

# Technology and profit maximization

Varian: Intermediate Microeconomics, 8e, chapters 18 and 19

# Theory of firm and market structure

Firms maximize profit.

The results of the interaction of profit-maximizing firms depends on:

- market structure
- properties of the product

The following four topics:

- theory of firm
- perfect competition
- monopoly and monopoly behavior
- oligopoly



## In this lecture you will learn

- what technology and a production function are
- what the difference between the short and long run is
- what it means that a firm maximizes profit and minimizes costs
- why fishermen in India need mobile phones



# Production and production plan

**Production** is the transforming inputs into outputs.

Inputs to production = factors of production (FP):

- labor
- land (resources)
- capital

**Production plan** is a combination of inputs and outputs.

Technological constraints – only certain combinations of inputs are *feasible* ways to produce a given amount of output.

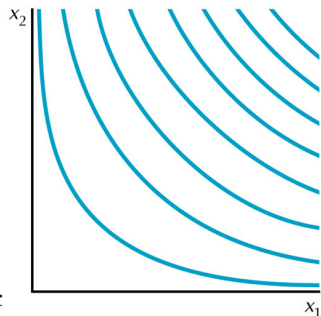
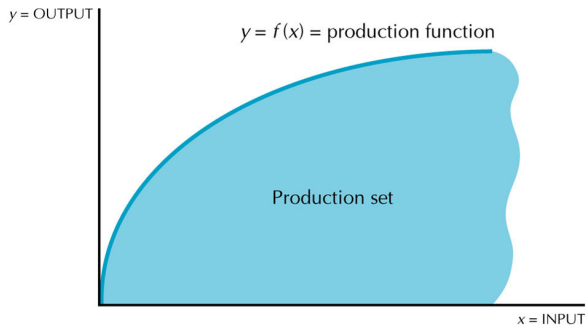
There is usually a number of feasible production plans.

## Describing technological constraints

**Production set** (= technology) – set of all feasible production plans.

**Production function** – maximum possible output that you can get from a given amount of input

**Isoquant** – set of all possible combinations of inputs 1 and 2 that are just sufficient to produce a given amount of output.



## Example – utility functions vs. production functions

Utility function – two consumers:

- consumer 1 –  $U_1(x_1, x_2) = x_1 + x_2$
- consumer 2 –  $U_2(x_1, x_2) = (x_1 + x_2)^2$

Both consumers have identical ICs and preferences  $\implies$  If they have the same BL, they choose the same consumption bundle.

Production function – two firms:

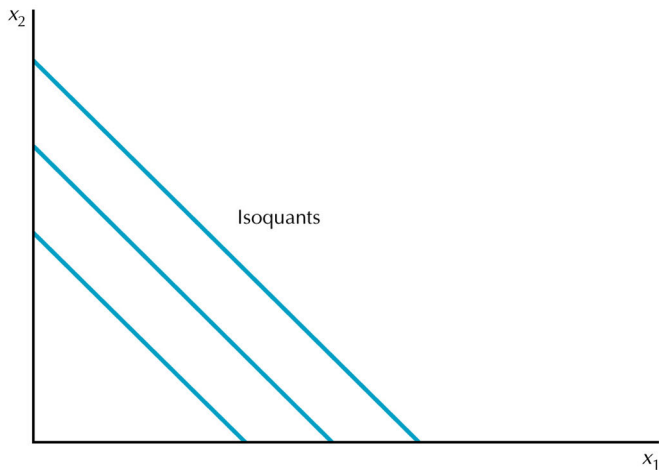
- firm 1 –  $f_1(x_1, x_2) = x_1 + x_2$
- firm 2 –  $f_2(x_1, x_2) = (x_1 + x_2)^2$

Both firms have identical isoquants but different technologies.  
At the same inputs they produce different outputs (if  $x_1 + x_2 \neq 1$ ).

## Examples of technology – perfect substitutes

Passport photographs – a photographer or a photo booth.

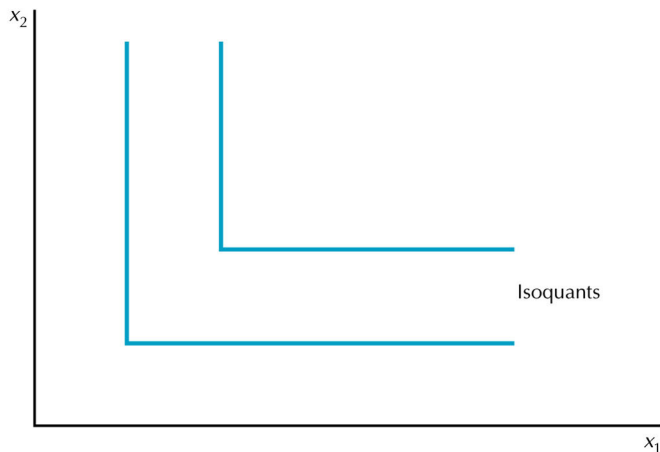
Production function –  $f(x_1, x_2) = x_1 + x_2$



## Examples of technology – fixed proportions

Ice cream cart – one sales man needs one cart.

Production function –  $f(x_1, x_2) = \min\{x_1, x_2\}$



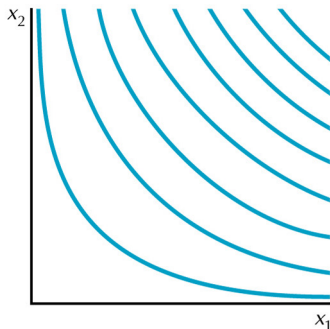


## Examples of technology – Cobb-Douglas

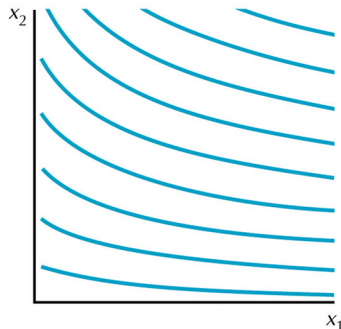
Cobb-Douglas production function –  $f(x_1, x_2) = Ax_1^c x_2^d$ :

- $A$  measures the scale of production
- $c$  and  $d$  measure how the output responds to changes in input

Cobb-Douglas *utility function* could be monotonically transformed – so we usually had  $A = 1$  and  $c + d = 1$ .



**A**  $A = 1$   $c = 1/2$   $d = 1/2$



**B**  $A = 1$   $c = 1/5$   $d = 4/5$

## Properties of technology: monotonicity and convexity

**Monotonicity** – if you increase at least one of the inputs, you should produce at least as much as before (nonincreasing isoquant).

Logics of monotonicity: *free disposal* = if the firm can costlessly dispose of any inputs, having extra inputs around can't hurt it.

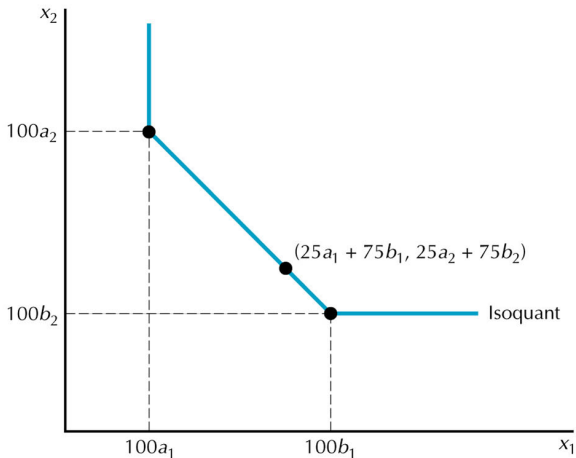
**Convexity** – if you have two ways to produce  $y$  units of output,  $(x_1, x_2)$  and  $(z_1, z_2)$ , then their weighted average will produce at least  $y$  units of output.

Logics of convexity: If there are more ways to produce the same output and the ways can be combined, we get a convex isoquant.

## Example: Deriving a convex isoquant – a 100 meter ditch

1 meter of ditch can be produced in two ways:

- using a small excavator: (hours of work, value of capital) =  $(a_1, a_2)$
- using a pickaxe: (hours of work, value of capital) =  $(b_1, b_2)$



## Marginal product

Marginal product of factor 1 ( $MP_1$ ) – what is the change in total product if we increase  $x_1$  by one unit and  $x_2$  remains constant:

$$MP_1(x_1, x_2) = \frac{\partial f(x_1, x_2)}{\partial x_1}$$

$MP$  is similar to  $MU$ , but the value of  $MP$  means something.



## Technical rate of substitution (TRS)

TRS = slope of isoquant (similar to MRS = slope of IC)

By how many units we can reduce  $x_2$ , if  $x_1$  increases by 1 unit and we want to produce the same output  $y$ ?

Calculation:

For changes in factors  $\Delta x_1$  and  $\Delta x_2$  on the same isoquant holds:

$$\Delta y = MP_1(x_1, x_2)\Delta x_1 + MP_2(x_1, x_2)\Delta x_2 = 0$$

By a simple manipulation of the equation, we get

$$TRS(x_1, x_2) = \frac{\Delta x_2}{\Delta x_1} = -\frac{MP_1(x_1, x_2)}{MP_2(x_1, x_2)}$$

**Diminishing TRS** = assumption of strict convexity – the absolute value of TRS decreases as we move along the isoquant to the right.

## Diminishing marginal product (law of diminishing MP)

Why can't we feed the world's population using one flower pot?

**Diminishing MP** – if the amount of input  $i$  increases (beyond a certain value) and other inputs remain constant,  $MP_i$  falls.

Is diminishing  $MP$  and diminishing TRS the same?

No, but the logics are similar:

- diminishing  $MP$  – as  $x_1$  rises,  $MP_1$  for a given  $x_2$  decreases
- diminishing TRS – as  $x_1$  rises moving along the isoquant to the right, the additional amount of input 1 necessary to compensate for the loss of 1 unit of input 2 is increasing.

## The short run and the long run

**The short run (SR)** – there will be some factors of production that are fixed at predetermined levels.

**The long run (LR)** – all the factors of production can be varied.

How long is the short run?

Cooper and Haltiwanger (RES, 2006) study the US manufacturing plants (1972 to 1988): 1 of 10 plants does not make any capital adjustment in a given year.

Plants make large changes and then wait. Depending on the size of the change needed, SR lasts from several months to more than a year.

Other sectors may differ (e.g. services or transport).

## Quasifixed inputs

**Quasifixed inputs** are used only if the output is positive, but the quantity is independent of the output.

Examples:

Electricity for lights in a production plant, some administrative positions

Quasifixed vs. fixed inputs:

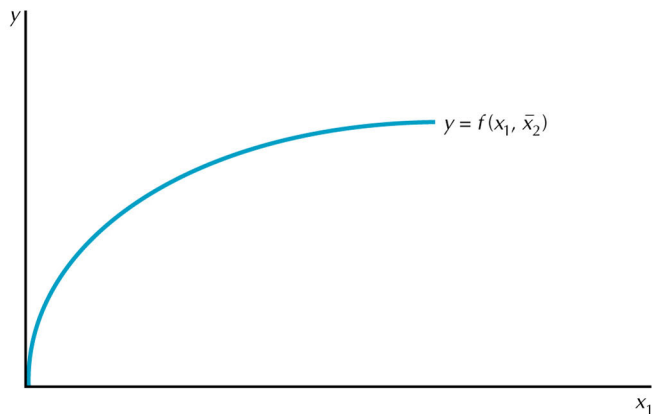
Quasifixed inputs exist in the SR and LR, fixed inputs only in the SR.



## Production function in the SR

Production function in the SR  $f(x_1, \bar{x}_2)$ , where input 1 is variable and input 2 is fixed.

The function in the graph has a diminishing MP. For low  $x_1$  the  $MP_1$  may be increasing (then the function would have an S-shape).



## Production function in the LR – returns to scale

What is the increase in output if all inputs increase  $t$  times ( $t > 1$ )?

If for all points on a production function  $f(x_1, x_2)$  holds that

- $f(tx_1, tx_2) = tf(x_1, x_2)$ ,  $f(\cdot)$  has **constant returns to scale**.
- $f(tx_1, tx_2) > tf(x_1, x_2)$ ,  $f(\cdot)$  has **increasing returns to scale**.
- $f(tx_1, tx_2) < tf(x_1, x_2)$ ,  $f(\cdot)$  has **decreasing returns to scale**.

### Examples:

- constant– replication the production technology, ...
- increasing – pin factory, airlines, plane production, ...
- decreasing – difficult to find real examples („organization“ does not change at the same rate)

## Examples – returns to scale of specific production functions

1) What are the returns to scale of  $f(x_1, x_2) = x_1^{1/2} x_2^{3/4}$ ?

Multiplying both inputs by  $t > 1$ , we get:

$$f(tx_1, tx_2) = (tx_1)^{1/2}(tx_2)^{3/4} = t^{5/4} x_1^{1/2} x_2^{3/4} = t^{5/4} f(x_1, x_2)$$

Increasing because  $f(tx_1, tx_2) > tf(x_1, x_2)$ .

2) What are the returns to scale of  $f(x_1, x_2) = \min\{x_1, x_2\}$ ?

Multiplying both inputs by  $t > 1$ , we get:

$$f(tx_1, tx_2) = \min\{tx_1, tx_2\} = t \times \min\{x_1, x_2\} = t \times f(x_1, x_2).$$

Constant because  $f(tx_1, tx_2) = tf(x_1, x_2)$ .

## EXAMPLE: Copy Exactly!

Intel operates dozens of plants produce and test computer chips.

Chip fabrication is such a delicate process that Intel found it difficult to manage quality in a heterogeneous environment. That is why Intel moved to its Copy Exactly! process

Each looks very much like another – constant returns to scale.

### 32nm Manufacturing Fabs



D1D Oregon - Now



D1C Oregon - 4Q 2009



Fab 32 Arizona - 2010



Fab 11X New Mexico - 2010



## EXAMPLE: Mobile technology in Indian fishing

Jensen (QJE, 2007) studied 15 markets in Kerala in southern India where fishermen sell their daily catch (no freezers available).

What was the contribution of mobile phones?

Coordination: before mobile phones there was often surplus of fish in one market and shortage of fish in the neighbouring market.

With mobile phones the supply matches the demand much better.

And the impact on prices?



# EXAMPLE: Mobile technology in Indian fishing (graph)

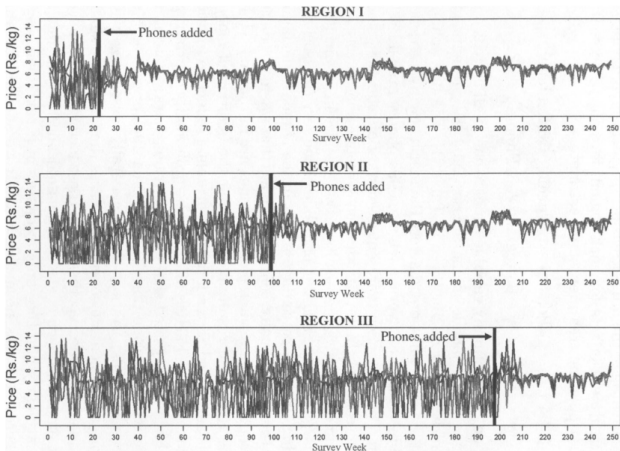


FIGURE IV

## Prices and Mobile Phone Service in Kerala

Data from the Kerala Fisherman Survey conducted by the author. The price series represent the average 7:30–8:00 A.M. beach price for average sardines. All prices in 2001 Rs.

## Profit maximization and cost minimization

**Profit maximization** – what production plan maximizes the firm's profit (for a given technology and input and output prices).

In this lecture we assume competitive input and output markets.  $\implies$   
Prices of inputs ( $\mathbf{w}$ ) and the price of output ( $p$ ) are given.

**Cost minimization** (next lecture) – what combination of inputs minimizes the cost of producing a given output (for a given technology and input prices) – derivation of the **cost function**.

In the second step the firm chooses the profit-maximizing output (for a given cost function and demand).

# The economic profit

We distinguish two types of costs:

- **Explicit costs** – accounting costs
- **Implicit costs** – opportunity costs of the firm-owned inputs

Example:

If the owner works in her firm and does not pay any wage to herself, she has no *accounting costs*, but the firm has an *implicit cost*.

Two types of profit:

- **Accounting profit** = revenues – explicit costs
- **Economic profit** = revenues – explicit costs – implicit costs

We will always use the *economic profit*.





## Profit maximization

A firm chooses quantities of inputs 1 and 2 to maximize its profit:

$$\max_{x_1, x_2} \pi = pf(x_1, x_2) - w_1x_1 - w_2x_2$$

It follows from the first order condition that :

$$pMP_1(x_1^*, x_2^*) = w_1$$

$$pMP_2(x_1^*, x_2^*) = w_2$$

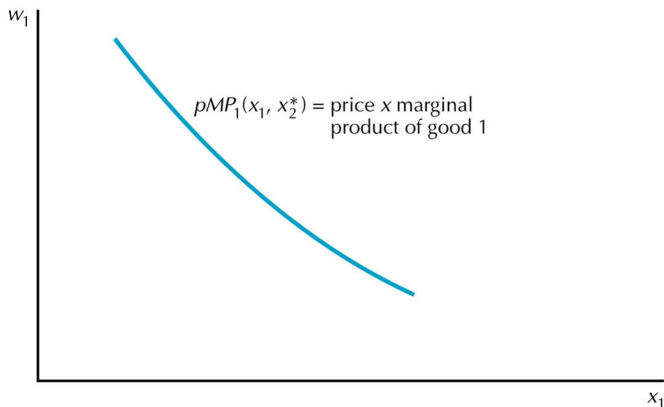
The firm maximizes profit if of all inputs the *value of their marginal product* equals to their *prices*.

## Demand for input

**Demand for input** – profit-maximizing quantities of input

The inverse demand for input 1 derived from the first order condition:

$$w_1 = pMP_1(x_1, x_2^*)$$



## Example of profit maximization in the SR

Production function:  $f(x_1, x_2) = x_1^{1/2} x_2^{1/2}$ , where  $\bar{x}_2 = 16$

Prices:  $(p, w_1, w_2) = (40, 10, 20)$

What is the optimum quantity of the variable input and profit?

Short-run production function:  $f(x_1, 16) = 4x_1^{1/2}$

Profit function:  $\pi = p4x_1^{1/2} - w_1x_1 - w_2\bar{x}_2$

Deriving with respect to  $x_1$  we get  $pMP_1 = w_1$  (FOC):

$$p \frac{2}{\sqrt{x_1}} = w_1$$

$$x_1 = \left( \frac{2p}{w_1} \right)^2$$

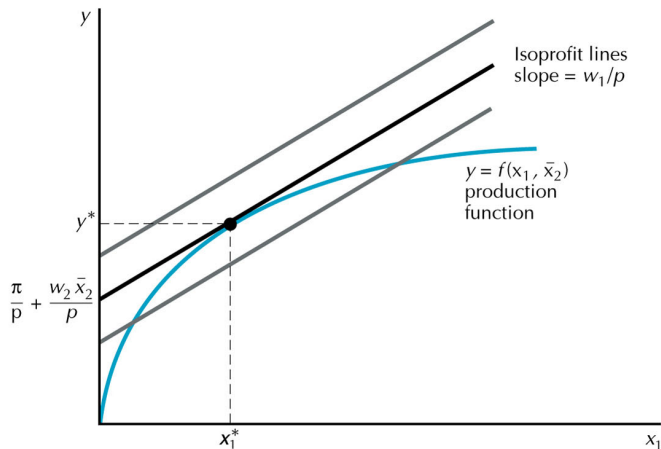
$$\underline{x_1 = 64}$$

Profit of the firm:  $\pi = p4x_1^{1/2} - w_1x_1 - w_2\bar{x}_2 = 320$

## Profit maximization in the SR (graph)

Isoprofit lines – combinations of  $x$  and  $y$  earning a constant profit  $\pi$ :

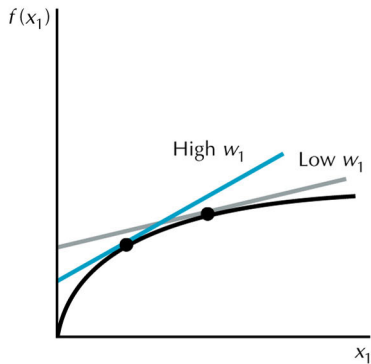
$$\pi = py - w_1x_1 - w_2\bar{x}_2 \iff y = \frac{\pi}{p} + \frac{w_2\bar{x}_2}{p} + \frac{w_1}{p}x_1$$



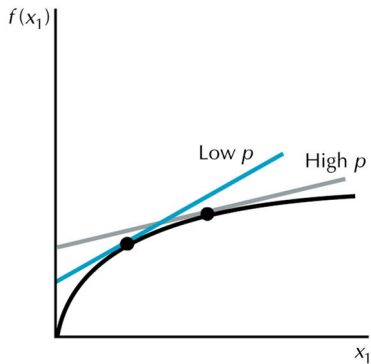
## Comparative statics

If the price of input  $w_1$  increases, the optimal  $x_1$  decreases (Fig. **A**).

If the price of output  $p$  increases, the optimal  $x_1$  increases (Fig. **B**).



**A**



**B**

## Revealed profitability

A profit-maximizing firm shows that the chosen combination of inputs and outputs represents a *feasible* production plan, which is at least as profitable as other feasible production plans.



## Revealed profitability – example

Two different choices the firm makes at different sets of prices:

- at prices at time  $t$  ( $p^t, w_1^t, w_2^t$ ) the firm chooses  $(y^t, x_1^t, x_2^t)$ ,
- at prices at time  $s$  ( $p^s, w_1^s, w_2^s$ ) the firm chooses  $(y^s, x_1^s, x_2^s)$ .

**Weak axiom of profit maximization (WAPM):** If the firm maximizes profit and the production function of the firm hasn't changed between times  $s$  and  $t$ , then we must have:

$$p^t y^t - w_1^t x_1^t - w_2^t x_2^t \geq p^t y^s - w_1^t x_1^s - w_2^t x_2^s \quad (1)$$

$$p^s y^s - w_1^s x_1^s - w_2^s x_2^s \geq p^s y^t - w_1^s x_1^t - w_2^s x_2^t \quad (2)$$

## Revealed profitability – example (cont'd)

We copy equation (1) and transpose the two sides of equation (2) to get

$$\begin{aligned} p^t y^t - w_1^t x_1^t - w_2^t x_2^t &\geq p^t y^s - w_1^t x_1^s - w_2^t x_2^s \\ -p^s y^t + w_1^s x_1^t + w_2^s x_2^t &\geq -p^s y^s + w_1^s x_1^s + w_2^s x_2^s. \end{aligned}$$

Since both equations have  $\geq$ , also the sum of the equations must have  $\geq$ :

$$\begin{aligned} (p^t - p^s)y^t - (w_1^t - w_1^s)x_1^t - (w_2^t - w_2^s)x_2^t \\ \geq (p^t - p^s)y^s - (w_1^t - w_1^s)x_1^s - (w_2^t - w_2^s)x_2^s. \end{aligned}$$

Rearranging this equation and substituting  $\Delta p$  for  $(p^t - p^s)$ ,  $\Delta y$  for  $(y^t - y^s)$ , and so on, we find

$$\Delta p \Delta y - \Delta w_1 \Delta x_1 - \Delta w_2 \Delta x_2 \geq 0.$$



## Revealed profitability – example (cont'd)

What follows from the result  $\Delta p \Delta y - \Delta w_1 \Delta x_1 - \Delta w_2 \Delta x_2 \geq 0$ ?

- If the output price  $p$  changes and  $w_1$  and  $w_2$  remain constant, then

$$\Delta p \Delta y \geq 0.$$

It never holds that  $\Delta p > 0$  and  $\Delta y < 0$ , or  $\Delta p < 0$  and  $\Delta y > 0$ .

$\implies$  *The supply curve of a competitive firm can't be decreasing.*

- If the price of input 1  $w_1$  changes and  $p$  and  $w_2$  remain constant, then

$$\Delta w_1 \Delta x_1 \leq 0.$$

It never holds that  $\Delta w_1 > 0$  and  $\Delta x_1 > 0$ , or  $\Delta w_1 < 0$  and  $\Delta x_1 < 0$ .

$\implies$  *The factor demand of a competitive firm can't be increasing.*

## Estimating technology using WAPM

### Can WAPM be used for estimating technology?

Yes, if the firm maximizes profit and its technology hasn't changed, we can use its choices to estimate its technology .

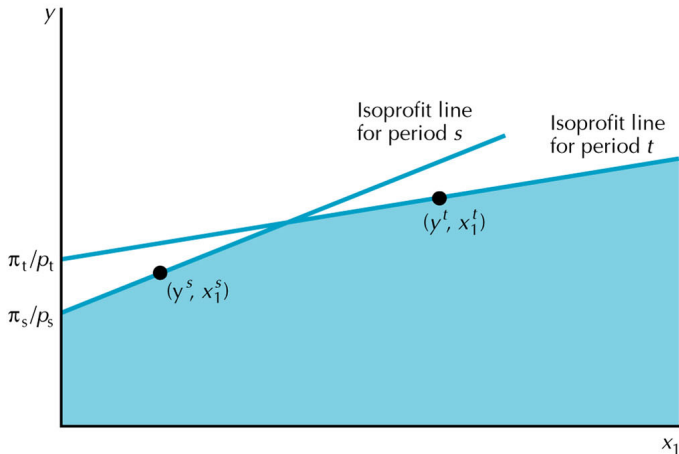
Suppose we have one input  $x_1$  and one output  $y$ . In period

- $t$ , the firm chooses a production plan  $(x_1^t, y^t)$  and the isoprofit function is  $\pi_t = p^t y - w_1^t x_1 \iff y = \pi_t/p^t + (w_1^t/p^t)x_1$ .
- $s$ , the firm chooses a production plan  $(x_1^s, y^s)$  and the isoprofit function is  $\pi_s = p^s y - w_1^s x_1 \iff y = \pi_s/p^s + (w_1^s/p^s)x_1$ .

It follows from WAPM that production plans above the isoprofit lines are not feasible. Otherwise the firm could increase its profit by choosing them.

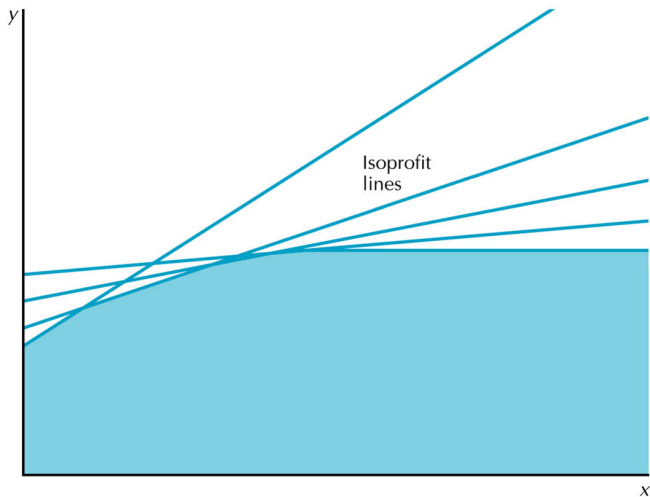
## Estimating technology using WAPM (cont'd)

White area = production plans above the isoprofit line. They are not *feasible*, so they can be part of the *technology*.



## Estimating technology using WAPM (cont'd)

The more production plans we observe at different prices, the tighter is the estimate of the *technology* (and the *production function*).



## APPLICATION: ATM withdrawal fees

Ishii (2005) “Compatibility, Competition, and Investment in Network Industries: ATM Networks in the Banking Industry”

People often pay higher withdrawal fees at ATMs of other banks (a surcharge). Does it reduce competition? Should this practice be banned by the government?

In order to answer these questions, we need to estimate a structural model and among other find out, what are the costs of ATM.

We do not observe the costs.  
We need to estimate them.



## APPLICATION: ATM withdrawal fees (cont'd)

We estimate the bank revenue as a function of the number of ATMs  $d$

It follows from the revealed profit maximization that the bank chooses  $d$  such that

$$MR \text{ of an ATM } d \geq MC \geq MR \text{ of an ATM } d + 1$$

Results:

- The size of fee surcharge influences the bank's demand.  
     $\implies$  Banks build an inefficiently large network of ATMs.
- $MC$  are higher than fee revenue. An additional ATM reduces the bank's revenue by more than by how much they increase his surplus.
- If the government banned the surcharge, competition would increase, profits decrease and consumer surplus increase = good for consumers.
- But banks would be motivated to reduce the network of ATMs, which would probably increase profits of banks and reduce CS.

## What should you know?

- Technology – all technologically feasible combinations of inputs and outputs.
- Production functions – the maximum output attainable with given inputs.
- We assume that isoquants are convex and monotonic.
- Technical rate of substitution measures the slope of the isoquant.
- SR: at least one input is fixed  
LR: all inputs are variable
- Returns to scale – how output changes if inputs are changed in the same proportion.



## What should you know? (cont'd)

- Profit maximization – what production plan maximizes firm's profit (for a given technology and input and output prices).  
**X** Cost minimization – what combination of inputs minimizes costs of a given output (for a given technology and input prices).
- Profit is the difference between revenues and costs. Implicit costs are included.
- If the firm is maximizing profits, then the value of the MP of each factor that it is free to vary must equal its factor price.
- The logic of profit maximization implies that the supply function of a competitive firm can't be decreasing and that each factor demand function can't be increasing.

