Demand and Slutsky equation

Varian: Intermediate Microeconomics, 8e, chapters 6 and 8

In this lecture, you will learn

- how a rational consumer reacts to changes in prices and income
- how animals react to changes in prices and income
- what substitution effect and income effect is
- why substitution effect cannot be positive
- what follows from this for the shape of the demand curve
- how to solve global warming and prevent blackouts

Demand

Demand function = relationship between optimal
quantity and prices and income:

$$x_1 = x_1(p_1, p_2, m)$$
$$x_2 = x_2(p_1, p_2, m)$$

Comparative statics in consumer theory –
what is the demand effect of changes in

- prices
- income

Change in price

Price elasticity of demand (ϵ) = percentage change in quantity (x_1) divided by percentage change in price of the same good (p_1):

$$\epsilon = \frac{\Delta x_1 / x_1}{\Delta p_1 / p_1} = \frac{\Delta x_1}{\Delta p_1} \cdot \frac{p_1}{x_1}$$

Dividing goods according to price elasticity:

- **ordinary good** – reduction in price increases quantity demanded:

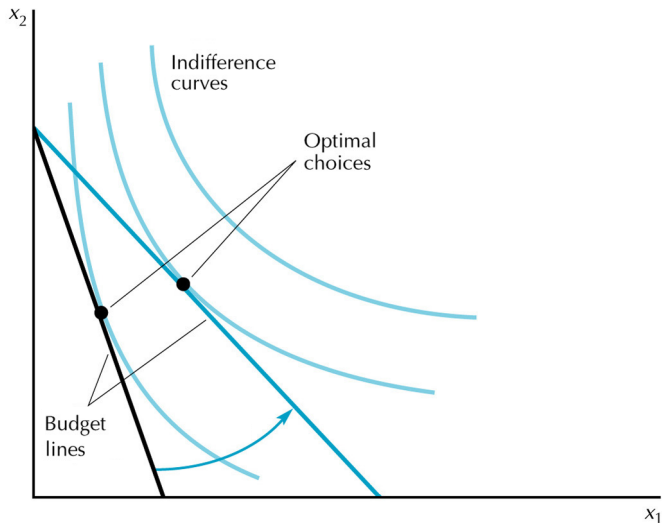$$\frac{\Delta x_1}{\Delta p_1} < 0, \quad \epsilon < 0$$

- **Giffen good** – reduction in price reduces quantity demanded:

$$\frac{\Delta x_1}{\Delta p_1} > 0, \quad \epsilon > 0$$

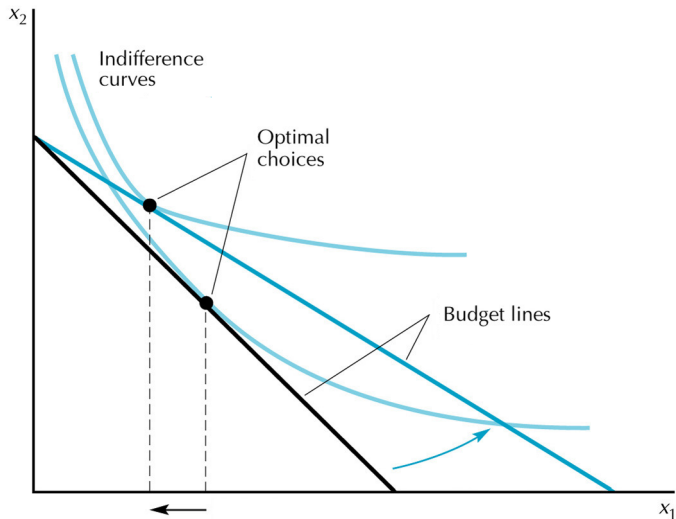Examples: rice and wheat in China (Jensen a Miller, AER, 2008)

Ordinary good

Good 1 in this graph is ordinary:
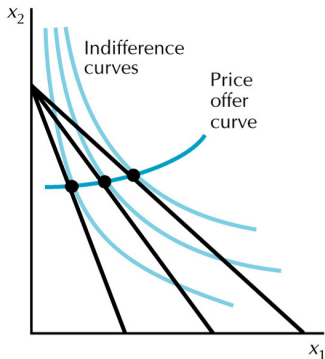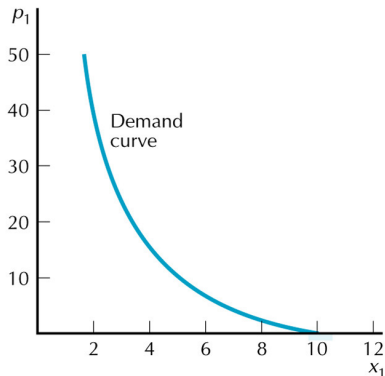
Giffen good

Good 1 in this graph is Giffen good:

Price consumption curve and demand curve

As price changes the optimal choice moves along the **price consumption curve** (PCC) or the **price offer curve** (POC).

Demand curve = the relationship between the optimal choice and a price, with income and the other price fixed.

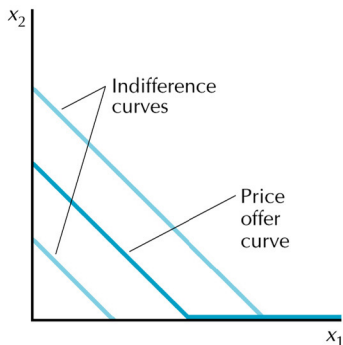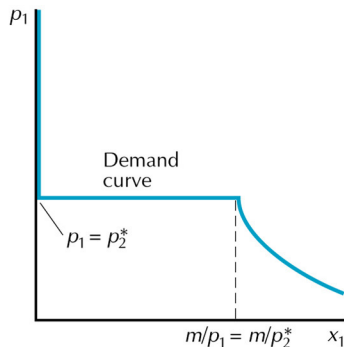**A** Price offer curve   **B** Demand curve

Examples of PCC and demand – perfect substitutes

Utility function $u(x_1, x_2) = x_1 + x_2$. Demand for good 1:

$$x_1 = \begin{cases} 0 & \text{when } p_1 > p_2 \\ \text{any number between 0 and } m/p_1 & \text{when } p_1 = p_2 \\ m/p_1 & \text{when } p_1 < p_2 \end{cases}$$
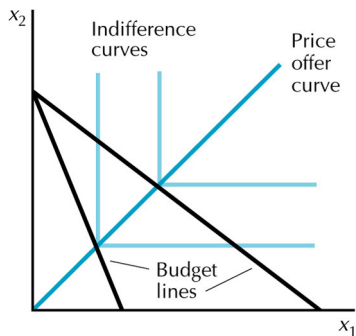

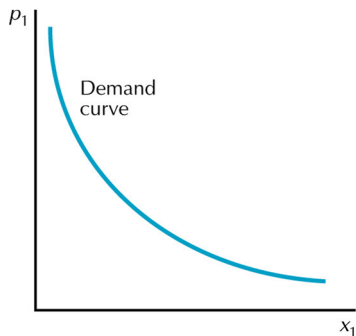**A** Price offer curve

B Demand curve

Examples of PCC and demand – perfect complements

Utility function $u(x_1, x_2) = \min\{x_1, x_2\}$. The consumer chooses $x_1 = x_2 = x$. $\implies$ PCC is a straight line. By substituting x into the BL, we get the demand function:

$$p_1 x + p_2 x = m \iff x_1 = x_2 = x = \frac{m}{p_1 + p_2}.$$

A Price offer curve

B Demand curve

Change in price of the other good

Cross elasticity of demand (ϵ_C) = percentage change in quantity of good 1 (x_1) divided by percentage change in price of good 2 (p_2):

$$\epsilon_C = \frac{\Delta x_1 / x_1}{\Delta p_2 / p_2} = \frac{\Delta x_1}{\Delta p_2} \cdot \frac{p_2}{x_1}$$

Dividing goods according to cross elasticity of demand:

- **substitutes** – rise in p_2 increases demand for good 1 x_1:

$$\frac{\Delta x_1}{\Delta p_2} > 0, \quad \epsilon_C > 0$$

- **complements** – rise in p_2 reduces demand for good 1 x_1:

$$\frac{\Delta x_1}{\Delta p_2} < 0, \quad \epsilon_C < 0$$

Change in income

Income elasticity of demand (ϵ_I) = percentage change in quantity demanded of good (x_1) divided by percentage change in income (m):

$$\epsilon_I = \frac{\Delta x_1 / x_1}{\Delta m / m} = \frac{\Delta x_1}{\Delta m} \cdot \frac{m}{x_1}$$

Dividing goods according to income elasticity of demand:

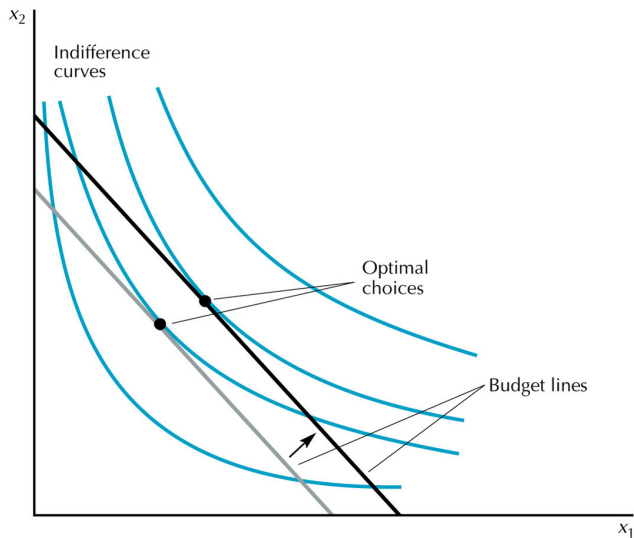- **normal good** – rise in income increases demand:

$$\frac{\Delta x_1}{\Delta m} > 0, \quad \epsilon_I > 0$$

  - **luxury good**: $\epsilon_I > 1$
  - **necessary good**: $\epsilon_I \in (0,1)$

- **inferior good** – rise in income decreases demand:

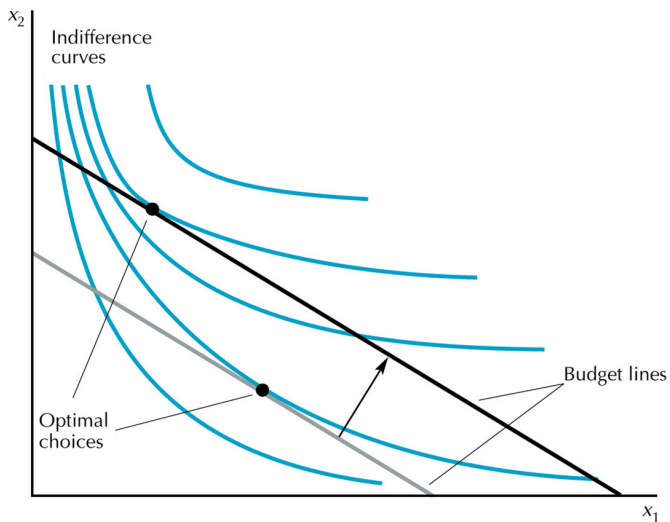$$\frac{\Delta x_1}{\Delta m} < 0, \quad \epsilon_I < 0$$

Normal good

Good 1 in this graph is normal:
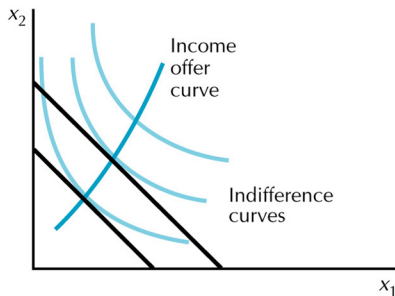
Inferior good
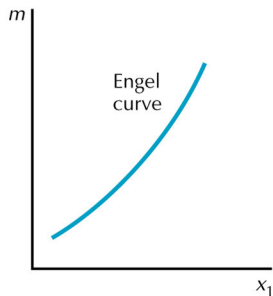
Good 1 in this graph is inferior:

Income consumption curve and Engel curve

As income changes the optimal choice moves along the **income consumption curve** (ICC) or **income expansion path** (IEP).

Engel curve (EC) = the relationship between optimal choice and income, with prices fixed.
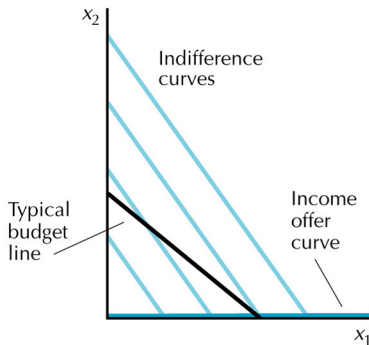

A Income offer curve

B Engel curve
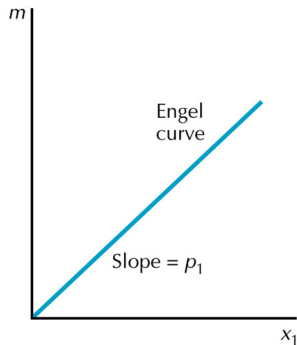
Examples of ICC and EC – perfect substitutes

Utility function $u(x_1, x_2) = x_1 + x_2$ a $p_1 < p_2$

Consumer buys only good 1 $\implies$ ICC is $x_2 = 0$.
Demand for good 1 is $x_1 = m/p_1 \implies$ EC is $m = p_1 x_1$.

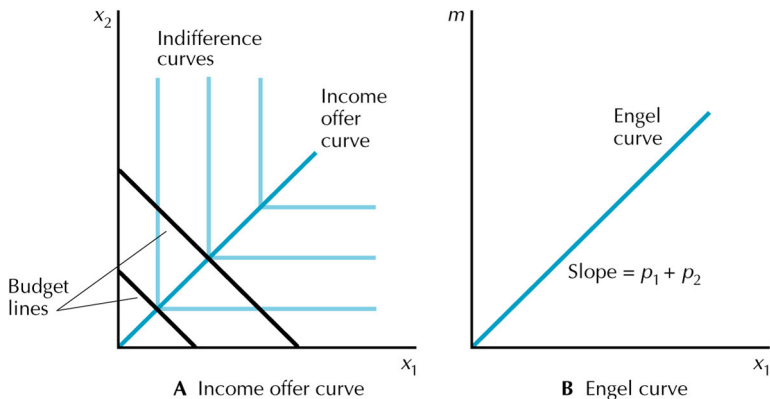**A** Income offer curve

B Engel curve

Examples of ICC and EC – perfect complements

Utility function $u(x_1, x_2) = \min\{x_1, x_2\}$ a $p_1 > 0$ a $p_2 > 0$

Consumer buys equal amounts of both goods – ICC is $x_2 = x_1$.
Demand for good 1 is $x_1 = \frac{m}{p_1 + p_2} \implies$ EC is $m = (p_1 + p_2)x_1$.

A Income offer curve   **B** Engel curve

Homotetické preference

Homotetic preferences = is $(x_1, x_2) \succ (y_1, y_2)$, then for all $t > 0$ holds that $(tx_1, tx_2) \succ (ty_1, ty_2)$.

Quantities demanded of goods change at the same ratio as income.
$\implies$ ICC and EC are straight lines from the origin.

Examples of homotetic preferences:

- perfect substitutes
- perfect complements
- Cobb-Douglas preferences

Homothetic preferences are not very realistic. If demand increases at a different rate than income, we have luxury or necessary goods.

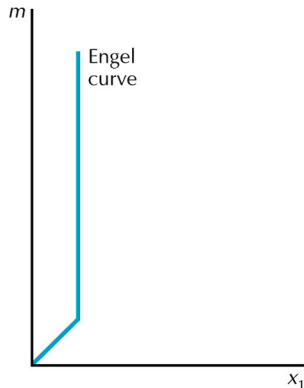# Examples of ICC and EC – quasilinear preferences

Utility function $u(x_1, x_2) = v(x_1) + x_2$ – ICs are vertically parallel.

With increasing income, ICC is first horizontal and then vertical.
Engel curve is increasing and then vertical.
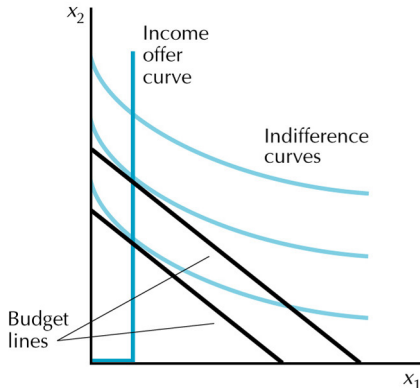

A Income offer curve   **B** Engel curve

EXAMPLE: Demand in animal kingdom

Kagel et al. (e.g. Econ. Inquiry, 1975) showed that rats and pigeons behave according to the laws of demand.

Chen et al. (JPE, 2006) found out that capuchin monkeys
- have stable preferences
- react rationally to changes in prices and income – their behavior follows the axioms of revealed preference (GARP)
- under uncertainty behave irrationally (same as people) – reference dependence, loss aversion
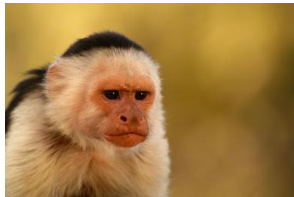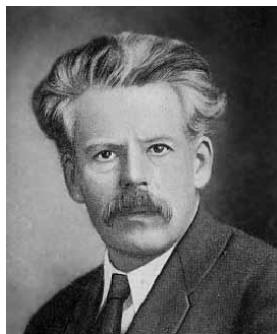
Slutsky equation

We concentrate only on the effect of a change in price.

We divide the effect of a change in price in
- pivot – **the Slutsky substitution effect**
- shift – **the income effect**
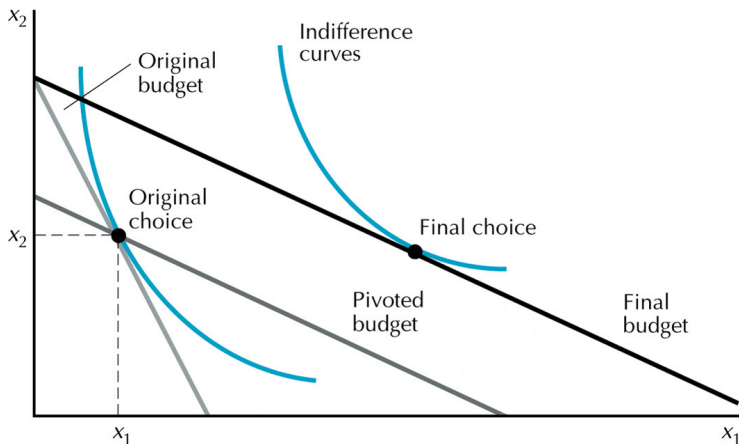
Substitution/income effect is
- positive when $\uparrow p \uparrow x$, or $\downarrow p \downarrow x$.
  (same direction)
- negative when $\uparrow p \downarrow x$, or $\downarrow p \uparrow x$.
  (opposite direction)

Pivoted budget line

The pivoted BL has the same slope as the final BL, but the income is compensated so that the original bundle (x_1, x_2) is just affordable.

Pivoted budget line (cont'd)

How to adjust the income so that (x_1, x_2) is just affordable?

The compensated income $m' = m + \Delta m$, where

$$\Delta m = x_1 \Delta p_1.$$

Example:

At my original income I consume 20 chocolates.

The price of chocolate increases from 20 to 30 CZK.

What should be the income compensation Δm?

The chocolates are just affordable if my income increases by

$$\Delta m = 20 \times 10 = 200 \text{ CZK}.$$

The Slutsky substitution effect

Substitution effect (SE) measures how demand changes when we change prices, keeping the purchasing power fixed (pivot):
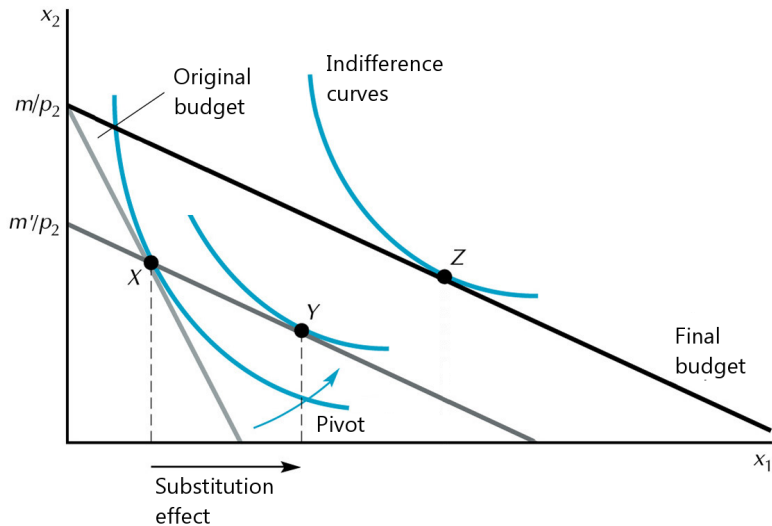
$$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m)$$

SE isolates the pure effect from changing the relative prices.

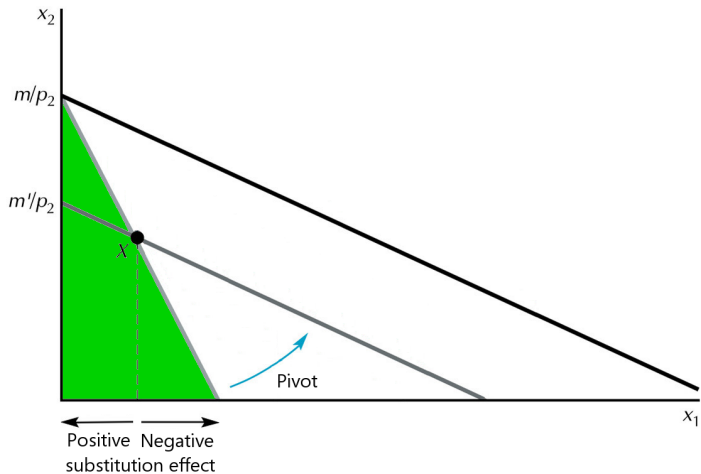Sometimes called the change in **compensated demand**:
Consumer's income is compensated for a change in price by Δm.

Slutsky substitution effect (graph)
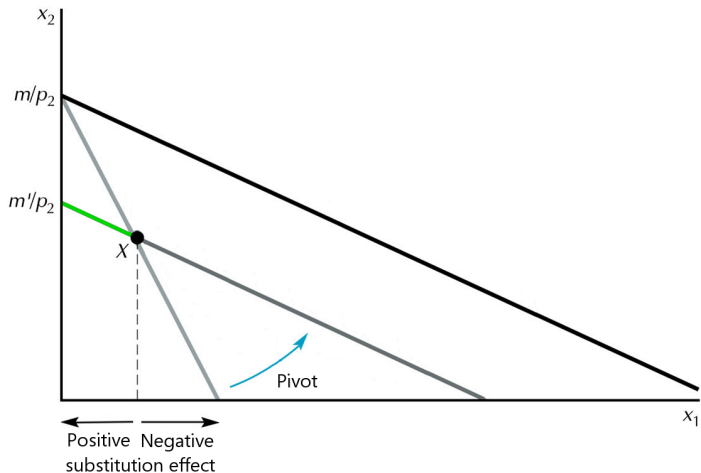
Sign of the substitution effect

Revealed preference – at the original price the consumer revealed, that he prefers the bundle X to all affordable bundles (green area).

Sign of the substitution effect

For a utility-maximizing consumer the bundle X is revealed preferred to all bundles in the green part of the pivoted BL $\implies$ the substitution effect is never positive.

The income effect

Income effect (IE) measures the change in demand the income changes from m' to m and prices remain fixed at (p_1', p_2) (shift):
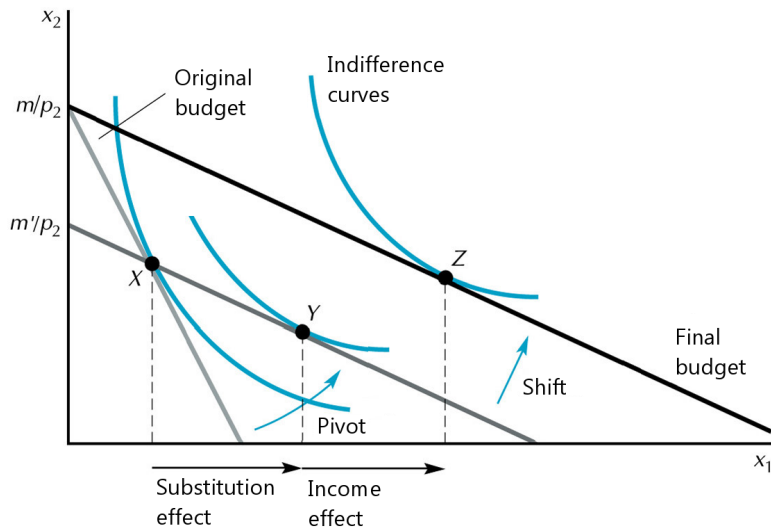
$$\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m')$$

IE isolates the pure effect of a change in purchasing power due to a price change.

Income effect is

- negative for normal goods (e.g. $\uparrow p \implies \downarrow m \implies \downarrow x$)
- positive for inferior goods (e.g. $\uparrow p \implies \downarrow m \implies \uparrow x$)

The substitution and income effect (graph)

The Slutsky identity

The Slutsky identity – the total change in demand TE = SE + IE:

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n$$

From Slutsky identity and the fact that SE is non-positive follows:

| Division of goods according to the effect of		
income		**price**
inferior good (positive IE)	$\leftarrow$	Giffen good (positive TE)
inferior good (positive IE)		ordinary good (negative TE)
normal good (negative IE)	$\rightarrow$	ordinary good (negative TE)

Giffen good must be inferior (plus: $|IE^+| > |SE^{0-}|$).
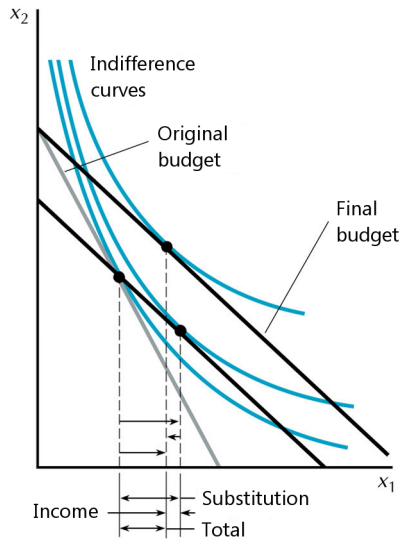Inferior good can be a Giffen or an ordinary good.
Ordinary good can be a inferior or a normal good.
Normal good must be a ordinary good (the law of demand).

SE a IE for inferior good (graph)
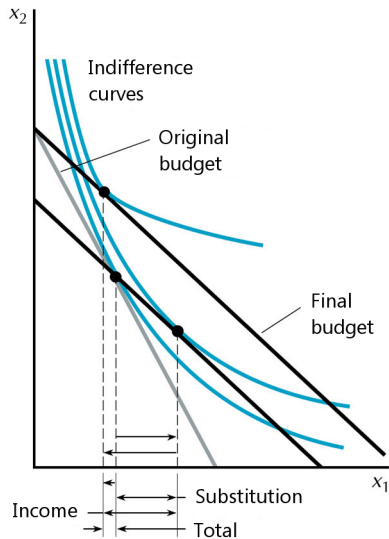
A Giffen good   **B** Inferior good (not Giffen)

The law of demand

The law of demand: If the demand for a good increases when income increases (normal good), then the demand for that good must decrease when its price increases.
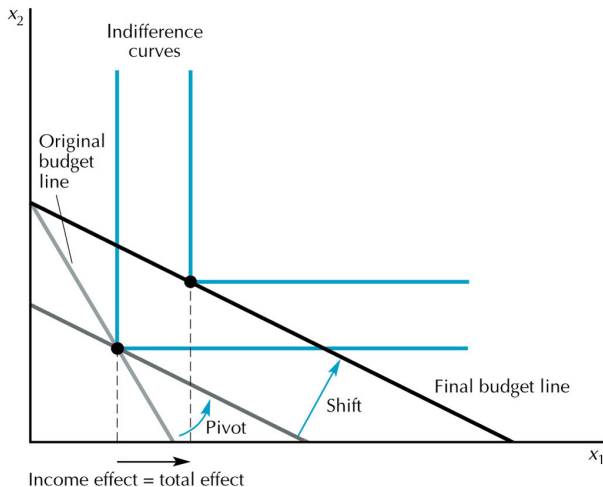
Reasoning:

1. A normal good must be ordinary ($TE^- = SE^{0-} + IE^-$).
2. An ordinary good has a decreasing demand curve.

SE and IE – perfect complements

TE = IE, SE is always 0.

Example – SE and IE – perfect complements

Utility function: $u(x_1, x_2) = \min\{2x_1, x_2\}$
Income: $m = 60$
Original prices: $(p_1, p_2) = (1, 1)$
Price of good 1 increases to $p_1' = 2$

What is the SE and IE of this change in price?

Demand function for good 1:

$$x_1 = \frac{m}{p_1 + 2p_2}$$

Original demand: $x_1(m, p_1) = 20$
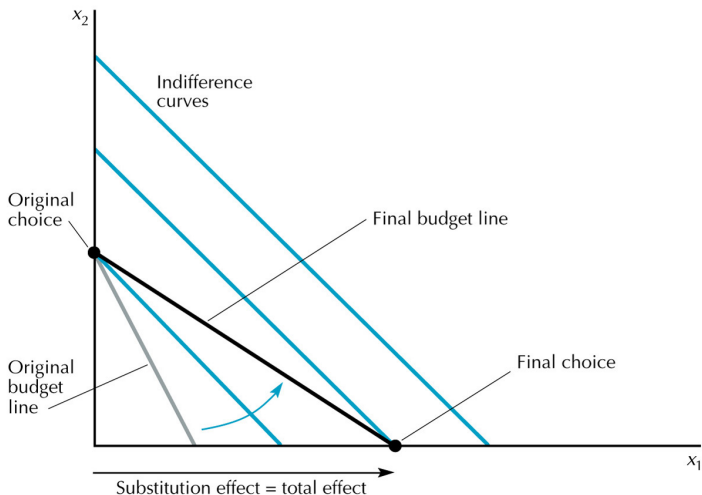Compensated income: $m' = m + x_1 \Delta p_1 = 60 + 20 = 80$

Using the demand function for good 1 we calculate the SE and IE:

$$\text{SE: } \Delta x_1^s = x_1(p_1', m') - x_1(p_1, m) = 20 - 20 = 0$$
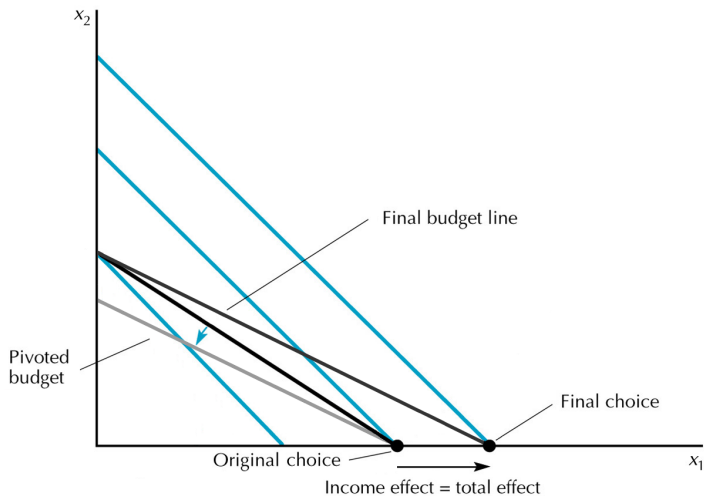$$\text{IE: } \Delta x_1^n = x_1(p_1', m) - x_1(p_1', m') = 15 - 20 = -5$$

SE and IE – perfect substitutes

Change in price $\rightarrow$ consumption of a different good: TE = SE, IE = 0

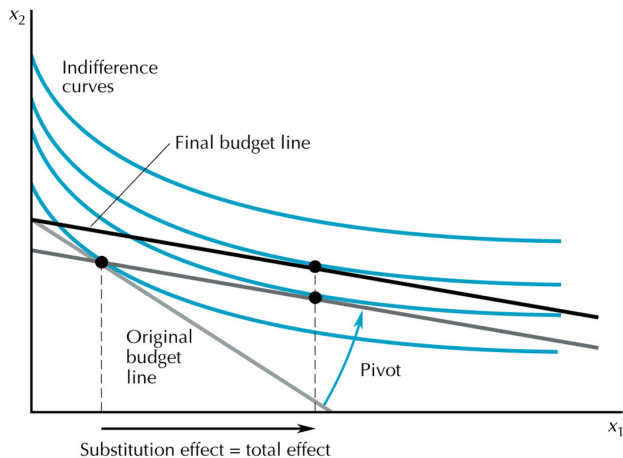# SE and IE – perfect substitutes (cont'd)

Change in price → consumption of the same good: TE = IE, SE = 0

SE and IE – quasilinear preferences

TE = SE, IE is always 0.

CASE: Solving the problem of global warming

One solution to global warming:

- tax emissions of CO_2
- return the tax revenue to consumers

Will the plan work?

CASE: Solving the problem of global warming (cont'd)

The original BL: $px + y = m$

The BL with a tax: $(p + t)x' + y' = m + tx' \iff px' + y' = m$

The bundle (x', y') is affordable at the original BL, but the consumer chooses the bundle (x, y). $\implies$ The consumer is worse off.

If instead of the transfer, government reduces the income tax, this might increase incentives to work so that people earn a higher income.

In this case consumers might be even better off.

CASE: Solving the problem of global warming (graph)

The plan works: people want less of CO_2 and more of other goods.
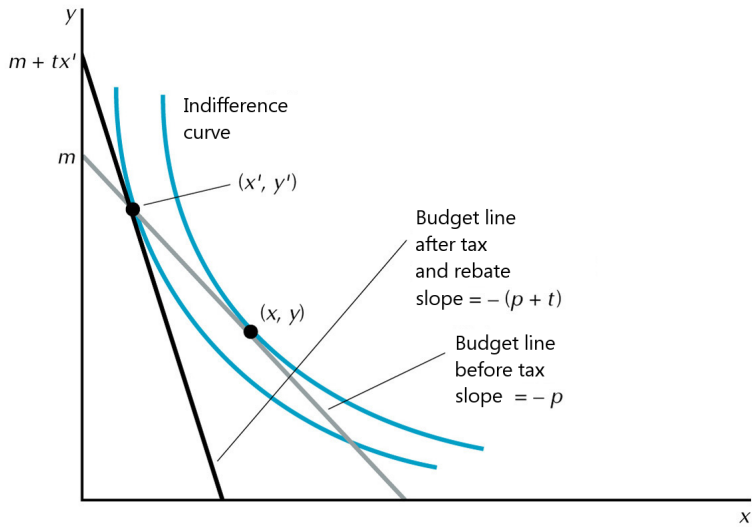
APPLICATION: Setting prices in energy markets

Electricity production suffers from an extreme capacity problem: it is relatively cheap to produce up to capacity, and impossible to produce more.

Building capacity is expensive $\implies$ necessary to reduce use in peak demand. The demand for electricity depends strongly on temperature which is easy to predict.

Question: How to set up pricing system so that those users who are able to cut back on their electricity use have incentive to do so.
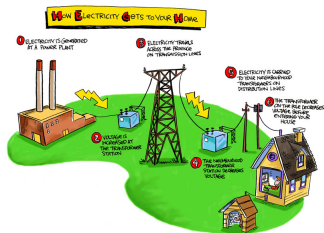
# APPLICATION: Setting prices in energy markets (cont'd)

One possible solution: Real Time Pricing (RTP)

RTP: large industrial users are equipped with special meters that allow the price of electricity to vary from minute to minute, depending on signals sent from the electricity generating company.

Georgia Power Company – the largest RTP program in the world:
In 1999 it was able to reduce demand on high-price days by inducing some large customers to cut their demand by as much as 60%.
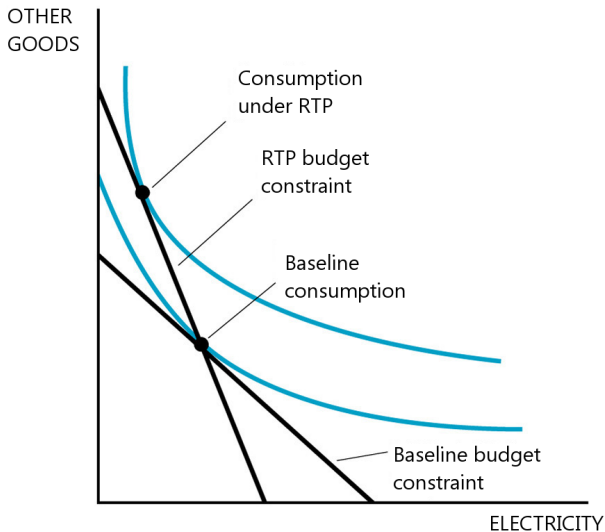
What should you know?

- Demand is a function of prices and income.

- Ordinary good: $\uparrow p_1 \implies \downarrow x_1$
  Giffen good: $\uparrow p_1 \implies \uparrow x_1$
  PCC connects the optima at different p_1
  Demand curve: relationship between p_1 and x_1

- Substitutes: $\uparrow p_2 \implies \uparrow x_1$
  Complements: $\uparrow p_2 \implies \downarrow x_1$

- Normal good: $\uparrow m \implies \uparrow x_1$
  Inferior good: $\uparrow m \implies \downarrow x_1$
  ICC connects optima at different m
  Engel curve: relationship between m and x_1

What should you know? (cont'd)

- Slutsky equation: $TE = SE + IE$
- SE: demand effect of a change in relative prices (at an original purchasing power)
- IE: demand effect of a change in purchasing power (at constant new relative prices)
- IE or SE is positive if $\uparrow p \uparrow x$ or $\downarrow p \downarrow x$.
  IE or SE is negative if $\uparrow p \downarrow x$ or $\downarrow p \uparrow x$.
- SE cannot be positive (revealed preference)
- Giffen goods are inferior: $TE^{+} = SE^{0-} + IE^{+}$
- Normal goods are ordinary: $TE^{-} = SE^{0-} + IE^{-}$
  $\implies$ Normal goods have a decreasing demand.