Repayment Plan

The plan of how to repay a loan. For more detailed analysis is created so-called redemption table.
It is always true that the installment *annuity "a"* is divided into **two components:**

$$a = I + M$$

where:

I . . . paid interest
M . . . debt amortization

In practice, we will deal with two basic scenarios of repayment plan. First, the **annuity** will be **constant**. Second, the **amortization** will be **constant**.

Constant annuity:

The main task here will be to estimate the regular payment for the financial institution that lent us money. To do so we will use the already known concept of after paid pension. The logic is; first you will take a credit by a financial institution and after a specific time you provide the first installment. That means we will work with the formula of after paid pension (we just change the letter **B** for Budged to **D** for Debt:

$$D = a * \frac{1-\frac{1}{(1+r)^n}}{r}$$

$$a = \frac{D*r}{1-\frac{1}{(1+r)^n}}$$

here:

D . . . Initial debt
r . . . interest rate

Repayment table

	Annuity	Interest	Amortization	Debt
0				D_0
1	a	I_1	M_1	D_1
2	a	I_2	M_2	D_2
3	a	I_3	M_3	D_3
⋮	⋮	⋮	⋮	⋮
n	a	I_n	M_n	D_n

The column of amortization represents a **geometric series**. So, it allows to calculate any row of the amortization table and also it allows sum up how much of the initial debt was already amortized or how much was paid on the interest. And so:

$$\sum_{i=1}^{k} M_i = M_1 * \frac{q^k - 1}{q - 1}$$

quotient q corresponds $(1 + r)$.

and then

$$\sum_{i=1}^{k} I_i = k * a - \sum_{i=1}^{k} M_i$$

For the concrete row:

$$M_k = M_1 * q^{k-1}$$

so

$$I_k = a - M_k$$

or

$$I_k = D_{k-1} * r$$

and finally

$$D_k = \frac{I_{k+1}}{r}$$

Constant amortization:

Repayment table

	Annuity	Interest	Amortization	Debt
0				D_0
1	a_1	I_1	M	D_1
2	a_2	I_2	M	D_2
3	a_3	I_3	M	D_3
⋮	⋮	⋮	⋮	⋮
n	a_n	I_n	M	D_n

For the case of constant amortization all three remaining columns are arithmetic series. So here could be also applied the sum or the concrete n-th member. In other words it is possible to estimate how much it was paid on interest, total or how much is remaining debt. Or it could be calculated the concrete row throughout the table.

Just to remind:

$$S = \frac{n}{2} * (a_1 + a_n)$$

where

$$a_n = a_1 + (n-1) * d$$

Illustrative example

Assume a loan in the amount of 40,000.00 that have to be totally repaid in six years. The regular installment will be provided four times a year. The bank ask for interest in the amount of 4.3 % p.a.. The interest is calculated on the quarterly basis like the installments. To analyze the debt we create an amortization table for a constant annuity and for a constant amortization. See the excel sheets.

How much will be paid in total in interest only after four years?

Determine the value of the 18th installment.

The solution - a redemption table is provided in Interactive Syllabus.