Annuities with focus on future values

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- The calculation can be focused on the **sum** of **present values**

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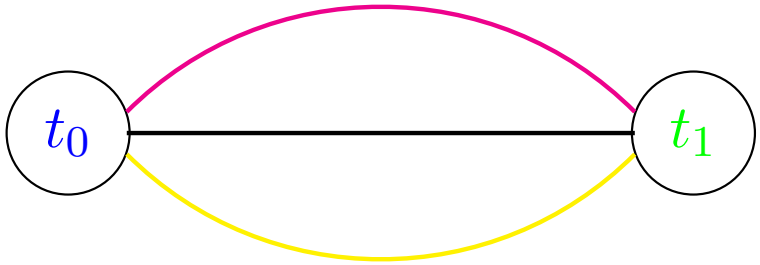

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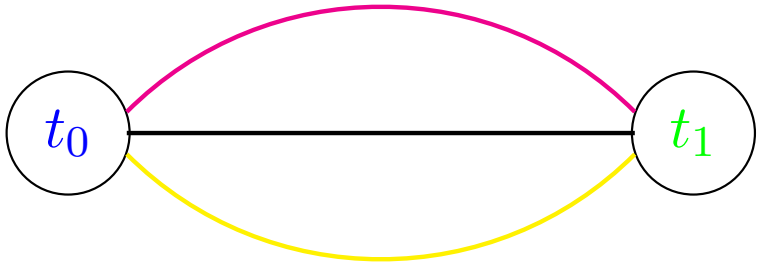

Note: The simplest case is if the interest and payment period are the same length. However, in practice the relationship may be different.

Graphic illustration of the annuity concept - FV´s

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Annuities over time:

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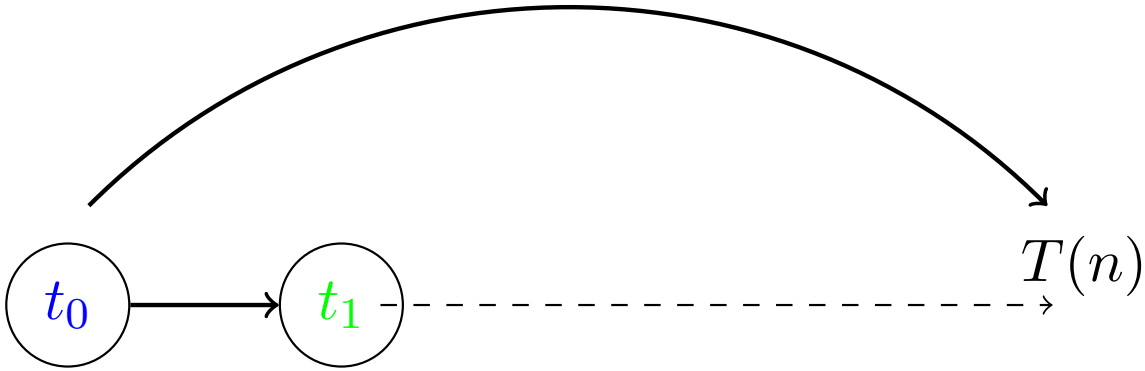

Sequence of annuities - FV´s

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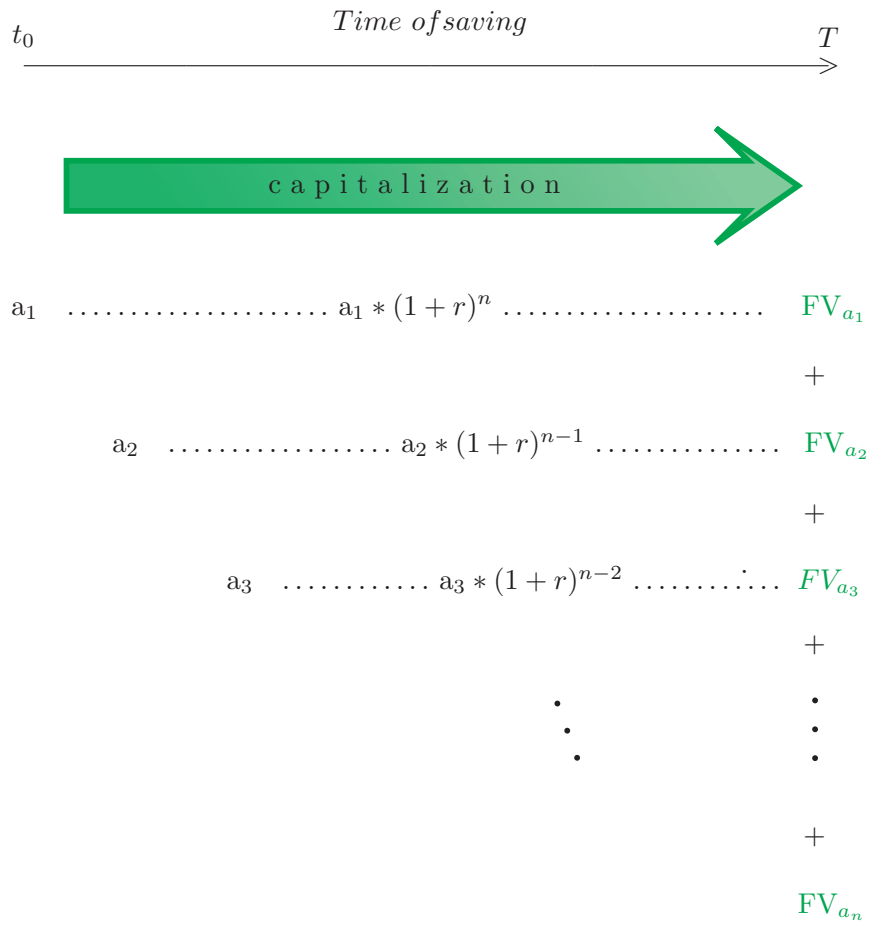

Saving: $S = \sum_{i=1}^{n} FV_i = FV_{a_1} + FV_{a_2} + \ldots + FV_{a_n}$

Saving plan

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Saving plan represents a practical use of the annuity calculation, where the aggregate sum of partial capitalized annuities is applied.

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Understanding the essence of Annuities

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G E O M E T R I C S E R I E S

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, and its properties . . .

Geometric serie

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In general:

a_1

$a_2 = a_1 * q$

$a_3 = a_2 * q$, **where** $q = \frac{a_n}{a_{n-1}}$, **and** $S_n = \frac{q^n - 1}{q - 1}$

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Note: A geometric serie can be **finite** *or* **infinite***.*

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Assuming $annuity = 1$ we get:

$$\frac{(1+r)^n}{(1+r)^{n-1}} = \frac{(1+r)^{n-1}}{(1+r)^{n-2}} = \cdots = (1+r) = q$$

Types of annuities

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The basic criterion in the division of annuities is the moment when the first annuity occurs. According to this concept, we distinguish between ordinary annuity and annuity-due.

Ordinary annuity - illustration

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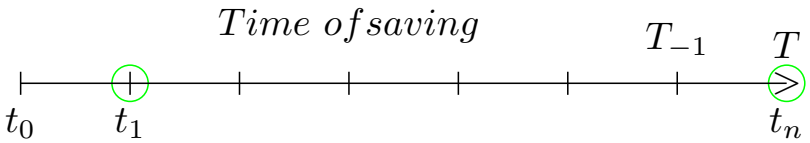

Annuity-ordinary (S^1):

$$S^1 = a * \frac{(1+r)^n - 1}{r}$$

Annuity-due

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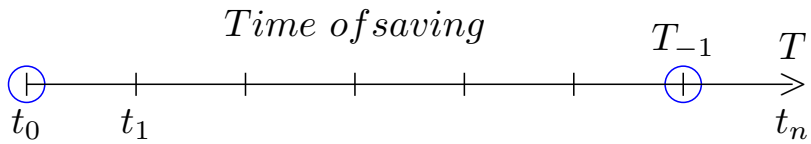

Annuity-due (S^0)**:**

$$S^0 = a * (1 + r) * \frac{(1 + r)^n - 1}{r}$$

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Saving plans

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Every parameter can be calculated in the annuity concept. Most often, it is the value of the amount saved. With the target amount *(the goal of the savings plan)*, it is possible to search for the amount of the annuity under the given conditions.

Or how long it is necessary to save, i.e. how many annuities must be saved. The most **complex** thing is to find out the **interest rate** when the annuity, the target amount and the savings period are known.

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Note: Every parameter except the interest rate can be easily expressed from the basic formula by algebraic modification.

Example 01 - Total amount of saving

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How much will be the **amount** on the saving account for a client that starts with saving in his age of **22** and will regularly save **until** his **50**. He will **save** regularly **750.00** at the **end** of each **month**. The bank offers an interest rate of **1.6% p.s.** for the entire savings period. The interest is calculated by every deposited annuity. **How much** would be saved if we used the **annuity-due** concept?

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$$S^1 = 666,059.2$$

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$$a = 2,045.56$$

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$$0.351409 + 1 = (1 + 0.019)^{2*n}$$

$$n = 8 \ \textit{years}$$

Literature:

GUTHRIE, Gary a Larry LEMON. *Mathematics of Interest Rates and Finance.* Pearson New International Edition, 2013. ISBN 978-1-292-03983-1.

DRAKE, P., FABOZZI, F.: Foundations and applications of the time value of money, John Wiley & Sons, 2009. ISBN 978-0-470-40736-3

DAHLQUIST, J., KNIGHT, R. *Principles of Finance.* OpenStax College, 2022.