Budget constraint, preferences, utility

Varian, Intermediate Microeconomics, 8e, chapters 2, 3, and 4

In this lecture, you will learn

- what budget set and budget line are
- how their shape is influenced by taxes and food stamps
- what preferences are and how they are derived
- what the basic types of preferences are – why some indiference curves are straight and some curved, or circle-shaped
- what we need a utility function for
- how to find out whether to reconstruct a stadium

Budget constraint

We assume that the consumer chooses a bundle (x_1, x_2), where x_1 and x_2 are quantities of goods 1 and 2.

Budget constraint is $p_1 x_1 + p_2 x_2 \leq m$:

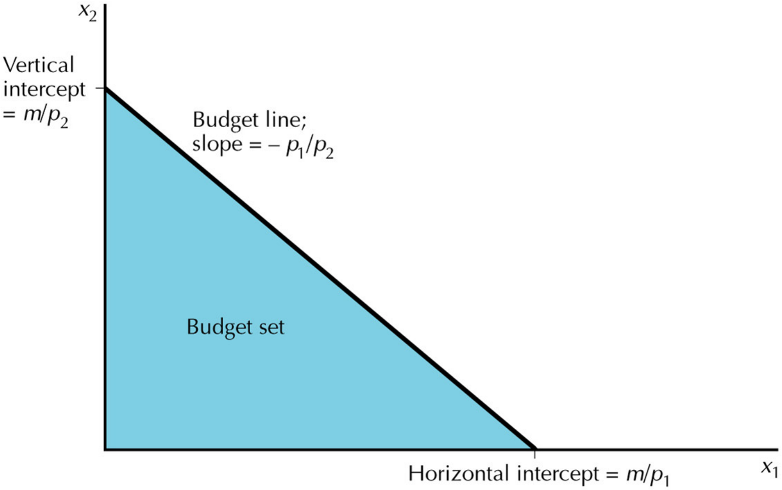
- p_1 and p_2 are prices of goods 1 and 2
- m is income

Budget set – bundles for which: $p_1 x_1 + p_2 x_2 \leq m$.

Budget line (BL) – bundles for which: $p_1 x_1 + p_2 x_2 = m$.

Budget set and budget line (graph)

Budget line: $p_1 x_1 + p_2 x_2 = m \iff x_2 = m/p_2 - (p_1/p_2)x_1$



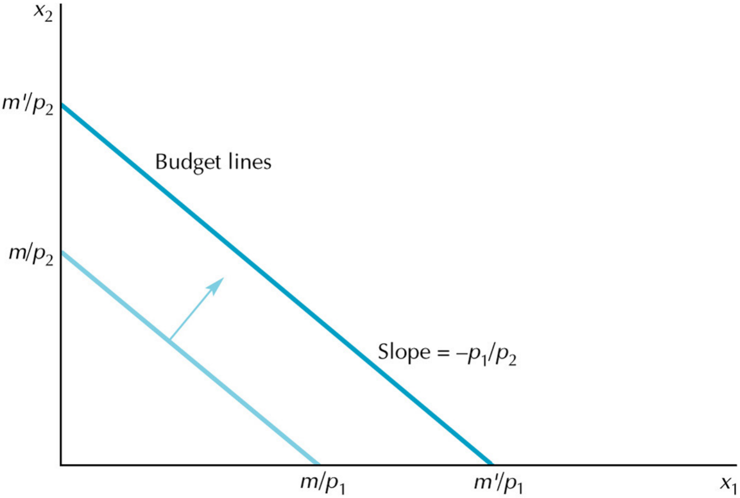
Composite good

The theory works for more than two goods.
How to plot it in a 2D graph?

On the y axis we can plot the **composite good**
$=$ money value of all other consumed goods.

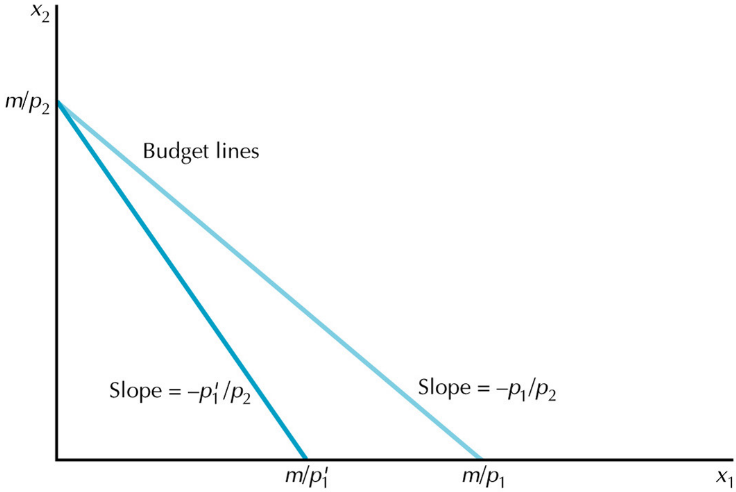
Change in income

A rise in income from m to m' $\Longrightarrow$ parallel shift out



Change in price

A rise in price from p_1 to p_1' $\implies$ pivot around the vertical intercept



Change in more variables

Multiplying all prices and income by t does not change BL:

$$tp_1x_1 + tp_2x_2 = tm \iff p_1x_1 + p_2x_2 = m$$

Multiplying all prices by t has the same effect as dividing income by t:

$$tp_1x_1 + tp_2x_2 = m \iff p_1x_1 + p_2x_2 = \frac{m}{t}$$

Numeraire

Any price or income can be normalized to 1 and adjust all variables so that the BL stays the same.

Numeraire = an item with its value normalized to 1

Budget line $p_1x_1 + p_2x_2 = m$:

- Good 1 is numeraire – the same BL:
$$x_1 + \frac{p_2}{p_1}x_2 = \frac{m}{p_1}$$
- Good 2 is numeraire – the same BL:
$$\frac{p_1}{p_2}x_1 + x_2 = \frac{m}{p_2}$$
- The income is numeraire – the same BL:
$$\frac{p_1}{m}x_1 + \frac{p_2}{m}x_2 = 1$$

Taxes and subsidies

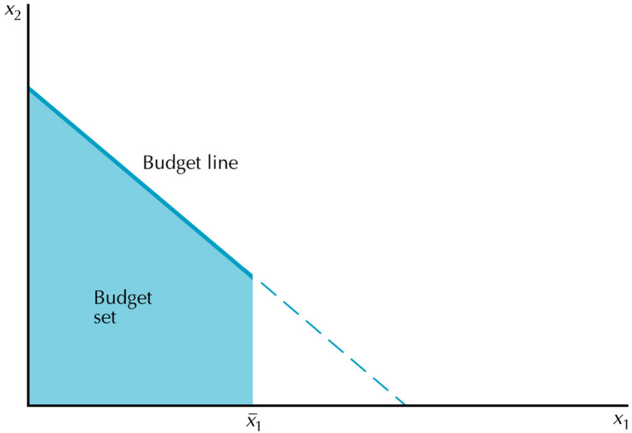
Three types of taxes:

- **Quantity tax** – consumer pays the amount t for each unit.
  $\rightarrow$ Price of good 1 increases to $p_1 + t$.
- **Value tax (ad valorem)** – consumer pays a share τ of price.
  $\rightarrow$ Price of good 1 increases to $p_1 + \tau p_1 = (1 + \tau)p_1$.
- **Lump-sum tax** – the value of the tax is independent from consumer's choice.
  $\rightarrow$ Consumer income decreases by the size of the tax.

Subsidy = a tax with a negative sign

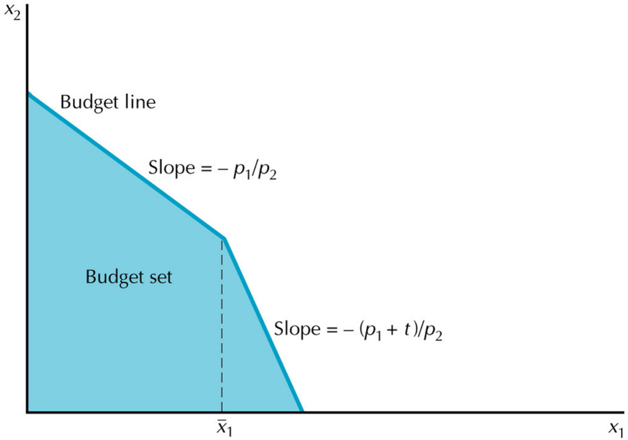
Rationing

If there is rationing imposed on good 1, no consumer is allowed to buy a higher quantity of good 1 than $\bar{x}_1$.



Taxing consumption greater than $\bar{x}_1$

If consumer pays a tax only on the consumption of good 1 that is in excess of $\bar{x}_1$..., budget line is steeper to the right of $\bar{x}_1$.

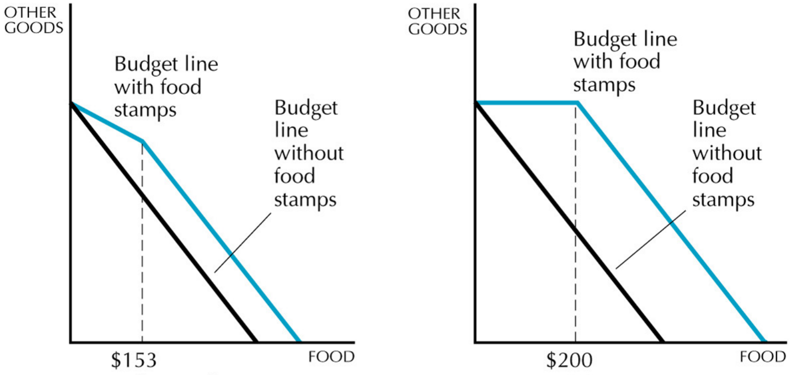


CASE: The food stamp program

Before 1979 (left graph):

- value subsidy – people pay a part of the value of the food stamp
- rationing – maximum value of stamps (e.g. 153 $)

After 1979 (right graph) – a specific number of food stamps for free



Preferences

Consumers compare bundles according to their preferences.

Preference relations – three symbols:

- bundle X is **strictly preferred** to bundle Y:

$$(x_1, x_2) \succ (y_1, y_2)$$

- bundle X is **weakly preferred** to bundle Y (bundle X is at least as good as bundle Y):

$$(x_1, x_2) \succeq (y_1, y_2)$$

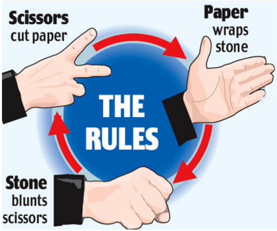
- consumer is **indiferent** between bundles X and Y:

$$(x_1, x_2) \sim (y_1, y_2)$$

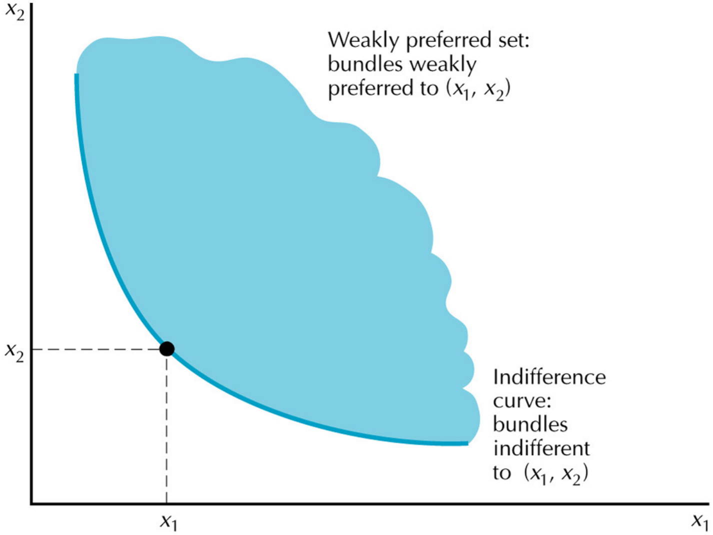
Assumptions about preferences

Assumptions that allow ordering of bundles according to preferences:

- **Completeness** — any two bundles can be compared: $(x_1, x_2) \succeq (y_1, y_2)$, or $(x_1, x_2) \preceq (y_1, y_2)$, or both
- **Reflexivity** — each bundle is at least as good itself: $(x_1, x_2) \succeq (x_1, x_2)$
- **Transitivity** — if $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (z_1, z_2)$, then $(x_1, x_2) \succeq (z_1, z_2)$

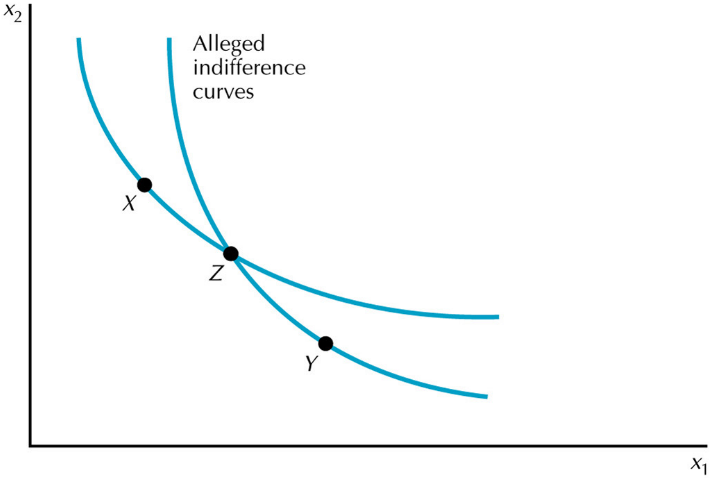


Weakly preferred set and indifference curves



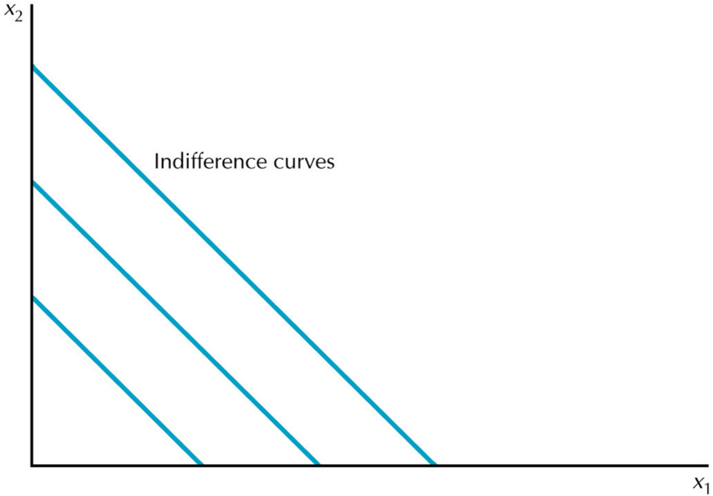
Two indifference curves cannot cross

Two different IC such that $X \succ Y$. Why cannot they cross? It follows from transitivity that if $X \sim Z$ and $Z \sim Y$ then $X \sim Y$.



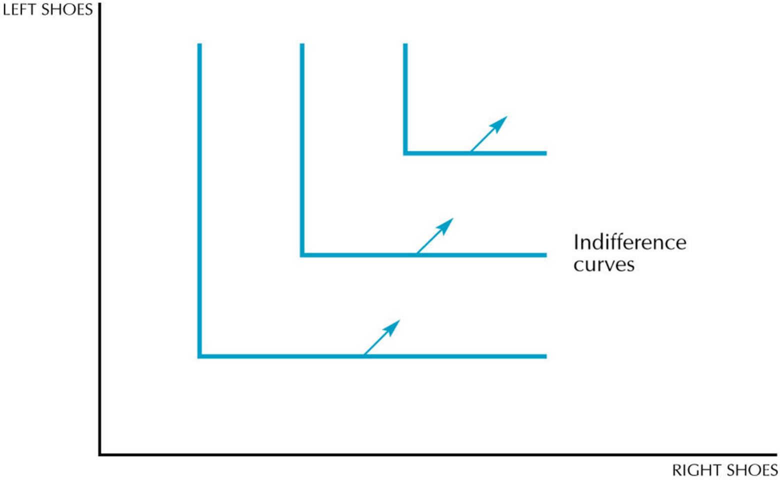
Examples of preferences – perfect substitutes

Willingness to substitute one good for the other at a constant rate $\Longrightarrow$ constant slope of the indifference curve (not necessarily -1).



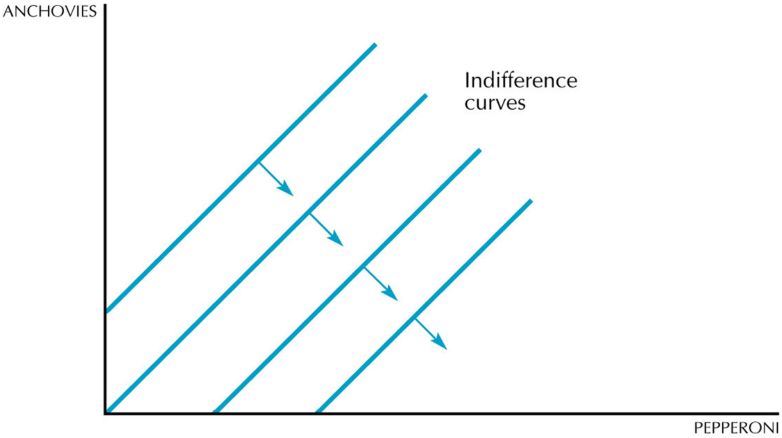
Examples of preferences – perfect complements

Consumption in fixed proportions (not necessarily 1:1).



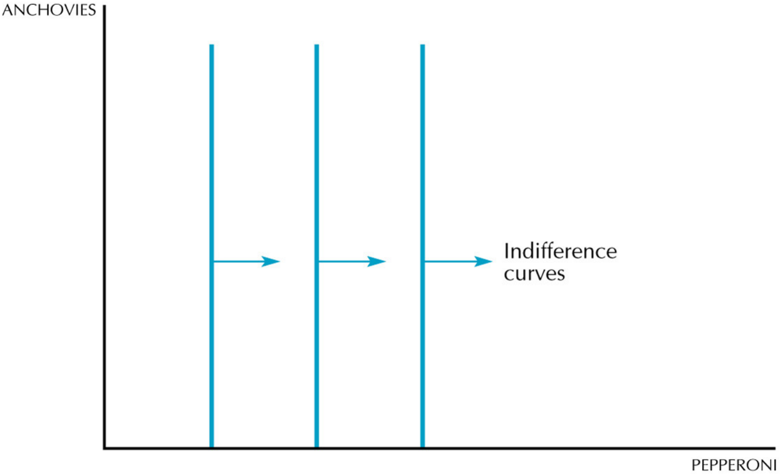
Examples of preferences – bads

The consumer likes pepperoni but does not like anchovies, they are a **bad** for her.



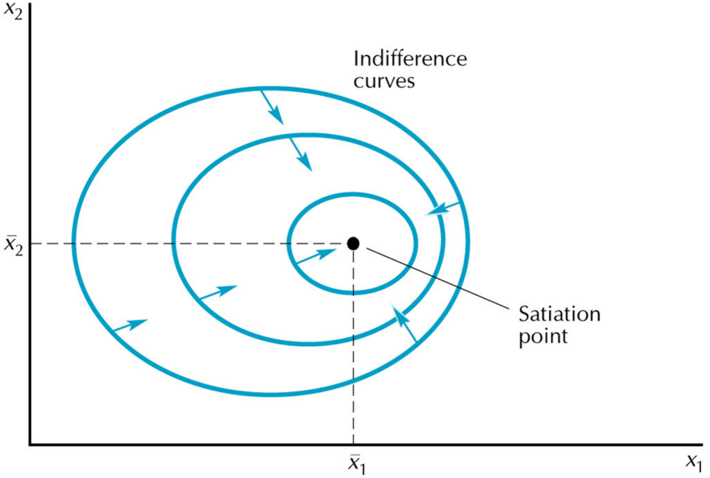
Examples of preferences – neutrals

The consumer likes pepperoni but is neutral about anchovies, they are a **neutral** for her.



Examples of preferences – satiation point

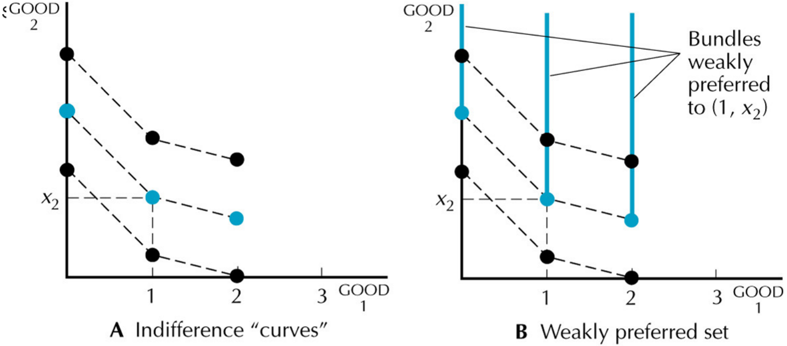
Satiation point is the most preferred point $(\bar{x}_1, \bar{x}_2)$. When the consumer has too much of one of the goods, it becomes a bad.



Examples of preferences – discrete goods

A **discrete good** is not divisible – consumption in integer amounts:

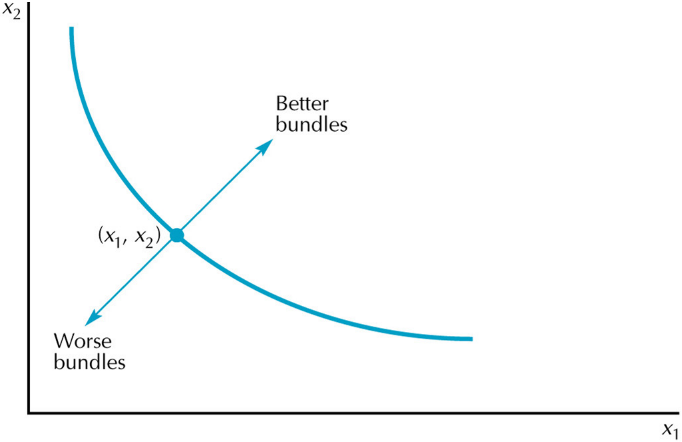
- indiference „curves“ – a set of discrete points
- a weakly preferred set – a set of line segments



Well-behaved preferences

Assumptions of well-behaved preferences: monotonicity and convexity

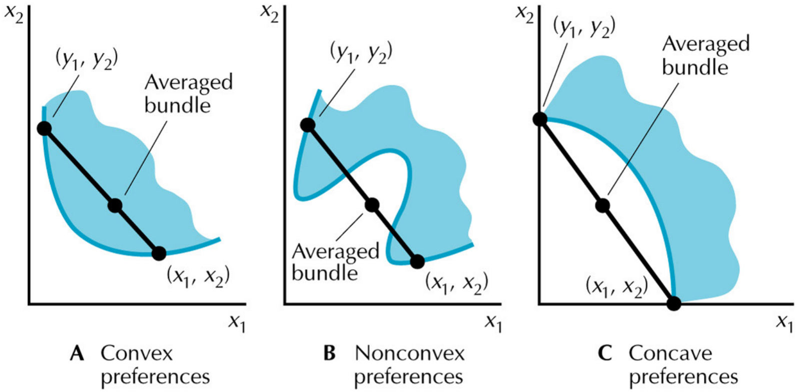
Monotonicity – more is better (it excludes bads)
$\implies$ indifference curves have negative slope.



Well-behaved preferences (cont'd)

Convexity – if $(x_1, x_2) \sim (y_1, y_2)$, then it holds for all $0 \leq t \leq 1$ that $(tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succeq (x_1, x_2)$.

Strict convexity – if $(x_1, x_2) \sim (y_1, y_2)$, then it holds for all $0 \leq t \leq 1$ that $(tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succ (x_1, x_2)$.

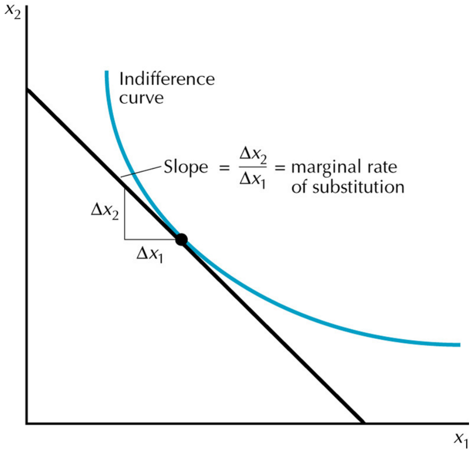


Marginal rate of substitution

Marginal rate of substitution (MRS) $=$ slope of the indifference curve:

$$\text{MRS} = \frac{\Delta x_2}{\Delta x_1} = \frac{dx_2}{dx_1}$$

Diminishing marginal rate of substitution – absolute value of MRS decreases as we increase x_1.



Interpretation of marginal rate of substitution

Interpretation of MRS:

- The amount of good 2 one is willing to pay for one unit of good 1.
- If good 2 is measured in money: MRS = **marginal willingness to pay** = how many dollars you would just be willing to give up for an additional unit of good 1.

APPLICATION: Build a stadium for Minnesota Vikings?

The club does not like the stadium – considers leaving Minnesota.

Fenn a Crooker (SEJ, 2009) measure how much households are willing to pay for Vikings staying in Minnesota = MRS between composite good and Vikings in Minnesota.

MRS of an average household: 531 \$
Value of the stadium: 531 \$ $\times$ 1,323 million households = 702 mil. \$

Estimated costs are 1 billion \$.

The new stadium opens in 2016
– the state provided 500 million \$.

Utility

Two concepts of utility:

Cardinal utility – attach a significance to the magnitude of utility:

- difficult to assign the magnitude
- not needed to describe choice behavior

Ordinal utility – important is only the order of preference:

- easy to set the utility – 1 rule: preferred bundle has a higher utility
- we can derive a complete theory of demand

We will use the ordinal utility.

Ordinal utility

Utility function is a way of assigning a number to every possible consumption bundle such that more-preferred bundles get assigned larger numbers than less-preferred bundles.

If $(x_1, x_2) \succ (y_1, y_2)$, then $u(x_1, x_2) > u(y_1, y_2)$.

Different ways to assign utilities that describe the same preferences:

Bundle	U_1	U_2	U_3
A	3	17	–1
B	2	10	–2
C	1	.002	–3

Monotonic transformation

Positive monotonic transformation $f(u)$ = any increasing function of u. Describes the same preferences as the original utility function u.

Examples of the function $f(u)$: $f(u) = 3u$, $f(u) = u + 3$, $f(u) = u^3$

Example:

Two bundles X and Y, preferences: $X \succ Y$

We assign utility so that $u(X) > u(Y)$, e.g. $u(X) = 1$, $u(Y) = -1$

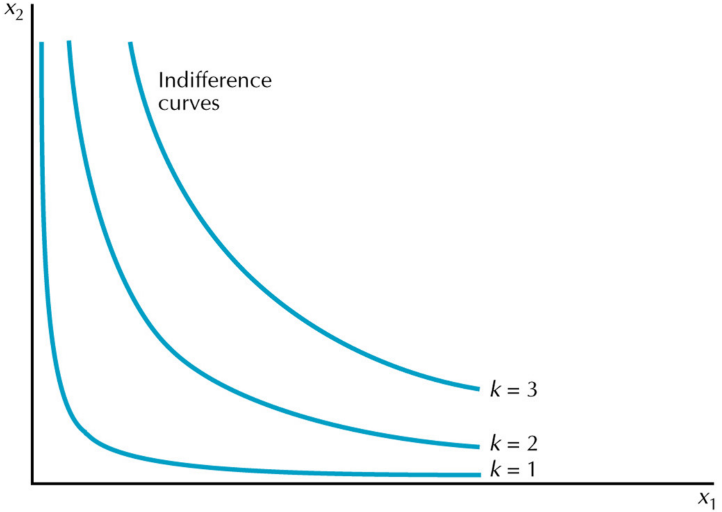
Do monotonic transformations $f_1(u) = 3u$ a $f_2(u) = u + 3$ represent the same preferences as the original utility function u?

Yes:

- $f_1(u) = 3u$: $\quad f_1(u(X)) = 3 > -3 = f_1(u(Y))$
- $f_2(u) = u + 3$: $\quad f_2(u(X)) = 4 > 2 = f_2(u(Y))$

Construction of indifference curves from utility function

Utility function $u(x_1, x_2) = x_1 x_2 \implies$ indifference curves $x_2 = \frac{k}{x_1}$



PROBLEM: The slope of indifference curves

The slope of indifference curves for two utility functions:

1. What is the slope of IC $x_2 = 4/x_1$ v point $(x_1, x_2) = (2, 2)$?

$$\text{Slope of indifference curves} = \text{MRS} = \frac{dx_2}{dx_1} = \frac{-4}{x_1^2} = -1$$

2. What is the slope of IC $x_2 = 10 - 6\sqrt{x_1}$ v point $(4, 5)$?

$$\text{Slope of indifference curves} = \text{MRS} = \frac{dx_2}{dx_1} = \frac{-3}{\sqrt{x_1}} = \frac{-3}{2}$$

Examples of utility functions – perfect substitutes

The consumer is willing to exchange

- coke and pepsi at a ratio 1:1

  important is the total number: e.g. $u(K, P) = K + P$
- 2 buns for 1 baguette

  baguette has a double weight: e.g. $u(R, H) = R + 2H$

Examples of utility functions – perfect complements

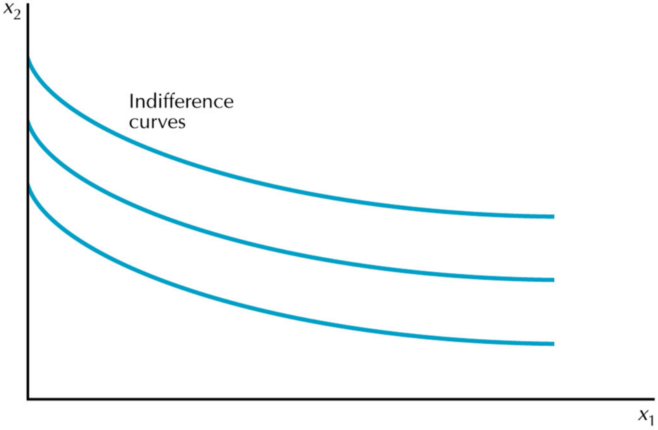
The consumer demands

- left and right shoes at a fixed ratio 1:1
  lower quantity matters: e.g. $u(L, P) = \min\{L, P\}$
- rum and coke at a fixed ratio 1:5
  goal: same numbers in the bracket – we need only 1/5 of coke:
  e.g. $u(R, K) = \min\{5R, K\}$

Examples of utility functions – quasilinear preferences

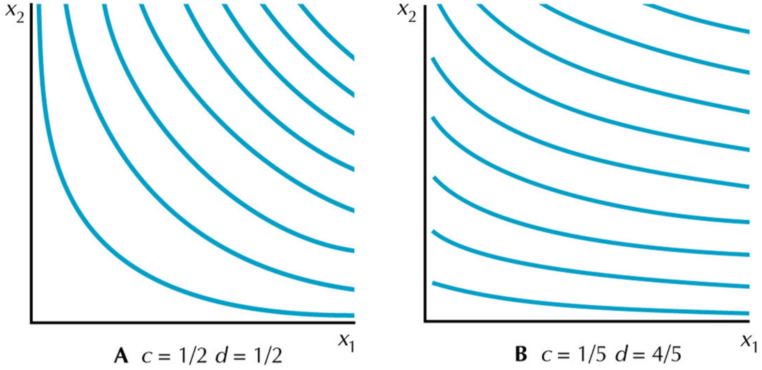
Indifference curves are vertically parallel (a practical property)

Utility function $u(x_1, x_2) = v(x_1) + x_2$, e.g. $u(x_1, x_2) = \sqrt{x_1} + x_2$



Examples of utility functions – Cobb-Douglas preferences

- A simple utility function representing well-behaved preferences.
- Utility function of the form $u(x_1, x_2) = x_1^c x_2^d$.
- More convenient to use the transformation $f(u) = u^{\frac{1}{c+d}}$ and write $x_1^a x_2^{1-a}$, where $a = c/(c+d)$.



Marginal utility

Marginal utility (MU) is the change in utility from an increase in consumption of one good, while the quantities of other goods are constant.

Partial derivatives of $u(x_1, x_2)$ with respect to x_1 or x_2.

Příklady:

- $u(x_1, x_2) = x_1 + x_2 \rightarrow MU_1 = \partial u / \partial x_1 = 1$
- $u(x_1, x_2) = x_1^a x_2^{1-a} \rightarrow MU_2 = \partial u / \partial x_2 = (1-a) x_1^a x_2^{-a}$

The value of MU changes with a monotonic transformation of the utility function. If we multiply utility times 2, MU increases times 2.

Relationship between MU and MRS

We want to measure MRS = slope of IC $u(x_1, x_2) = k$,
where k is a constant.

We are interested in (Δx_1, Δx_2), for which the utility is constant:

$$MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0$$

$$\text{MRS} = \frac{\Delta x_2}{\Delta x_1} = -\frac{MU_1}{MU_2}$$

We can calculate MRS from the utility function. E.g. for $u = \sqrt{x_1 x_2}$:

$$\text{MRS} = -\frac{\partial u / \partial x_1}{\partial u / \partial x_2} = -\frac{0{,}5 x_1^{-0{,}5} x_2^{0{,}5}}{0{,}5 x_1^{0{,}5} x_2^{-0{,}5}} = -\frac{x_2}{x_1}$$

The value of MRS does not change with monotonic transformation.
If we multiply utility function times 2, MRS$= -\frac{2MU_1}{2MU_2} = -\frac{MU_1}{MU_2}$.

APPLICATION: Utility from commuting

People decide whether to take bus or car.

Each type of transport represents a bundle with different characteristics, e.g.:

- x_1 is walking time
- x_2 is time taking a bus or car
- x_3 is the total cost of commuting
- ...

Assume that the utility function has a linear form $U(x_1, ..., x_n) = \beta_1 x_1 + ... + \beta_n x_n$.

Then we use statistical techniques to estimate the parameters β_i that best describe choices.

APPLICATION: Utility from commuting (cont'd)

Domenich and McFadden (1975) estimated the following utility function:

$$U(TW, TT, C) = -0{,}147\,TW - 0{,}0411\,TT - 2{,}24\,C$$

- TW = total walking time in minutes
- TT = total driving time in minutes
- C = total cost in dollars

The parameters can be used for different purposes.

For instance, we can:

- calculate the marginal rate of substitution between two characteristics
- forecast consumer response to proposed changes
- estimate whether a change is worthwhile in a benefit-cost sense

What should you know?

- Budget set = consumption bundles available at given prices and income
- Budget line are bundles for which the entire income is spent.
- If the preference relation is complete, reflexive and transitive, consumer can order bundles according to preferences.
- Monotonicity and convexity are reasonable assumptions – easier to find the optimum bundle.

What should you know? (cont'd)

- Utility function assigns numbers to different bundles so that the bundles are ordered according to preferences.
- The numbers have no meaning in itself. Monotonic transformation of u represents the same preferences..
- MRS measures the slope of IC.
- The slope of IC measures the willingness to pay for good 1 (in units of good 2)
- The slope of BL measures the opportunity cost of good 1(in units of good 2)