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Applied Financial Econometrics

Class 10: EWMA volatility model

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Alternative volatility model: EWMA

- Exponentially weighted moving average.
- Capable to address heteroskedasticity and volatility clustering
- This model assigns different weights to the past information, where more recent lags receive more importance than old observations.
- The weight assignation decays exponentially at a rate $0<\lambda<1$.

$$\hat{\sigma}^2_{i,t+1} = (1-\lambda)\sum_{n=0}^{\infty} \lambda^n r^2_{i,t-n}$$

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EWMA model

- If λ moves away from 1, the EWMA assigns higher weights to the recent than the past observations.
- Then, the quality of the results depends on the election of the parameter λ.
- A value greater (lower) than the optimal goes to an under-reaction (over-reaction) to the new information input.
- Equivalently, the model could be expressed by the following recurrence relation:

$$\hat{\sigma}_{i,t+1}^2 = (1-\lambda)\, r_{i,t}^2 + \lambda \sigma_{i,t}^2$$

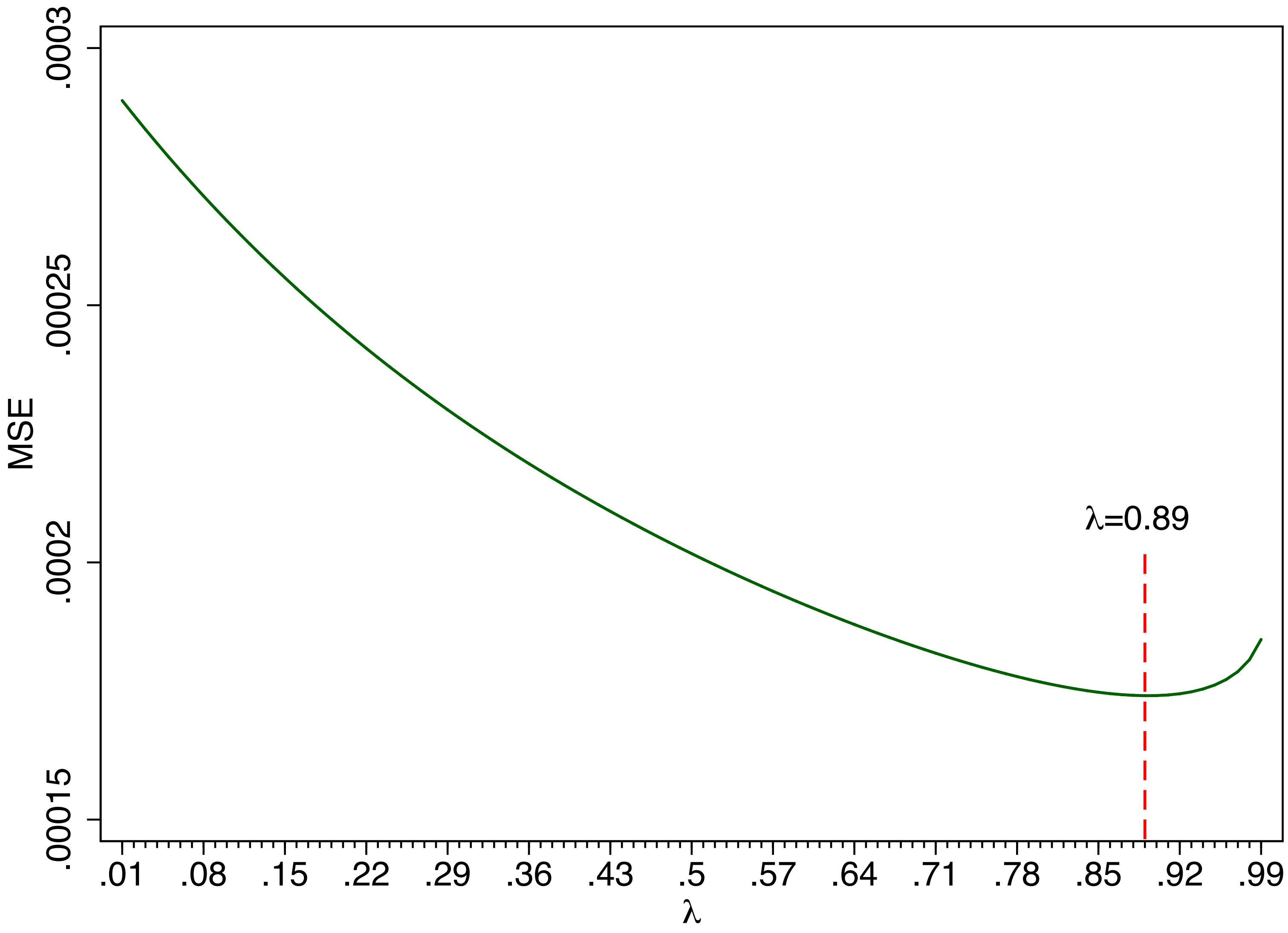
EWMA model

- The first RHS term of the previous equation updates the variance due to the new information, while the second one represents the persistence effect
- It is considered as a particular case of the of Engle and Bollerslev (1986) IGARCH(1,1), which at the same time is a restricted case of the Bollerslev (1986) standard GARCH(1,1) such that the coefficients of both lagged squared-returns and lagged variances sum one (i.e., a unit-root GARCH).

Alternative model 1: EWMA

- The EWMA method was popularized by RiskMetrics (1996), who recommends a smoothing factor equal to:
  - 0.94 for dailiy forecast.
  - 0.97 for 1-month forecast.
- These values are obtained minimizing the average squared errors for a large number of time-series

Optimal (in-sample) lambda for DJIA components for daily forecasting



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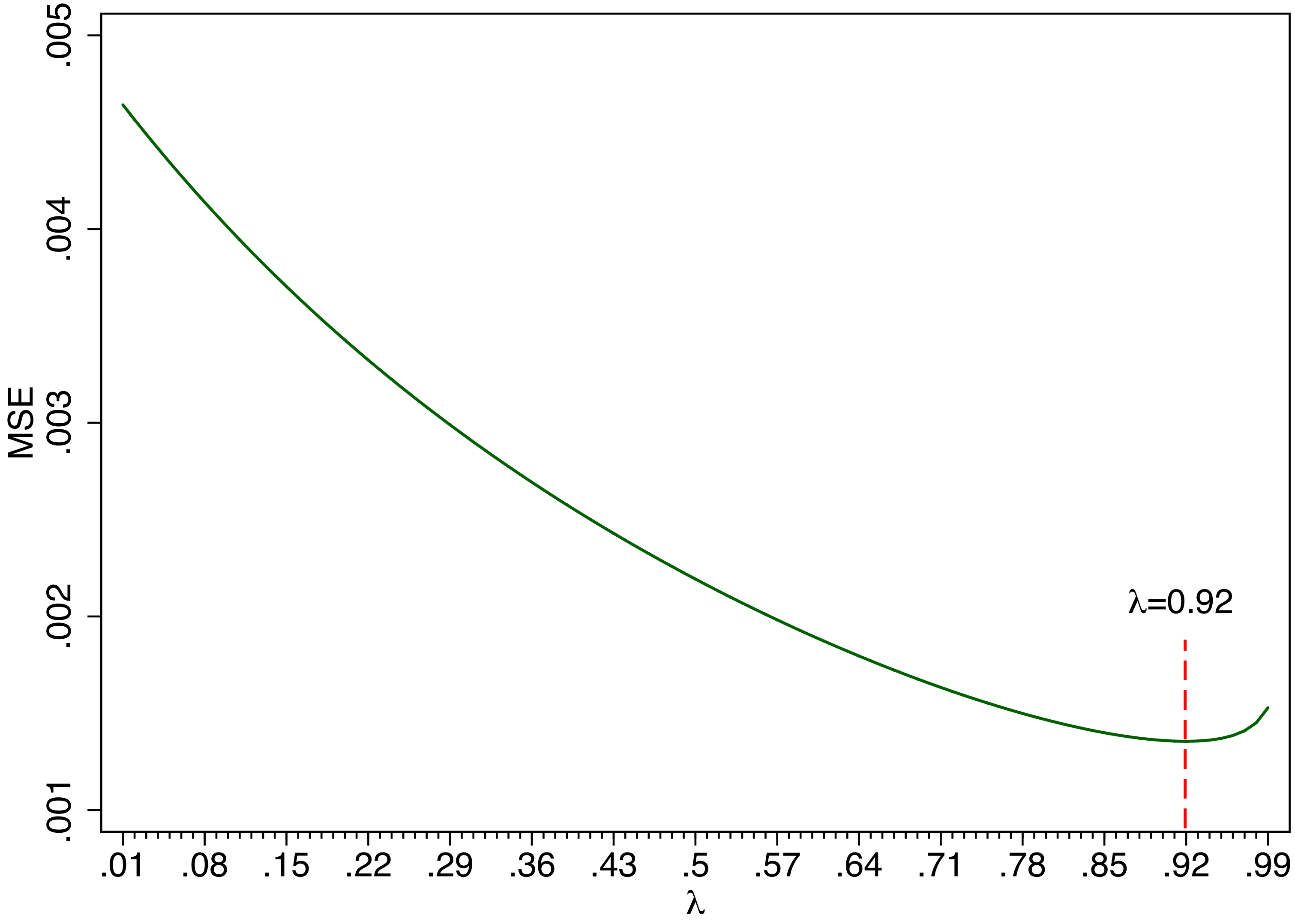
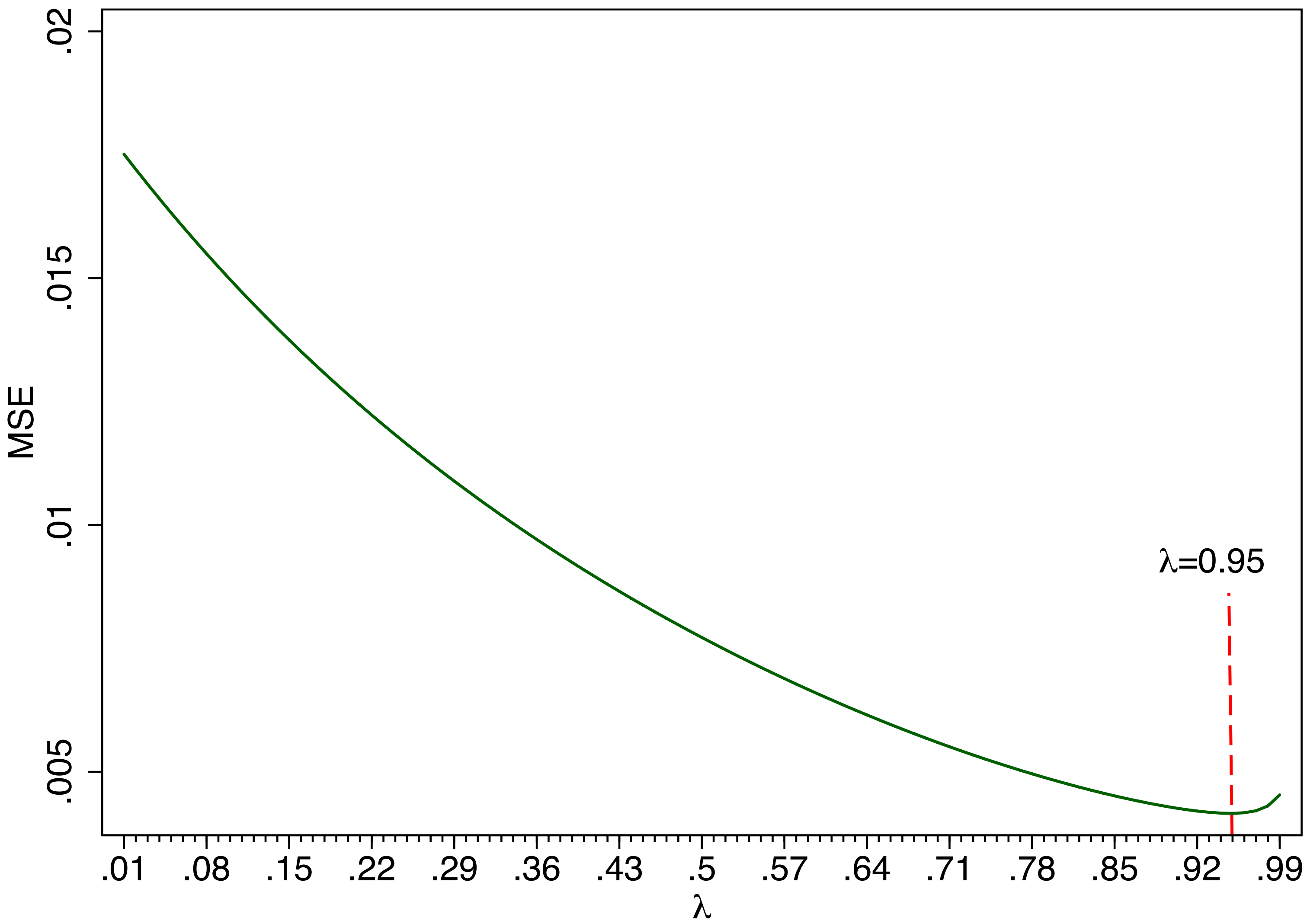
Optimal (in-sample) lambda for DJIA components for weekly forecasting



Optimal (in-sample) lambda for DJIA components for bi-weekly forecasting



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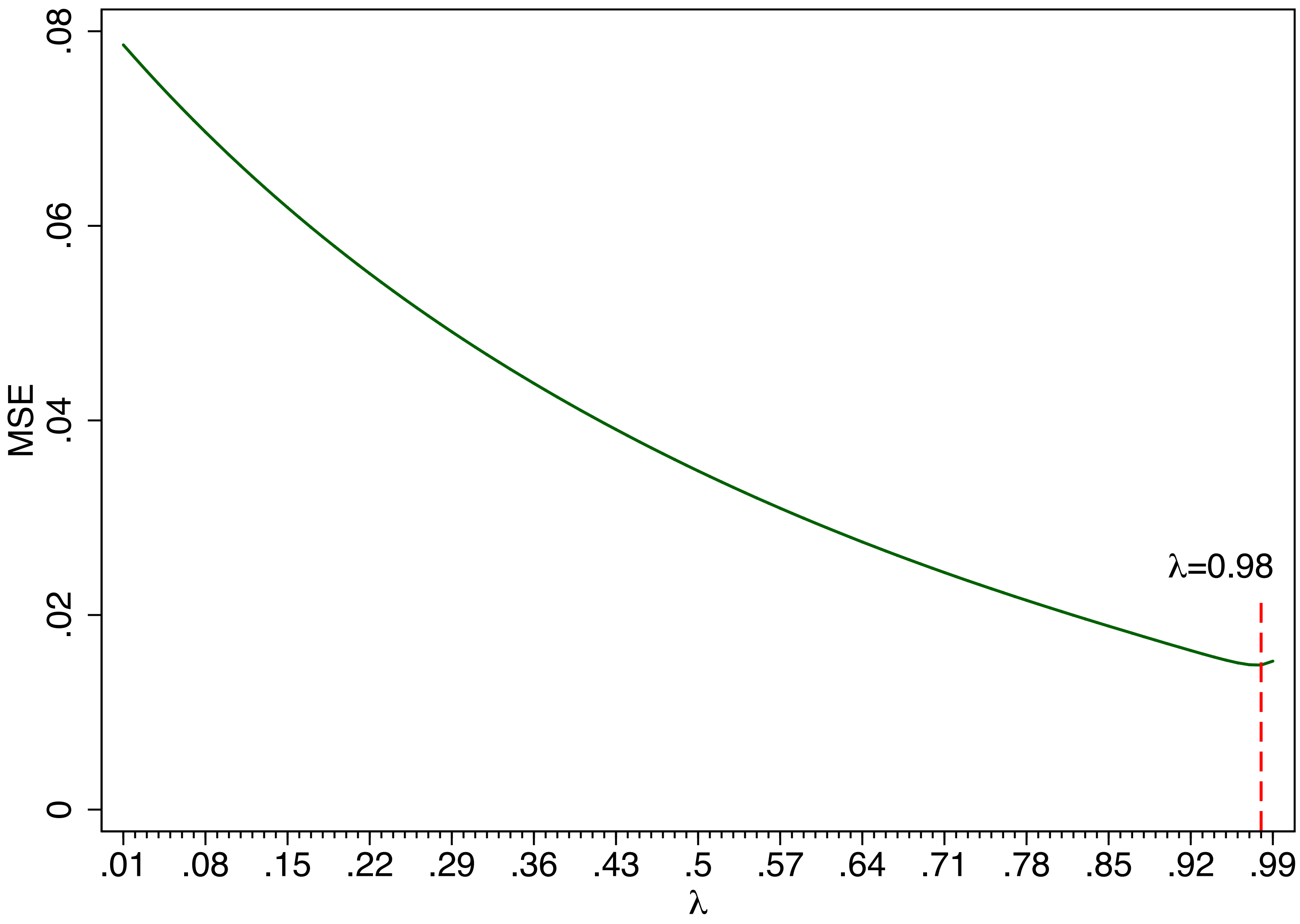
Optimal (in-sample) lambda for DJIA components for monthly forecasting



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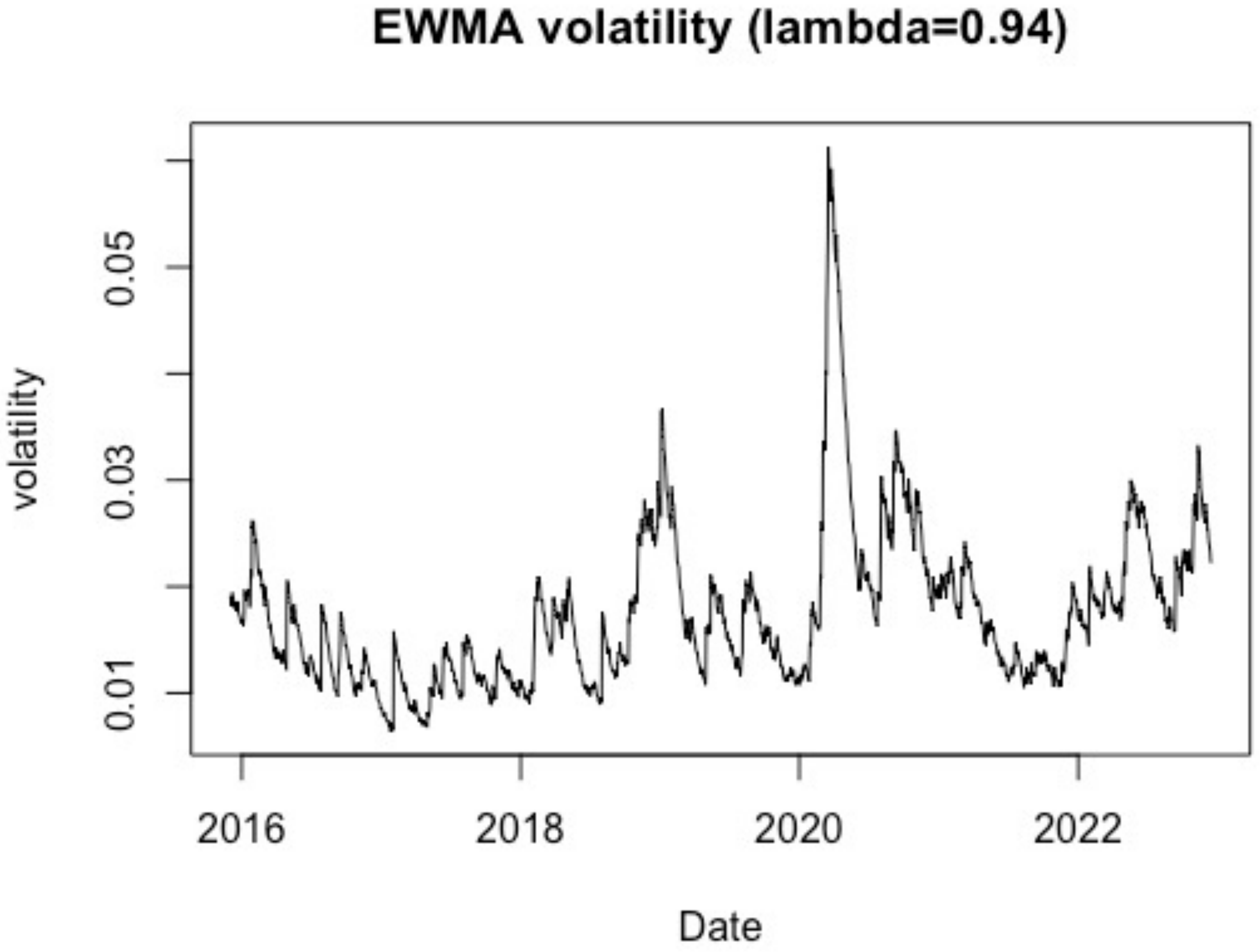
Implementing EWMA (constant lambda)

– Option 1: Quarks package

```
library('quantmod')
getSymbols(c('AAPL','^GSPC'),src='yahoo',
           from="2015-12-01",periodicity = 'daily')
r.AAPL<-diff(log(AAPL[,6]))
r.AAPL<-na.omit(r.AAPL)
```

```
install.packages("quarks")
library('quarks')
EWMA_var<-ewma(r.AAPL, lambda = 0.94)
EWMA_vol<-sqrt(EWMA_VAR)
plot(time(r.AAPL),EWMA_vol,type='l',
     main='EWMA volatility (lambda=0.94)',
     xlab='Date',ylab='Volatility')
```



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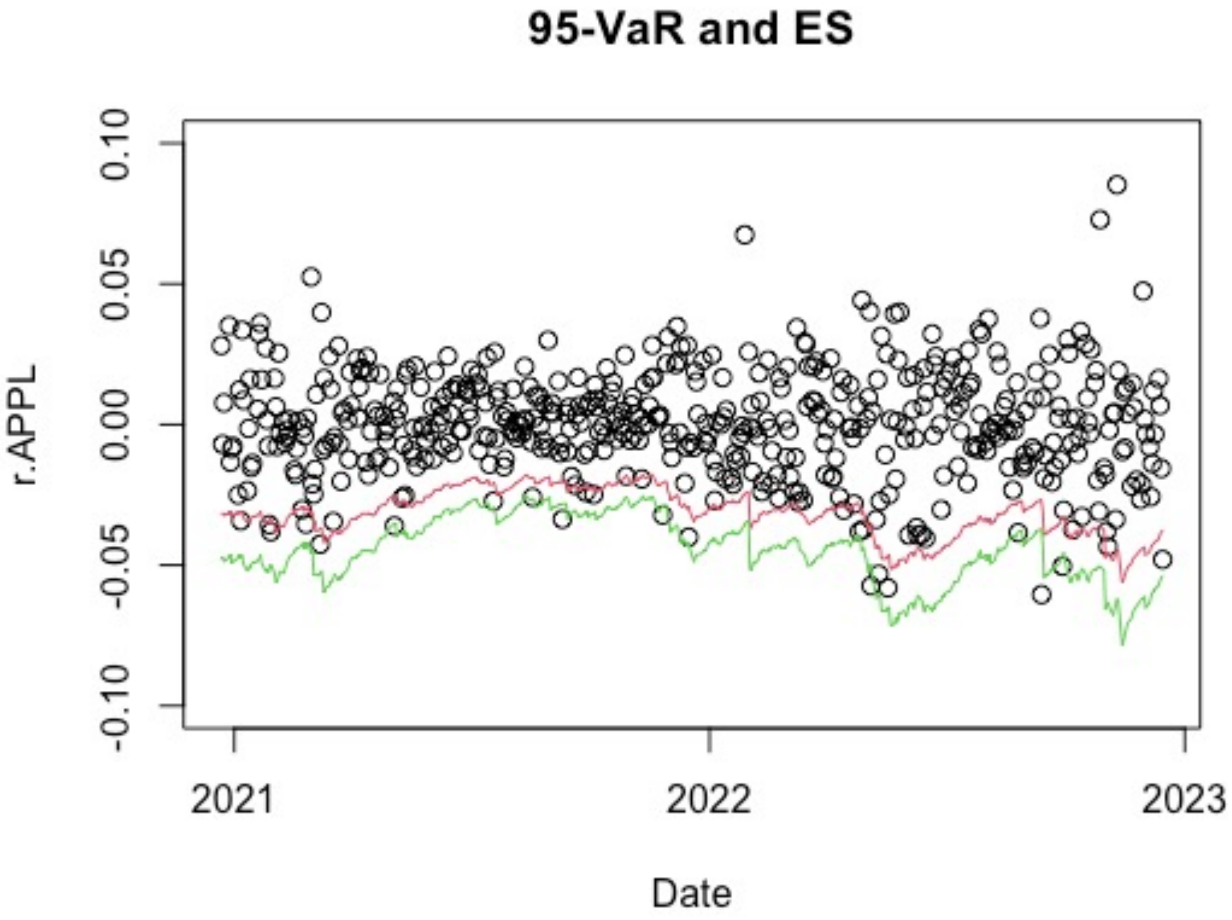
VaR and ES calculation

```
R<-as.numeric(r.AAPL)
n <- length(r.AAPL)
Rolling <- rollcast(R, p = 0.95, model = 'EWMA',
                     method = 'vwhs', lambda = 0.94,
                    nwin = 1000, nout=500)
#nout: number of obs. for out-of-sample forecasting
#nwin:  window size for rolling forecasts
#vwhs: volatility weighted historical simulation
VaR95<-Rolling$VaR
ES95<-Rolling$ES

plot(time(r.AAPL[(n-499):n]),Rolling$xout,ylim=c(-0.1,0.1),
     ylab='r.APPL',xlab='Date',
     main='95-VaR and ES')
lines(time(r.AAPL[(n-499):n]),-VaR95,type='l',col=2)
lines(time(r.AAPL[(n-499):n]),-ES95,type='l',col=3)
```



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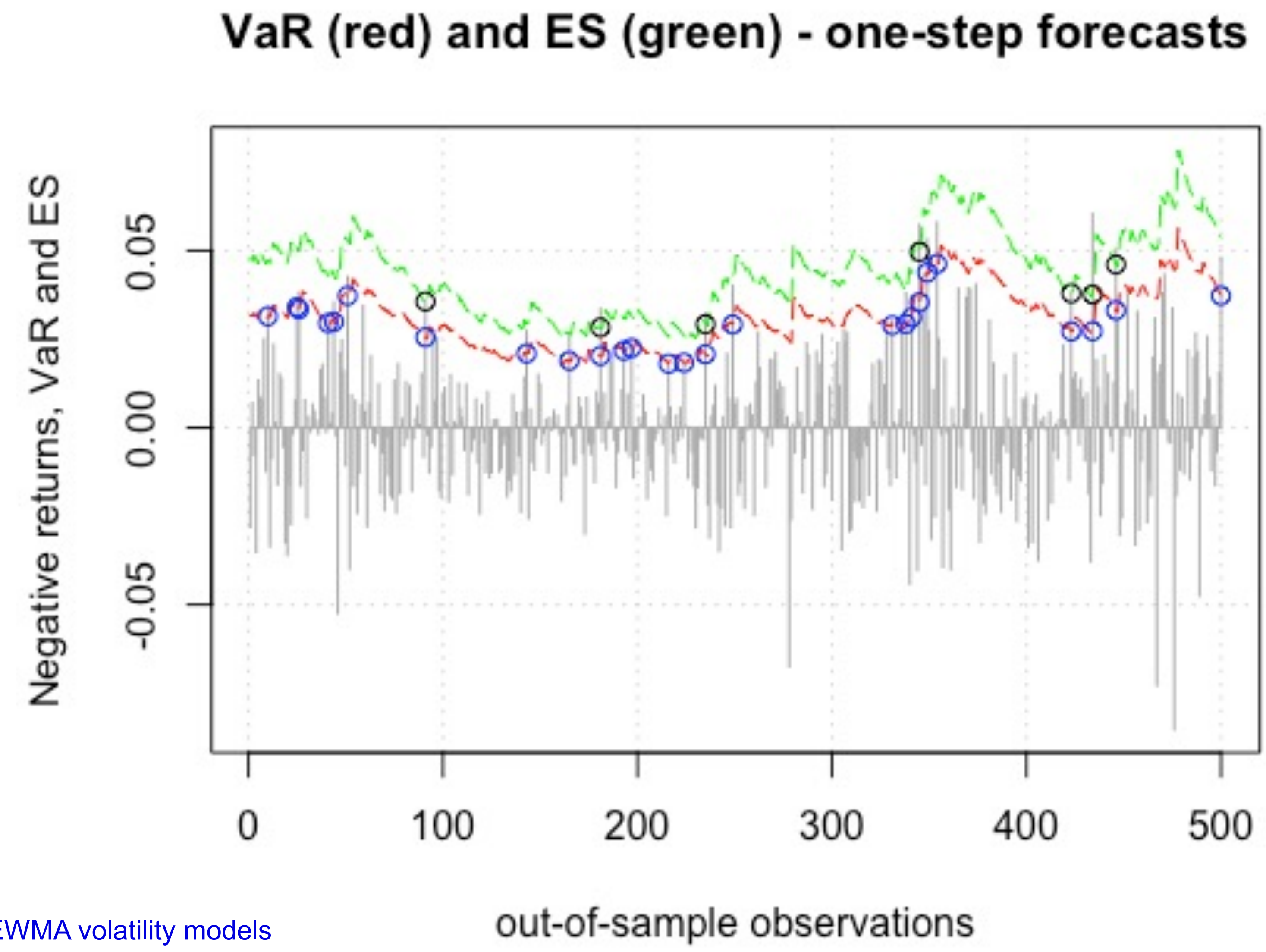
VaR and ES calculation

```
plot(Rolling)
```



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Backtesting VaR

```
cvgtest(Rolling, conflvl = 0.95)
```

```
-----------------------------------------
|             Test results              |
-----------------------------------------
 Method: Volatility Weighting
 Model:  EWMA
-----------------------------------------
|      Unconditional coverage test      |
-----------------------------------------
H0: w = 0.95

p_[uc] = 0.8384

Decision: Fail to reject H0
```

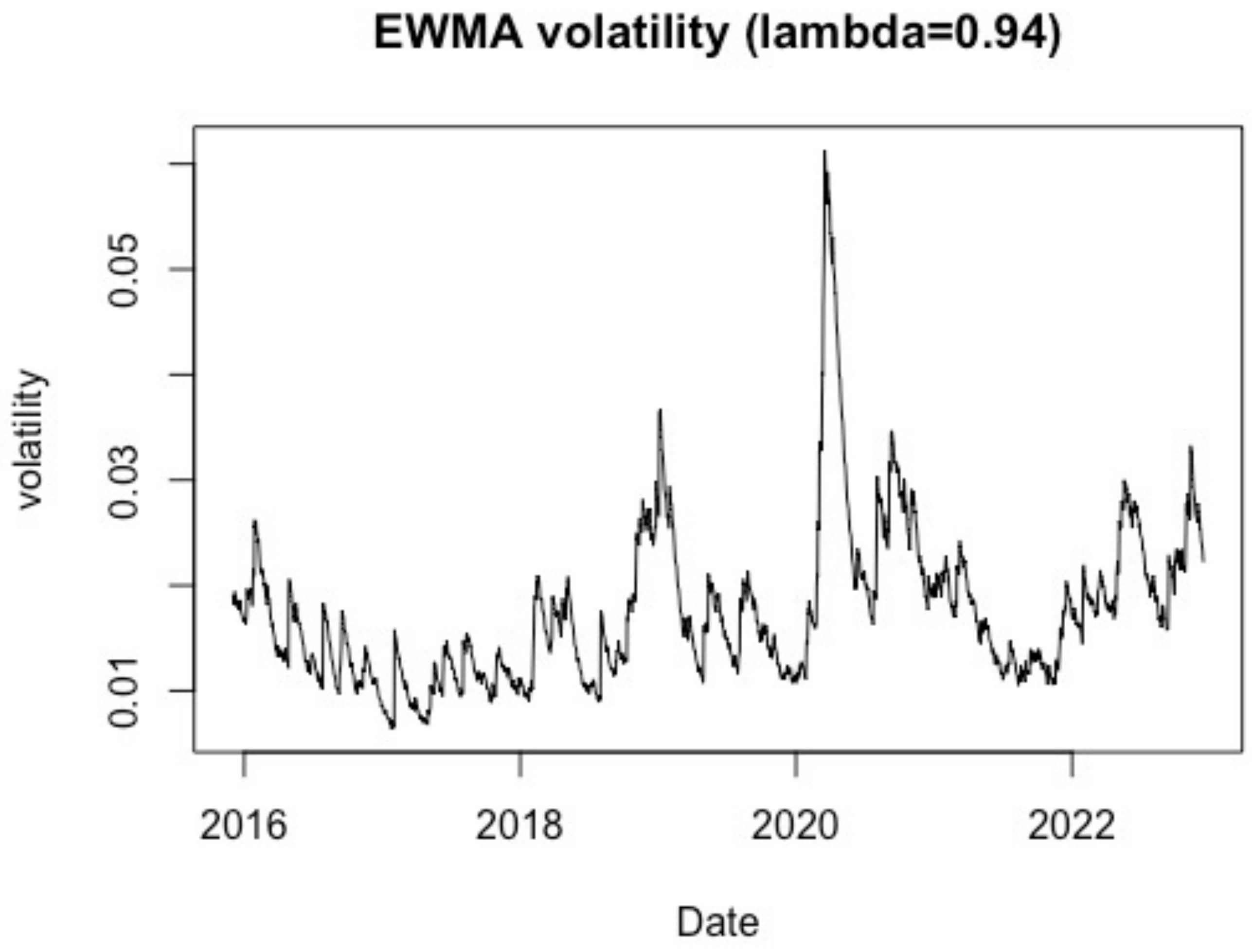
```
-----------------------------------------
|           Independence test           |
-----------------------------------------
H0: w_[00] = w_[10]

p_[ind] = 0.0137

Decision: Reject H0
```

```
-----------------------------------------
|       Conditional coverage test       |
-----------------------------------------
H0: w_[00] = w_[10] = 0.95

p_[cc] = 0.0469

Decision: Reject H0
-----------------------------------------
```

Implementing EWMA (constant lambda)

– Option 2: Rugarch package

```
library('rugarch')
spec = ugarchspec(mean.model=list(armaOrder=c(0,0),
      include.mean=FALSE),
      variance.model = list(model = "iGARCH",
      garchOrder = c(1,1)),
      distribution.model = "norm",
      fixed.pars = list(omega=0, alpha1=1-0.94))
EWMA2<-ugarchfilter(spec,r.AAPL)
EWMA2_vol<-EWMA2@filter$sigma
plot(time(r.AAPL),EWMA2_vol,type='l',
     main='EWMA volatility (lambda=0.94)',
     xlab='Date',ylab='Volatility')
```



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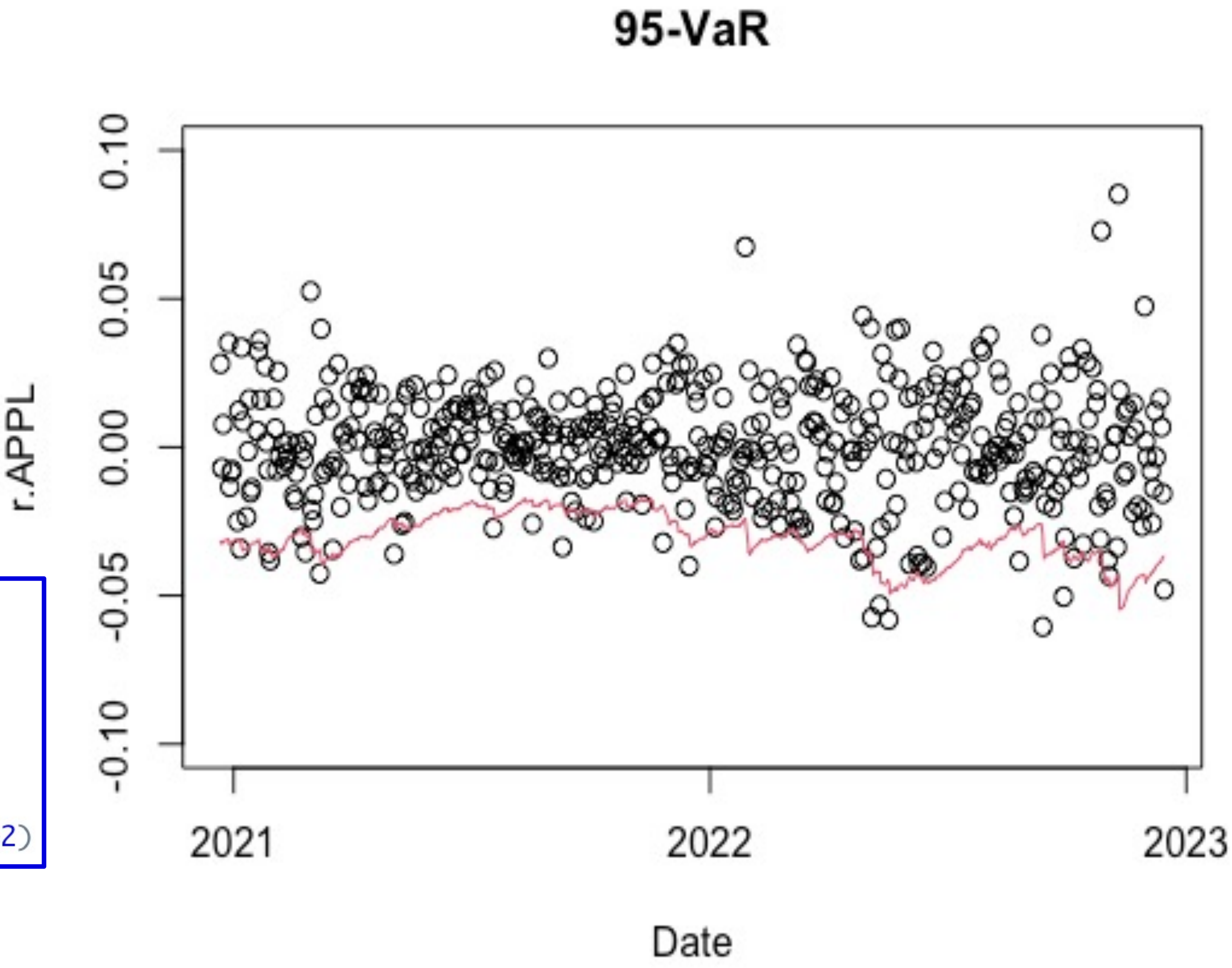
VaR Calculation

```
Q<-quantile(EWMA2,prob=0.05)
VaR_95_2<-Q[(n-499):n]
plot(time(r.AAPL[(n-499):n]),r.AAPL[(n-499):n],
     ylim=c(-0.1,0.1),
     ylab='r.APPL',xlab='Date',
     main='95-VaR')
lines(time(r.AAPL[(n-499):n]),VaR_95_2,type='l',col=2)
```



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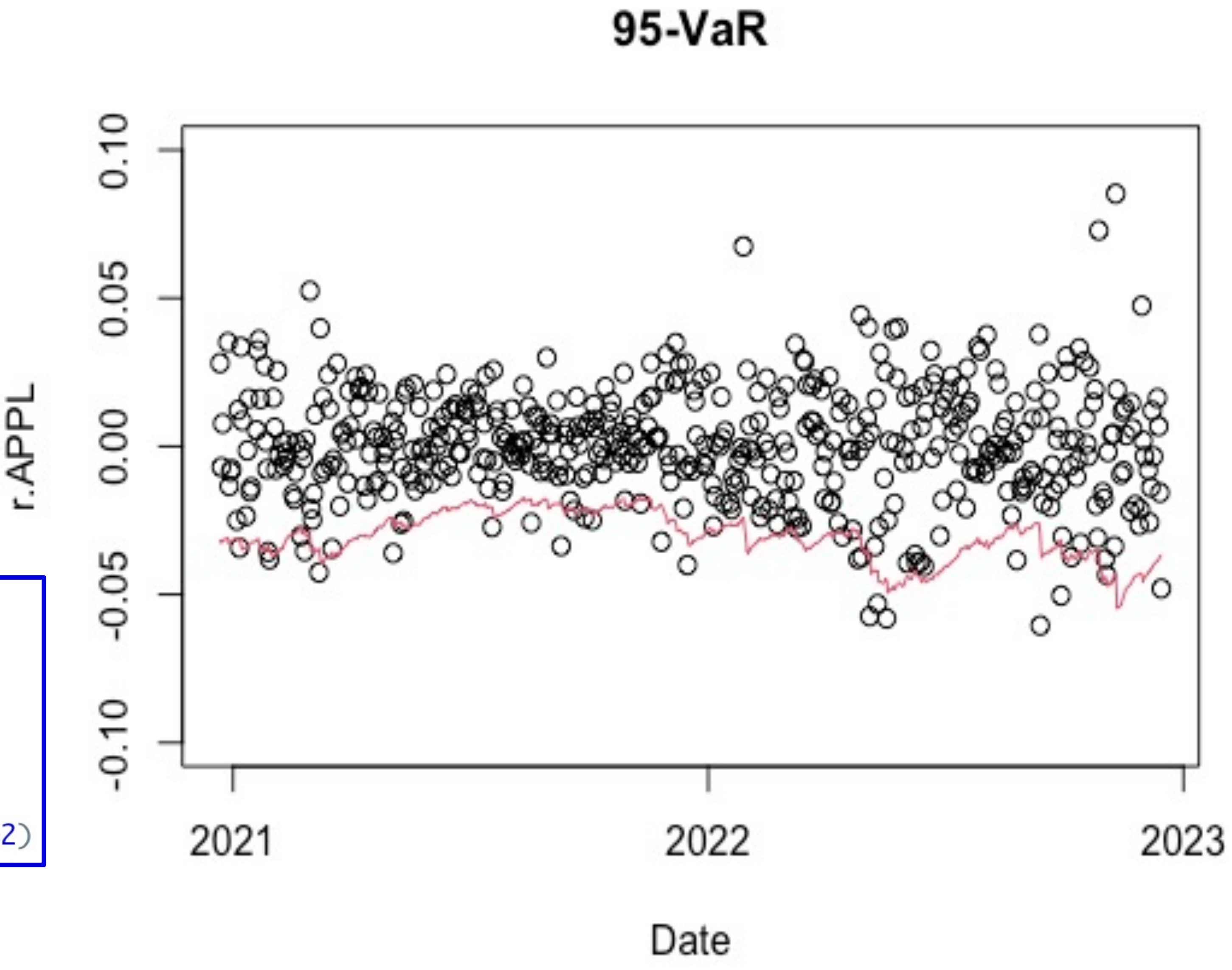
VaR Calculation

```
Q<-quantile(EWMA2,prob=0.05)
VaR_95_2<-Q[(n-499):n]
plot(time(r.AAPL[(n-499):n]),r.AAPL[(n-499):n],
     ylim=c(-0.1,0.1),
     ylab='r.APPL',xlab='Date',
     main='95-VaR')
lines(time(r.AAPL[(n-499):n]),VaR_95_2,type='l',col=2)
```



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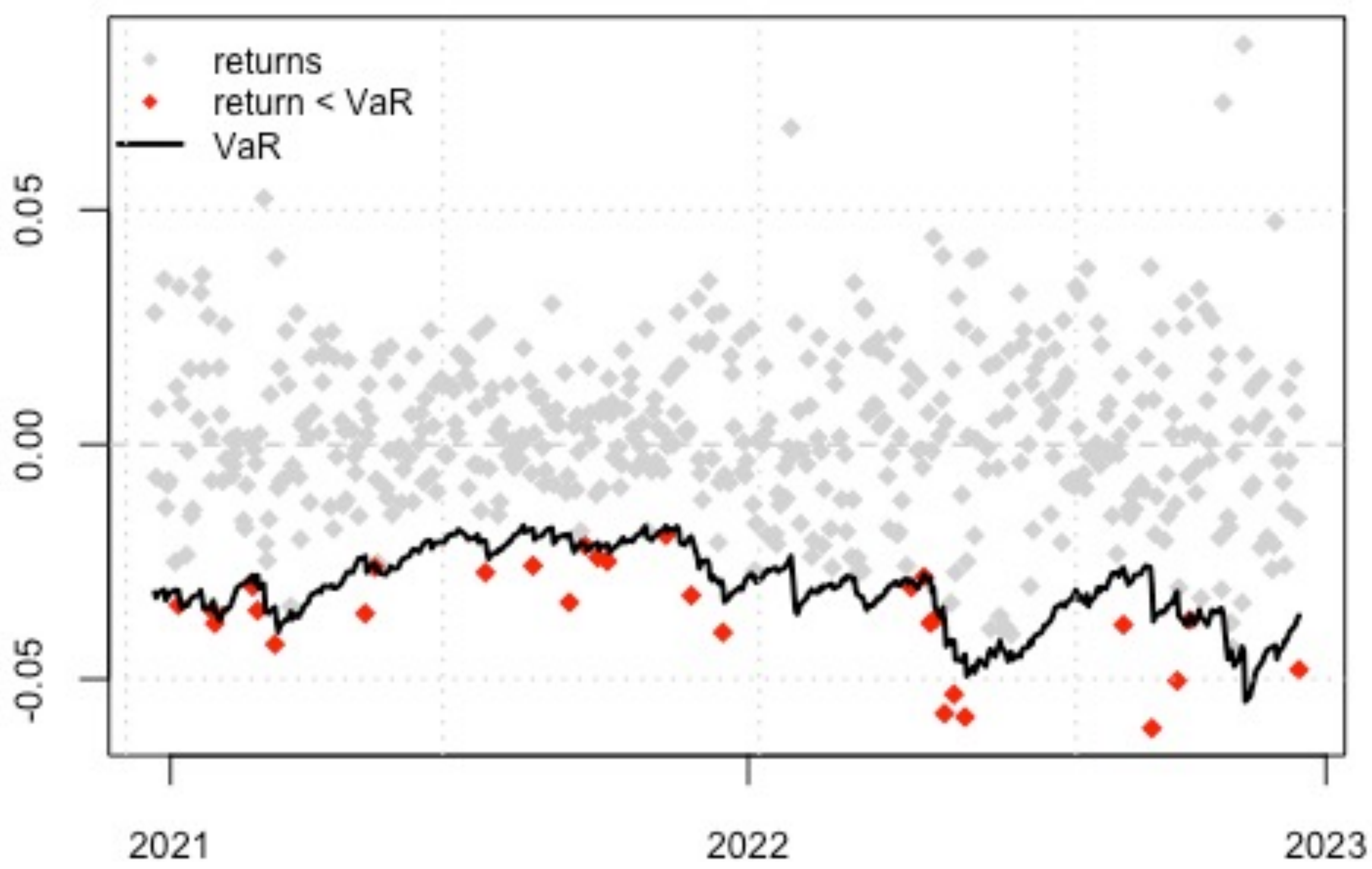
VaR Calculation

```
VaRplot(0.05,r.AAPL[(n-499):n],VaR_95_2)
```



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Backtesting VaR

```
VaRTest(0.05,r.AAPL[(n-499):n],VaR_95_2, conf.level = 0.95)
```

```
$expected.exceed
[1] 25

$actual.exceed
[1] 29

$uc.H0
[1] "Correct Exceedances"

$uc.LRstat
[1] 0.6421395

$uc.critical
[1] 3.841459

$uc.LRp
[1] 0.4229371

$uc.Decision
[1] "Fail to Reject H0"
```

```
$cc.H0
[1] "Correct Exceedances & Independent"

$cc.LRstat
[1] 0.9528143

$cc.critical
[1] 5.991465

$cc.LRp
[1] 0.6210106

$cc.Decision
[1] "Fail to Reject H0"
```

References

– Tim Bollerslev. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31 (3):307–327, 1986.
– Robert F. Engle and Tim Bollerslev. Modelling the persistence of conditional variances. *Econometric Reviews*, 5(1):1–50, 1986.
– RiskMetrics. Technical Document, J.P.Morgan/Reuters, New York, 1996. Fourth Edition.

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