CHAPTER 2

CAPITAL BUDGETING

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LEARNING OUTCOMES

After completing this chapter, you will be able to do the following:

- Describe the capital budgeting process, including the typical steps of the process, and distinguish among the various categories of capital projects.
- Describe the basic principles of capital budgeting, including cash flow estimation.
- Explain how the evaluation and selection of capital projects is affected by mutually exclusive projects, project sequencing, and capital rationing.
- Calculate and interpret the results using each of the following methods to evaluate a single capital project: net present value (NPV), internal rate of return (IRR), payback period, discounted payback period, average accounting rate of return (AAR), and profitability index (PI).
- Explain the NPV profile, compare NPV and IRR methods when evaluating independent and mutually exclusive projects, and describe the problems associated with each of the evaluation methods.
- Describe the relative popularity of the various capital budgeting methods and explain the relation between NPV and company value and stock price.
- Describe the expected relations among an investment's NPV, company value, and stock price.
- Calculate the yearly cash flows of an expansion capital project and a replacement capital project, and evaluate how the choice of depreciation method affects those cash flows.
- Explain the effects of inflation on capital budgeting analysis.
- Evaluate and select the optimal capital project in situations of (1) mutually exclusive projects with unequal lives, using either the least common multiple of lives approach or the equivalent annual annuity approach, and (2) capital rationing.
- Explain how sensitivity analysis, scenario analysis, and Monte Carlo simulation can be used to estimate the standalone risk of a capital project.

- Explain the procedure for determining the discount rate to be used in valuing a capital project and calculate a project's required rate of return using the capital asset pricing model (CAPM).
- Describe the types of real options and evaluate the profitability of investments with real options.
- Explain capital budgeting pitfalls.
- Calculate and interpret accounting income and economic income in the context of capital budgeting.
- Distinguish among and evaluate a capital project using the economic profit, residual income, and claims valuation models.

1. INTRODUCTION

Capital budgeting is the process that companies use for decision making on capital projects—those projects with a life of a year or more. This is a fundamental area of knowledge for financial analysts for many reasons:

- First, capital budgeting is very important for corporations. Capital projects, which make up the long-term asset portion of the balance sheet, can be so large that sound capital budgeting decisions ultimately decide the future of many corporations. Capital decisions cannot be reversed at a low cost, so mistakes are very costly. Indeed, the real capital investments of a company describe a company better than its working capital or capital structures, which are intangible and tend to be similar for many corporations.
- Second, the principles of capital budgeting have been adapted for many other corporate decisions, such as investments in working capital, leasing, mergers and acquisitions, and bond refunding.
- Third, the valuation principles used in capital budgeting are similar to the valuation principles used in security analysis and portfolio management. Many of the methods used by security analysts and portfolio managers are based on capital budgeting methods. Conversely, there have been innovations in security analysis and portfolio management that have also been adapted to capital budgeting.
- Finally, although analysts have a vantage point outside the company, their interest in valuation coincides with the capital budgeting focus of maximizing shareholder value. Because capital budgeting information is not ordinarily available outside the company, the analyst may attempt to estimate the process, within reason, at least for companies that are not too complex. Further, analysts may be able to appraise the quality of the company's capital budgeting process; for example, on the basis of whether the company has an accounting focus or an economic focus.

This chapter is organized as follows: Section 2 presents the steps in a typical capital budgeting process. After introducing the basic principles of capital budgeting in Section 3, in Section 4 we discuss the criteria by which a decision to invest in a project may be made. Section 5 presents a crucial element of the capital budgeting process: organizing the cash flow information that is the raw material of the analysis. Section 6 looks further at cash flow analysis. Section 7 demonstrates methods to extend the basic investment criteria to address economic alternatives and risk. Finally, Section 8 compares other income measures and valuation models that analysts use to the basic capital budgeting model.

2. THE CAPITAL BUDGETING PROCESS

The specific capital budgeting procedures that a manager uses depend on the manager's level in the organization, the size and complexity of the project being evaluated, and the size of the organization. The typical steps in the capital budgeting process are as follows:

- *Step 1, Generating Ideas*. Investment ideas can come from anywhere, from the top or the bottom of the organization, from any department or functional area, or from outside the company. Generating good investment ideas to consider is the most important step in the process.
- *Step 2, Analyzing Individual Proposals*. This step involves gathering the information to forecast cash flows for each project and then evaluating the project's profitability.
- *Step 3, Planning the Capital Budget*. The company must organize the profitable proposals into a coordinated whole that fits within the company's overall strategies, and it also must consider the projects' timing. Some projects that look good when considered in isolation may be undesirable strategically. Because of financial and real resource issues, scheduling and prioritizing projects is important.
- *Step 4, Monitoring and Post-Auditing*. In a post-audit, actual results are compared to planned or predicted results, and any differences must be explained. For example, how do the revenues, expenses, and cash flows realized from an investment compare to the predictions? Post-auditing capital projects is important for several reasons. First, it helps monitor the forecasts and analysis that underlie the capital budgeting process. Systematic errors, such as overly optimistic forecasts, become apparent. Second, it helps improve business operations. If sales or costs are out of line, it will focus attention on bringing performance closer to expectations if at all possible. Finally, monitoring and post-auditing recent capital investments will produce concrete ideas for future investments. Managers can decide to invest more heavily in profitable areas and scale down or cancel investments in areas that are disappointing.

Planning for capital investments can be very complex, often involving many persons inside and outside of the company. Information about marketing, science, engineering, regulation, taxation, finance, production, and behavioral issues must be systematically gathered and evaluated. The authority to make capital decisions depends on the size and complexity of the project. Lower-level managers may have discretion to make decisions that involve less than a given amount of money, or that do not exceed a given capital budget. Larger and more complex decisions are reserved for top management, and some are so significant that the company's board of directors ultimately has the decision-making authority.

Like everything else, capital budgeting is a cost–benefit exercise. At the margin, the benefits from the improved decision making should exceed the costs of the capital budgeting efforts.

Companies often put capital budgeting projects into some rough categories for analysis. One such classification would be as follows:

1. *Replacement projects*. These are among the easier capital budgeting decisions. If a piece of equipment breaks down or wears out, whether to replace it may not require careful analysis. If the expenditure is modest and if not investing has significant implications for production, operations, or sales, it would be a waste of resources to overanalyze the decision. Just make the replacement. Other replacement decisions involve replacing

existing equipment with newer, more efficient equipment, or perhaps choosing one type of equipment over another. These replacement decisions are often amenable to very detailed analysis, and you might have a lot of confidence in the final decision.
2. *Expansion projects*. Instead of merely maintaining a company's existing business activities, expansion projects increase the size of the business. These expansion decisions may involve more uncertainties than replacement decisions, and these decisions will be more carefully considered.
3. *New products and services*. These investments expose the company to even more uncertainties than expansion projects. These decisions are more complex and will involve more people in the decision-making process.
4. *Regulatory, safety, and environmental projects*. These projects are frequently required by a governmental agency, an insurance company, or some other external party. They may generate no revenue and might not be undertaken by a company maximizing its own private interests. Often, the company will accept the required investment and continue to operate. Occasionally, however, the cost of the regulatory/safety/environmental project is sufficiently high that the company would do better to cease operating altogether or to shut down any part of the business that is related to the project.
5. *Other*. The projects above are all susceptible to capital budgeting analysis, and they can be accepted or rejected using the net present value (NPV) or some other criterion. Some projects escape such analysis. These are either pet projects of someone in the company (such as the CEO buying a new aircraft) or so risky that they are difficult to analyze by the usual methods (such as some research and development decisions).

3. BASIC PRINCIPLES OF CAPITAL BUDGETING

Capital budgeting has a rich history and sometimes employs some pretty sophisticated procedures. Fortunately, capital budgeting relies on just a few basic principles. Capital budgeting usually uses the following five assumptions:

1. *Decisions are based on cash flows*. The decisions are not based on accounting concepts, such as net income. Furthermore, intangible costs and benefits are often ignored because, if they are real, they should result in cash flows at some other time.
2. *Timing of cash flows is crucial*. Analysts make an extraordinary effort to detail precisely when cash flows occur.
3. *Cash flows are based on opportunity costs*. What are the incremental cash flows that occur with an investment compared to what they would have been without the investment?
4. *Cash flows are analyzed on an after-tax basis*. Taxes must be fully reflected in all capital budgeting decisions.
5. *Financing costs are ignored*. This may seem unrealistic, but it is not. Most of the time, analysts want to know the after-tax operating cash flows that result from a capital investment. Then, these after-tax cash flows and the investment outlays are discounted at the "required rate of return" to find the net present value (NPV). Financing costs are reflected in the required rate of return. If we included financing costs in the cash flows and in the discount rate, we would be double-counting the financing costs. So even though a project may be financed with some combination of debt and equity, we ignore these costs, focusing on the operating cash flows and capturing the costs of debt (and other capital) in the discount rate.

Capital budgeting cash flows are not accounting net income. Accounting net income is reduced by noncash charges such as accounting depreciation. Furthermore, to reflect the cost of debt financing, interest expenses are also subtracted from accounting net income. (No subtraction is made for the cost of equity financing in arriving at accounting net income.) Accounting net income also differs from economic income, which is the cash inflow plus the change in the market value of the company. Economic income does not subtract the cost of debt financing, and it is based on the changes in the market value of the company, not changes in its book value (accounting depreciation). We will further consider cash flows, accounting income, economic income, and other income measures at the end of this chapter.

In assumption 5 above, we referred to the rate used in discounting the cash flows as the "required rate of return." The required rate of return is the discount rate that investors should require given the riskiness of the project. This discount rate is frequently called the "opportunity cost of funds" or the "cost of capital." If the company can invest elsewhere and earn a return of r, or if the company can repay its sources of capital and save a cost of r, then r is the company's opportunity cost of funds. If the company cannot earn more than its opportunity cost of funds on an investment, it should not undertake that investment. Unless an investment earns more than the cost of funds from its suppliers of capital, the investment should not be undertaken. The cost-of-capital concept is discussed more extensively elsewhere. Regardless of what it is called, an economically sound discount rate is essential for making capital budgeting decisions.

Although the principles of capital budgeting are simple, they are easily confused in practice, leading to unfortunate decisions. Some important capital budgeting concepts that managers find very useful are given below.

- A **sunk cost** is one that has already been incurred. You cannot change a sunk cost. Today's decisions, on the other hand, should be based on current and future cash flows and should not be affected by prior, or sunk, costs.
- An **opportunity cost** is what a resource is worth in its next-best use. For example, if a company uses some idle property, what should it record as the investment outlay: the purchase price several years ago, the current market value, or nothing? If you replace an old machine with a new one, what is the opportunity cost? If you invest \$10 million, what is the opportunity cost? The answers to these three questions are, respectively: the current market value, the cash flows the old machine would generate, and \$10 million (which you could invest elsewhere).
- An **incremental cash flow** is the cash flow that is realized because of a decision: the cash flow *with* a decision minus the cash flow *without* that decision. If opportunity costs are correctly assessed, the incremental cash flows provide a sound basis for capital budgeting.
- An **externality** is the effect of an investment on other things besides the investment itself. Frequently, an investment affects the cash flows of other parts of the company, and these externalities can be positive or negative. If possible, these should be part of the investment decision. Sometimes externalities occur outside of the company. An investment might benefit (or harm) other companies or society at large, and yet the company is not compensated for these benefits (or charged for the costs). **Cannibalization** is one externality. Cannibalization occurs when an investment takes customers and sales away from another part of the company.
- **Conventional** versus **nonconventional cash flows**. A conventional cash flow pattern is one with an initial outflow followed by a series of inflows. In a nonconventional cash flow pattern, the initial outflow is not followed by inflows only, but the cash flows can flip from positive to negative again (or even change signs several times). An investment that involved outlays (negative cash flows) for the first couple of years that were then followed by positive

cash flows would be considered to have a conventional pattern. If cash flows change signs once, the pattern is conventional. If cash flows change signs two or more times, the pattern is nonconventional.

Several types of project interactions make the incremental cash flow analysis challenging. The following are some of these interactions:

- **Independent** versus **mutually exclusive projects**. Independent projects are projects whose cash flows are independent of each other. Mutually exclusive projects compete directly with each other. For example, if Projects A and B are mutually exclusive, you can choose A or B, but you cannot choose both. Sometimes there are several mutually exclusive projects, and you can choose only one from the group.
- **Project sequencing**. Many projects are sequenced through time, so that investing in a project creates the option to invest in future projects. For example, you might invest in a project today and then in one year invest in a second project if the financial results of the first project or new economic conditions are favorable. If the results of the first project or new economic conditions are not favorable, you do not invest in the second project.
- **Unlimited funds** versus **capital rationing**. An unlimited funds environment assumes that the company can raise the funds it wants for all profitable projects simply by paying the required rate of return. Capital rationing exists when the company has a fixed amount of funds to invest. If the company has more profitable projects than it has funds for, it must allocate the funds to achieve the maximum shareholder value subject to the funding constraints.

4. INVESTMENT DECISION CRITERIA

Analysts use several important criteria to evaluate capital investments. The two most comprehensive measures of whether a project is profitable or unprofitable are the net present value (NPV) and internal rate of return (IRR). In addition to these, we present four other criteria that are frequently used: the payback period, discounted payback period, average accounting rate of return (AAR), and profitability index (PI). An analyst must fully understand the economic logic behind each of these investment decision criteria as well as its strengths and limitations in practice.

4.1. Net Present Value

For a project with one investment outlay, made initially, the **net present value** (**NPV**) is the present value of the future after-tax cash flows minus the investment outlay, or

$$\text{NPV} = \sum_{t=1}^{n} \frac{\text{CF}_t}{(1+r)^t} - \text{Outlay} \qquad (2\text{-}1)$$

where

CF_t = after-tax cash flow at time t
r = required rate of return for the investment
Outlay = investment cash flow at time zero

To illustrate the net present value criterion, we will take a look at a simple example. Assume that Gerhardt Corporation is considering an investment of €50 million in a capital project that will return after-tax cash flows of €16 million per year for the next four years plus another €20 million in year five. The required rate of return is 10 percent.

For the Gerhardt example, the NPV would be

$$\text{NPV} = \frac{16}{1.10^1} + \frac{16}{1.10^2} + \frac{16}{1.10^3} + \frac{16}{1.10^4} + \frac{20}{1.10^5} - 50$$

$$\text{NPV} = 14.545 + 13.223 + 12.021 + 10.928 + 12.418 - 50$$

$$\text{NPV} = 63.136 - 50 = €13.136 \text{ million}^1$$

The investment has a total value, or present value of future cash flows, of €63.136 million. Since this investment can be acquired at a cost of €50 million, the investing company is giving up €50 million of its wealth in exchange for an investment worth €63.136 million. The investor's wealth increases by a net of €13.136 million.

Because the NPV is the amount by which the investor's wealth increases as a result of the investment, the decision rule for the NPV is as follows:

Invest if	$\text{NPV} > 0$
Do not invest if	$\text{NPV} < 0$

Positive NPV investments are wealth-increasing, while negative NPV investments are wealth-decreasing.

Many investments have cash flow patterns in which outflows may occur not only at time zero, but also at future dates. It is useful to consider the NPV to be the present value of all cash flows:,

$$\text{NPV} = \text{CF}_0 + \frac{\text{CF}_1}{(1+r)^1} + \frac{\text{CF}}{(1+r)^2} + \cdots + \frac{\text{CF}_n}{(1+r)^n}, \text{ or}$$

$$\text{NPV} = \sum_{t=0}^{n} \frac{\text{CF}_t}{(1+r)^t} \tag{2-2}$$

In Equation 2-2, the investment outlay, CF_0, is simply a negative cash flow. Future cash flows can also be negative.

4.2. Internal Rate of Return

The internal rate of return (IRR) is one of the most frequently used concepts in capital budgeting and in security analysis. The IRR definition is one that all analysts know by heart. For a project with one investment outlay, made initially, the IRR is the discount rate that

[1]Occasionally, you will notice some rounding errors in our examples. In this case, the present values of the cash flows, as rounded, add up to 63.135. Without rounding, they add up to 63.13627, or 63.136. We will usually report the more accurate result, the one that you would get from your calculator or computer without rounding intermediate results.

makes the present value of the future after-tax cash flows equal that investment outlay. Written out in equation form, the IRR solves this equation:

$$\sum_{t=1}^{n} \frac{CF_t}{(1+\mathrm{IRR})^t} = \text{Outlay}$$

where IRR is the internal rate of return. The left-hand side of this equation is the present value of the project's future cash flows, which, discounted at the IRR, equals the investment outlay. This equation will also be seen rearranged as

$$\sum_{t=1}^{n} \frac{CF_t}{(1+\mathrm{IRR})^t} - \text{Outlay} = 0 \tag{2-3}$$

In this form, Equation 2-3 looks like the NPV equation, Equation 2-1, except that the discount rate is the IRR instead of r (the required rate of return). Discounted at the IRR, the NPV is equal to zero.

In the Gerhardt Corporation example, we want to find a discount rate that makes the total present value of all cash flows, the NPV, equal zero. In equation form, the IRR is the discount rate that solves this equation:

$$-50 + \frac{16}{(1+\mathrm{IRR})^1} + \frac{16}{(1+\mathrm{IRR})^2} + \frac{16}{(1+\mathrm{IRR})^3} + \frac{16}{(1+\mathrm{IRR})^4} + \frac{20}{(1+\mathrm{IRR})^5} = 0$$

Algebraically, this equation would be very difficult to solve. We normally resort to trial and error, systematically choosing various discount rates until we find one, the IRR, that satisfies the equation. We previously discounted these cash flows at 10 percent and found the NPV to be €13.136 million. Since the NPV is positive, the IRR is probably greater than 10 percent. If we use 20 percent as the discount rate, the NPV is –€0.543 million, so 20 percent is a little high. One might try several other discount rates until the NPV is equal to zero; this approach is illustrated in Exhibit 2-1.

EXHIBIT 2-1 Trial and Error Process for Finding IRR

Discount Rate	NPV
10%	13.136
20%	–0.543
19%	0.598
19.5%	0.022
19.51%	0.011
19.52%	0.000

The IRR is 19.52 percent. Financial calculators and spreadsheet software have routines that calculate the IRR for us, so we do not have to go through this trial and error procedure ourselves. The IRR, computed more precisely, is 19.5197 percent.

The decision rule for the IRR is to invest if the IRR exceeds the required rate of return for a project:

Invest if $\quad$ IRR $> r$

Do not invest if $\quad$ IRR $< r$

In the Gerhardt example, since the IRR of 19.52 percent exceeds the project's required rate of return of 10 percent, Gerhardt should invest.

Many investments have cash flow patterns in which the outlays occur at time zero and at future dates. Thus, it is common to define the IRR as the discount rate that makes the present values of all cash flows sum to zero:

$$\sum_{t=0}^{n} \frac{\mathrm{CF}_t}{(1+\mathrm{IRR})^t} = 0 \tag{2-4}$$

Equation 2-4 is a more general version of Equation 2-3.

4.3. Payback Period

The payback period is the number of years required to recover the original investment in a project. The payback is based on cash flows. For example, if you invest $10 million in a project, how long will it be until you recover the full original investment? Exhibit 2-2 illustrates the calculation of the payback period by following an investment's cash flows and cumulative cash flows.

EXHIBIT 2-2 Payback Period Example

Year	0	1	2	3	4	5
Cash flow	–10,000	2,500	2,500	3,000	3,000	3,000
Cumulative cash flow	–10,000	–7,500	–5,000	–2,000	1,000	4,000

In the first year, the company recovers 2,500 of the original investment, with 7,500 still unrecovered. You can see that the company recoups its original investment between Year 3 and Year 4. After three years, 2,000 is still unrecovered. Since the Year 4 cash flow is 3,000, it would take two-thirds of the Year 4 cash flow to bring the cumulative cash flow to zero. So, the payback period is three years plus two-thirds of the Year 4 cash flow, or 3.67 years.

The drawbacks of the payback period are transparent. Since the cash flows are not discounted at the project's required rate of return, the payback period ignores the time value of money and the risk of the project. Additionally, the payback period ignores cash flows after the payback period is reached. In Exhibit 2-2, for example, the Year 5 cash flow is completely ignored in the payback computation!

Example 2-1 is designed to illustrate some of the implications of these drawbacks of the payback period.

EXAMPLE 2-1 Drawbacks of the Payback Period

The cash flows, payback periods, and NPVs for Projects A through F are given in Exhibit 2-3. For all of the projects, the required rate of return is 10 percent.

EXHIBIT 2-3 Examples of Drawbacks of the Payback Period

Year	Cash Flows					
	Project A	Project B	Project C	Project D	Project E	Project F
0	–1,000	–1,000	–1,000	–1,000	–1,000	–1,000
1	1,000	100	400	500	400	500
2		200	300	500	400	500
3		300	200	500	400	10,000
4		400	100		400	
5		500	500		400	
Payback period	1.0	4.0	4.0	2.0	2.5	2.0
NPV	–90.91	65.26	140.60	243.43	516.31	7,380.92

Comment on why the payback period provides misleading information about the following:

1. Project A
2. Project B versus Project C
3. Project D versus Project E
4. Project D versus Project F

Solutions:

1. Project A does indeed pay itself back in one year. However, this result is misleading because the investment is unprofitable, with a negative NPV.
2. Although Projects B and C have the same payback period and the same cash flow after the payback period, the payback period does not detect the fact that Project C's cash flows within the payback period occur earlier and result in a higher NPV.
3. Projects D and E illustrate a common situation. The project with the shorter payback period is the less profitable project. Project E has a longer payback and higher NPV.
4. Projects D and F illustrate an important flaw of the payback period—that the payback period ignores cash flows after the payback period is reached. In this case, Project F has a much larger cash flow in Year 3, but the payback period does not recognize its value.

The payback period has many drawbacks—it is a measure of payback and not a measure of profitability. By itself, the payback period would be a dangerous criterion for evaluating capital projects. Its simplicity, however, is an advantage. The payback period is very easy to calculate and to explain. The payback period may also be used as an indicator of project liquidity. A project with a two-year payback may be more liquid than another project with a longer payback.

Because it is not economically sound, the payback period has no decision rule like that of the NPV or IRR. If the payback period is being used (perhaps as a measure of liquidity), analysts should also use an NPV or IRR to ensure that their decisions also reflect the profitability of the projects being considered.

4.4. Discounted Payback Period

The discounted payback period is the number of years it takes for the cumulative discounted cash flows from a project to equal the original investment. The discounted payback period partially addresses the weaknesses of the payback period. Exhibit 2-4 gives an example of calculating the payback period and discounted payback period. The example assumes a discount rate of 10 percent.

EXHIBIT 2-4 Payback Period and Discounted Payback Period

Year	0	1	2	3	4	5
Cash flow (CF)	–5,000	1,500.00	1,500.00	1,500.00	1,500.00	1,500.00
Cumulative CF	–5,000	–3,500.00	–2,000.00	–500.00	1,000.00	2,500.00
Discounted CF	–5,000	1,363.64	1,239.67	1,126.97	1,024.52	931.38
Cumulative discounted CF	–5,000	–3,636.36	–2,396.69	–1,269.72	–245.20	686.18

The payback period is 3 years plus $500/1500 =$ one-third of the fourth year's cash flow, or 3.33 years. The discounted payback period is between four and five years. The discounted payback period is four years plus $245.20/931.38 = 0.26$ of the fifth year's discounted cash flow, or 4.26 years.

The discounted payback period relies on discounted cash flows, much as the NPV criterion does. If a project has a negative NPV, it will usually not have a discounted payback period since it never recovers the initial investment.

The discounted payback does account for the time value of money and risk within the discounted payback period, but it ignores cash flows after the discounted payback period is reached. This drawback has two consequences. First, the discounted payback period is not a good measure of profitability (like the NPV or IRR) because it ignores these cash flows. Second, another idiosyncrasy of the discounted payback period comes from the possibility of negative cash flows after the discounted payback period is reached. It is possible for a project to have a negative NPV but to have a positive cumulative discounted cash flow in the middle of its life and, thus, a reasonable discounted payback period. The NPV and IRR, which consider all of a project's cash flows, do not suffer from this problem.

4.5. Average Accounting Rate of Return

The average accounting rate of return (AAR) can be defined as

$$\text{AAR} = \frac{\text{Average net income}}{\text{Average book value}}$$

To understand this measure of return, we will use a numerical example.

Assume a company invests \$200,000 in a project that is depreciated straight-line over a five-year life to a zero salvage value. Sales revenues and cash operating expenses for each year are as shown in Exhibit 2-5. The table also shows the annual income taxes (at a 40 percent tax rate) and the net income.

EXHIBIT 2-5 Net Income for Calculating an Average Accounting Rate of Return

	Year 1	Year 2	Year 3	Year 4	Year 5
Sales	\$100,000	\$150,000	\$240,000	\$130,000	\$80,000
Cash expenses	50,000	70,000	120,000	60,000	50,000
Depreciation	40,000	40,000	40,000	40,000	40,000
Earnings before taxes	10,000	40,000	80,000	30,000	–10,000
Taxes (at 40 percent)	4,000	16,000	32,000	12,000	–4,000*
Net income	6,000	24,000	48,000	18,000	–6,000

*Negative taxes occur in Year 5 because the earnings before taxes of –\$10,000 can be deducted against earnings on other projects, thus reducing the tax bill by \$4,000.

For the five-year period, the average net income is \$18,000. The initial book value is \$200,000, declining by \$40,000 per year until the final book value is \$0. The average book value for this asset is (\$200,000 – \$0)/2 = \$100,000. The average accounting rate of return is

$$\text{AAR} = \frac{\text{Average net income}}{\text{Average book value}} = \frac{18{,}000}{100{,}000} = 18\%$$

The advantages of the AAR are that it is easy to understand and easy to calculate. The AAR has some important disadvantages, however. Unlike the other capital budgeting criteria discussed here, the AAR is based on accounting numbers and not based on cash flows. This is an important conceptual and practical limitation. The AAR also does not account for the time value of money, and there is no conceptually sound cutoff for the AAR that distinguishes between profitable and unprofitable investments. The AAR is frequently calculated in different ways, so the analyst should verify the formula behind any AAR numbers that are supplied by someone else. Analysts should know the AAR and its potential limitations in practice, but they should rely on more economically sound methods like the NPV and IRR.

4.6. Profitability Index

The profitability index (PI) is the present value of a project's future cash flows divided by the initial investment. It can be expressed as

$$\text{PI} = \frac{\text{PV of future cash flows}}{\text{Initial investment}} = 1 + \frac{\text{NPV}}{\text{Initial investment}} \tag{2-5}$$

You can see that the PI is closely related to the NPV. The PI is the *ratio* of the PV of future cash flows to the initial investment, while an NPV is the *difference* between the PV of future cash flows and the initial investment. Whenever the NPV is positive, the PI will be greater than 1.0, and conversely, whenever the NPV is negative, the PI will be less than 1.0. The investment decision rule for the PI is as follows:

Invest if $\text{PI} > 1.0$

Do not invest if $\text{PI} < 1.0$

Because the PV of future cash flows equals the initial investment plus the NPV, the PI can also be expressed as 1.0 plus the ratio of the NPV to the initial investment, as shown in Equation 2-5 earlier. Example 2-2 illustrates the PI calculation.

EXAMPLE 2-2 Example of a PI Calculation

The Gerhardt Corporation investment (discussed earlier) had an outlay of €50 million, a present value of future cash flows of €63.136 million, and an NPV of €13.136 million. The profitability index is

$$\text{PI} = \frac{\text{PV of future cash flows}}{\text{Initial investment}} = \frac{63.136}{50.000} = 1.26$$

The PI can also be calculated as

$$\text{PI} = 1 + \frac{\text{NPV}}{\text{Initial investment}} = 1 + \frac{13.136}{50.000} = 1.26$$

Because the $\text{PI} > 1.0$, this is a profitable investment.

The PI indicates the value you are receiving in exchange for one unit of currency invested. Although the PI is used less frequently than the NPV and IRR, it is sometimes used as a guide in capital rationing. The PI is usually called the profitability index in corporations, but it is commonly referred to as a "benefit–cost ratio" in governmental and not-for-profit organizations.

4.7. NPV Profile

The NPV profile shows a project's NPV graphed as a function of various discount rates. Typically, the NPV is graphed vertically (on the *y*-axis) and the discount rates are graphed horizontally (on the *x*-axis). The NPV profile for the Gerhardt capital budgeting project is shown in Example 2-3.

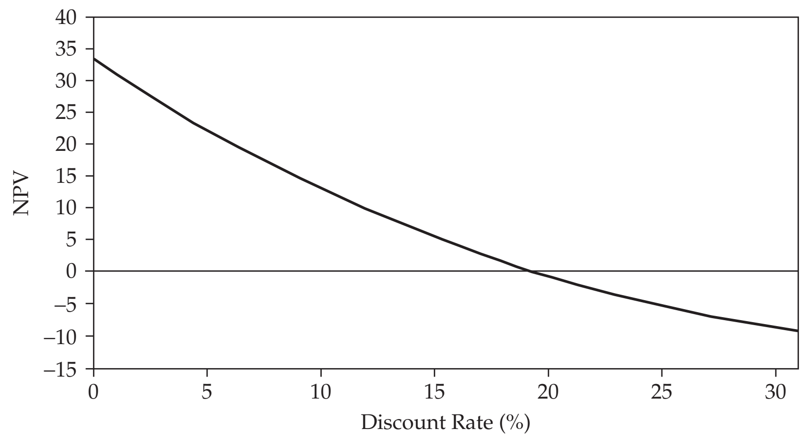
EXAMPLE 2-3 NPV Profile

For the Gerhardt example, we have already calculated several NPVs for different discount rates. At 10 percent the NPV is €13.136 million; at 20 percent the NPV is –€0.543 million; and at 19.52 percent (the IRR), the NPV is zero. What is the NPV if the discount rate is 0 percent? The NPV discounted at 0 percent is €34 million, which is simply the sum of all of the undiscounted cash flows. Exhibits 2-6 and 2-7 show the NPV profile for the Gerhardt example for discount rates between 0 percent and 30 percent.

EXHIBIT 2-6 Gerhardt NPV Profile

Discount Rate	NPV € millions
0%	34.000
5.00%	22.406
10.00%	13.136
15.00%	5.623
19.52%	0.000
20.00%	–0.543
25.00%	–5.661
30.00%	–9.954

EXHIBIT 2-7 Gerhardt NPV Profile



Three interesting points on this NPV profile are where the profile goes through the vertical axis (the NPV when the discount rate is zero), where the profile goes through the horizontal axis (where the discount rate is the IRR), and the NPV for the required rate of return (NPV is €13.136 million when the discount rate is the 10 percent required rate of return).

The NPV profile in Exhibit 2-7 is very well-behaved. The NPV declines at a decreasing rate as the discount rate increases. The profile is convex from the origin (convex from below). You will shortly see some examples in which the NPV profile is more complicated.

4.8. Ranking Conflicts between NPV and IRR

For a single conventional project, the NPV and IRR will agree on whether to invest or to not invest. For independent, conventional projects, no conflict exists between the decision rules for the NPV and IRR. However, in the case of two mutually exclusive projects, the two criteria will sometimes disagree. For example, Project A might have a larger NPV than Project B, but Project B has a higher IRR than Project A. In this case, should you invest in Project A or in Project B?

Differing cash flow patterns can cause two projects to rank differently with the NPV and IRR. For example, suppose Project A has shorter-term payoffs than Project B. This situation is presented in Example 2-4.

Whenever the NPV and IRR rank two mutually exclusive projects differently, as they do in the example above, you should choose the project based on the NPV. Project B, with the higher NPV, is the better project because of the reinvestment assumption. Mathematically, whenever you discount a cash flow at a particular discount rate, you are implicitly assuming

EXAMPLE 2-4 Ranking Conflict Due to Differing Cash Flow Patterns

Projects A and B have similar outlays but different patterns of future cash flows. Project A realizes most of its cash payoffs earlier than Project B. The cash flows as well as the NPV and IRR for the two projects are shown in Exhibit 2-8. For both projects, the required rate of return is 10 percent.

EXHIBIT 2-8 Cash Flows, NPV, and IRR for Two Projects with Different Cash Flow Patterns

	Cash Flows						
Year	0	1	2	3	4	NPV	IRR
Project A	–200	80	80	80	80	53.59	21.86%
Project B	–200	0	0	0	400	73.21	18.92%

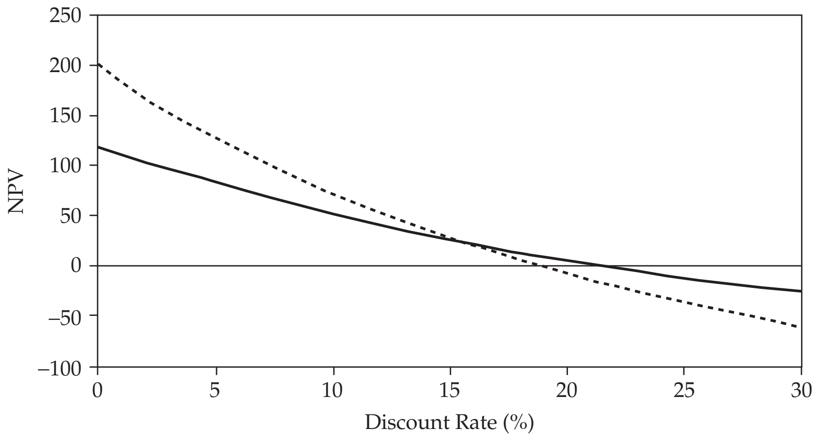
If the two projects were not mutually exclusive, you would invest in both because they are both profitable. However, you can choose either A (which has the higher IRR) or B (which has the higher NPV).

Exhibits 2-9 and 2-10 show the NPVs for Project A and Project B for various discount rates between 0 percent and 30 percent.

EXHIBIT 2-9 NPV Profiles for Two Projects with Different Cash Flow Patterns

Discount Rate	NPV for Project A	NPV for Project B
0%	120.00	200.00
5.00%	83.68	129.08
10.00%	53.59	73.21
15.00%	28.40	28.70
15.09%	27.98	27.98
18.92%	11.41	0.00
20.00%	7.10	–7.10
21.86%	0.00	–18.62
25.00%	–11.07	–36.16
30.00%	–26.70	–59.95

EXHIBIT 2-10 NPV Profiles for Two Projects with Different Cash Flow Patterns



Note that Project B (broken line) has the higher NPV for discount rates between 0 percent and 15.09 percent. Project A (solid line) has the higher NPV for discount rates exceeding 15.09 percent. The crossover point of 15.09 percent in Exhibit 2-10 corresponds to the discount rate at which both projects have the same NPV (of 27.98). Project B has the higher NPV below the crossover point, and Project A has the higher NPV above it.

that you can reinvest a cash flow at that same discount rate.[2] In the NPV calculation, you use a discount rate of 10 percent for both projects. In the IRR calculation, you use a discount rate equal to the IRR of 21.86 percent for Project A and 18.92 percent for Project B.

Can you reinvest the cash inflows from the projects at 10 percent, or 21.86 percent, or 18.92 percent? When you assume the required rate of return is 10 percent, you are assuming an opportunity cost of 10 percent—you are assuming that you can either find other projects that pay a 10 percent return or pay back your sources of capital that cost you 10 percent. The fact that you earned 21.86 percent in Project A or 18.92 percent in Project B does not mean that you can reinvest future cash flows at those rates. (In fact, if you can reinvest future cash flows at 21.86 percent or 18.92 percent, these should have been used as your required rate of return instead of 10 percent.) Because the NPV criterion uses the most realistic discount rate—the opportunity cost of funds—the NPV criterion should be used for evaluating mutually exclusive projects.

Another circumstance that frequently causes mutually exclusive projects to be ranked differently by NPV and IRR criteria is project scale—the sizes of the projects. Would you rather have a small project with a higher rate of return or a large project with a lower rate of return? Sometimes, the larger, low rate of return project has the better NPV. This case is developed in Example 2-5.

EXAMPLE 2-5 Ranking Conflicts Due to Differing Project Scale

Project A has a much smaller outlay than Project B, although they have similar future cash flow patterns. The cash flows as well as the NPVs and IRRs for the two projects are shown in Exhibit 2-11. For both projects, the required rate of return is 10 percent.

EXHIBIT 2-11 Cash Flows, NPV, and IRR for Two Projects of Differing Scale

	Cash Flows						
Year	0	1	2	3	4	NPV	IRR
Project A	–100	50	50	50	50	58.49	34.90%
Project B	–400	170	170	170	170	138.88	25.21%

If they were not mutually exclusive, you would invest in both projects because they are both profitable. However, you can choose either Project A (which has the higher IRR) or Project B (which has the higher NPV).

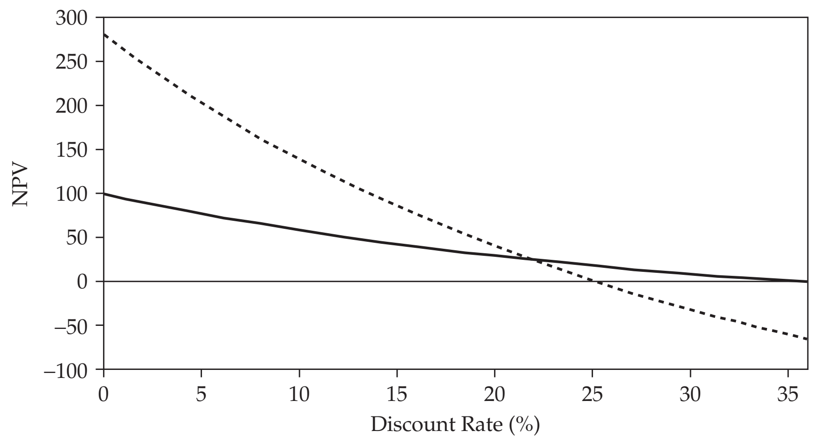
[2]For example, assume that you are receiving \$100 in one year discounted at 10 percent. The present value is \$100/1.10 = \$90.91. Instead of receiving the \$100 in one year, invest it for one additional year at 10 percent, and it grows to \$110. What is the present value of \$110 received in two years discounted at 10 percent? It is the same \$90.91. Because both future cash flows are worth the same, you are implicitly assuming that reinvesting the earlier cash flow at the discount rate of 10 percent has no effect on its value.

Exhibits 2-12 and 2-13 show the NPVs for Project A and Project B for various discount rates between 0 percent and 30 percent.

EXHIBIT 2-12 NPV Profiles for Two Projects of Differing Scale

Discount Rate	NPV for Project A	NPV for Project B
0%	100.00	280.00
5.00%	77.30	202.81
10.00%	58.49	138.88
15.00%	42.75	85.35
20.00%	29.44	40.08
21.86%	25.00	25.00
25.00%	18.08	1.47
25.21%	17.65	0.00
30.00%	8.31	–31.74
34.90%	0.00	–60.00
35.00%	–0.15	–60.52

EXHIBIT 2-13 NPV Profiles for Two Projects of Differing Scale



Note that Project B (broken line) has the higher NPV for discount rates between 0 percent and 21.86 percent. Project A has the higher NPV for discount rates exceeding 21.86 percent. The crossover point of 21.86 percent in Exhibit 2-13 corresponds to the discount rate at which both projects have the same NPV (of 25.00). Below the crossover point, Project B has the higher NPV, and above it, Project A has the higher NPV. When cash flows are discounted at the 10 percent required rate of return, the choice is clear—Project B, the larger project, which has the superior NPV.

The good news is that the NPV and IRR criteria will usually indicate the same investment decision for a given project. They will usually both recommend acceptance or rejection of the project. When the choice is between two mutually exclusive projects and the NPV and IRR rank the two projects differently, the NPV criterion is strongly preferred. There are good reasons for this preference. The NPV shows the amount of gain, or wealth increase, as a currency amount. The reinvestment assumption of the NPV is the more economically realistic. The IRR does give you a rate of return, but the IRR could be for a small investment or for only a short period of time. As a practical matter, once a corporation has the data to calculate the NPV, it is fairly trivial to go ahead and calculate the IRR and other capital budgeting criteria. However, the most appropriate and theoretically sound criterion is the NPV.

4.9. The Multiple IRR Problem and the No IRR Problem

A problem that can arise with the IRR criterion is the "multiple IRR problem." We can illustrate this problem with the following nonconventional cash flow pattern:[3]

Time	0	1	2
Cash Flow	–1,000	5,000	–6,000

The IRR for these cash flows satisfies this equation:

$$-1{,}000 + \frac{5{,}000}{(1+\mathrm{IRR})^1} + \frac{-6{,}000}{(1+\mathrm{IRR})^2} = 0$$

It turns out that there are two values of IRR that satisfy the equation: $\mathrm{IRR} = 1 = 100$ percent and $\mathrm{IRR} = 2 = 200$ percent. To further understand this problem, consider the NPV profile for this investment, which is shown in Exhibits 2-14 and 2-15.

As you can see in the NPV profile, the NPV is equal to zero at $\mathrm{IRR} = 100$ percent and $\mathrm{IRR} = 200$ percent. The NPV is negative for discount rates below 100 percent, positive between 100 percent and 200 percent, and then negative above 200 percent. The NPV reaches its highest value when the discount rate is 140 percent.

It is also possible to have an investment project with no IRR. The "no-IRR problem" occurs with this cash flow pattern:[4]

Time	0	1	2
Cash Flow	100	–300	250

The IRR for these cash flows satisfies this equation:

$$100 + \frac{-300}{(1+\mathrm{IRR})^1} + \frac{250}{(1+\mathrm{IRR})^2} = 0$$

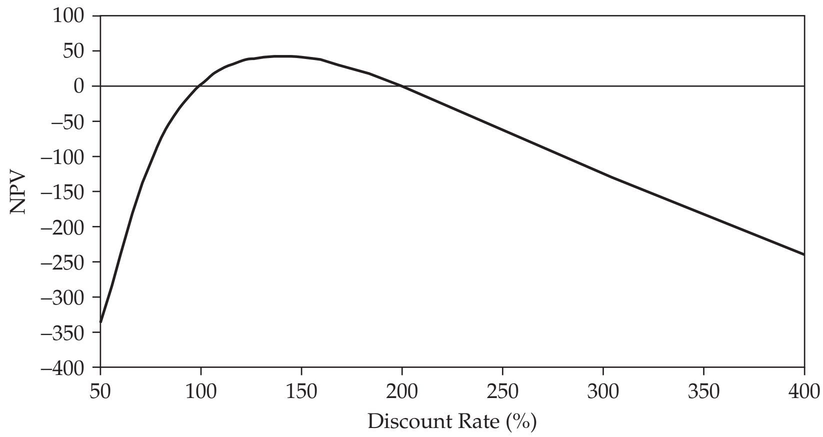
For these cash flows, no discount rate exists that results in a zero NPV. Does that mean this project is a bad investment? In this case, the project is actually a good investment. As

[3]This example is adapted from Hirschleifer (1958).
[4]This example is also adapted from Hirschleifer.

EXHIBIT 2-14 NPV Profile for a Multiple IRR Example

Discount Rate	NPV
0%	–2,000.00
25%	–840.00
50%	–333.33
75%	–102.04
100%	0.00
125%	37.04
140%	41.67
150%	40.00
175%	24.79
200%	0.00
225%	–29.59
250%	–61.22
300%	–125.00
350%	–185.19
400%	–240.00
500%	–333.33
1,000%	–595.04
2,000%	–775.51
3,000%	–844.95
4,000%	–881.62
10,000%	–951.08
1,000,000%	–999.50

EXHIBIT 2-15 NPV Profile for a Multiple IRR Example

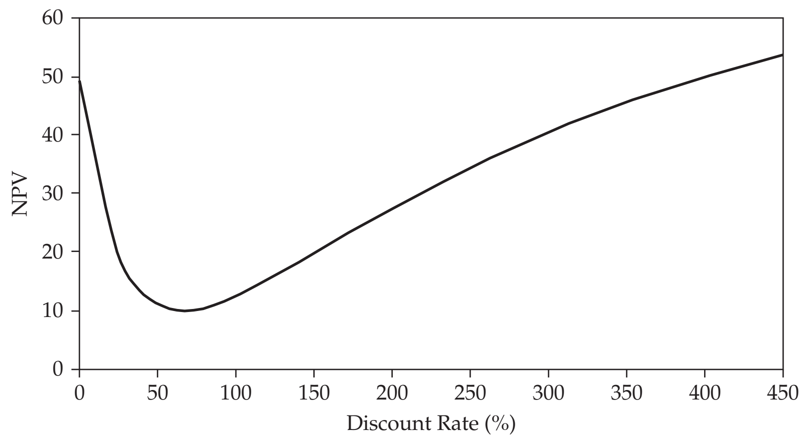


Exhibits 2-16 and 2-17 show, the NPV is positive for all discount rates. The lowest NPV, of 10, occurs for a discount rate of 66.67 percent, and the NPV is always greater than zero. Consequently, no IRR exists.

EXHIBIT 2-16 NPV Profile for a Project with No IRR

Discount Rate	NPV
0%	50.00
25%	20.00
50%	11.11
66.67%	10.00
75%	10.20
100%	12.50
125%	16.05
150%	20.00
175%	23.97
200%	27.78
225%	31.36
250%	34.69
275%	37.78
300%	40.63
325%	43.25
350%	45.68
375%	47.92
400%	50.00

EXHIBIT 2-17 NPV Profile for a Project with No IRR



For conventional projects that have outlays followed by inflows—negative cash flows followed by positive cash flows—the multiple IRR problem cannot occur. However, for nonconventional projects, as in the example above, the multiple IRR problem can occur. The IRR equation is essentially an nth degree polynomial. An nth degree polynomial can have up to n solutions, although it will have no more real solutions than the number of cash flow sign changes. For example, a project with two sign changes could have zero, one, or two IRRs. Having two sign changes does not mean that you *will* have multiple IRRs; it just means that you *might*. Fortunately, most capital budgeting projects have only one IRR. Analysts should always be aware of the unusual cash flow patterns that can generate the multiple IRR problem.

4.10. Popularity and Usage of the Capital Budgeting Methods

Analysts need to know the basic logic of the various capital budgeting criteria as well as the practicalities involved in using them in real corporations. Before delving into the many issues involved in applying these models, we would like to present some feedback on their popularity.

The usefulness of any analytical tool always depends on the specific application. Corporations generally find these capital budgeting criteria useful. Two recent surveys by Graham and Harvey (2001) and Brounen, De Jong, and Koedijk (2004) report on the frequency of their use by U.S. and European corporations. Exhibit 2-18 gives the mean responses of executives in five countries to the question, "How frequently does your company use the following techniques when deciding which projects or acquisitions to pursue?"

EXHIBIT 2-18 Mean Responses about Frequency of Use of Capital Budgeting Techniques

	United States	United Kingdom	Netherlands	Germany	France
Internal rate of return*	3.09	2.31	2.36	2.15	2.27
Net present value*	3.08	2.32	2.76	2.26	1.86
Payback period*	2.53	2.77	2.53	2.29	2.46
Hurdle rate	2.13	1.35	1.98	1.61	0.73
Sensitivity analysis	2.31	2.21	1.84	1.65	0.79
Earnings multiple approach	1.89	1.81	1.61	1.25	1.70
Discounted payback period*	1.56	1.49	1.25	1.59	0.87
Real options approach	1.47	1.65	1.49	2.24	2.20
Accounting rate of return*	1.34	1.79	1.40	1.63	1.11
Value at risk	0.95	0.85	0.51	1.45	1.68
Adjusted present value	0.85	0.78	0.78	0.71	1.11
Profitability index*	0.85	1.00	0.78	1.04	1.64

Respondents used a scale ranging from 0 (never) to 4 (always).
*These techniques were described in this section of the chapter. You will encounter the others elsewhere.

Although financial textbooks preach the superiority of the NPV and IRR techniques, it is clear that several other methods are heavily used.[5] In the four European countries, the payback period is used as often as, or even slightly more often than, the NPV and IRR. In these two studies, larger companies tended to prefer the NPV and IRR over the payback period. The fact that the U.S. companies were larger, on average, partially explains the greater U.S. preference for the NPV and IRR. Other factors influence the choice of capital budgeting techniques. Private corporations used the payback period more frequently than did public corporations. Companies managed by an MBA had a stronger preference for the discounted cash flow techniques. Of course, any survey research also has some limitations. In this case, the persons in these large corporations responding to the surveys may not have been aware of all of the applications of these techniques.

These capital budgeting techniques are essential tools for corporate managers. Capital budgeting is also relevant to external analysts. Because a corporation's investing decisions ultimately determine the value of its financial obligations, the corporation's investing processes are vital. The NPV criterion is the criterion most directly related to stock prices. If a corporation invests in positive NPV projects, these should add to the wealth of its shareholders. Example 2-6 illustrates this scenario.

EXAMPLE 2-6 NPVs and Stock Prices

Freitag Corporation is investing €600 million in distribution facilities. The present value of the future after-tax cash flows is estimated to be €850 million. Freitag has 200 million outstanding shares with a current market price of €32.00 per share. This investment is new information, and it is independent of other expectations about the company. What should be the effect of the project on the value of the company and the stock price?

Solution. The NPV of the project is €850 million – €600 million = €250 million. The total market value of the company prior to the investment is €32.00 × 200 million shares = €6,400 million. The value of the company should increase by €250 million to €6,650 million. The price per share should increase by the NPV per share, or €250 million/200 million shares = €1.25 per share. The share price should increase from €32.00 to €33.25.

The effect of a capital budgeting project's positive or negative NPV on share price is more complicated than Example 6 above, in which the value of the stock increased by the project's NPV. The value of a company is the value of its existing investments plus the net present values of all of its future investments. If an analyst learns of an investment, the impact of that investment on the stock price will depend on whether the investment's profitability is

[5]Analysts often refer to the NPV and IRR as "discounted cash flow techniques" because they accurately account for the timing of all cash flows when they are discounted.

more or less than expected. For example, an analyst could learn of a positive NPV project, but if the project's profitability is less than expectations, this stock might drop in price on the news. Alternatively, news of a particular capital project might be considered as a signal about other capital projects underway or in the future. A project that by itself might add, say, €0.25 to the value of the stock might signal the existence of other profitable projects. News of this project might increase the stock price by far more than €0.25.

The integrity of a corporation's capital budgeting processes is important to analysts. Management's capital budgeting processes can demonstrate two things about the quality of management: the degree to which management embraces the goal of shareholder wealth maximization, and its effectiveness in pursuing that goal. Both of these factors are important to shareholders.

5. CASH FLOW PROJECTIONS

In Section 4, we presented the basic capital budgeting models that managers use to accept or reject capital budgeting proposals. In that section, we assumed the cash flows were given, and we used them as inputs to the analysis. In Section 5, we detail how these cash flows are found for an "expansion" project. An expansion project is an independent investment that does not affect the cash flows for the rest of the company. In Section 6, we will deal with a "replacement" project, in which the cash flow analysis is more complicated. A replacement project must deal with the differences between the cash flows that occur with the new investment and the cash flows that would have occurred for the investment being replaced.

5.1. Table Format with Cash Flows Collected by Year

The cash flows for a conventional expansion project can be grouped into (1) the investment outlays, (2) after-tax operating cash flows over the project's life, and (3) terminal year after-tax nonoperating cash flows. Exhibit 2-19 gives an example of the cash flows for a capital project where all of the cash flows are collected by year.

The investment outlays include a \$200,000 outlay for fixed capital items. This outlay includes \$25,000 for nondepreciable land, plus \$175,000 for equipment that will be depreciated straight-line to zero over five years. The investment in net working capital is the net investment in short-term assets required for the investment. This is the investment in receivables and inventory needed, less the short-term payables generated by the project. In this case, the project required \$50,000 of current assets but generated \$20,000 in current liabilities, resulting in a total investment in net working capital of \$30,000. The total investment outlay at time zero is \$230,000.

Each year, sales will be \$220,000 and cash operating expenses will be \$90,000. Annual depreciation for the \$175,000 depreciable equipment is \$35,000 (one-fifth of the cost). The result is an operating income before taxes of \$95,000. Income taxes at a 40 percent rate are $0.40 \times \$95{,}000 = \$38{,}000$. This leaves operating income after taxes of \$57,000. Adding back the depreciation charge of \$35,000 gives the annual after-tax operating cash flow of \$92,000.[6]

[6]Examining the operating cash flows in Exhibit 19, we have a \$220,000 inflow from sales, a \$90,000 outflow for cash operating expenses, and a \$38,000 outflow for taxes. This is an after-tax cash flow of \$92,000.

EXHIBIT 2-19 Capital Budgeting Cash Flows Example (Cash Flows Collected by Year)

Year	0	1	2	3	4	5
Investment outlays:						
Fixed capital	–200,000					
Net working capital	–30,000					
Total	–230,000					
Annual after-tax operating cash flows:						
Sales		220,000	220,000	220,000	220,000	220,000
Cash operating expenses		90,000	90,000	90,000	90,000	90,000
Depreciation		35,000	35,000	35,000	35,000	35,000
Operating income before taxes		95,000	95,000	95,000	95,000	95,000
Taxes on operating income		38,000	38,000	38,000	38,000	38,000
Operating income after taxes		57,000	57,000	57,000	57,000	57,000
Add back: Depreciation		35,000	35,000	35,000	35,000	35,000
After-tax operating cash flow		92,000	92,000	92,000	92,000	92,000
Terminal year after-tax nonoperating cash flows:						
After-tax salvage value						40,000
Return of net working capital						30,000
Total						70,000
Total after-tax cash flow	–230,000	92,000	92,000	92,000	92,000	162,000
Net present value at 10 percent required rate of return	162,217					
Internal rate of return	32.70%					

At the end of Year 5, the company will sell off the fixed capital assets. In this case, the fixed capital assets (including the land) are sold for $50,000, which represents a gain of $25,000 over the remaining book value of $25,000. The gain of $25,000 is taxed at 40 percent, resulting in a tax of $10,000. This leaves $40,000 for the fixed capital assets after taxes. Additionally, the net working capital investment of $30,000 is recovered, as the short-term assets (such as inventory and receivables) and short-term liabilities (such as payables) are no longer needed for the project. Total terminal year nonoperating cash flows are then $70,000.

The investment project has a required rate of return of 10 percent. Discounting the future cash flows at 10 percent and subtracting the investment outlay gives an NPV of $162,217. The internal rate of return is 32.70 percent. Because the investment has a positive NPV, this project should be accepted. The IRR investment decision criterion would also recommend accepting the project because the IRR is greater than the required rate of return.

5.2. Table Format with Cash Flows Collected by Type

In the layout in Exhibit 2-19, we essentially collected the cash flows in the columns, by *year*, and then found the NPV by summing the present values of the annual cash flows (at the bottom of each column). There is another way of organizing the same information. We could also find the NPV by finding the present values of the cash flows in Exhibit 2-19 by rows, which are the *types* of cash flows. This approach is shown in Exhibit 2-20.

EXHIBIT 2-20 Capital Budgeting Cash Flows Example (Cash Flows Collected by Type)

Time	Type of Cash Flow	Before-Tax Cash Flow	After-Tax Cash Flow	PV at 10%
0	Fixed capital	−200,000	−200,000	−200,000
0	Net working capital	−30,000	−30,000	−30,000
1−5	Sales minus cash expenses	220,000 − 90,000 = 130,000	130,000(1 − 0.40) = 78,000	295,681
1−5	Depreciation tax savings	None	0.40(35,000) = 14,000	53,071
5	After-tax salvage value	50,000	50,000 − 0.40(50,000 − 25,000) = 40,000	24,837
5	Return of net working capital	30,000	30,000	18,628
			NPV =	162,217

As Exhibit 2-20 shows, the outlays in fixed capital and in net working capital at time zero total \$230,000. For Years 1 through 5, the company realizes an after-tax cash flow for sales minus cash expenses of \$78,000, which has a present value of \$295,681. The depreciation charge results in a tax savings of \$14,000 per year, which has a present value of \$53,071. The present values of the after-tax salvage and of the return of net working capital are also shown in the table. The present value of all cash flows is an NPV of \$162,217. Obviously, collecting the after-tax cash flows by year, as in Exhibit 2-19, or by type, as in Exhibit 2-20, results in the same NPV.

5.3. Equation Format for Organizing Cash Flows

The capital budgeting cash flows in the example project above were laid out in one of two alternative tabular formats. Analysts may wish to take even another approach. Instead of producing a table, you can also look at the cash flows using equations such as the following:

1. Initial outlay:
   For a new investment

$$\text{Outlay} = \text{FCInv} + \text{NWCInv}$$

where

$$\text{FCInv} = \text{Investment in new fixed capital}$$
$$\text{NWCInv} = \text{Investment in net working capital}$$

The above equation can be generalized for a replacement project (covered in Section 6.2), in which existing fixed capital is sold and provides some of the funding for the new fixed capital purchased. The outlay is then

$$\text{Outlay} = \text{FCInv} + \text{NWCInv} - \text{Sal}_0 + \text{T}(\text{Sal}_0 - \text{B}_0) \tag{2-6}$$

where

Sal_0 = Cash proceeds (salvage value) from sale of old fixed capital
T = Tax rate
B_0 = Book value of old fixed capital

2. Annual after-tax operating cash flow:

$$\text{CF} = (\text{S} - \text{C} - \text{D})(1 - \text{T}) + \text{D, or} \tag{2-7}$$

$$\text{CF} = (\text{S} - \text{C})(1 - \text{T}) + \text{TD} \tag{2-8}$$

where

S = sales
C = cash operating expenses
D = depreciation charge

3. Terminal year after-tax nonoperating cash flow:

$$\text{TNOCF} = \text{Sal}_\text{T} + \text{NWCInv} - \text{T}(\text{Sal}_\text{T} - \text{B}_\text{T}) \tag{2-9}$$

where

Sal_T = Cash proceeds (salvage value) from sale of fixed capital on termination date
B_T = Book value of fixed capital on termination date

The outlay in the example is found with Equation 2-6:

$$\text{Outlay} = 200{,}000 + 30{,}000 - 0 + 0 = \$230{,}000$$

For a replacement project, the old fixed capital would be sold for cash (Sal_0) and then there would be taxes paid on the gain (if $\text{Sal}_0 - \text{B}_0$ were positive) or a tax saving (if $\text{Sal}_0 - \text{B}_0$ were negative). In this example, Sal_0 and $\text{T}(\text{Sal}_0 - \text{B}_0)$ are zero because no existing fixed capital is sold at time zero.

Using Equation 2-7, we find that the annual after-tax operating cash flow is

$$\begin{aligned} \text{CF} &= (\text{S} - \text{C} - \text{D})(1 - \text{T}) + \text{D} \\ &= (220{,}000 - 90{,}000 - 35{,}000)(1 - 0.40) + 35{,}000 = 95{,}000(0.60) + 35{,}000 \\ &= 57{,}000 + 35{,}000 = \$92{,}000 \end{aligned}$$

Equation 2-7 is the project's net income plus depreciation. An identical cash flow results if we use Equation 2-8:

$$\begin{aligned} CF &= (S - C)(1 - T) + TD \\ &= (220{,}000 - 90{,}000)(1 - 0.40) + 0.40(35{,}000) \\ &= 130{,}000(0.60) + 0.40(35{,}000) = 78{,}000 + 14{,}000 = \$92{,}000 \end{aligned}$$

Equation 2-8 is the after-tax sales and cash expenses plus the depreciation tax savings. The analyst can use either equation.

Equation 2-9 provides the terminal year nonoperating cash flow:

$$\begin{aligned} TNOCF &= Sal_T + NWCInv - T(Sal_T - B_T) \\ &= 50{,}000 + 30{,}000 - 0.40(50{,}000 - 25{,}000) \\ &= 50{,}000 + 30{,}000 - 10{,}000 = \$70{,}000 \end{aligned}$$

The old fixed capital (including land) is sold for \$50,000, but \$10,000 of taxes must be paid on the gain. Including the \$30,000 return of net working capital gives a terminal year nonoperating cash flow of \$70,000.

The NPV of the project is the present value of the cash flows—an outlay of \$230,000 at time zero, an annuity of \$92,000 for five years, plus a single payment of \$70,000 in five years:

$$NPV = -230{,}000 + \sum_{t=1}^{5} \frac{92{,}000}{(1.10)^t} + \frac{70{,}000}{(1.10)^5} = -230{,}000 + 348{,}752 + 43{,}465 = \$162{,}217$$

We obtain an identical NPV of \$162,217 whether we use a tabular format collecting cash flows by year, a tabular format collecting cash flows by type, or an equation format using Equations 2-6 through 2-9. The analyst usually has some flexibility in choosing how to solve a problem. Furthermore, the analysis that an analyst receives from someone else could be in varying formats. The analyst must interpret this information correctly regardless of format. An analyst may need to present information in alternative formats, depending on what the client or user of the information wishes to see. All that is important is that the cash flows are complete (with no cash flows omitted and none double-counted), that their timing is recognized, and that the discounting is done correctly.

6. MORE ON CASH FLOW PROJECTIONS

Cash flow analysis can become fairly complicated. Section 6 extends the analysis of the previous section to include more details on depreciation methods, replacement projects (as opposed to simple expansion projects), the use of spreadsheets, and the effects of inflation.

6.1. Straight-Line and Accelerated Depreciation Methods

Before going on to more complicated investment decisions, we should mention the variety of depreciation methods that are in use. The example in Section 5.1 assumed straight-line depreciation down to a zero salvage value. Most accounting texts give a good description of the straight-line method, the sum-of-years digits method, the double-declining balance method (and the 150 percent declining balance method), and the units-of-production and service hours method.[7]

Many countries specify the depreciation methods that are acceptable for tax purposes in their jurisdictions. For example, in the U.S., corporations use the MACRS (modified accelerated cost recovery system) for tax purposes. Under MACRS, real property (real estate) is usually depreciated straight-line over a 27.5- or 39-year life, and other capital assets are usually grouped into MACRS asset classes and subject to a special depreciation schedule in each class. These MACRS classes and the depreciation rates for each class are shown in Exhibit 2-21.

EXHIBIT 2-21 Depreciation Rates under U.S. MACRS

	Recovery Period Class					
Year	3-Year	5-Year	7-Year	10-Year	15-Year	20-Year
1	33.33%	20.00%	14.29%	10.00%	5.00%	3.75%
2	44.45	32.00	24.49	18.00	9.50	7.22
3	14.81	19.20	17.49	14.40	8.55	6.68
4	7.41	11.52	12.49	11.52	7.70	6.18
5		11.52	8.93	9.22	6.93	5.71
6		5.76	8.93	7.37	6.23	5.28
7			8.93	6.55	5.90	4.89
8			4.45	6.55	5.90	4.52
9				6.55	5.90	4.46
10				6.55	5.90	4.46
11				3.29	5.90	4.46
12					5.90	4.46
13					5.90	4.46
14					5.90	4.46
15					5.90	4.46
16					2.99	4.46
17						4.46
18						4.46
19						4.46
20						4.46
21						2.25

For the first four MACRS classes (3-year, 5-year, 7-year, and 10-year), the depreciation is double-declining-balance with a switch to straight-line when optimal and with a half-year convention. For the last two classes (15-year and 20-year), the depreciation is 150 percent-declining-balance with a switch to straight-line when optimal and with a half-year convention.

[7]White, Sondhi, and Fried (2003) is a good example. Consult their chapter 8, "Analysis of Long-Lived Assets: Part II—Analysis of Depreciation and Impairment" for review and examples.

Take 5-year property in Exhibit 2-21 as an example. With double-declining-balance, the depreciation each year is $2/5 = 40$ percent of the beginning-of-year book value. However, with a half-year convention, the asset is assumed to be in service for only six months during the first year, and only one half of the depreciation is allowed the first year. After the first year, the depreciation rate is 40 percent of the beginning balance until Year 4, when straight-line depreciation would be at least as large, so we switch to straight-line. In Year 6, we have one-half of a year of the straight-line depreciation remaining because we assumed the asset was placed in service halfway through the first year.

Accelerated depreciation generally improves the NPV of a capital project compared to straight-line depreciation. For an example of this effect, we will assume the same capital project as in Exhibit 2-19, except that the depreciation is MACRS 3-year property. When using straight-line, the depreciation was 20 percent per year ($35,000). The depreciation percentages for MACRS 3-year property are given in Exhibit 2-21. The first-year depreciation is $0.3333 \times 175{,}000 = \$58{,}327.50$, second-year depreciation is $0.4445 \times 175{,}000 = \$77{,}787.50$, third-year depreciation is $0.1481 \times 175{,}000 = \$25{,}917.50$, fourth-year depreciation is $0.0741 \times 175{,}000 = \$12{,}967.50$, and fifth-year depreciation is zero. The impact on the NPV and IRR of the project is shown in Exhibit 2-22.

EXHIBIT 2-22 Capital Budgeting Example with MACRS

Year	0	1	2	3	4	5
Investment outlays:						
Fixed capital	–200,000					
Net working capital	–30,000					
Total	–230,000					
Annual after-tax operating cash flows:						
Sales		220,000	220,000	220,000	220,000	220,000
Cash operating expenses		90,000	90,000	90,000	90,000	90,000
Depreciation		58,328	77,788	25,918	12,968	0
Operating income before taxes		71,673	52,213	104,083	117,033	130,000
Taxes on operating income (40%)		28,669	20,885	41,633	46,813	52,000
Operating income after taxes		43,004	31,328	62,450	70,220	78,000
Add back: Depreciation		58,328	77,788	25,918	12,968	0
After-tax operating cash flow		101,331	109,115	88,367	83,187	78,000
Terminal year after-tax nonoperating cash flows:						
After-tax salvage value						40,000
Return of net working capital						30,000
Total						70,000
Total after-tax cash flows	–230,000	101,331	109,115	88,367	83,187	148,000
Net present value at 10% required rate of return	$167,403					
Internal rate of return	34.74%					

As the table shows, the depreciation charges still sum to \$175,000 (except for \$2 of rounding), but they are larger in Years 1 and 2 and smaller in Years 3, 4, and 5. Although this method reduces operating income after taxes in Years 1 and 2 (and increases it in Years 3, 4, and 5), it reduces tax outflows in Years 1 and 2 and increases them later. Consequently, the after-tax operating cash flows (which were \$92,000 per year) increase in early years and decrease in later years. This increases the NPV from \$162,217 to \$167,403, a difference of \$5,186. The IRR also increases from 32.70 percent to 34.74 percent.[8]

The impact of accelerated depreciation can be seen without going through the complete analysis in Exhibit 2-22. We previously showed in Exhibit 2-20 that the present value of the depreciation tax savings (which was an annuity of $0.40 \times \$35{,}000 = \$14{,}000$ a year for five years) was \$53,071. The present value of the tax savings from accelerated depreciation is shown in Exhibit 2-23.

EXHIBIT 2-23 Present Value of Tax Savings from Accelerated Depreciation

Year	Depreciation	Tax Savings	PV at 10%
1	\$58,327.50	$0.40 \times \$58{,}327.5 = \$23{,}331$	\$21,210
2	\$77,787.50	$0.40 \times \$77{,}787.5 = \$31{,}115$	\$25,715
3	\$25,917.50	$0.40 \times \$25{,}917.5 = \$10{,}367$	\$7,789
4	\$12,967.50	$0.40 \times \$12{,}967.5 = \$5{,}187$	\$3,543
5	\$0	$0.40 \times \$0 = \$0$	\$0
	Total present value		\$58,257

By using the accelerated depreciation schedule, we increase the present value of the tax savings from \$53,071 (from Exhibit 2-20) to \$58,257, an increase of \$5,186. The tax deferral associated with the accelerated depreciation (compared to straight-line) adds \$5,186 to the NPV of the project.

There are a myriad of tax and depreciation schedules that apply to investment projects around the world. These tax and depreciation schedules are also subject to change from year to year. To accurately assess the profitability of a particular capital project, it is vital to identify and apply the schedules that are relevant to the capital budgeting decision at hand.

6.2. Cash Flows for a Replacement Project

In Section 5.1, we evaluated the cash flows for an expansion project, basing our after-tax cash flows on the outlays, annual operating cash flows after tax, and salvage value for the project by itself. In many cases, however, investing in a project will be more complicated. Investing could affect many of the company's cash flows. In principle, the cash flows relevant to an investing decision are the incremental cash flows: the cash flows the company realizes *with* the investment compared to the cash flows the company would realize *without* the investment. For

[8]This example assumes that the investment occurs on the first day of the tax year. If the outlay occurs later in the tax year, the depreciation tax savings for the tax years are unchanged, which means that the cash savings occur sooner, increasing their present values. The result is a higher NPV and IRR.

example, suppose we are investing in a new project with an outlay of \$100,000 and we sell off existing assets that the project replaces for \$30,000. The incremental outlay is \$70,000.

A very common investment decision is a replacement decision, in which you replace old equipment with new equipment. This decision requires very careful analysis of the cash flows. The skills required to detail the replacement decision cash flows are also useful for other decisions in which an investment affects other cash flows in the company. We use the term "replacement" loosely, primarily to indicate that the cash flow analysis is more complicated than it was for the simpler expansion decision.

Assume we are considering the replacement of old equipment with new equipment that has more capacity and is less costly to operate. The characteristics of the old and new equipment are given below:

Old Equipment		New Equipment	
Current book value	\$400,000		
Current market value	\$600,000	Acquisition cost	\$1,000,000
Remaining life	10 years	Life	10 years
Annual sales	\$300,000	Annual sales	\$450,000
Cash operating expenses	\$120,000	Cash operating expenses	\$150,000
Annual depreciation	\$40,000	Annual depreciation	\$100,000
Accounting salvage value	\$0	Accounting salvage value	\$0
Expected salvage value	\$100,000	Expected salvage value	\$200,000

If the new equipment replaces the old equipment, an additional investment of \$80,000 in net working capital will be required. The tax rate is 30 percent, and the required rate of return is 8 percent.

The cash flows can be found by carefully constructing tables like Exhibit 2-19 or by using Equations 2-6 through 2-9. The initial outlay is the investment in the new equipment plus the additional investment in net working capital less the after-tax proceeds from selling the old equipment:

$$\text{Outlay} = \text{FCInv} + \text{NWCInv} - \text{Sal}_0 + \text{T}(\text{Sal}_0 - \text{B}_0)$$

$$\text{Outlay} = 1{,}000{,}000 + 80{,}000 - 600{,}000 + 0.3(600{,}000 - 400{,}000) = \$540{,}000$$

In this case, the outlay of \$540,000 is \$1,080,000 for new equipment and net working capital minus the after-tax proceeds of \$540,000 the company receives from selling the old equipment. The incremental operating cash flows are

$$\begin{aligned}\text{CF} &= [\text{S} - \text{C} - \text{D}](1 - \text{T}) + \text{D}\\ &= [(450{,}000 - 300{,}000) - (150{,}000 - 120{,}000) - (100{,}000 - 40{,}000)]\\ &\quad (1 - 0.30) + (100{,}000 - 40{,}000)\\ &= (150{,}000 - 30{,}000 - 60{,}000)(1 - 0.30) + 60{,}000 = \$102{,}000\end{aligned}$$

The incremental sales are \$150,000, incremental cash operating expenses are \$30,000, and incremental depreciation is \$60,000. The incremental after-tax operating cash flow is \$102,000 per year.

At the project termination, the new equipment is expected to be sold for \$200,000, which constitutes an incremental cash flow of \$100,000 over the \$100,000 expected salvage price of the old equipment. Since the accounting salvage values for both the new and old equipment were zero, this gain is taxable at 30 percent. The company also recaptures its investment in net working capital. The terminal year after-tax nonoperating cash flow is

$$\begin{aligned}\text{TNOCF} &= \text{Sal}_T + \text{NWCInv} - T(\text{Sal}_T - B_T)\\ &= (200{,}000 - 100{,}000) + 80{,}000 - 0.30[(200{,}000 - 100{,}000) - (0 - 0)]\\ &= \$150{,}000\end{aligned}$$

Once the cash flows are identified, the NPV and IRR are readily found. The NPV, found by discounting the cash flows at the 8 percent required rate of return, is

$$\text{NPV} = -540{,}000 + \sum_{t=1}^{10} \frac{102{,}000}{1.08^t} + \frac{150{,}000}{1.08^{10}} = \$213{,}907$$

The IRR, found with a financial calculator, is 15.40 percent. Because the NPV is positive, this equipment replacement decision is attractive. The fact that the IRR exceeds the 8 percent required rate of return leads to the same conclusion.

The key to estimating the incremental cash flows for the replacement is to compare the cash flows that occur with the new investment to the cash flows that would have occurred without the new investment. The analyst is comparing the cash flows with a particular course of action to the cash flows with an alternative course of action.

6.3. Spreadsheet Modeling

Although the examples in this book can be readily solved with a financial calculator, capital budgeting is usually done with the assistance of personal computers and spreadsheets such as Microsoft Excel®. Spreadsheets are heavily used for several reasons. Spreadsheets provide a very effective way of building even complex models. Built-in spreadsheet functions (such as those for finding rates of return) are easy to use. The model's assumptions can be changed and solved easily. Models can be shared with other analysts, and they also help in presenting the results of the analysis. Example 2-7 shows how a spreadsheet can be used to solve a capital budgeting problem.

EXAMPLE 2-7 Capital Budgeting with a Spreadsheet

Lawton Enterprises is evaluating a project with the following characteristics:

- Fixed capital investment is \$2,000,000.
- The project has an expected six-year life.

- The initial investment in net working capital is $200,000. At the end of each year, net working capital must be increased so that the cumulative investment in net working capital is one-sixth of the next year's projected sales.
- The fixed capital is depreciated 30 percent in Year 1, 35 percent in Year 2, 20 percent in Year 3, 10 percent in Year 4, 5 percent in Year 5, and 0 percent in Year 6.
- Sales are $1,200,000 in Year 1. They grow at a 25 percent annual rate for the next two years, and then grow at a 10 percent annual rate for the last three years.
- Fixed cash operating expenses are $150,000 for Years 1–3 and $130,000 for Years 4–6.
- Variable cash operating expenses are 40 percent of sales in Year 1, 39 percent of sales in Year 2, and 38 percent in Years 3–6.
- Lawton's marginal tax rate is 30 percent.
- Lawton will sell its fixed capital investments for $150,000 when the project terminates and recapture its cumulative investment in net working capital. Income taxes will be paid on any gains.
- The project's required rate of return is 12 percent.
- If taxable income on the project is negative in any year, the loss will offset gains elsewhere in the corporation, resulting in a tax savings.

1. Determine whether this is a profitable investment using the NPV and IRR.
2. If the tax rate increases to 40 percent and the required rate of return increases to 14 percent, is the project still profitable?

Solution to 1.

EXHIBIT 2-24 Cash Flows for Lawton Investment (rounded to nearest $1,000)

Year	0	1	2	3	4	5	6
Fixed capital investment	–2,000						
NWC investments	–200	–50	–63	–31	–34	–38	
Sales		1,200	1,500	1,875	2,063	2,269	2,496
Fixed cash expenses		150	150	150	130	130	130
Variable cash expenses		480	585	713	784	862	948
Depreciation		600	700	400	200	100	0
Operating income before taxes		–30	65	613	949	1177	1417
Taxes on operating income		–9	20	184	285	353	425
Operating income after taxes		–21	45	429	664	824	992
Add back: Depreciation		600	700	400	200	100	0
After-tax operating cash flow		579	745	829	864	924	992
Salvage value							150
Taxes on salvage value							–45
Return of NWC							416
Total after-tax cash flows	–2,200	529	682	798	830	886	1,513
NPV (at $r = 12$ percent)	1,181						
IRR	26.60%						

Because the NPV of $1,181,000 is positive, the project is profitable for Lawton to undertake. The IRR investment decision rule also indicates that the project is profitable because the IRR of 26.60 percent exceeds the 12 percent required rate of return.

Solution to 2. The tax rate and required return can be changed in the spreadsheet model. When these changes are made, the NPV becomes $736,000 and the IRR becomes 24.02 percent. (The revised spreadsheet is not printed here.) Although profitability is lower, the higher tax rate and required rate of return do not change the investment decision.

6.4. Effects of Inflation on Capital Budgeting Analysis

Inflation affects capital budgeting analysis in several ways. The first decision the analyst must make is whether to do the analysis in "nominal" terms or in "real" terms. Nominal cash flows include the effects of inflation, while real cash flows are adjusted downward to remove the effects of inflation. It is perfectly acceptable to do the analysis in either nominal or real terms, and sound decisions can be made either way. However, inflation creates some issues regardless of the approach.

The cash flows and discount rate used should both be nominal or both be real. In other words, nominal cash flows should be discounted at a nominal discount rate, and real cash flows should be discounted at a real rate. The real rate, just like real cash flows, has had the effect of inflation taken out. In general, the relationship between real and nominal rates is

$$(1 + \text{Nominal rate}) = (1 + \text{Real rate})(1 + \text{Inflation rate})$$

Inflation reduces the value of depreciation tax savings (unless the tax system adjusts depreciation for inflation). The effect of expected inflation is captured in the discounted cash flow analysis. If inflation is higher than expected, the profitability of the investment is correspondingly lower than expected. Inflation essentially shifts wealth from the taxpayer to the government. Higher-than-expected inflation increases the corporation's real taxes because it reduces the value of the depreciation tax shelter. Conversely, lower-than-expected inflation reduces real taxes (the depreciation tax shelters are more valuable than expected).

Inflation also reduces the value of fixed payments to bondholders. When bonds are originally issued, bondholders pay a price for the bonds reflecting their inflationary expectations. If inflation is higher than expected, the real payments to bondholders are lower than expected. Higher-than-expected inflation shifts wealth from bondholders to the issuing corporations. Conversely, if inflation is lower than expected, the real interest expenses of the corporation increase, shifting wealth from the issuing corporation to its bondholders.

Finally, inflation does not affect all revenues and costs uniformly. The company's after-tax cash flows will be better or worse than expected depending on how particular sales outputs or cost inputs are affected. Furthermore, contracting with customers, suppliers, employees, and sources of capital can be complicated as inflation rises.

The capital budgeting model accommodates the effects of inflation, although inflation complicates the capital budgeting process (and the operations of a business, in general).

7. PROJECT ANALYSIS AND EVALUATION

Assessing the opportunity costs and analyzing the risks of capital investments becomes more complex and sophisticated as you examine real cases. The first project interaction we examine in this section is that of comparing mutually exclusive projects with unequal lives. We will briefly describe other project interactions, but will not examine them in detail. We also examine the process of capital budgeting under capital rationing.

Up to this point, we have largely ignored the issue of accounting for risk. We will introduce risk analysis in two ways. The first is accounting for risk on a standalone basis. The second is accounting for risk on a systematic basis.

7.1. Mutually Exclusive Projects with Unequal Lives

We have previously looked at mutually exclusive projects and decided that the best project is the one with the greatest NPV. However, if the mutually exclusive projects have differing lives and the projects will be replaced (or replicated) repeatedly when they wear out, the analysis is more complicated. The analysis of a one-shot (one time only) investment differs from that of an investment chain (in which the asset is replaced regularly in the future).

For example, assume we have two projects with unequal lives of two and three years, with the following after-tax cash flows:

Project S					
	CF_t	–100	60	90	
	t (time)	0	1	2	3

Project L					
	CF_t	–140	80	70	60
	t (time)	0	1	2	3

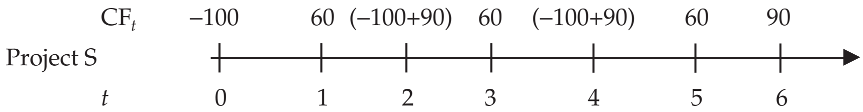
Both projects have a 10 percent required rate of return. The NPV of Project S is \$28.93 and the NPV of Project L is \$35.66. Given that the two projects are mutually exclusive, Project L, with the greater NPV, should be chosen.

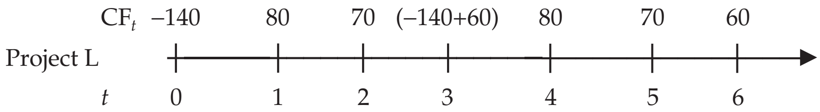
However, let us now assume that these are not one-shot investments, but investments in assets that the company will need to replace when they wear out. Project S would be replaced every two years and Project L every three years. This situation is often referred to as a replacement chain. In this type of problem, you should examine the entire chain and not just the first link in the chain. If the projects are part of a replacement chain, examining the cash flows for only the initial investment for Projects S and L is improper because Project L provides cash flows during Year 3, when Project S provides none.

There are two logically equivalent ways of comparing mutually exclusive projects in a replacement chain. They are the "least common multiple of lives" approach and the "equivalent annual annuity" approach.

7.1.1. Least Common Multiple of Lives Approach

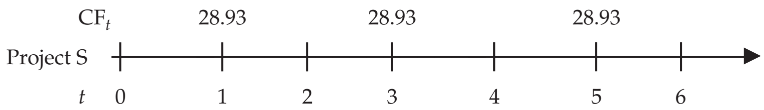
For the least common multiple of lives approach, the analyst extends the time horizon of analysis so that the lives of both projects will divide exactly into the horizon. For Projects S and L, the least common multiple of 2 and 3 is 6: The two-year project would be replicated three times over the six-year horizon and the three-year project would be replicated two times over the six-year horizon.[9] The cash flows for replicating Projects S and L over a six-year horizon are shown below.

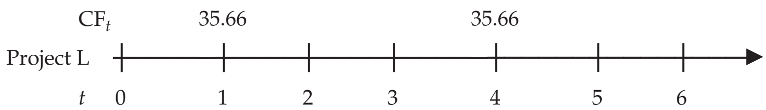




Discounting the cash flows for the six-year horizon results in an NPV for Project S of \$72.59 and an NPV for Project L of \$62.45. Apparently, investing in Project S and replicating the investment over time has a greater NPV than choosing Project L and replicating it. This decision is the reverse of the one we made when looking solely at the NPVs of the initial investments!

Because the NPV of a single investment represents the present values of its cash flows, you can also visualize the NPV of a replacement chain as the present value of the NPVs of each investment (or link) in the chain. For Projects S and L, the NPVs of each investment are shown on the timelines below:





Investing in Project S is equivalent to receiving values of \$28.93 at times 0, 2, and 4, while investing in Project L is equivalent to receiving values of \$35.66 at times 0 and 3. The present values of these cash flow patterns are \$72.59 for Project S and \$62.45 for Project L. Discounting the NPVs of each investment in the chain is equivalent to discounting all of the individual cash flows in the chain.

7.1.2. Equivalent Annual Annuity Approach

The other method for properly evaluating a replacement chain is called the equivalent annual annuity (EAA) approach. The name for this approach is very descriptive. For an investment

[9]The least common multiple of lives is not necessarily the product of the two lives, as in the case of Projects S and L. For example, if two projects have lives of 8 and 10 years, the least common multiple of lives is 40 years, not 80. Both 8 and 10 are exactly divisible into 40.

project with an outlay and variable cash flows in the future, the project NPV summarizes the equivalent value at time zero. For this same project, the EAA is the annuity payment (series of equal annual payments over the project's life) that is equivalent in value to the NPV.

Analysts can use a simple two-step procedure to find the EAA. The first step is to find the present value of all of the cash flows for an investment—the investment's NPV. The second step is to calculate an annuity payment that has a value equivalent to the NPV. For Project S above, we already calculated the NPV of the project over its two-year life to be $28.93. The second step is to find an annuity payment for the two-year life that is equivalent. For a two-year life and a 10 percent discount rate, a payment of $16.66 is the equivalent annuity.

The EAA for Project L is found by annuitizing its $35.66 NPV over three years, so the EAA for Project L is $14.34.

The decision rule for the EAA approach is to choose the investment chain that has the highest EAA, which in this case is Project S.

Given these two approaches to comparing replacement chains, which one should the analyst use? As a practical matter, the two approaches are logically equivalent and will result in the same decision.[10] Consequently, the analyst can choose one approach over the other based on personal preference. Or, if the audience for the analyst's work prefers to see the analysis using one approach, the analyst can simply produce the analysis in that format.

7.2. Capital Rationing

Capital rationing is the case in which the company's capital budget has a size constraint. For example, the capital budget is a fixed money amount. A fixed capital budget can place the company in several interesting situations. To illustrate these, we will assume that the company has a fixed $1,000 capital budget and has the opportunity to invest in four projects. The projects are of variable profitability.

In the first situation, the budget is adequate to invest in all profitable projects. Consider the four projects in Exhibit 2-25.

EXHIBIT 2-25 First Capital Rationing Example

	Investment Outlay	NPV	PI	IRR
Project 1	600	220	1.37	15%
Project 2	200	70	1.35	16%
Project 3	200	–60	0.70	10%
Project 4	400	–100	0.75	8%

In this case, the company has two positive-NPV projects, Projects 1 and 2, which involve a total outlay of $800. Their total NPV is $290. The company should choose these projects, and it will have $200 in its capital budget left over. These excess funds can be used elsewhere

[10]For Projects S and L, the NPVs of a replacement chain over the least common multiple of lives (six years) were $72.59 for Project S and $62.45 for Project L. If we discount the EAA for Project S ($16.66) and the EAA for Project L ($14.34) for six years (treating each as a six-year annuity), we have the same NPVs. Hence, the least common multiple of lives and EAA approaches are consistent with each other.

in the company (moved to someone else's budget, used to pay dividends or repurchase shares, or used to pay down debt). If a manager is afraid to return the excess funds and chooses to invest in Project 3, the manager will consume the whole capital budget but reduce the total NPV to $230, essentially destroying $60 of wealth for the company.

A second case exists in which the company has more profitable projects than it can choose, but it is able to invest in the most profitable ones available. Continuing with the $1,000 capital budget, this second case is illustrated in Exhibit 2-26.

EXHIBIT 2-26 Second Capital Rationing Example

	Investment Outlay	NPV	PI	IRR
Project 5	600	300	1.50	16%
Project 6	200	80	1.40	18%
Project 7	200	60	1.30	12%
Project 8	200	40	1.20	14%

When the analyst has a fixed budget, the PI is especially useful because it shows the profitability of each investment per currency unit invested. If we rank these projects by their PIs, Projects 5, 6, and 7 are the best projects and we are able to select them. This selection results in a total NPV of $440. The IRRs, shown in the last column, are not a reliable guide to choosing projects under capital rationing because a high-IRR project may have a low NPV. Wealth maximization is best guided by the NPV criterion.

A third case exists in which the company has more profitable projects than it can choose, but it is not able to invest in the most profitable ones available. Assume the company cannot invest in fractional projects: It must take all or none of each project it chooses. Continuing with the $1,000 capital budget, this case is illustrated in Exhibit 2-27.

EXHIBIT 2-27 Third Capital Rationing Example

	Investment Outlay	NPV	PI	IRR
Project 9	600	300	1.50	15%
Project 10	600	270	1.45	16%
Project 11	200	80	1.40	12%
Project 12	400	100	1.25	11%

In this example, an unlimited budget of $1,800 would generate a total NPV of $750. However, when the budget constraint is imposed, the highest NPV results from choosing Projects 9 and 12. The company is forced to choose its best project and its fourth-best project, as indicated by their relative PIs. Any other combination of projects either violates the budget or has a lower total NPV.

Capital rationing has the potential to misallocate resources. Capital markets are supposed to allocate funds to their highest and best uses, with the opportunity cost of funds (used as the discount rate for NPVs or the hurdle rate for IRRs) guiding this allocation process. Capital rationing violates market efficiency if society's resources are not allocated where they will generate the best returns. Companies that use capital rationing may be doing either "hard" or "soft" capital rationing. Under hard capital rationing, the budget is fixed and the managers cannot go beyond it. Under soft capital rationing, managers may be allowed to over-spend their budgets if they argue effectively that the additional funds will be deployed profitably.

In the case of hard rationing, choosing the optimal projects that fit within the budget and maximize the NPV of the company can be computationally intensive. Sometimes, managers use estimates and trial and error to find the optimal set of projects. The PI can be used as a guide in this trial and error process. Other times, the number of possibilities is so daunting that mathematical programming algorithms are used.

7.3. Risk Analysis of Capital Investments—Standalone Methods

So far, we have evaluated projects by calculating a single NPV to decide whether a project is profitable. We took a single value, or point estimate, of each input into the model and combined the values to calculate the NPV.

Risk is usually measured as a dispersion of outcomes. In the case of standalone risk, we typically measure the riskiness of a project by the dispersion of its NPVs or the dispersion of its IRRs. Sensitivity analysis, scenario analysis, and simulation analysis are very popular standalone risk analysis methods. These risk measures depend on the variation of the project's cash flows.

To illustrate the standalone risk tools, we will use the following "base case" capital project:

Unit price	\$5.00
Annual unit sales	40,000
Variable cost per unit	\$1.50
Investment in fixed capital	\$300,000
Investment in working capital	\$50,000
Project life	6 years
Depreciation (straight-line)	\$50,000
Expected salvage value	\$60,000
Tax rate	40 percent
Required rate of return	12 percent

The outlay, from Equation 2-6, is \$300,000 plus \$50,000, or \$350,000. The annual after-tax operating cash flow, from Equation 2-7, is

$$\begin{aligned} CF &= (S - C - D)(1 - T) + D \\ &= [(5 \times 40{,}000) - (1.50 \times 40{,}000) - (50{,}000)](1 - 0.40) + 50{,}000 \\ &= \$104{,}000 \end{aligned}$$

The terminal year after-tax nonoperating cash flow, from Equation 2-9, is

$$\begin{aligned} \text{TNOCF} &= \text{Sal}_6 + \text{NWCInv} - \text{T}(\text{Sal}_6 - \text{B}_6) \\ &= 60{,}000 + 50{,}000 - 0.40(60{,}000 - 0) = \$86{,}000 \end{aligned}$$

The project NPV is

$$\text{NPV} = -350{,}000 + \sum_{t=1}^{6} \frac{104{,}000}{1.12^t} + \frac{86{,}000}{1.12^6} = -350{,}000 + 471{,}157 = \$121{,}157$$

7.3.1. Sensitivity Analysis

Sensitivity analysis calculates the effect on the NPV of changes in one input variable at a time. The base case above has several input variables. If we wish to do a sensitivity analysis of several of them, we must specify the changes in each that we wish to evaluate. Suppose we want to consider the following:

	Base Value	Low Value	High Value
Unit price	\$5.00	\$4.50	\$5.50
Annual unit sales	40,000	35,000	45,000
Variable cost per unit	\$1.50	\$1.40	\$1.60
Expected salvage value	\$60,000	\$30,000	\$80,000
Tax rate	40%	38%	42%
Required rate of return	12%	10%	14%

We have changed each of six input variables. Exhibit 2-28 shows the NPV calculated for the base case. Then the NPV is recalculated by changing one variable from its base case value to its high or low value.

EXHIBIT 2-28 Sensitivity of Project NPV to Changes in a Variable

	Project NPV			
Variable	Base Case	With Low Estimate	With High Estimate	Range of Estimates
Unit price	\$121,157	\$71,820	\$170,494	\$98,674
Annual unit sales	\$121,157	\$77,987	\$164,326	\$86,339
Cost per unit	\$121,157	\$131,024	\$111,289	\$19,735
Salvage value	\$121,157	\$112,037	\$127,236	\$15,199
Tax rate	\$121,157	\$129,165	\$113,148	\$16,017
Required return	\$121,157	\$151,492	\$93,602	\$57,890

As Exhibit 2-28 shows, the project's NPV is most sensitive to changes in the unit price variable. The project's NPV is least sensitive to changes in the salvage value. Roughly speaking, the project's NPV is most sensitive to changes in unit price and in unit sales. It is least affected by changes in cost per unit, salvage value, and the tax rate. Changes in the required rate of return also have a substantial effect, but not as much as changes in price or unit sales.

In a sensitivity analysis, the manager can choose which variables to change and by how much. Many companies have access to software that can be instructed to change a particular variable by a certain amount—for example, to increase or decrease unit price, unit sales, and cost per unit by 10 percent. The software then produces the changes in NPV for each of these changes. Sensitivity analysis can be used to establish which variables are most influential on the success or failure of a project.

7.3.2. Scenario Analysis

Sensitivity analysis calculates the effect on the NPV of changes in one variable at a time. In contrast, scenario analysis creates scenarios that consist of changes in several of the input variables and calculates the NPV for each scenario. Although corporations could do a large number of scenarios, in practice they usually do only three. They can be labeled variously, but we will present an example with "pessimistic," "most likely," and "optimistic" scenarios. Continuing with the basic example from the section above, the values of the input variables for the three scenarios are given in the table in Exhibit 2-29.

EXHIBIT 2-29 Input Variables and NPV for Scenario Analysis

	Scenario		
Variable	Pessimistic	Most Likely	Optimistic
Unit price	$4.50	$5.00	$5.50
Annual unit sales	35,000	40,000	45,000
Variable cost per unit	$1.60	$1.50	$1.40
Investment in fixed capital	$320,000	$300,000	$280,000
Investment in working capital	$50,000	$50,000	$50,000
Project life	6 years	6 years	6 years
Depreciation (straight-line)	$53,333	$50,000	$46,667
Salvage value	$40,000	$60,000	$80,000
Tax rate	40%	40%	40%
Required rate of return	13%	12%	11%
NPV	–$5,725	$121,157	$269,685
IRR	12.49%	22.60%	34.24%

The most likely scenario is the same as the base case we used above for sensitivity analysis, and the NPV for the most likely scenario is $121,157. To form the pessimistic and optimistic scenarios, managers change several of the assumptions for each scenario. For the pessimistic scenario, several of the input variables are changed to reflect higher costs, lower revenues, and a higher required rate of return. As the table shows, the result is a negative NPV for the pessimistic scenario and an IRR that is less than the pessimistic scenario's 13 percent required rate of return. For the optimistic scenario, the more favorable revenues, costs, and required rate of return result in very good NPV and IRR.

For this example, the scenario analysis reveals the possibility of an unprofitable investment, with a negative NPV and with an IRR less than the cost of capital. The range for the NPV is fairly large compared to the size of the initial investment, which indicates that the investment is fairly risky. This example included three scenarios for which management wants to know the profitability of the investment for each set of assumptions. Other scenarios can be investigated if management chooses to do so.

7.3.3. Simulation (Monte Carlo) Analysis

Simulation analysis is a procedure for estimating a probability distribution of outcomes, such as for the NPV or IRR for a capital investment project. Instead of assuming a single value (a point estimate) for the input variables in a capital budgeting spreadsheet, the analyst can assume several variables to be stochastic, following their own probability distributions. By simulating the results hundreds or thousands of times, the analyst can build a good estimate of the distributions for the NPV or IRR. Because of the volume of computations, analysts and corporate managers rely heavily on their personal computers and specialized simulation software such as @RISK.[11] Example 2-8 presents a simple simulation analysis.

EXAMPLE 2-8 Capital Budgeting Simulation

Gouhua Zhang has made the following assumptions for a capital budgeting project:

- Fixed capital investment is 20,000; no investment in net working capital is required.
- The project has an expected five-year life.
- The fixed capital is depreciated straight-line to zero over a five-year life. The salvage value is normally distributed with an expected value of 2,000 and a standard deviation of 500.
- Unit sales in Year 1 are normally distributed with a mean of 2,000 and a standard deviation of 200.
- Unit sales growth after Year 1 is normally distributed with a mean of 6 percent and standard deviation of 4 percent. Assume the same sales growth rate for Years 2–5.
- The sales price is 5.00 per unit, normally distributed with a standard deviation of 0.25 per unit. The same price holds for all five years.
- Cash operating expenses as a percentage of total revenue are normally distributed with a mean and standard deviation of 30 percent and 3 percent, respectively.
- The discount rate is 12 percent and the tax rate is 40 percent.

[11]@RISK is a popular and powerful risk analysis tool sold by Palisade Corporation. @RISK is an add-in for Microsoft Excel that allows simulation techniques to be incorporated into spreadsheet models.

1. What are the NPV and IRR using the expected values of all input variables?
2. Perform a simulation analysis and provide probability distributions for the NPV and IRR.

Solution to 1.

EXHIBIT 2-30 Expected Cash Flows for Simulation Example

Time	0	1	2	3	4	5
Fixed capital	–20,000					
After-tax salvage value						1,200
Price		5.00	5.00	5.00	5.00	5.00
Output		2,000	2,120	2,247	2,382	2,525
Revenue		10,000	10,600	11,236	11,910	12,625
Cash operating expenses		3,000	3,180	3,371	3,573	3,787
Depreciation		4,000	4,000	4,000	4,000	4,000
Operating income before taxes		3,000	3,420	3,865	4,337	4,837
Taxes on operating income		1,200	1,368	1,546	1,735	1,935
Operating income after taxes		1,800	2,052	2,319	2,602	2,902
Depreciation		4,000	4,000	4,000	4,000	4,000
Total after-tax cash flow	–20,000	5,800	6,052	6,319	6,602	8,102
NPV (at $r = 12$ percent)	3,294					
IRR	18.11%					

Based on the point estimates for each variable (the mean values for each), which are shown in Exhibit 2-30 above, Zhang should find the NPV to be 3,294 and the IRR to be 18.11 percent.

Solution to 2. Zhang performs a simulation using @RISK with 10,000 iterations. For each iteration, values for the five stochastic variables (price, output, output growth rate, cash expense percentage, and salvage value) are selected from their assumed distributions and the NPV and IRR are calculated. After the 10,000 iterations, the resulting information about the probability distributions for the NPV and IRR is shown in Exhibits 2-31 and 2-32.

As shown, the distributions for the NPV and IRR are somewhat normal looking. The means and standard deviations for each are given in Exhibit 2-32. Both distributions have a slight positive skewness, which means the distributions are skewed to the right. The two kurtosis values are fairly close to 3.0, which means that the distributions are not peaked or fat-tailed compared to the standard normal distribution. The median is the value at which 50 percent of the 10,000 outcomes fall on either side. The 90 percent confidence intervals show that 90 percent of the observations fall between –379 and 7,413 for the NPV and between 11.38 percent and 25.13 percent for the IRR. Although not shown in the table, 7.04 percent of the observations had a negative NPV and an IRR less than the 12 percent discount rate.

The means of the NPV and IRR from the simulation (in Exhibit 2-32) are fairly close to their values calculated using point estimates for all of the input variables (in Exhibit 2-30). This is not always the case, but it is here. The additional information from a simulation is the dispersions of the NPV and IRR. Given his assumptions and model, the simulation results show Zhang the distributions of NPV and IRR outcomes that should be expected. Managers and analysts often prefer to know these total distributions rather than just their mean values.

EXHIBIT 2-31 Probability Distributions for NPV and IRR

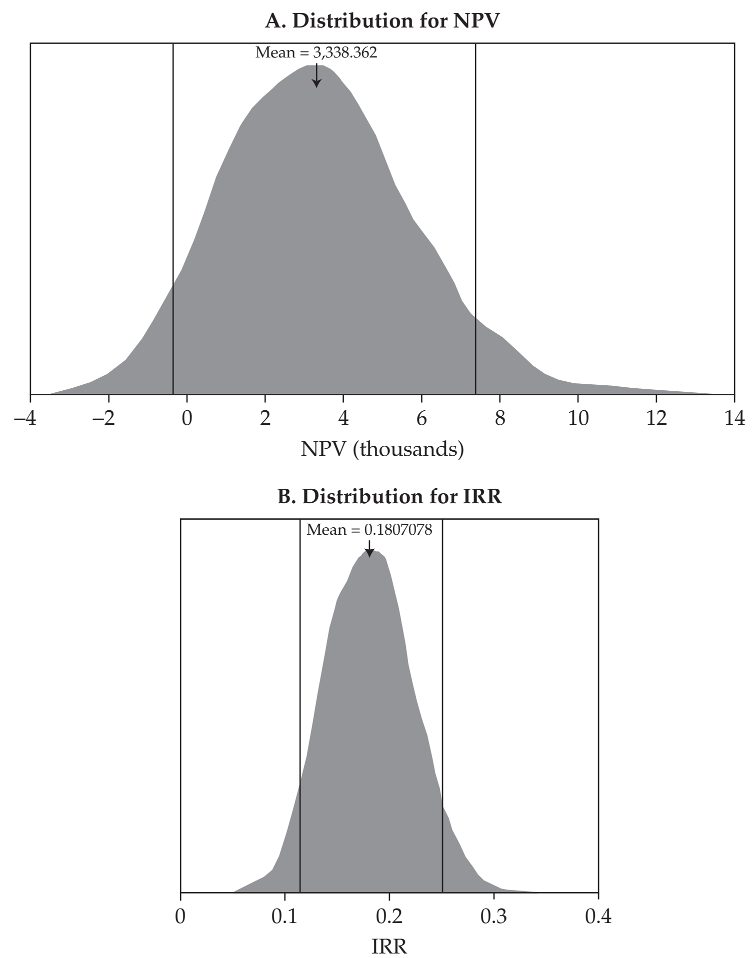


EXHIBIT 2-32 Summary Statistics for NPV and IRR

Statistic	NPV	IRR
Mean	3,338	18.07%
Standard deviation	2,364	4.18%
Skewness	0.2909	0.1130
Kurtosis	3.146	2.996
Median	3,236	18.01%
90% confidence interval	–379 to 7,413	11.38% to 25.13%

Correlations between Input Variables and NPV and IRR

Input Variable	NPV	IRR
Output	0.71	0.72
Output growth rate	0.49	0.47
Price	0.34	0.34
Cash expense proportion	–0.28	–0.29
Salvage value	0.06	0.05

The correlations in Exhibit 2-32 can be interpreted as sensitivity measures. Changes in the "output" variable have the highest correlation with NPV and IRR outcomes. The salvage value has the lowest (absolute value) correlation.

This capital budgeting simulation example was not very complex, with only five stochastic variables. The example's five input variables were assumed to be normally distributed—in reality, many other distributions can be employed. Finally, the randomly chosen values for each variable were assumed to be independent. They can be selected jointly instead of independently. Simulation techniques have proved to be a boon for addressing capital budgeting problems.

Sensitivity analysis, scenario analysis, and simulation analysis are well-developed standalone risk analysis methods. These risk measures depend on the variation of the project's cash flows. Market risk measures, presented in the next section, depend not only on the variation of a project's cash flows, but also on how those cash flows covary with (or correlate with) market returns.

7.4. Risk Analysis of Capital Investments—Market Risk Methods

When using market risk methods, the discount rate to be used in evaluating a capital project is the rate of return required on the project by a diversified investor. The discount rate should thus be a risk-adjusted discount rate, which includes a premium to compensate investors for risk.[12] This risk premium should reflect factors that are priced or valued in the marketplace. The two

[12]Our approach to capital budgeting is to discount expected cash flows at a risk-adjusted cost of capital. An alternative approach, which is also conceptually sound, is the "certainty-equivalent method." In this method, certainty-equivalent cash flows (expected cash flows that are reduced to certainty equivalents) are valued by discounting them at a risk-free discount rate. The use of risk-adjusted discount rates is more intuitive and much more popular.

equilibrium models for estimating this risk premium are the capital asset pricing model (CAPM) and arbitrage pricing theory (APT). We will discuss the CAPM as a way of finding risk-adjusted discount rates, although you should be aware that other methods can be used.

In the CAPM, total risk can be broken into two components: systematic risk and unsystematic risk. Systematic risk is the portion of risk that is related to the market and that cannot be diversified away. Unsystematic risk is nonmarket risk, risk that is idiosyncratic and that can be diversified away. Diversified investors can demand a risk premium for taking systematic risk, but not unsystematic risk.[13] Hence, the standalone risk measures—total risk measured by the dispersion of the NPV or the IRR—are inappropriate when the corporation is diversified, or, as is more likely, when the corporation's investors are themselves diversified.

In the capital asset pricing model, a project's or asset's "beta," or β, is generally used as a measure of systematic risk. The security market line (SML) expresses the asset's required rate of return as a function of β:

$$r_j = R_F + \beta_i[E(R_M) - R_F] \tag{2-10}$$

where

r_i = required return for project or asset i

R_F = risk-free rate of return

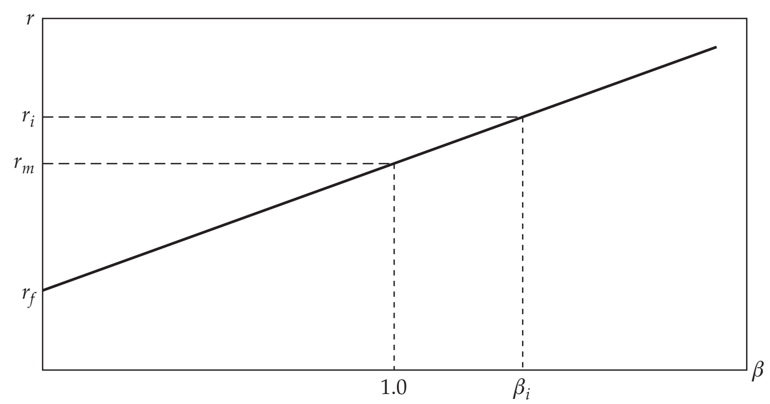
β_i = beta of project or asset i

$[\mathrm{E}(R_M) - R_F]$ = market risk premium, the difference between the expected market return and the risk-free rate of return

The project's required rate of return is equal to the risk-free rate plus a risk premium, where the risk premium is the product of the project beta and the market risk premium.

Here, the required rate of return (sometimes called a hurdle rate) is specific to the risk of the project. There is no one hurdle rate appropriate for all projects.

EXHIBIT 2-33 SML for Capital Budgeting Projects



[13]The capital asset pricing model uses this intuition to show how risky assets should be priced relative to the market. While the CAPM assigns a single market risk premium for each security, the APT develops a set of risk premia. The CAPM and APT are developed in detail elsewhere in the CFA curriculum.

The security market line (SML) is graphed in Exhibit 2-33. This line indicates the required rate of return for a project, given its beta. The required rate of return can be used in two ways:

1. The SML is used to find the required rate of return. The required rate of return is then used to find the NPV. Positive NPV projects are accepted and negative NPV projects are rejected.
2. The SML is used to find the required rate of return. The project's IRR is compared to the required rate of return. If the IRR is greater than the required return, the project is accepted (this point would plot above the SML in Exhibit 2-33). If the IRR is less than the required rate of return (below the SML), the project is rejected.

Example 2-9 illustrates how the capital asset pricing model and the security market line are used as part of the capital budgeting process.

EXAMPLE 2-9 Using the SML to Find the Project Required Rate of Return

Premont Systems is evaluating a capital project with the following characteristics:

- The initial outlay is €150,000.
- Annual after-tax operating cash flows are €28,000.
- After-tax salvage value at project termination is €20,000.
- Project life is 10 years.
- The project beta is 1.20.
- The risk-free rate is 4.2 percent and the expected market return is 9.4 percent.

1. Compute the project NPV. Should the project be accepted?
2. Compute the project IRR. Should the project be accepted?

Solution to 1. The project required rate of return is

$$r_i = R_F + \beta_i[E(R_M) - R_F] = 4.2\% + 1.20(9.4\% - 4.2\%) = 4.2\% + 6.24\%$$
$$= 10.44\%$$

The cash flows discounted at 10.44 percent give an NPV of

$$\text{NPV} = -150{,}000 + \sum_{t=1}^{10} \frac{28{,}000}{1.1044^t} + \frac{20{,}000}{1.1044^{10}} = €26{,}252$$

The project should be accepted because it has a positive NPV.

Solution to 2. The IRR, found with a financial calculator, is 14.24 percent. The required rate of return, established with the SML as in the solution to Question 1 above, is 10.44 percent. Since the IRR exceeds the required rate of return, the project should be accepted. For a beta of 1.20, the IRR of 14.24 percent would plot above the SML.

Using project betas to establish required rates of return for capital projects is especially important when a project's risk differs from that of the company. The cost of capital for a company is estimated for the company as a whole—it is based on the average riskiness of the company's assets as well as its financial structure. The required rates of return of debt and equity are used to estimate the weighted (overall) average cost of capital (WACC) for the company. When a project under consideration is more risky or less risky than the company, the WACC should not be used as the project required rate of return.

For example, assume that the risk-free rate of return is 3 percent, the market return is 8 percent, and the company beta is 0.9. Assume also that the company is considering three projects: Project A with a 0.5 beta, Project B with a 0.9 beta, and Project C with a 1.1 beta. The required rates of return for the company and for each project are as follows:

Company:	$3\% + 0.9(8\% - 3\%) = 7.5\%$
Project A:	$3\% + 0.5(8\% - 3\%) = 5.5\%$
Project B:	$3\% + 0.9(8\% - 3\%) = 7.5\%$
Project C:	$3\% + 1.1(8\% - 3\%) = 8.5\%$

If management uses the company WACC as the required return for all projects, this rate is too high for Project A, making it less likely that Project A would be accepted. Project B has the same risk as the company, so it would be evaluated fairly. Using the WACC for Project C makes the error of using a discount rate that is too low, which would make it more likely that this high-risk project would be accepted. Whenever possible, it is desirable to use project-specific required rates of return instead of the company's overall required rate of return.

Market returns are readily available for publicly traded companies. The stock betas of these companies can then be calculated, and this calculation assists in estimating the companies' betas and WACC. Unfortunately, however, the returns for specific capital projects are not directly observable, and we have to use proxies for their betas. Frequently, we can employ the pure-play method, in which the analyst identifies other publicly traded stocks in the same business as the project being considered. The betas for the stocks of these companies are used to estimate a project beta. In the pure-play method, these proxy companies need to be relatively focused in the same line of business as the project. When the pure-play method is not possible, other methods, such as estimating accounting betas or cross-sectional regression analysis, are used.

7.5. Real Options

Real options are capital budgeting options that allow managers to make decisions in the future that alter the value of capital budgeting investment decisions made today. Instead of making all capital budgeting decisions now, at time zero, managers can wait and make additional decisions at future dates when these future decisions are contingent upon future economic events or information. These sequential decisions, in which future decisions depend on the decisions made today as well as on future economic events, are very realistic capital budgeting applications.

Real options are like financial options—they just deal with real assets instead of financial assets. A simple financial option could be a call option on a share of stock. Suppose the stock is selling for \$50, the exercise (strike) price is \$50, and the option expires in one year. If the stock goes up to \$60, you exercise the option and have a gain of \$10 in one year. If the stock goes down to $40, you do not exercise, and you have no gain. However, no gain is better than the $10 loss you would have had if you had purchased the stock at the beginning of the year. Real options, like financial options, entail the right to make a decision, but not the obligation. The corporation should exercise a real option only if it is value-enhancing.

Just as financial options are contingent on an underlying asset, real options are contingent on future events. The flexibility that real options give to managers can greatly enhance the NPV of the company's capital investments. The following are several types of these real options:

Timing options. Instead of investing now, the company can delay investing. Delaying an investment and basing the decision on hopefully improved information that you might have in, say, a year could help improve the NPV of the projects selected.

Sizing options. If after investing, the company can abandon the project when the financial results are disappointing, it has an **abandonment option**. At some future date, if the cash flow from abandoning a project exceeds the present value of the cash flows from continuing the project, managers should exercise the abandonment option. Conversely, if the company can make additional investments when future financial results are strong, the company has a **growth option** or an **expansion option**.

Flexibility options. Once an investment is made, other operational flexibilities may be available besides abandonment or expansion. For example, suppose demand exceeds capacity. Management may be able to exercise a **price-setting option**. By increasing prices, the company could benefit from the excess demand, which it cannot do by increasing production. There are also **production-flexibility** options. Even though it is expensive, the company can profit from working overtime or from adding additional shifts. The company can also work with customers and suppliers for their mutual benefit whenever a demand–supply mismatch occurs. This type of option also includes the possibility of using different inputs or producing different outputs.

Fundamental options. In cases like those above, there are options embedded in a project that can raise its value. In other cases, the whole investment is essentially an option. The payoffs from the investment are contingent on an underlying asset, just like most financial options. For example, the value of an oil well or refinery investment is contingent upon the price of oil. The value of a gold mine is contingent upon the price of gold. If oil prices are low, you may not drill a well. If oil prices are high, you go ahead and drill. Many R&D (research and development) projects also look like options.

There are several approaches to evaluating capital budgeting projects with real options. One of the difficulties with real options is that the analysis can be very complicated. Although some of the problems are simple and can be readily solved, many of them are so complex that they are expensive to evaluate or you may not have much confidence in the analysis. Four commonsense approaches to real options analysis are presented below.

1. Use discounted cash flow (DCF) analysis without considering options. If the NPV is positive without considering real options, and the project has real options that would simply add more value, it is unnecessary to evaluate the options. Just go ahead and make the investment.
2. Consider the Project NPV = NPV (based on DCF alone) – Cost of options + Value of options. Go ahead and calculate the NPV based on expected cash flows. Then simply add the value associated with real options. For example, if a project has a negative NPV based on DCF alone of $50 million, will the options add at least that much to its value?

3. Use decision trees. Although they are not as conceptually sound as option pricing models, decision trees can capture the essence of many sequential decision making problems.
4. Use option pricing models. Except for simple options, the technical requirements for solving these models may require you to hire special consultants or "quants." Some large companies have their own specialists.

The analyst is confronted with (1) a variety of real options that investment projects may possess and (2) a decision about how to reasonably value these options. Example 2-10 deals with production flexibility; in this case, an additional investment outlay gives the company an option to use alternative fuel sources.

EXAMPLE 2-10 Production-Flexibility Option

Sackley AquaFarms estimated the NPV of the expected cash flows from a new processing plant to be –\$0.40 million. Sackley is evaluating an incremental investment of \$0.30 million that would give management the flexibility to switch between coal, natural gas, and oil as an energy source. The original plant relied only on coal. The option to switch to cheaper sources of energy when they are available has an estimated value of \$1.20 million. What is the value of the new processing plant including this real option to use alternative energy sources?

Solution. The NPV, including the real option, should be:

$$\text{Project NPV} = \text{NPV (based on DCF alone)} - \text{Cost of options} + \text{Value of options}$$

$$\text{Project NPV} = -0.40 \text{ million} - 0.30 \text{ million} + 1.20 \text{ million} = \$0.50 \text{ million}$$

Without the flexibility offered by the real option, the plant is unprofitable. The real option to adapt to cheaper energy sources adds enough to the value of this investment to give it a positive NPV.

Two of the most valuable options are to abandon or expand a project at some point after the original investment. Example 2-11 illustrates the abandonment option.

EXAMPLE 2-11 Abandonment Option

Nyberg Systems is considering a capital project with the following characteristics:

- The initial outlay is €200,000.
- Project life is four years.
- Annual after-tax operating cash flows have a 50 percent probability of being €40,000 for the four years and a 50 percent probability of being €80,000.

- Salvage value at project termination is zero.
- The required rate of return is 10 percent.
- In one year, after realizing the first-year cash flow, the company has the option to abandon the project and receive the salvage value of €150,000.

1. Compute the project NPV assuming no abandonment.
2. What is the optimal abandonment strategy? Compute the project NPV using that strategy.

Solution to 1. The expected annual after-tax operating cash flow is 0.50(40,000) + 0.50 (80,000) = €60,000. The cash flows discounted at 10 percent give an NPV of

$$\text{NPV} = -200{,}000 + \sum_{t=1}^{4} \frac{60{,}000}{1.10^t} = -€9{,}808$$

The project should be rejected because it has a negative NPV.

Solution to 2. The optimal abandonment strategy would be to abandon the project in one year if the subsequent cash flows are worth less than the abandonment value. If at the end of the first year the low cash flow occurs, you can abandon for €150,000 and give up €40,000 for the following three years. The €40,000 annual cash flow, discounted for three years at 10 percent, has a present value of only €99,474, so you should abandon. Three years of the higher €80,000 cash flow has a present value of €198,948, so you should not abandon. After the first year, abandon if the low cash flow occurs, and do not abandon if the high cash flow occurs.

If the high cash flow occurs and you do not abandon, the NPV is

$$\text{NPV} = -200{,}000 + \sum_{t=1}^{4} \frac{80{,}000}{1.10^t} = €53{,}589$$

If you abandon when the low cash flow occurs, you receive the first year cash flow and the abandonment value and then no further cash flows. In that case, the NPV is

$$\text{NPV} = -200{,}000 + \frac{40{,}000 + 150{,}000}{1.10} = €27{,}273$$

The expected NPV is then

$$\text{NPV} = 0.50(53{,}589) + 0.50(-27{,}273) = €13{,}158$$

Optimal abandonment raises the NPV by 13,158 – (–€9,808) = €22,966.

A fundamental real option could be a gold mine or an oil well. Example 2-12 looks at the possibility of purchasing the rights to a gold mining property.

EXAMPLE 2-12 Erichmann Gold Mine

The Erichmann family has offered a five-year option on one of its small gold mining properties for $10 million. The current price of gold is $400 per ounce. The mine holds an estimated 500,000 ounces that could be mined at an average cost of $450 per ounce. The maximum production rate is 200,000 ounces per year. How would you assess the Erichmann family's offer?

Solution. A binomial option model can be built for the underlying price of gold. These binomial models are very common in assessing the value of financial options such as puts and calls on stocks, callable bonds, or mortgages with prepayment options. Whenever the price path for gold is above $450 per ounce, it might be attractive to commence mining. Of course, you would cease mining whenever the price is lower. With additional information about the volatility of gold prices and the risk-free interest rate, an expert could build this binomial model and value the real option. Comparing the value of this real option to its $10 million cost would enable you to make an investment decision.

A critical assumption of many applications of traditional capital budgeting tools is that the investment decision is made now, with no flexibility considered in future decisions. A more reasonable approach is to assume that the corporation is making sequential decisions, some now and some in the future. A combination of optimal current and future decisions is what will maximize company value. Real options analysis tries to incorporate rational future decisions into the assessment of current investment decision making. This future flexibility, exercised intelligently, enhances the value of capital investments. Some real options can be valued with readily available option pricing models, such as the binomial model or the Black–Scholes–Merton option pricing model.[14] Unfortunately, many real options are very complex and hard to value, which poses a challenge as the analyst tries to lay out the economic contingencies of an investment and assess their values. A real option, with the future flexibility it provides, can be an important piece of the value of many projects.

7.6. Common Capital Budgeting Pitfalls

Although the principles of capital budgeting may be easy to learn, applying the principles to real world investment opportunities can be challenging. Some of the common mistakes that managers make are listed in Exhibit 2-34.

Economic responses. Economic responses to an investment often affect its profitability, and these responses have to be correctly anticipated. For example, in response to a successful investment, competitors can enter and reduce the investment's profitability. Similarly, vendors, suppliers, and employees may want to gain from a profitable enterprise. Companies that

[14]Chapter 4 of Chance (2003) gives an excellent overview of option pricing models.

EXHIBIT 2-34 Common Capital Budgeting Pitfalls

Not incorporating economic responses into the investment analysis
Misusing capital budgeting templates
Pet projects
Basing investment decisions on EPS, net income, or return on equity
Using IRR to make investment decisions
Bad accounting for cash flows
Overhead costs
Not using the appropriate risk-adjusted discount rate
Spending all of the investment budget just because it is available
Failure to consider investment alternatives
Handling sunk costs and opportunity costs incorrectly

make highly profitable investments often find that a competitive marketplace eventually causes profitability to revert to normal levels.

Template errors. Because hundreds or even thousands of projects need to be analyzed over time, corporations have standardized capital budgeting templates for managers to use in evaluating projects. This situation creates risks in that the template model may not match the project, or employees may input inappropriate information.

Pet projects. Pet projects are projects that influential managers want the corporation to invest in. Ideally, pet projects will receive the normal scrutiny that other investments receive and will be selected on the strength of their own merits. Often, unfortunately, pet projects are selected without undergoing normal capital budgeting analysis. Or the pet project receives the analysis, but overly optimistic projections are used to inflate the project's profitability.

EPS, net income, or ROE. Managers sometimes have incentives to boost EPS, net income, or ROE. Many investments, even those with strong NPVs, do not boost these accounting numbers in the short run and may even reduce them. Paying attention to short-run accounting numbers can result in choosing projects that are not in the long-run economic interests of the business.

Basing decisions on the IRR. The NPV criterion is economically sound. The IRR criterion is also sound for independent projects (with conventional cash flow patterns). If projects are mutually exclusive or competitive with each other, investing in projects based on the IRR will tend to result in choosing smaller, short-term projects with high IRRs at the expense of larger, longer-term, high NPV projects. Basing decisions on paybacks or accounting rates of return is even more dangerous. These measures can be economically unsound.

Bad accounting for cash flows. In analyzing a complicated project, it is easy to omit relevant cash flows, double count cash flows, and mishandle taxes.

Overhead costs. In large companies, the cost of a project must include the overhead it generates for such things as management time, information technology support, financial systems, and other support. Although these items are hard to estimate, over- or under-estimating these overhead costs can lead to poor investment decisions.

Discount rate errors. The required rate of return for a project should be based on its risk. If a project is being financed with debt (or with equity), you should still use the project's required rate of return and not the cost of debt (or the cost of equity). Similarly, a high-risk project should not be discounted at the company's overall cost of capital, but at the project's required rate of return. Discount rate errors have a huge impact on the computed NPVs of long-lived projects.

Overspending and underspending the capital budget. Politically, many managers will spend all of their budget and argue that their budget is too small. In a well-run company, managers will return excess funds whenever their profitable projects cost less than their budget, and managers will make a sound case for extra funds if their budget is too small.

Failure to consider investment alternatives. Generating good investment ideas is the most basic step in the capital budgeting process, and many good alternatives are never even considered.

Sunk costs and opportunity costs. Ignoring sunk costs is difficult for managers to do. Furthermore, not identifying the economic alternatives (real and financial) that are the opportunity costs is probably the biggest failure in much analysis. Only costs that change with the decision are relevant.

8. OTHER INCOME MEASURES AND VALUATION MODELS

Capital budgeting was one of the first widespread applications of discounted cash flow analysis. In the basic capital budgeting model, the analyst values an investment by discounting future after-tax cash flows at the rate of return required by investors. Subtracting the initial investment results in the project's NPV. The future cash flows consist of after-tax operating cash flows plus returns of investment (such as salvage value and sale of working capital).

Analysts will employ and encounter other concepts of income and other valuation approaches besides this basic capital budgeting model. Because some of these other approaches are economically sound and widely employed, we will briefly describe some of them here. By considering these approaches, you can see the distinguishing features of each approach and that they should result in consistent valuations (if they are used correctly).

To facilitate the comparison of income measures and valuation models, we will employ as an example a simple company (the Granite Corporation) that invests in one project. The company goes out of business when that project expires. After evaluating that project with the NPV and IRR capital budgeting models, we will examine that same project using the following alternative methods:

- Economic income and accounting income
- Economic profit valuation
- Residual income valuation
- Claims valuation

Our purpose is to show how the various income measures and valuation methods are related to each other.

8.1. The Basic Capital Budgeting Model

The basic capital budgeting model (presented earlier) identifies the after-tax operating cash flows from an investment as well as nonoperating cash flows (such as the initial investment or future recovery of invested capital or net working capital). Then, these cash flows are discounted at the required rate of return for the asset to establish the NPV.

The base-case capital budgeting project is the following. The company is going to invest \$150,000 and generate sales for the next five years as shown in Exhibit 2-35. Variable cash operating expenses will be 50 percent of sales each year, and fixed cash operating expenses are \$20,000. Depreciation is straight-line to zero, \$30,000 per year with a zero book value at the end of five years. The income tax rate is 40 percent. Salvage value is \$10,000, which is taxable at 40 percent, leaving an after-tax salvage value of \$6,000 at the end of five years. The required rate of return is 10 percent.

EXHIBIT 2-35 Basic Capital Budgeting Example for Granite Corporation

Year	0	1	2	3	4	5
Fixed capital investment	–150,000					
Sales		150,000	200,000	250,000	200,000	150,000
Variable cash expenses		75,000	100,000	125,000	100,000	75,000
Fixed cash expenses		20,000	20,000	20,000	20,000	20,000
Depreciation		30,000	30,000	30,000	30,000	30,000
Operating income before taxes		25,000	50,000	75,000	50,000	25,000
Taxes at 40 percent		10,000	20,000	30,000	20,000	10,000
Operating income after taxes		15,000	30,000	45,000	30,000	15,000
After-tax operating cash flow		45,000	60,000	75,000	60,000	45,000
Salvage value						10,000
Taxes on salvage value						4,000
After-tax salvage value						6,000
Total after-tax cash flow	–150,000	45,000	60,000	75,000	60,000	51,000
NPV (at $r = 10$ percent)	69,492					
IRR	26.27%					

The present value of the after-tax cash flows for Years 1–5 is \$219,492. Subtracting the investment of \$150,000 results in the NPV of \$69,492. The IRR for the investment is 26.27 percent.

8.2. Economic and Accounting Income

Economic income and accounting income differ from the after-tax operating cash flows used in the basic capital budgeting model.

Economic income is the profit realized from an investment. For a given year, economic income is the investment's after-tax cash flow plus the change in the market value:

$$\text{Economic income} = \text{Cash flow} + \text{Change in market value}$$

$$\text{Economic income} = \text{Cash flow} + (\text{Ending market value} - \text{Beginning market value}) \tag{2-11}$$

or

$$\text{Economic income} = \text{Cash flow} - (\text{Beginning market value} - \text{Ending market value})$$

$$\text{Economic income} = \text{Cash flow} - \text{Economic depreciation}^{15}$$

For the Granite Corporation, the cash flows are already calculated in Exhibit 2-35. The beginning market value at time zero is the present value of the future after-tax cash flows at the 10 percent required rate of return, or $219,492. The market value at any future date is the present value of subsequent cash flows discounted back to that date. For the Granite Corporation, the cash flows, changes in market value, and economic incomes are shown in Exhibit 2-36.

EXHIBIT 2-36 Economic Income for Granite Corporation

Year	1	2	3	4	5
Beginning market value	219,492	196,441	156,086	96,694	46,364
Ending market value	196,441	156,086	96,694	46,364	0
Change in market value	–23,051	–40,356	–59,391	–50,331	–46,364
After-tax cash flow	45,000	60,000	75,000	60,000	51,000
Economic income	21,949	19,644	15,609	9,669	4,636
Economic rate of return	10%	10%	10%	10%	10%

In Year 1, the beginning value is $219,492 and the ending value is $196,441, so the change in value is –$23,051. The economic income is the cash flow plus the change in value, or $45,000 + (–$23,051) = $21,949. The economic income for Years 2–5 is found similarly. The economic rate of return is the year's economic income divided by its beginning market value. Notice that the economic rate of return is precisely 10 percent each year, which was the required rate of return on the project.

Accounting income for this company will differ from the economic income for two reasons. First, the accounting depreciation is based on the original cost of the investment (not the market value of the investment). Consequently, the accounting depreciation schedule does not follow the declines in the market value of an asset. Besides being based on accounting depreciation instead of economic depreciation, accounting net income is the after-tax income remaining after paying interest expenses on the company's debt obligations. In contrast, interest expenses are ignored when computing the economic income for an asset or the after-tax operating cash flows in the basic capital budgeting model. As explained in Section 3, the effects of financing costs are captured in the discount rate, not in the cash flows. In the capital

[15]These equations are conceptually identical because economic depreciation is the negative of the change in market value. For example, assume the cash flow is 10, the beginning market value is 30, and the ending market value is 25. Cash flow + Change in market value = Cash flow + (Ending market value – Beginning market value) = 10 + (25 – 30) = 5. Or, Cash flow – Economic depreciation = Cash flow – (Beginning market value – Ending market value) = 10 – (30 – 25) = 5.

budgeting model, if we included interest expenses in the cash flows, we would be double counting them.

To illustrate these differences, we will assume that the company borrows an amount equal to one-half of the value of the company, which is 50 percent of \$219,492, or \$109,746, and that it pays $8^1/_3$ percent interest each year on the beginning balance. With a 40 percent tax rate, the after-tax interest cost is $8^1/_3$ percent $(1 - 0.40) = 5.0$ percent. Because the Granite Corporation has a five-year life, it does not need to borrow or retain earnings for the future,

EXHIBIT 2-37 Condensed Financial Statements for Granite Corporation

Year	0	1	2	3	4	5
Balance Sheets:						
Assets	150,000	120,000	90,000	60,000	30,000	0
Liabilities	109,746	98,221	78,043	48,347	23,182	0
Net worth	40,254	21,779	11,957	11,653	6,818	0
Income Statements:						
Sales		150,000	200,000	250,000	200,000	150,000
Variable cash expenses		75,000	100,000	125,000	100,000	75,000
Fixed cash expenses		20,000	20,000	20,000	20,000	20,000
Depreciation		30,000	30,000	30,000	30,000	30,000
EBIT		25,000	50,000	75,000	50,000	25,000
Interest expense		9,146	8,185	6,504	4,029	1,932
EBT		15,854	41,815	68,496	45,971	23,068
Taxes at 40 percent		6,342	16,726	27,399	18,388	9,227
Net income before salvage		9,513	25,089	41,098	27,583	13,841
After-tax salvage value						6,000
Net income		9,513	25,089	41,098	27,583	19,841
Statements of Cash Flows:						
Operating cash flows:						
Net income		9,513	25,089	41,098	27,583	19,841
Depreciation		30,000	30,000	30,000	30,000	30,000
Total		39,513	55,089	71,098	57,583	49,841
Financing cash flows:						
Debt repayment		–11,525	–20,178	–29,696	–25,165	–23,182
Dividends/repurchases		–27,987	–34,911	–41,402	–32,417	–26,659
Total		–39,513	–55,089	–71,098	–57,583	–49,841
Investing cash flows		0	0	0	0	0
Total cash flows		0	0	0	0	0

and all cash flows will be distributed to bondholders and stockholders. Granite will maintain a 50 percent debt/value ratio on the company's debt, so bondholders will receive $8^1/_3$ percent interest on their beginning bond balance and the debt will also be amortized (paid down) whenever the value of the company goes down. Furthermore, after all operating costs, interest expenses, and taxes are paid, stockholders will receive all remaining cash flows each year as a cash dividend or share repurchase.[16]

The financial statements for the Granite Corporation are shown in Exhibit 2-37.

The income statement for financial reporting purposes differs from that used in the capital budgeting model because the interest on debt obligations is now taken out as an expense before arriving at net income. The book value of the company's assets is based on the original accounting cost minus accumulated accounting depreciation. Note that the liabilities and net worth are also declining in the balance sheet. The liabilities decline each year, reflecting the amounts that were paid annually to reduce the principal of the loan. Notice, also, that the net worth is declining. Normally, the net worth of a company increases because beginning equity is increased by net retentions—the excess of net income over dividends paid. In this case, the company is shrinking and going out of business in five years, so the distributions to shareholders (which can be either cash dividends or share repurchases) exceed net income and net worth declines. The amounts that are paid each year to reduce debt and for dividends/share repurchases are shown in the financing section of the statement of cash flows.

Accounting measures of performance also can differ from economic measures of performance. Exhibit 2-38 repeats the economic income and accounting income from Exhibits 2-36 and 2-37. The exhibit also shows the economic rate of return each year and two popular accounting measures of performance: the return on equity (ROE = net income divided by beginning equity) and return on assets (ROA = EBIT divided by beginning assets).

EXHIBIT 2-38 Economic Income, Accounting Income, and Rates of Return for Granite Corporation

Year	1	2	3	4	5
Economic income	21,949	19,644	15,609	9,669	4,636
Accounting income	9,513	25,089	41,098	27,583	19,841
Economic rate of return	10.00%	10.00%	10.00%	10.00%	10.00%
Return on equity (ROE)	23.63%	115.20%	343.71%	236.70%	291.00%
Return on assets (ROA)	16.67%	41.67%	83.33%	83.33%	83.33%

As Exhibit 2-38 illustrates, economic and accounting incomes differ substantially. Over the five years, economic income is much less than accounting income, and the patterns certainly differ. In addition, the accounting rates of return, the ROE and ROA, for this admittedly unusual company are quite different from the economic rate of return.

[16]The assumptions may be unrealistic, but this is a very simple corporation.

8.3. Economic Profit, Residual Income, and Claims Valuation

Although the capital budgeting model is widely employed, analysts have used other procedures to divide up the cash flows from a company or project and then value them using discounted cash flow methods. We present three of these alternative models here: the economic profit model, the residual income model, and the claims valuation model. Used correctly, they are all consistent with the basic capital budgeting model and with each other.

8.3.1. Economic Profit

The first alternative method for measuring income and valuing assets is based on economic profit (EP).[17] Economic profit has been used in asset valuation as well as in performance measurement and management compensation. Its calculation is loosely as follows:

$$EP = NOPAT - \$WACC \tag{2-12}$$

where

$$\begin{aligned}
EP &= \text{Economic profit} \\
NOPAT &= \text{Net operating profit after tax} = EBIT(1 - \text{Tax rate}) \\
EBIT &= \text{Operating income before taxes, or Earnings before interest and taxes} \\
\$WAC &= \text{Dollar cost of capital} = WACC \times \text{Capital} \\
WACC &= \text{Weighted average (or overall) cost of capital} \\
\text{Capital} &= \text{Investment}
\end{aligned}$$

EP is a periodic measure of profit above and beyond the dollar cost of the capital invested in the project. The dollar cost of capital is the dollar return that the company must make on the project in order to pay the debt holders and the equity holders their respective required rates of return.[18]

For the Granite Corporation, for the first year, we have the following:

$$\begin{aligned}
NOPAT &= EBIT(1 - \text{Tax rate}) = 25{,}000(1 - 0.40) = \$15{,}000 \\
\$WACC &= WACC \times \text{Capital} = 10\% \times 150{,}000 = \$15{,}000 \\
EP &= NOPAT - \$WACC = 15{,}000 - 15{,}000 = \$0
\end{aligned}$$

Exhibit 2-39 shows the EP for all five years for the Granite Corporation.

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[17]Economic Value Added® or EVA, trademarked by the consulting firm Stern Stewart & Company, is a well-known commercial application of the economic profit approach. See Stewart (1991) and Peterson and Peterson (1996) for complete discussion.

[18]In the chapter on cost of capital, we will explain the relationship between the required rate of return on the project or WACC (here 10 percent), the rate of return required by debtholders (here 8⅓ percent), and the rate of return required by equityholders (here 15 percent).

EXHIBIT 2-39 EP for Granite Corporation

Year	1	2	3	4	5**
Capital*	150,000	120,000	90,000	60,000	30,000
NOPAT	15,000	30,000	45,000	30,000	21,000
$WACC	15,000	12,000	9,000	6,000	3,000
EP	0	18,000	36,000	24,000	18,000

*Depreciation is $30,000 per year.
**The $6,000 after-tax gain from salvage is included in NOPAT in Year 5.

EP is readily applied to valuation of an asset or a security. The NPV found by discounted cash flow analysis in the basic capital budgeting model will be equal to the present value of future EP discounted at the weighted average cost of capital.

$$\text{NPV} = \sum_{t=1}^{\infty} \frac{\text{EP}_t}{(1 + \text{WACC})^t} \tag{2-13}$$

This NPV is also called the market value added (MVA).[19] So we have

$$\text{NPV} = \text{MVA} = \sum_{t=1}^{\infty} \frac{\text{EP}_t}{(1 + \text{WACC})^t} \tag{2-14}$$

Discounting the five years of EP for the Granite Corporation at the 10 percent WACC gives an NPV (and MVA) of $69,492. The total value of the company (of the asset) is the original investment of $150,000 plus the NPV of $69,492, or $219,492. The valuation using EP is the same as that found with the basic capital budgeting model.

8.3.2. Residual Income

Another method for estimating income and valuing an asset is the residual income method.[20] This method focuses on the returns to equity, where

$$\text{Residual income} = \text{Net income} - \text{Equity charge}$$

or

$$\text{RI}_t = \text{NI}_t - r_e \text{B}_{t-1} \tag{2-15}$$

[19]Peterson and Peterson define MVA as the market value of the company minus the capital invested, which is an NPV.
[20]See Chapter 5 in Pinto, Henry, Robinson, and Stowe (2010) and Edwards and Bell (1961) for treatments of residual income analysis.

where

RI_t = Residual income during period t

NI_t = Net income during period t

$r_e B_{t-1}$ = Equity charge for period t, which is the required rate of return on equity, r_e, times the beginning-of-period book value of equity, B_{t-1}

For the first year for the Granite Corporation, the net income is $9,513. The beginning book value of equity is $40,254 (from the balance sheet in Exhibit 2-37), and the required rate of return on equity is 15 percent. Consequently, the residual income for Year 1 is:

$$RI_t = NI_t - r_e B_{t-1} = 9{,}513 - 0.15(40{,}254) = 9{,}513 - 6{,}038 = \$3{,}475$$

The residual income for all five years for Granite is shown in Exhibit 2-40.

EXHIBIT 2-40 Residual Income for Granite Corporation

Year	1	2	3	4	5*
NI_t	9,513	25,089	41,098	27,583	19,841
$r_e B_{t-1}$	6,038	3,267	1,794	1,748	1,023
RI_t	3,475	21,822	39,304	25,835	18,818

*The $6,000 after-tax gain from salvage is included in NI in Year 5.

Residual income, like EP, can also be applied to valuation of an asset or security. The NPV of an investment is the present value of future residual income discounted at the required rate of return on equity.

$$NPV = \sum_{t=1}^{\infty} \frac{RI_t}{(1 + r_e)^t} \tag{2-16}$$

Discounting the residual income for the Granite Corporation at the 15 percent required rate of return on equity gives an NPV of $69,492. The total value of the company (of the asset) is the present value of the residual income, the original equity investment, plus the original debt investment:

PV of residual income	$69,492
Equity investment	40,254
Debt investment	109,746
Total value	$219,492

The value of the company is the original book value of its debt and equity plus the present value of the residual income (which is the project's NPV). Again, this is the same value we found with the basic capital budgeting model and with the EP model.

8.3.3. Claims Valuation

To value a company, the EP valuation approach essentially adds the present value of EP to the original investment. The residual income approach adds the present value of residual income to the original debt and equity investments in the company. Since the EP approach is from the perspective of all suppliers of capital, EP is discounted at the overall WACC. The residual income approach takes the perspective of equity investors, so residual income is discounted at the cost of equity.

The third and final alternative valuation approach that we present is to divide the operating cash flows between security-holder classes (in this example, debt and equity), and then value the debt and equity cash flows separately.

Balance Sheet

Assets	Liabilities
	Equity

The basic capital budgeting approach is to value the asset, which is on the left-hand side of the balance sheet above. The claims valuation approach values the liabilities and equity, the claims against the assets, which are on the right-hand side of the balance sheet. The value of the claims should equal the value of the assets.

For the Granite Corporation, the cash flows to debtholders are the interest payments and principal payments. These are valued by discounting them at the cost of debt, which is $8^1/_3$ percent. The cash flows to stockholders are the dividends and share repurchases, which are valued by discounting them at the 15 percent cost of equity. Exhibit 2-41 below lists the future cash flows for debt and equity.

EXHIBIT 2-41 Payments to Bondholders and Stockholders of Granite Corporation

Year	1	2	3	4	5
Interest payments	9,146	8,185	6,504	4,029	1,932
Principal payments	11,525	20,178	29,696	25,165	23,182
Total debt payments	20,671	28,363	36,199	29,194	25,114
Equity distributions	27,987	34,911	41,402	32,417	26,659

The present value of the total debt payments, discounted at the cost of debt, is $109,746. The value of the equity distributions, discounted at the cost of equity, is $109,746. The total value of the company is the combined value of debt and equity, which is $219,492.

In our example, the basic capital budgeting model, the economic profit model, the residual income model, and the claims valuation model all result in the same valuation of the company. In the real world, analysts must deal with many accounting complications. Some of these complications may include pension liability adjustments, valuations of marketable securities held, exchange rate gains and losses, and adjustments for leases, inventories, goodwill, deferred taxes, and so forth. In theory, all of the valuation models are equivalent. In practice, even with due diligence and care, analysts may prefer one approach over others and disagree about valuations.

There are other approaches to valuation that analysts use and run across. Two common ones are the free cash flow to the firm **(FCFF)** and free cash flow to equity **(FCFE)** approaches.[21] The free cash flow to the firm approach is fundamentally the same as the basic capital budgeting approach. The free cash flow to equity approach is related to the claims valuation approach. In corporate finance, corporate managers usually value an asset by valuing its total after-tax cash flows. Security analysts typically value equity by valuing the cash flows to stockholders. Real estate investors often evaluate real estate investments by valuing the cash flows to the equity investor after payments to creditors, which is like the claims valuation approach.

9. SUMMARY

Capital budgeting is the process that companies use for decision making on capital projects—those projects with a life of a year or more. This chapter developed the principles behind the basic capital budgeting model, the cash flows that go into the model, and several extensions of the basic model.

- Capital budgeting undergirds the most critical investments for many corporations—their investments in long-term assets. The principles of capital budgeting have been applied to other corporate investing and financing decisions and to security analysis and portfolio management.
- The typical steps in the capital budgeting process are: (1) generating ideas, (2) analyzing individual proposals, (3) planning the capital budget, and (4) monitoring and post-auditing.
- Projects susceptible to capital budgeting process can be categorized as: (1) replacement, (2) expansion, (3) new products and services, and (4) regulatory, safety, and environmental.
- Capital budgeting decisions are based on incremental after-tax cash flows discounted at the opportunity cost of funds. Financing costs are ignored because both the cost of debt and the cost of other capital are captured in the discount rate.
- The net present value (NPV) is the present value of all after-tax cash flows, or

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t}$$

  where the investment outlays are negative cash flows included in the CF_ts and where r is the required rate of return for the investment.
- The IRR is the discount rate that makes the present value of all future cash flows sum to zero. This equation can be solved for the IRR:

$$\sum_{t=0}^{n} \frac{CF_t}{(1+IRR)^t} = 0$$

- The payback period is the number of years required to recover the original investment in a project. The payback is based on cash flows.

[21] The free cash flow to the firm and free cash flow to equity approaches are developed in Chapter 4 of Pinto, Henry, Robinson, and Stowe (2010).