

Regression Discontinuity Design

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Nature does not make jumps.

(Natura non facit saltus)

Some things do not occur naturally and requires **some** explanation.



Example: Campbell (1960)

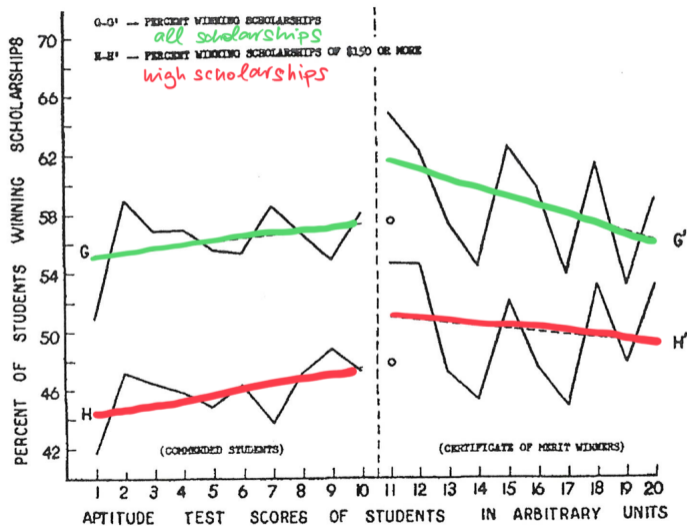


FIG. 2. Regression of success in winning scholarships on exposure determiner.

- "Certificate of merit" is given to students above some threshold in an aptitude test.
- High-scoring students went for "National merit scholarship" which is not included on the vertical axis \implies downward slope.

Students **right below** and **right above** the thresholds are similar.

It is as if they were assigned to **group below** and the **group above** randomly.

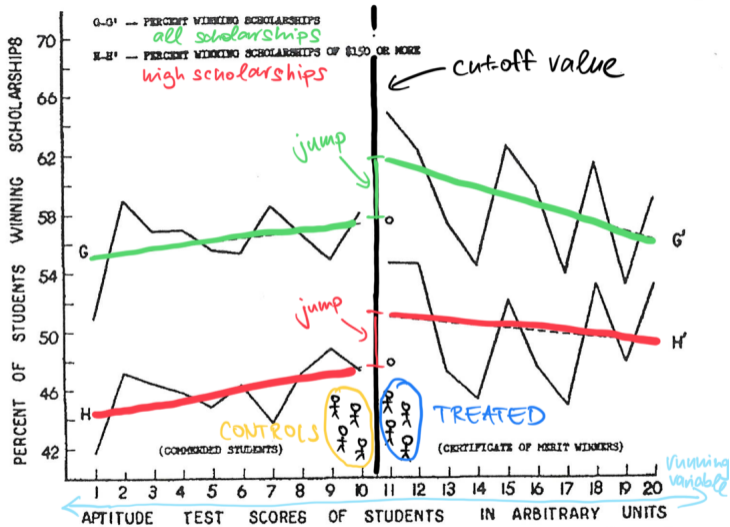
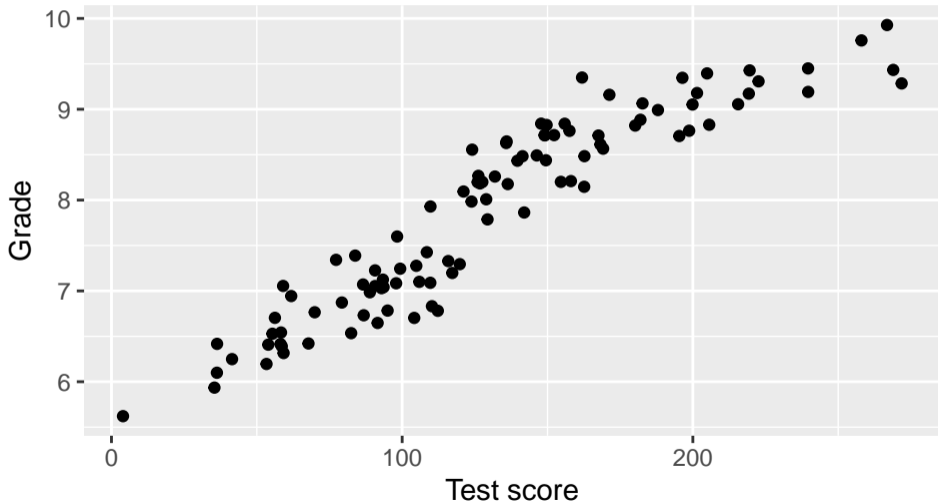
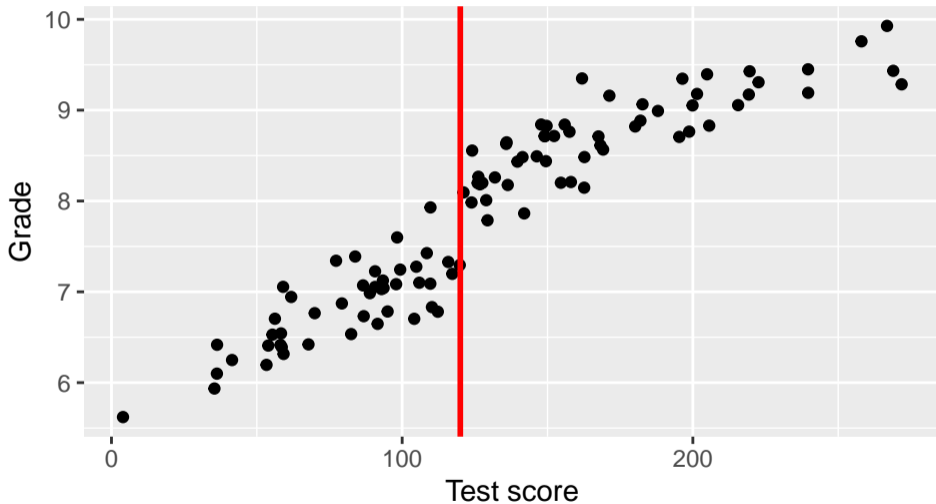


FIG. 2. Regression of success in winning scholarships on exposure determiner.

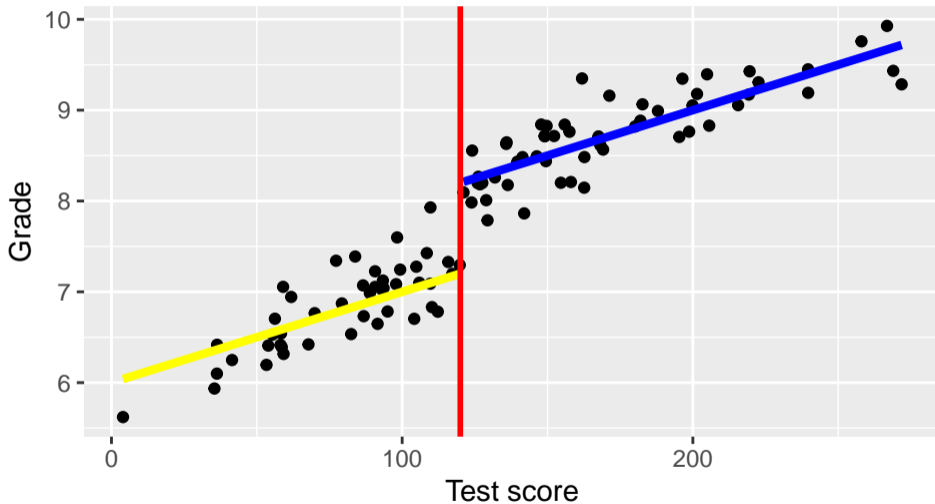
Regression Discontinuity Design Demonstration



Regression Discontinuity Design Demonstration



Regression Discontinuity Design Demonstration



Sharp RDD

$$\hat{\delta} = \underbrace{\lim_{z \downarrow 0} \hat{E}[Y|Z = z]}_{\text{treated}} - \underbrace{\lim_{z \uparrow 0} \hat{E}[Y|Z = z]}_{\text{control}}$$

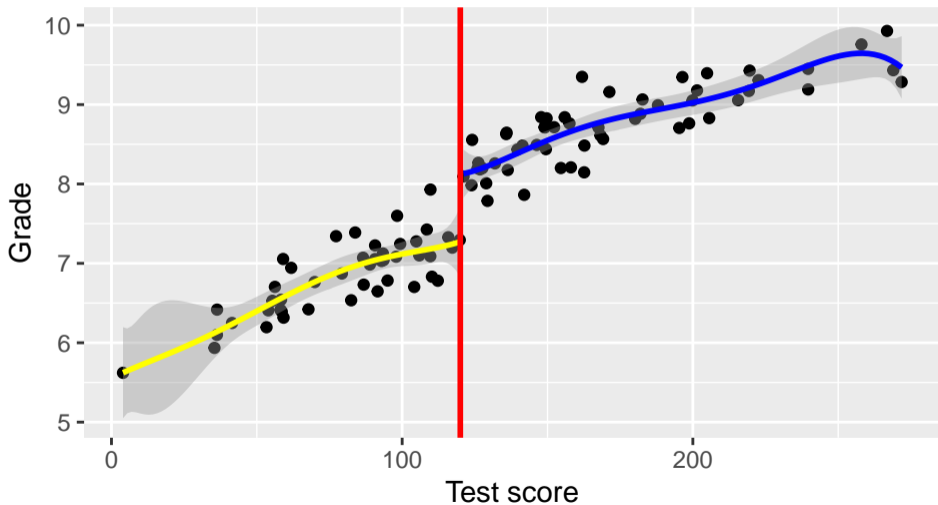
- Random variation is coming from the real world constraints/rules
- Treatment $D = 1$ or $D = 0$ if defined in terms of **running variable** Z :
 $D = 1 \iff Z > 0$

Sharp RDD

- In the previous example we used linear model.
- Yet in practice we have only seldom reasons to believe that such model is correct
- We wish to have a sufficiently flexible model.

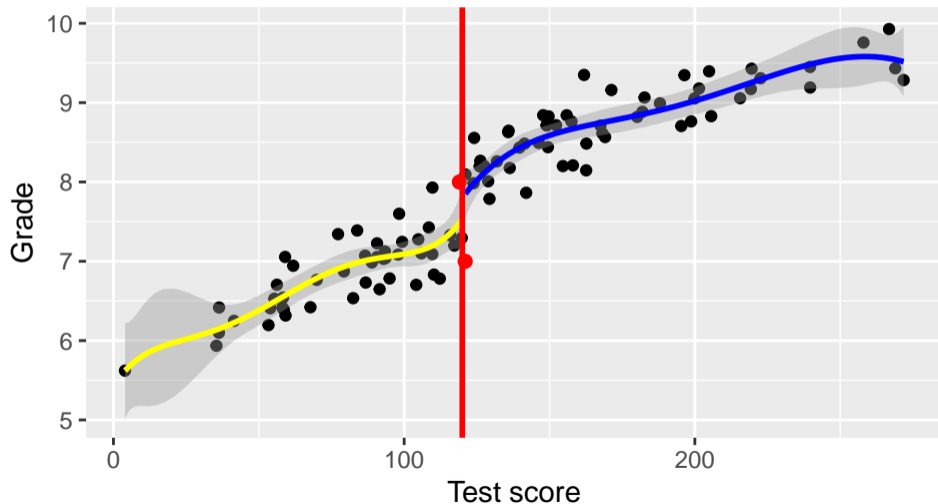
Too flexible?

RDD Demonstration (5th degree polynomial)



Sensitive? (add 2 points)

RDD Demonstration (5th degree polynomial) + 2 obs



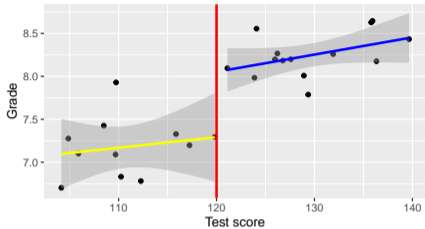
Sharp RDD

- It is the **threshold** that is important
- This is where all the action takes place
- You have to defend that there is no manipulation around the threshold

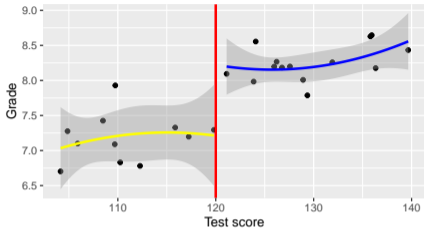
It's all about the threshold



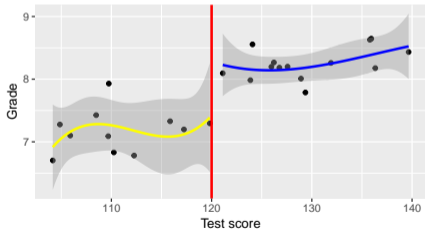
RDD Demonstration (linear)



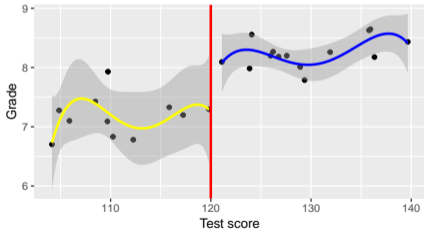
RDD Demonstration (quadratic)



RDD Demonstration (cubic)



RDD Demonstration (quartic)



Sharp RDD

- But how do we choose the model for $\hat{E}[Y|Z = z]$ on **the left** and on **the right**?
- Clearly, there are many many ways how we can do this!
- Gelman and Imbens (2019) warns against the higher order polynomials. Obviously: you can cook up the results according to your liking.
- And that is always a bad thing.
- Just look at the title: **"Why high-order polynomials should not be used in regression discontinuity designs."**

Gelman and Imbens (2019)

- Issue 1: Some observations are given excessive weights → Check the weights.
- Issue 2: Results are sensitive to the degree of polynomial (see below) → Use local regression.
- Issue 3: Confidence intervals are too narrow. → Global regressions are simply not precise enough **at the cut-off point**, use local regression instead.

	Order of polynomial	DATA 1 estimate (se)	Order of polynomial	DATA 2 estimate (se)
global	1	0.167 (0.008)	global	0.024 (0.008)
global	2	0.079 (0.010)	global	0.176 (0.012)
global	3	0.112 (0.011)	global	0.209 (0.015)
global	4	0.077 (0.013)	global	0.174 (0.018)
global	5	0.069 (0.016)	global	0.164 (0.021)
global	6	0.104 (0.018)	global	0.197 (0.025)
local	1	0.080 (0.012)	local	0.196 (0.018)
local	2	0.063 (0.017)	local	0.176 (0.027)

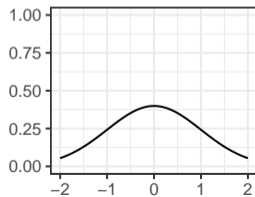
Local linear regression (loess)

$$\left(\hat{\beta}_0(x), \hat{\beta}_1(x)\right) = \arg \min_{\beta_0, \beta_1} \underbrace{\sum_{i=1}^n}_{\text{Sum of}} \underbrace{K\left(\frac{x_i - x}{h}\right)}_{\text{weighted}} \cdot \underbrace{\left(y_i - (\beta_0 + \beta_1 x_i)\right)^2}_{\text{squared errors}}$$

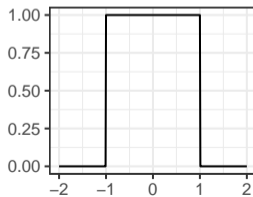
$$\hat{E}[Y|X = x] = \hat{\beta}_0(x) + \hat{\beta}_1(x) \cdot x$$

Kernel functions: 

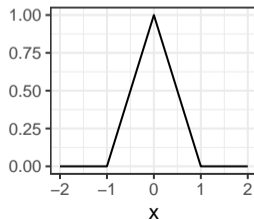
Gaussian



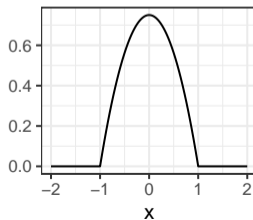
Uniform



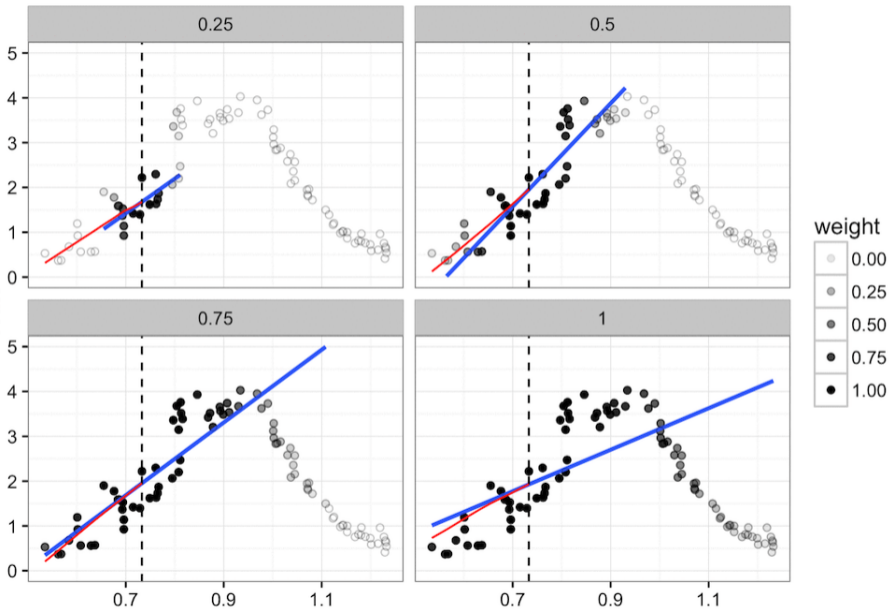
Triangular

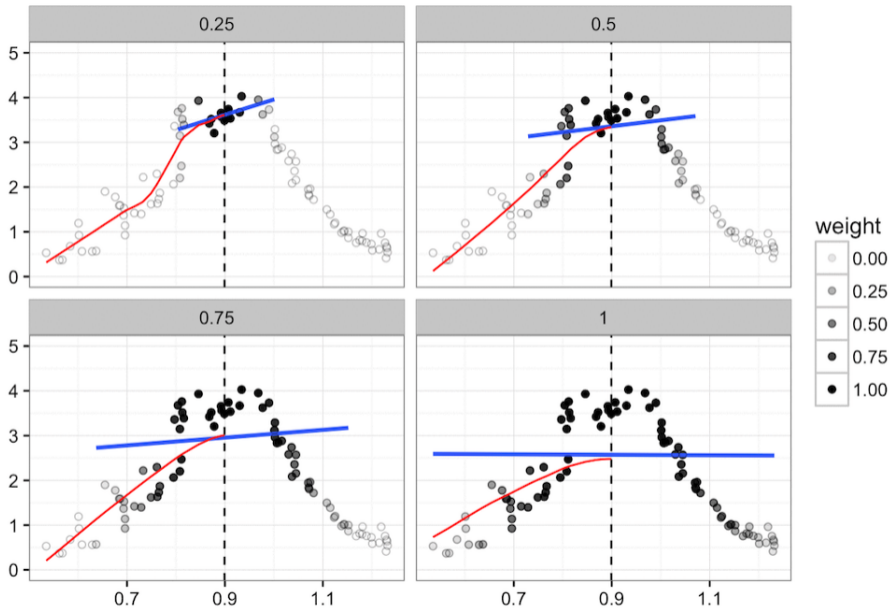


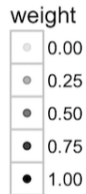
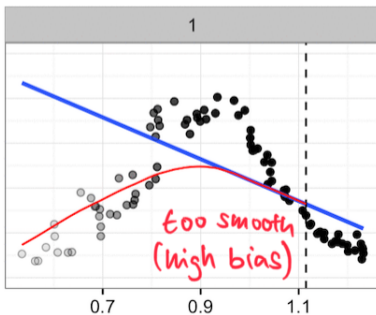
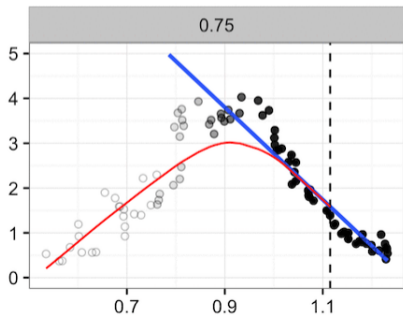
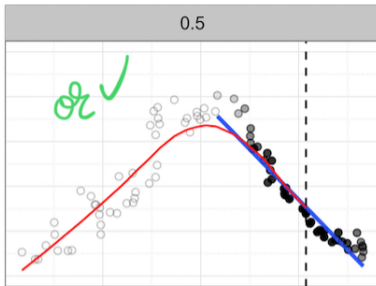
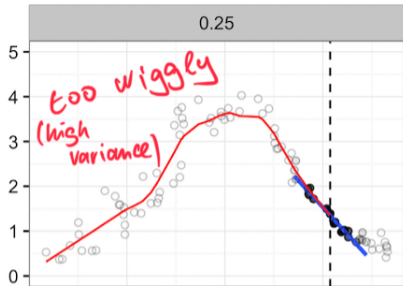
Epanechnikov



Bandwidth h : bias vs variance tradeoff.

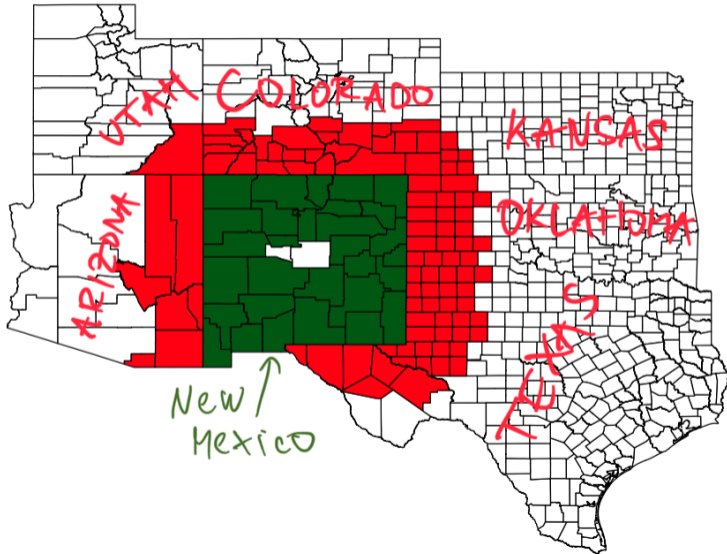






- In practice, the choice of the bandwidth is more important than the choice of the **kernel** (weighting function)
- Local linear regression methods (and other semi/non-parametrics methods) have problems at the boundary, fewer points \rightarrow more bias. There are different ways (fixes) how to deal with this (Noack and Rothe, 2021).
- If the running variable is discrete, one should account for it (Kolesar and Rothe, 2018)

Example: Mask mandates

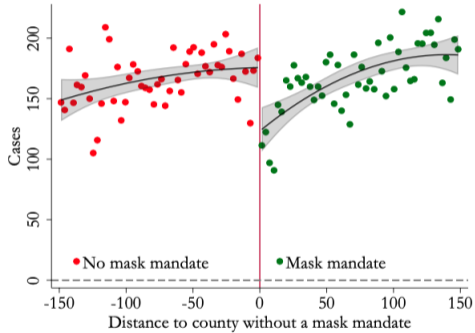


$$y_{c,s,t+k} = \delta mask_{st} + \beta dist_{cst} + \gamma dist_{cst}^2 + \theta X_{c,s,t} + \tau_t + \tau_c + \epsilon_{cst}$$

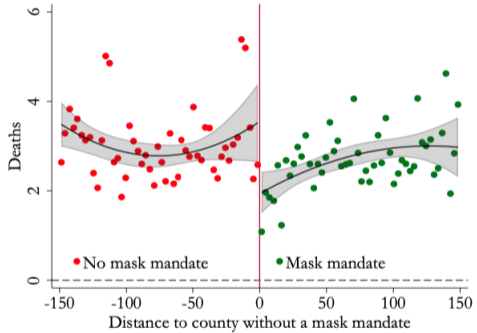
- c - county
- s - state
- t - week
- k lag ($k = 1$ for cases, $k = 2$ for hospital admissions, $k = 4$ for deaths)
- $mask_{st}$ - mask mandate in state s at time t
- $dist_{cst}$ - minimum distance of country c is state s to a county with a different mask policy at time t
- $X_{c,s,t}$ - controls using lagged mobility

Discontinuity?

Figure 3: New weekly COVID-19 cases and deaths per 100,000 inhabitants



(a) Cases



(b) Deaths

Results

Table 1: Results per 100,000 inhabitants

(a) Unconditional results

	(1) Cases	(2) Admissions	(3) Deaths
State mask mandate	-55.22*** [17.68]	-11.46*** [4.24]	-0.74** [0.34]
Observations	45577	22042	41034
R2	0.487	0.417	0.215
Mean of dep. variable	166.44	23.57	2.64
Linear term	Yes	Yes	Yes
Quadratic term	Yes	Yes	Yes
Lagged mobility	Yes	Yes	Yes

Fuzzy RDD

- Random variation is coming from the real world constraints/rules
- Treatment probability $Pr(D = 1|Z = z)$ here discontinuously changes at the cut-off value $Z = 0$ (it is just an IV!!!)
- Very specific subpopulation - those marginal people **at the cut-off** (compliers)
- The question is: How do we extrapolate??

Fuzzy RDD as an IV

Wald estimator:

$$\hat{\beta}_{Wald} = \frac{\hat{E}[Y|Z = 1] - \hat{E}[Y|Z = 0]}{\hat{E}[D|Z = 1] - \hat{E}[D|Z = 0]}$$

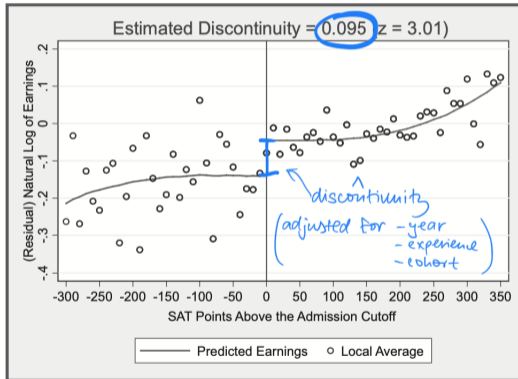
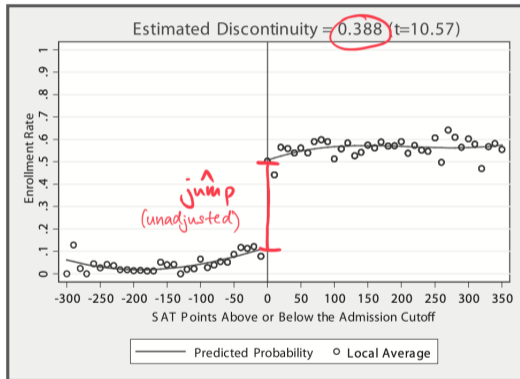
Or

$$\hat{\beta}_{IV} = (\mathbf{X}^T P_Z \mathbf{X})^{-1} \mathbf{X}^T P_Z y$$

where $\mathbf{X} = [\mathbf{1}, X, D]$ $\mathbf{Z} = [\mathbf{1}, X, Z]$ $y_i = \mathbf{X}_i \beta + e_i$

Example: Hoekstra (2009)

$$\log(\text{earnings}) = \psi_{\text{year}} + \phi_{\text{experience}} + \theta_{\text{cohort}} + \varepsilon$$



We have exogenous variation in the prob. of enrollment rate. What is the effect on earnings 15 years later?

How to quantify the effect?

We wish to estimate this:

$$\text{Outcome} = \beta_0 + \beta_1(\text{Above}) + \gamma_1 \left(\overbrace{h(\text{Adjusted SAT score})}^{\text{Some flexible function}} \right) + \gamma_2 \text{GPA} + \gamma_3(\text{SAT Score}) + \varepsilon$$

Cut-off set to 0.

But taking into account variation in year of admission, experience and cohort. Hoekstra did it in two steps:

- Step 1** For every individual, regress outcome and independent variables on $\psi_{\text{year}}, \phi_{\text{experience}}, \theta_{\text{cohort}}$ and obtain residuals. Average these residuals across different years for every single individual.
- Step 2** Plug in residuals from these regressions in this equation and estimate it via OLS.
- Step 3** Scale the estimate of coefficient of interest $\hat{\beta}_1$ by the size of the discontinuous jump.

Two stage estimator. Standard errors? Bootstrap.

TABLE 1.—EARNINGS DISCONTINUITIES AND CORRESPONDING INTENT-TO-TREAT AND ENROLLMENT ESTIMATES FOR WHITE MEN

Regression Specification	Function of Adjusted SAT	Flexible Polynomial?	Additional Controls	Discontinuity	Treatment Effect	
				Estimated Earnings Discontinuity	Intent-to-Treat Effect	Enrollment Effect
(1) Plotted in Figure 2	Cubic	No	No	0.095*** (0.032) [0.003]	0.135*** (0.046) [0.004]	0.223*** (0.079) [0.005]
(2)	Cubic	No	Yes	0.092*** (0.033) [0.005]	0.131*** (0.048) [0.006]	0.216*** (0.081) [0.008]
(3)	Quadratic	Yes	Yes	0.111** (0.045) [0.014]	0.170** (0.073) [0.019]	0.281** (0.121) [0.021]
(4) (includes only applicants within 200 points of cutoff)	Quadratic	No	Yes	0.081** (0.038) [0.034]	0.116** (0.056) [0.038]	0.192** (0.094) [0.041]
(5) (includes only applicants within 100 points of cutoff)	Linear	No	Yes	0.074** (0.038) [0.050]	0.110* (0.058) [0.060]	0.181* (0.099) [0.067]

Three well-known papers

- Angrist and Lavy (1999) - Maimonides rule - what is the effect of classroom size on academic achievements. Classrooms cannot have more than 40 students.
- Black (1999) - used impact of school districts on the house prices. Houses at the district boundaries, similar but differ in terms of elementary school that the child attends. "...parents are willing to pay 2.5 percent more for a 5 percent increase in test scores"
- Van Der Klaauw (2002) - effect of financial aid on student enrollments. Exploiting the discontinuity in financial aid eligibility in student's test performance.

RDD



The image shows a Google Scholar search interface. The search bar contains the text "regression discontinuity design". Below the search bar, the results are displayed as "Articles" with "About 26,300 results (0.06 sec)". The number "26,300" is circled in red, and a red arrow points from the circle to three red exclamation marks "!!!".

- Only popular since 2000s.
- Visually very appealing. The details matters. Work hard on compelling figures, they do matter a lot.
- You should not see a jump in the density of a running variable around the threshold - it may suggest some manipulation.
- There seems to be many choices to consider: estimation, bandwidth, kernel. Stick to defaults unless you have very good reasons to not to.
- There are many many situations with cut-offs.

RDD

- Data hungry - we need many points around the threshold to provide credible evidence on the jump.
- Cunningham (2021) suggests you to work on relationships with people who have access to this kind of data. These are built on trust and honesty.
- Korting et al. (2020) - people are quite good at spotting the discontinuities - they conducted some tests.

Thank you for your attention!

References

- Thistlethwaite, D. L., & Campbell, D. T. (1960). Regression-discontinuity analysis: An alternative to the ex post facto experiment. *Journal of Educational Psychology*, 51(6), 309–317. <https://doi.org/10.1037/h0044319>
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- Gelman, Andrew, and Guido Imbens. "Why high-order polynomials should not be used in regression discontinuity designs." *Journal of Business & Economic Statistics* 37.3 (2019): 447-456.
- Niels-Jakob H Hansen ; Rui C. Mano: "Mask Mandates Save Lives." *IMF Working Papers* 2021.205 (2021).
- Noack, Claudia, and Christoph Rothe. *Bias-Aware Inference in Fuzzy Regression Discontinuity Designs*. arXiv. org, 2021.
- Kolesár, Michal, and Christoph Rothe. "Inference in regression discontinuity designs with a discrete running variable." *American Economic Review* 108.8 (2018): 2277-2304.
- Korting, Christina, et al. "Visual Inference and Graphical Representation in Regression Discontinuity Designs." (2020).
- The chapter in Cunningham, Scott. *Causal Inference*. Yale University Press, 2021 is very engaging and fun to read.