,mean=mean(r.AAPL), sd=sqrt(var(r.AAPL)))
qdist(distribution = 'std' ,mu=pars[1], sigma=pars[2],
      shape = pars[3] , p = 0.01)
```

	Uncond.	Normal	t-student
5%	-0.027507	-0.02934347	-0.02626725
1%	-0.05298592	-0.04185024	-0.0500444

VaR computation

– Delta normal VaR

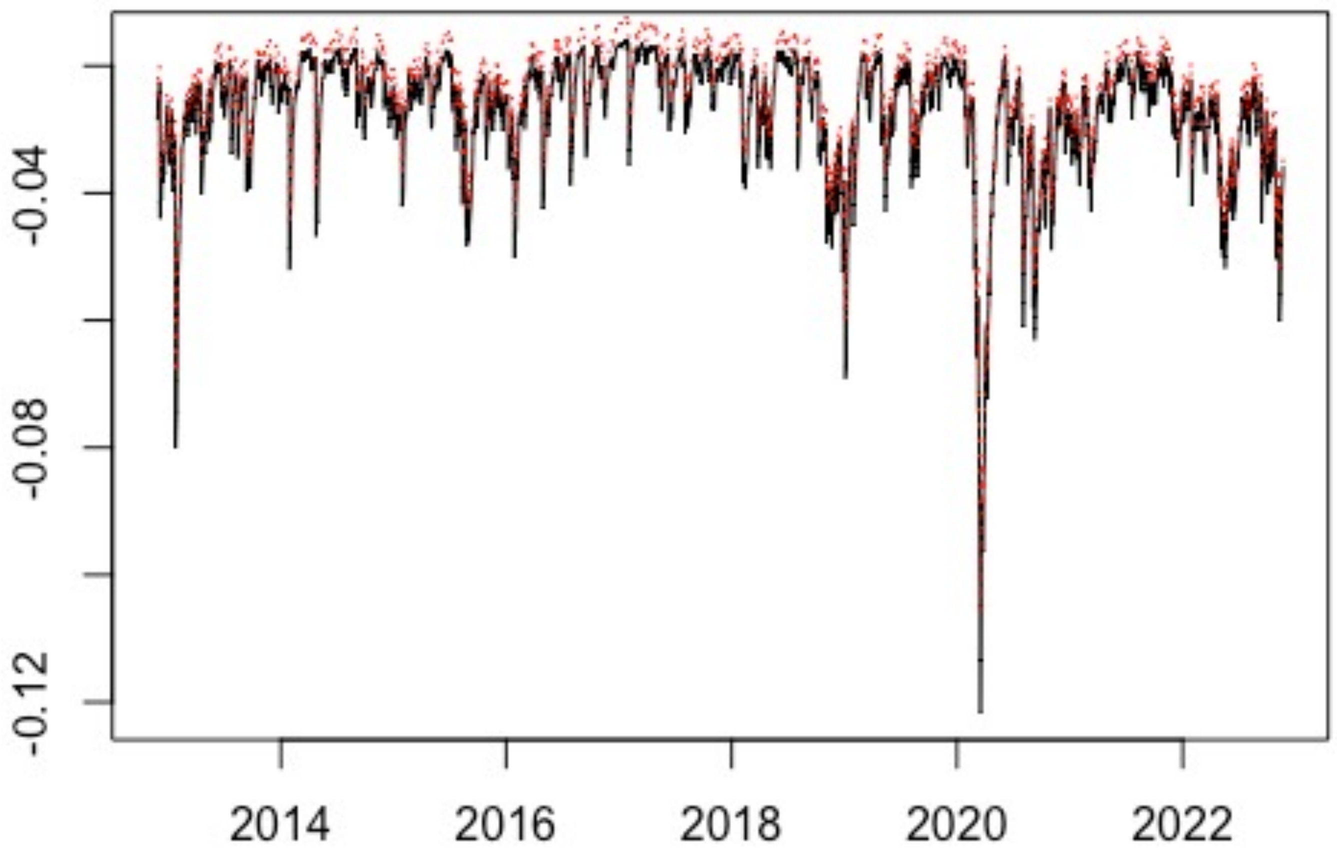
$$\begin{aligned}\mathrm{VaR}(a) &= N^{-1}(a,\mu,\sigma) \\ &= \mu + \sigma * N^{-1}(a)\end{aligned}$$

– GARCH(1,1) VaR with student's t-distribution in the underlying:

$$\mathrm{VaR}(a)_t = \mu + \sigma_{t|t-1} * F^{-1}(a)$$

VaR computation

```
garchSpec2 <- ugarchspec(
  variance.model=list(model="sGARCH",
                      garchOrder=c(1,1)),
  mean.model=list(armaOrder=c(0,0)),
  distribution.model="std")
garchFit2<- ugarchfit(spec=garchSpec2, data=r.AAPL)
AAPL_hat2 <- garchFit2@fit$fitted.values
AAPL_vol_hat2 <- ts(garchFit2@fit$sigma)
# 95-VAR under normality
VAR_AAPL_N<-AAPL_hat+AAPL_vol_hat*qnorm(p=.05)
# 95-VAR under t-student
VAR_AAPL_t<-AAPL_hat2+AAPL_vol_hat2*qdist(
  distribution = 'std', p = 0.05,shape = pars[3])
#plot
plot(time(r.AAPL),VAR_AAPL_N,type='line')
lines(time(r.AAPL),VAR_AAPL_t,col='red',lty='dotted')
```



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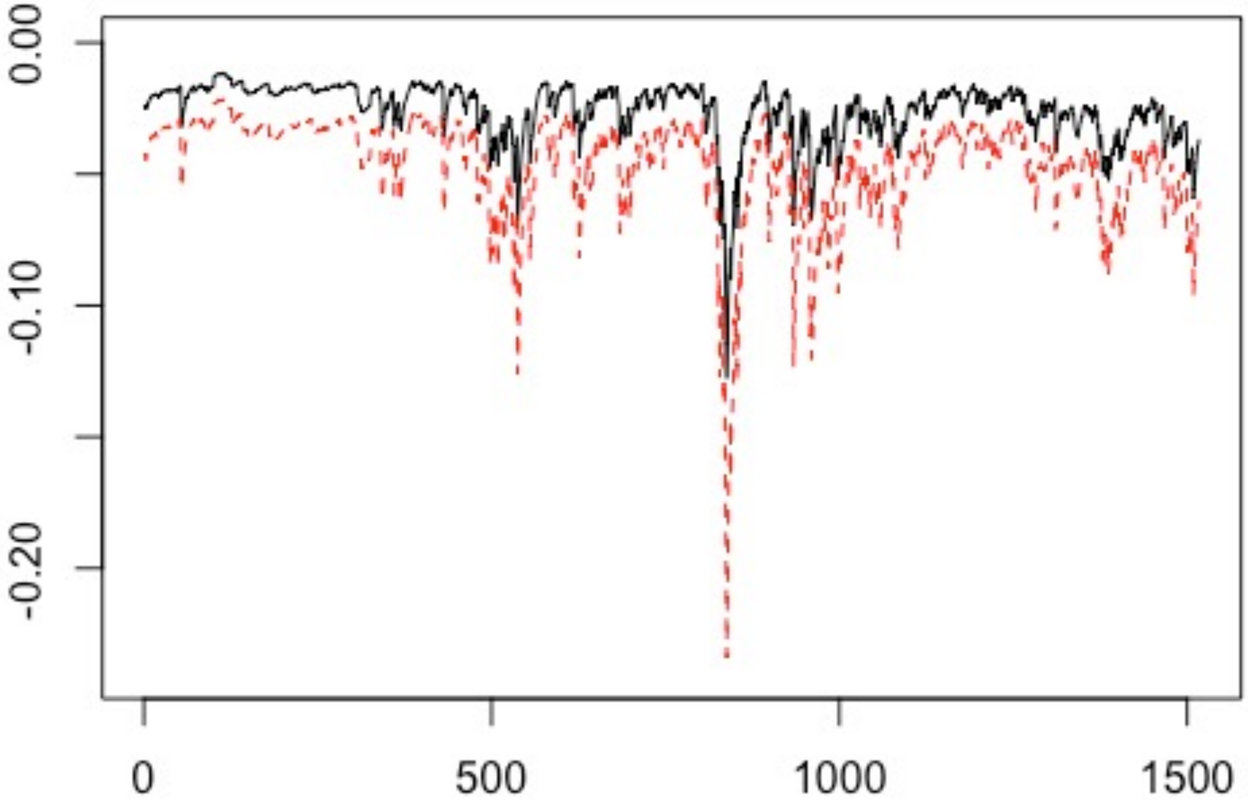
Out of sample GARCH/VaR forecasting

- ugarchroll: Rolling estimation and forecasting via GARCH family.

```r
rolling_est<- ugarchroll(garchSpec2,
            r.AAPL, n.start=1000,
            refit.every = 25,
            refit.window = "moving",
            VaR.alpha = c(0.01, 0.05))
Var95<-rolling_est@forecast$VaR[,2]
Var99<-rolling_est@forecast$VaR[,1]
plot(Var95,type='l',ylim=c(-0.24,0))
lines(Var99,lty='dashed',col='red')
```



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Errors of the model

```
prediction<-as.data.frame(rolling_est)
head(prediction)
#Prediction error for the mean
error_mean<-prediction$Realized-prediction$Mu
#Prediction error for the variance
error_var<-error_mean^2-prediction$Sigma^2
mean(error_mean^2)
mean(error_var^2)
```

	Mu	Sigma	Skew	Shape	Shape(GIG)	Realized
2016-11-15	0.000747833	0.01650391	0	4.214219	0	0.0131567844
2016-11-16	0.000747833	0.01623713	0	4.214219	0	0.0265334138
2016-11-17	0.000747833	0.01720848	0	4.214219	0	-0.0003638326
2016-11-18	0.000747833	0.01648021	0	4.214219	0	0.0009998037
2016-11-21	0.000747833	0.01584315	0	4.214219	0	0.0150597476
2016-11-22	0.000747833	0.01580330	0	4.214219	0	0.0006262995

```
> mean(error_mean^2)
[1] 0.00038391
> mean(error_var^2)
[1] 0.0003852829
```

Assignment 7

- Considere two assets (S&P500 and one stock of your preference) compute the 95-VaR & 99-VaR, and compare the out-of-sample accuracy for the following models
  - Constant and normal volatility.
  - GARCH (1,1) with normal distributional assumption for the underlying.
  - GARCH (1,1) with t-student distributional assumption for the underlying.
  - GARCH-GJR with t-student distributional assumption for the underlying.
  - E-GARCH with t-student distributional assumption for the underlying.

Note: For all the cases assume an ARIMA(0,0,0) model for the log-returns.