MPF_ASAN Security Analysis and Derivatives

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- Information about the course in the Interactive sylabus

Literature

- **Equity asset valuation** / Jerald E. Pinto, Elaine Henry, Thomas R. Robinson, John D. Stowe ; with a contribution by Raymond D. Rath Hoboken, New Jersey : Wiley, 2010.
- **Analysis of investments & management of portfolios** / Frank K. Reilly, Keith C. Brown Australia : South-Western Cengage Learning, 2012.

The Time value of money

Decomposing Interest Rates

We often view interest rates as compensation for bearing risk.

- Interest rates can then be viewed as compensation for
  - Delaying consumption "risklessly" (the risk-free real rate, R_f)
  - Bearing inflation risk over the life of the instrument (inflation risk premium,
    or IRP)
  - The possibility that the borrower will not make the promised payments at the promised time (default risk premium, or DRP)
  - The possibility that the investor will need to convert the investment to cash quickly and will not receive a fair value (liquidity risk premium, or LRP)
  - The increased sensitivity of longer-maturity instruments to changes in prevailing market rates (maturity risk premium, or MRP)

$$r = \underbrace{R_f + \text{IRP}}_{\text{Nominal Risk-Free Rate (approximately)}} + \text{DRP} + \text{LRP} + \text{MRP}$$

Nominal Risk-Free Rate (approximately)

The Time value of Money

Compounding is the process of moving cash flows forward in time.
Discounting is the process of moving cash flows back in time.

- Time value of money problems help us assess equivalency of differing cash flow streams across time, including
  - The value today (present value, or PV) of a single amount we will receive in the future (future value, or FV)
  - The value today (PV) of a stream of equally sized cash flows to be received at uniform increments of time in the future (payments or annuity, PMT or A)
  - The value today (PV) of a stream of unequally sized and/or timed cash flows in the future (CF)
  - The future values of the above
  - The annuitized values of the above

Comparing Interest Rates

- You may encounter investments with different stated rates of interest and different compounding frequencies.
- To compare such rates, you need a common reference time period and a method for combining the rates and compounding periods such that a comparison is accurate.
- The equivalent annual rate (EAR) is just such a rate. Once calculated, **EAR represents the interest rate across one year that would have been earned on an equivalent stated rate of interest with no intrayear compounding.**

$$\text{EAR} = (1 + \text{Periodic interest rate})^{m} - 1$$

where m is the number of times compounding will occur in one year

Comparing Interest Rates

Focus On: Calculations

Stated Annual Rate	Periodic Rate	No. Compounding Periods	EAR
10% monthly compounding	0.8333%	12	10.4713%
10% quarterly compounding	2.5%	4	10.3813%
10% semiannual compounding	5%	2	10.25%
10% annual compounding	10%	1	10%

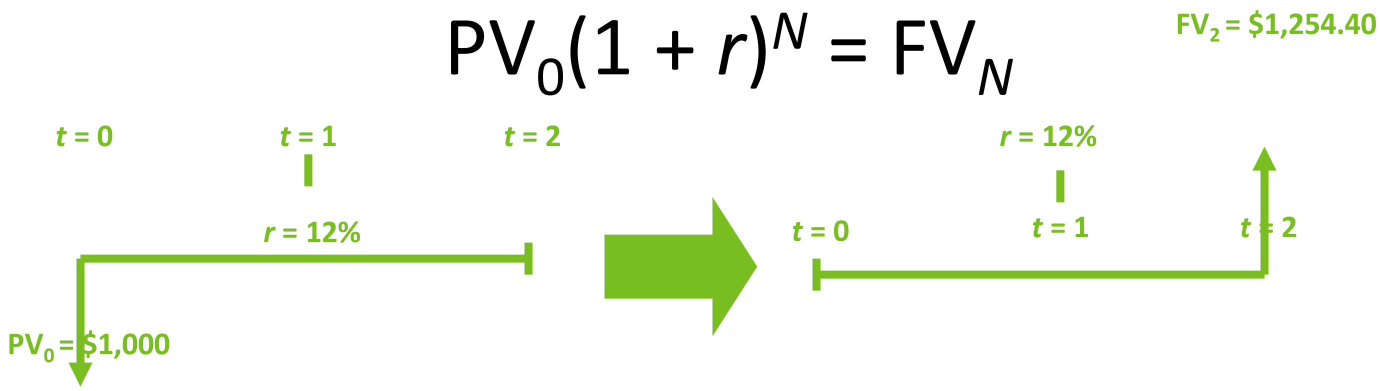
$$\text{EAR} = (1 + \text{Periodic interest rate})^{m} - 1$$

Future Value (FV)

Given a present value (PV), we can compound to return a future value (FV).

- If we have \$1,000 today that we put in an exchange-traded fund that will generate 12% per year for the next two years, how much will we have in the account in two years?

$$PV_0(1 + r)^N = FV_N$$



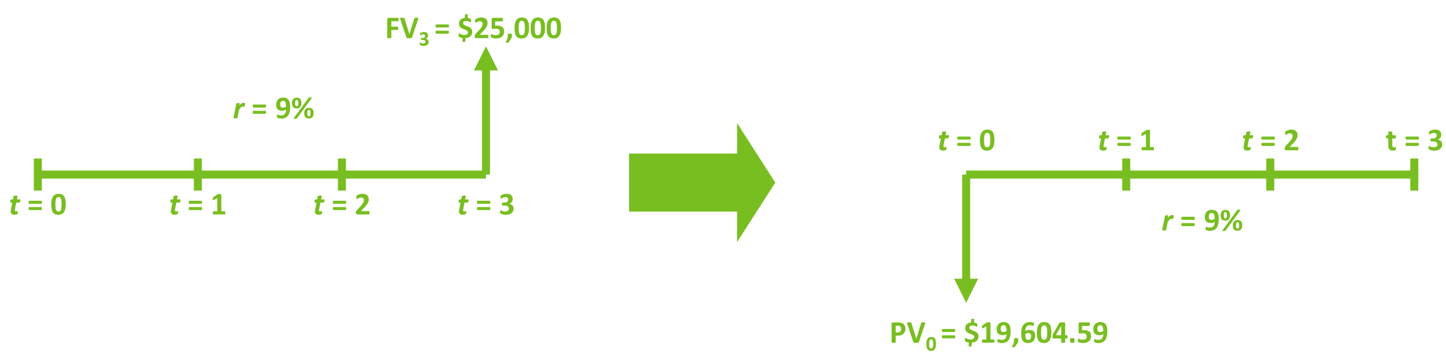
$$\$1{,}000(1 + 0.12)^2 = \$1{,}254.40$$

Present Value (PV)

Given a future value (FV), we can discount it to return a present value (PV).

- If we expect to receive a check for \$25,000 in three years and our opportunity cost of funds is 9%, the present value of the future payment is

$$\text{PV}_0 = \frac{\text{FV}_N}{(1+r)^N}$$

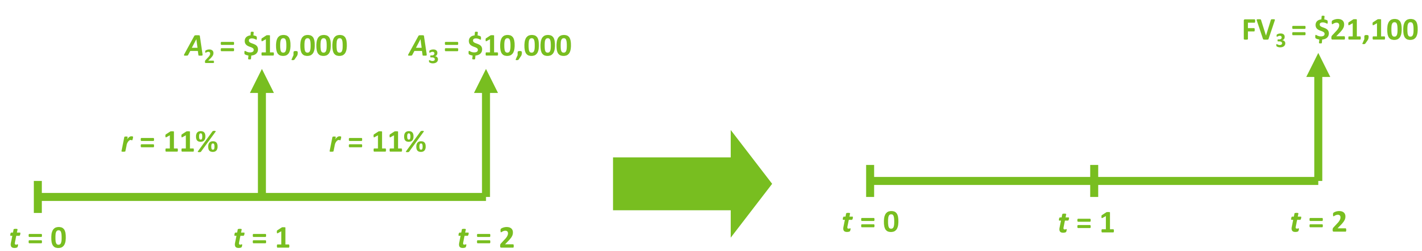


$$\$19{,}304.59 = \frac{\$25{,}000}{(1+0.09)^3}$$

FV of an Annuity (A)

Calculate the future value of a series of regular payments at regular intervals.

- If you invest $10,000 each year for the next two years at 11% starting in one year, how much will you have at the end of the two years?



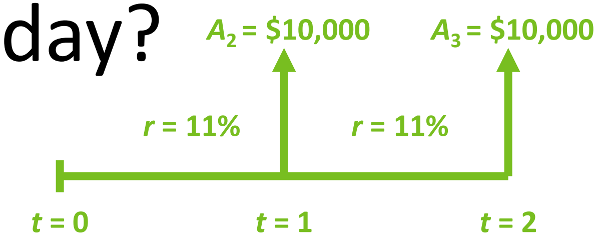
$$\mathrm{FV}_N = A\left[\frac{(1+r)^N - 1}{r}\right]$$

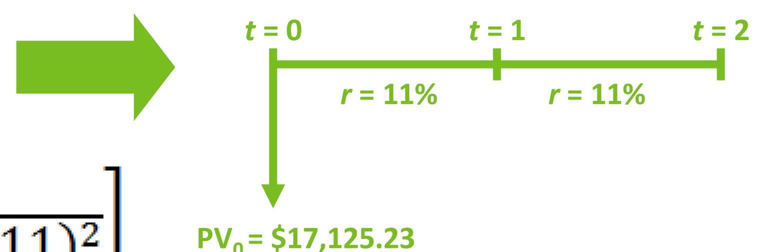
$$\$21{,}100 = \$10{,}000\left[\frac{(1+0.11)^2 - 1}{0.11}\right]$$

PV of an annuity (a)

Calculate the present value of a series of regular payments received at regular intervals.

- If you expect to receive $10,000 each year for two years starting in one year, and your opportunity cost is 11%, how much is it worth today?





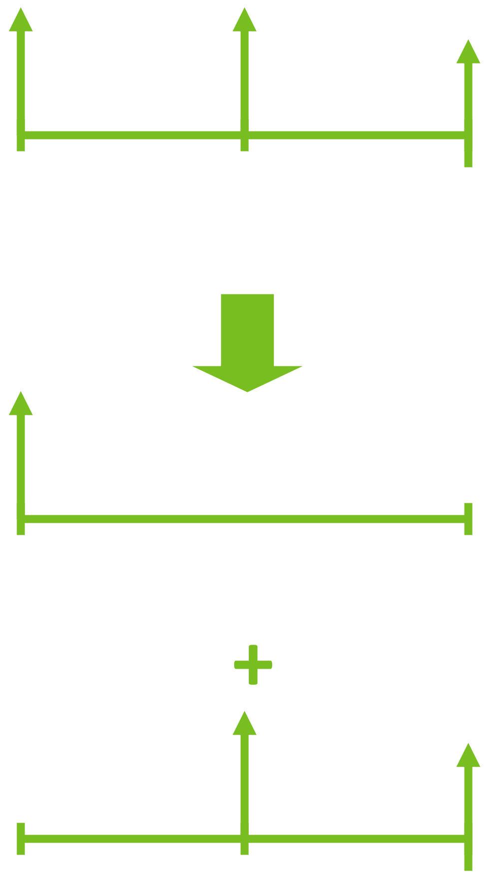
$$\$17{,}125.23 = \$10{,}000\left[\frac{1-\dfrac{1}{(1+0.11)^2}}{0.11}\right]$$

$$\text{PV}_0 = A\left[\frac{1-\dfrac{1}{(1+r)^N}}{r}\right]$$

Annuity Due Values

An annuity due is just like an annuity except that the first payment is received (paid) at the beginning of a period rather than the end.

- You can find the PV (FV) of an annuity due in several ways:
  - Take the PV (FV) of each individual part to a common point in time and use value additivity to combine them.
  - Treat it as an annuity, combine the cash flows at the annuity origin in time, and then move the resulting cash flow to the desired point in time
  - Treat it as a single lump sum and an ordinary annuity of one period shorter, and then calculate the PV (FV) of each component and add them together, again using value additivity.
    - Depicted: Three-period annuity due as a two-period annuity and a single lump sum.



Annuity Due Values

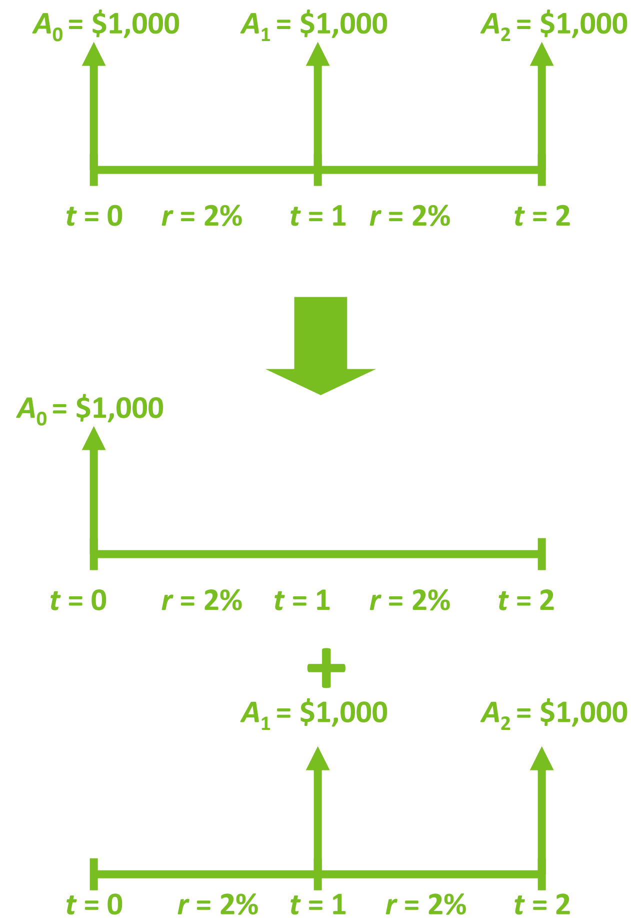
Focus On: Calculations

- You currently own a rental house that yields annual rent of $1,000. There is a rent payment due today and one each at the end of the next two years. If you deposit all three into your bank account, which earns 2%, how much money will you have at the end of Year 2?

$$FV_2 = PV_0(1+r)^N + A\left[\frac{(1+r)^N - 1}{r}\right]$$

$$FV_2 = \$1000(1+0.02)^2 + \$1000\left[\frac{(1+0.02)^2 - 1}{0.02}\right]$$

$$FV_2 = \$3060.40$$

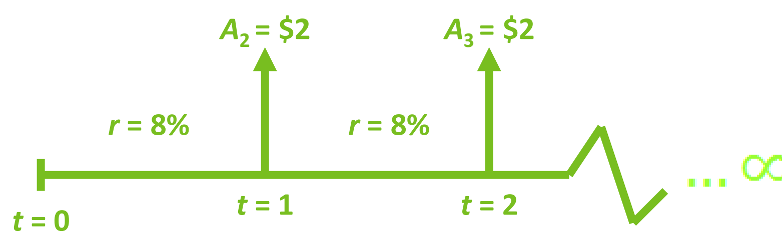


Present Value of a perpetuity

Cash flows that never end are known as perpetuities.

- These can occur with many types of investments, including stocks and bonds.
  - A type of perpetual bond is a "consol."
- Suppose you plan to invest in a utility stock that will pay a $2 dividend for the life of the company. You don't expect the dividend to ever grow, and similar stocks have an 8% required rate of return. How much should the stock be worth today?

$$\text{PV}_0 = A\sum_{i=1}^{\infty}\frac{1}{(1+r)^i} \cong \frac{A}{r}$$

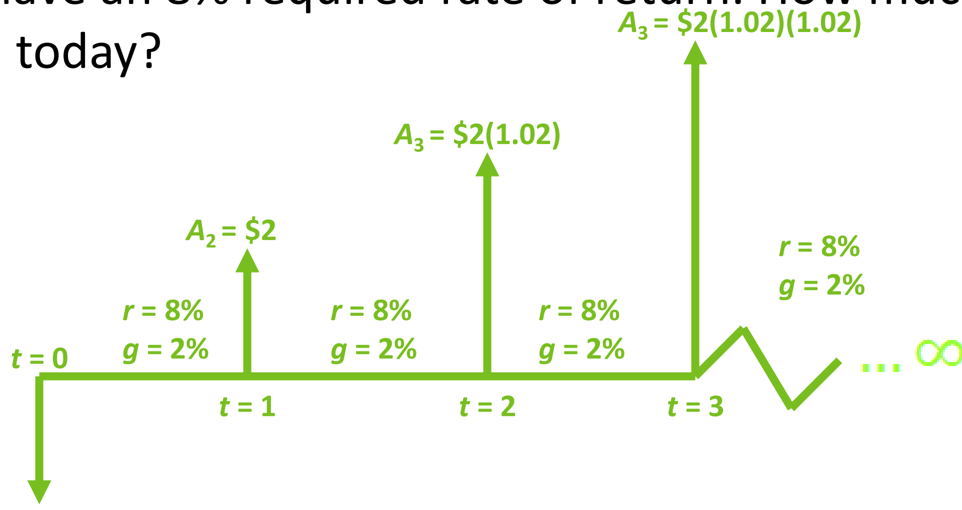


$$\$25 \cong \frac{2}{0.08}$$

Present Value of a Growing Perpetuity

If we assume growth stays constant and it is less than the discount rate, then we can calculate the present value of a growing perpetuity.

- Suppose you plan to invest in a different utility stock that will paid a $2 dividend last year. You expect the dividend to grow (*g*) by 2% per year, and similar stocks have an 8% required rate of return. How much should the stock be worth today?

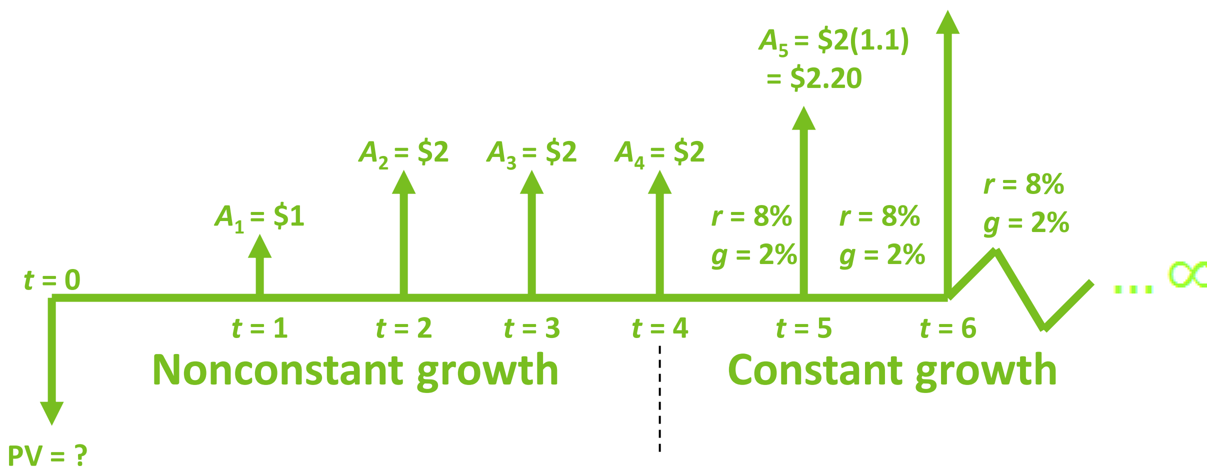


$$\mathrm{PV}_0 = A\sum_{i=1}^{\infty}\frac{(1+g)^i}{(1+r)^i} \cong \frac{A_1}{r-g} = \frac{2(1+0.02)}{0.08-0.02} = \$34$$

Solving complex TVM problems

We can use value additivity and cash flow diagrams to solve complex TVM problems more easily.

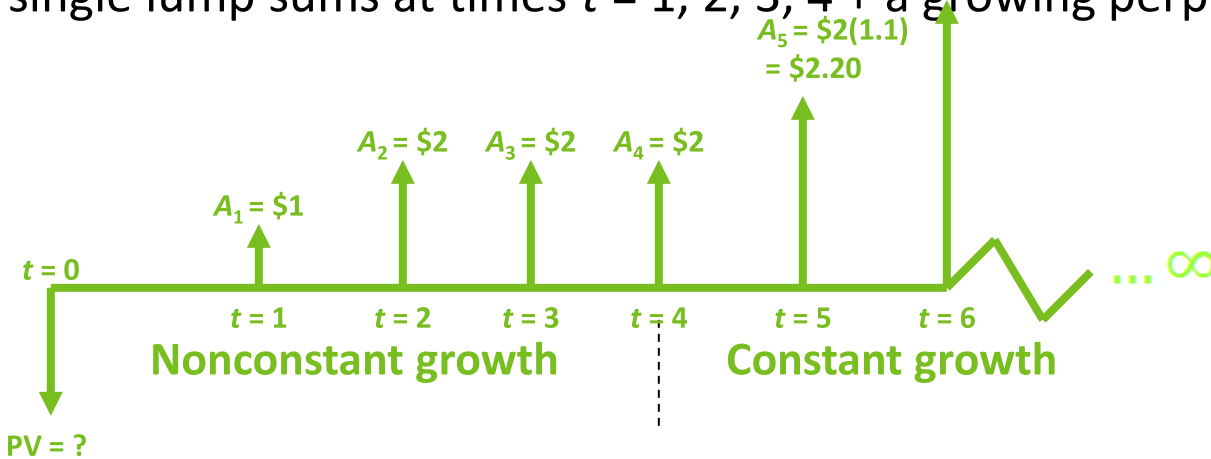
- Consider a stock that currently pays no dividend. In one year, it is expected to pay a $1 dividend. The year after, it will pay $2 for three years. After that, the dividends will grow at a constant rate of 2% per year forever. If you require a 8% rate of return on the stock, what is its value to you today?



Solving complex TVM problems

We can use value additivity and cash flow diagrams to solve complex TVM problems more easily.

- This can be viewed as
  1) A single lump sum at t = 1 + a 3-period annuity from t = 2 to 4 + growing perpetuity or
  2) Four single lump sums at times t = 1, 2, 3, 4 + a growing perpetuity.



Solving Complex TVM Problems

Focus On: Calculations

Solution for approach 1:

$$PV_0 = \frac{\$1}{(1+0.08)^1} + \frac{\$2\left[\frac{1-\frac{1}{(1+0.08)^3}}{0.08}\right]}{(1+0.08)^1} + \frac{\frac{\$2(1.02)}{0.08-0.02}}{(1+0.08)^4}$$

$$PV_0 = \frac{\$1}{(1+0.08)^1} + \frac{\$5.1542}{(1+0.08)^1} + \frac{\$34}{(1+0.08)^4} = \$30.6893$$

$$PV_0 = \frac{\$1}{(1+0.08)^1} + \frac{\$2}{(1+0.08)^2} + \frac{\$2}{(1+0.08)^3} + \frac{\$2}{(1+0.08)^4} + \frac{\frac{\$2(1.02)}{0.08-0.02}}{(1+0.08)^4}$$

- Solution for approach 2:

$$PV_0 = \$30.6893$$

Basic Principles of TVM

- You **cannot** add or subtract cash flows that occur at different times without first compounding or discounting them to the same point in time.
  - Once at the same point in time, we can add or subtract the resulting equivalency cash flows.
  - This is known as the **cash flow additivity** principle: If two or more cash flows occur at the same point in time, we can add or subtract them together.
- The period of time associated with the cash flows must match the period of time associated with the discounting or compounding rate (use **periodic rates** for compounding and discounting).

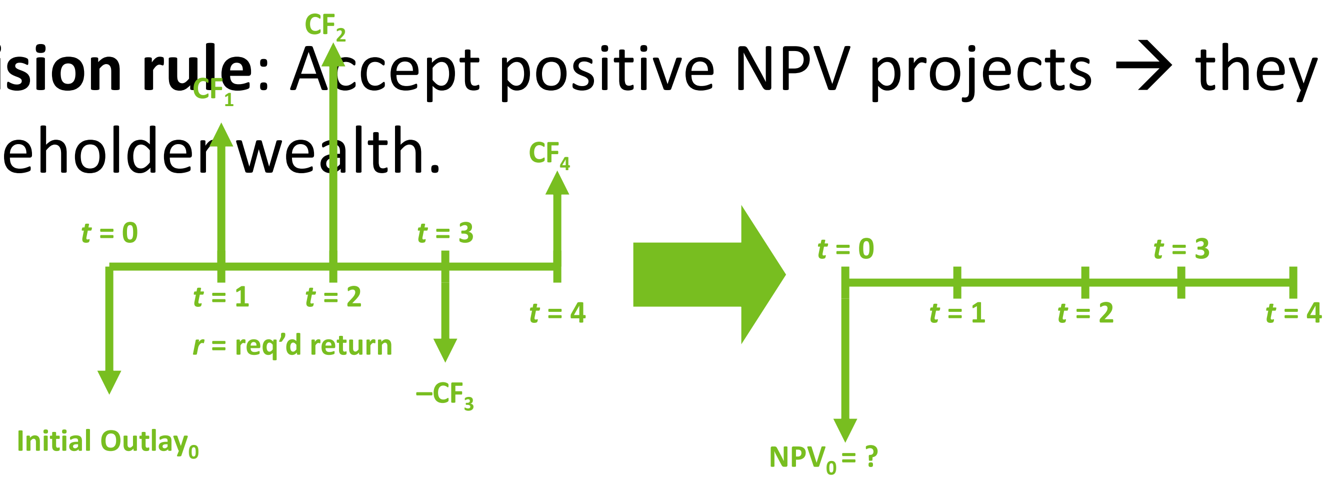
Discounted Cash Flow applications

Investment decision criteria

- Net Present Value (NPV)
- Internal Rate of Return (IRR)
- Payback Period
- Discounted Payback Period

Net present value (NPV)

- **Net present value** is the sum of the present values of all the positive cash flows minus the sum of the present values of all the negative cash flows.
- **Interpretation**: When the discount rate applied is an appropriate hurdle rate, it measures the contribution of the project to shareholder wealth.
- **Decision rule**: Accept positive NPV projects → they increase shareholder wealth.



Net present value (NPV)

Focus On: Calculations

Steps in calculating NPV

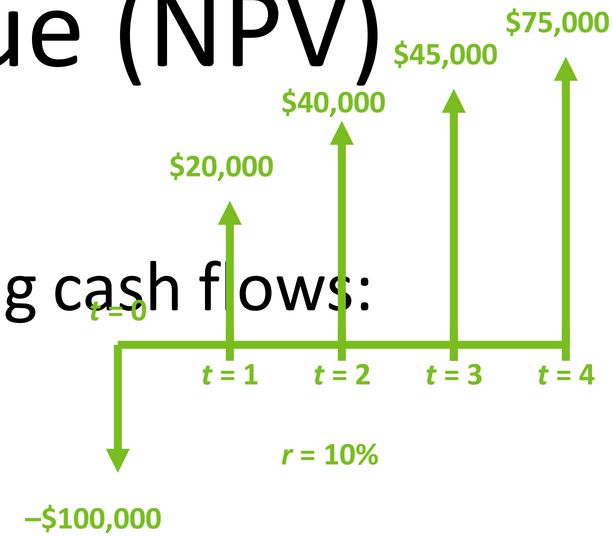
1. Identify all the incremental cash flows associated with the project.
2. Determine the appropriate discount rate.
3. Using that discount rate, calculate the present value of all of the inflows (positive sign) and outflows (negative sign).
4. Sum the present values together → the result is the project's NPV.
5. Apply the NPV decision rule.
   - If you have mutually exclusive projects → accept the one with the highest NPV.

$$\mathrm{NPV} = \sum_{t=0}^{N} \frac{\mathrm{CF}_t}{(1+r)^t}$$

Net present value (NPV)

Focus On: Calculations

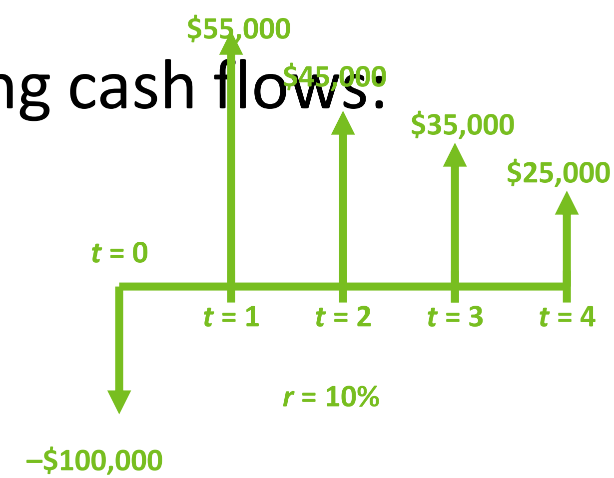
Consider Project A with the following cash flows:



The NPV for this project is…?

Decision?

Consider Project B with the following cash flows:



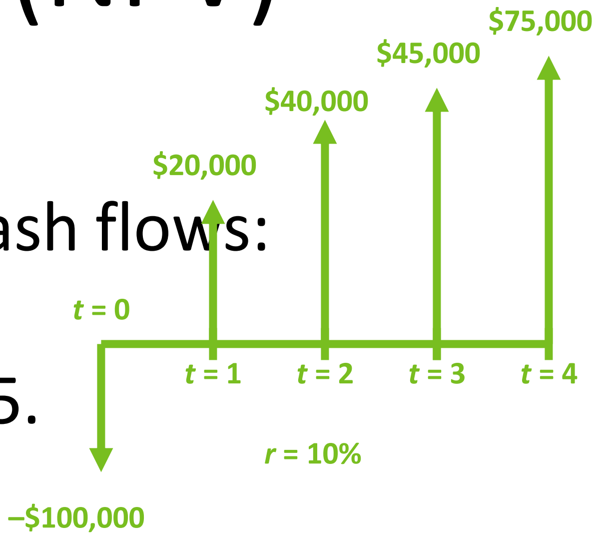
The NPV for this project is…?

Decision?

Net present value (NPV)

Focus On: Calculations

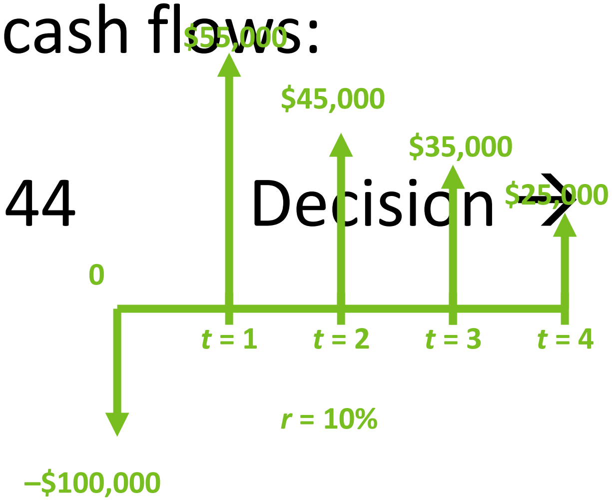
Consider Project A with the following cash flows:



The NPV for this project is $36,274.85.

Decision → Accept the project.

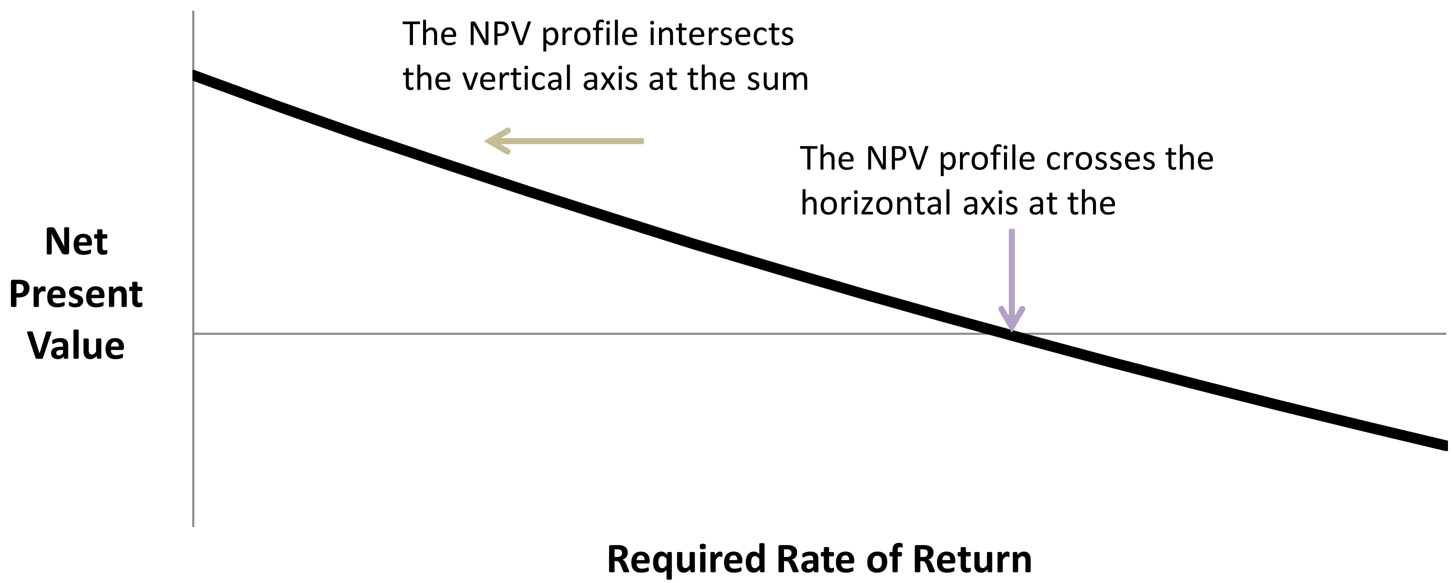
Consider Project B with the following cash flows:



The NPV for this project is $30,561.44 Decision → Accept the project.

Net present value profile

The **net present value profile** is the graphical illustration of the NPV of a project at different required rates of return.



Internal rate of return (IRR)

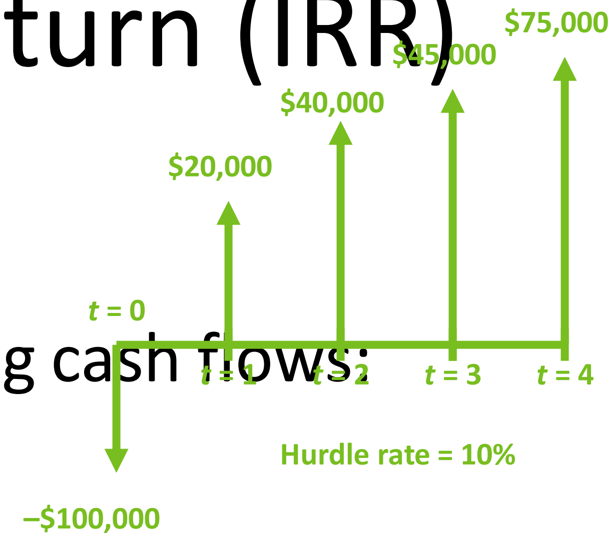
- The **internal rate of return** is the discount rate that sets the sum of the present value of the positive cash flows equal to the sum of the present value of the negative cash flows.
  - The discount rate at which NPV = 0
- **Interpretation**: IRR is the expected compound return when all intervening cash flows in the project can be reinvested at the IRR and the investment will be held until maturity.
- **Calculation**: In practice, use a spreadsheet, financial software, or financial calculator to determine IRR.
- **Decision**: Accept projects for which IRR > hurdle rate → increases shareholder wealth.

$$\text{NPV} = \sum_{t=0}^{N} \frac{CF_t}{(1 + \text{IRR})^t} = 0$$

Internal rate of return (IRR)

Focus On: Calculations

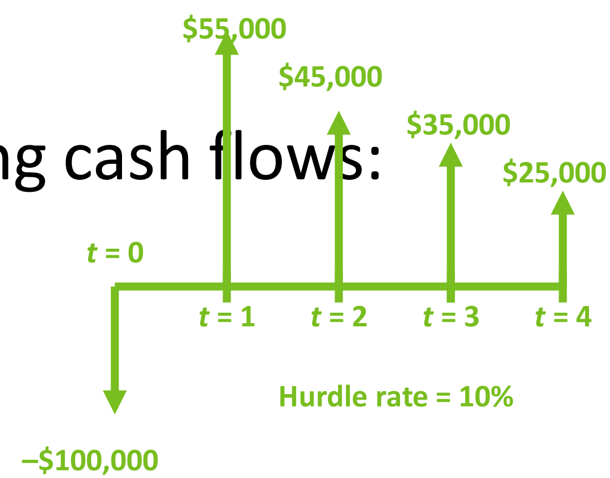
Consider Project A with the following cash flows:



The IRR for this project is…?

Decision?

Consider Project B with the following cash flows:



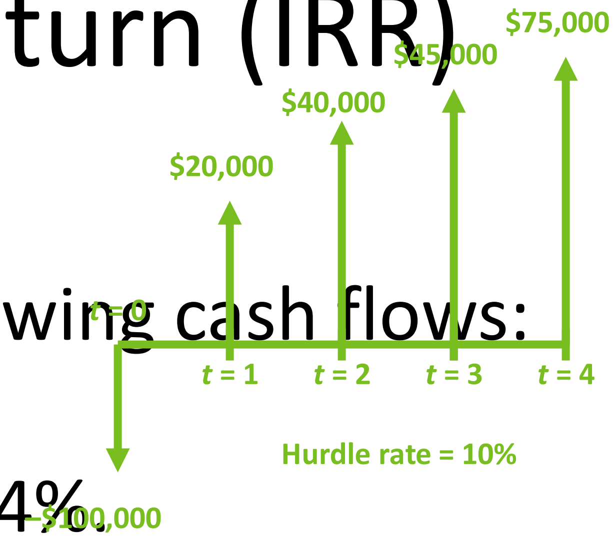
The IRR for this project is…?

Decision?

Internal rate of return (IRR)

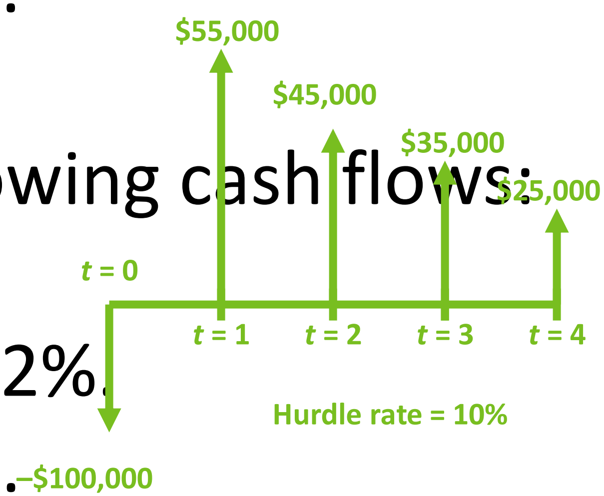
Focus On: Calculations

Consider Project A with the following cash flows:



The IRR for this project is 22.84%.
Decision→ Accept the project.

Consider Project B with the following cash flows:



The IRR for this project is 25.62%.
Decision→ Accept the project.

The multiple IRR problem

- If cash flows change sign more than once during the life of the project, there may be more than one rate that can force the present value of the cash flows to be equal to zero.
  - This scenario is called the “multiple IRR problem.”
  - In other words, there is no unique IRR if the cash flows are nonconventional.

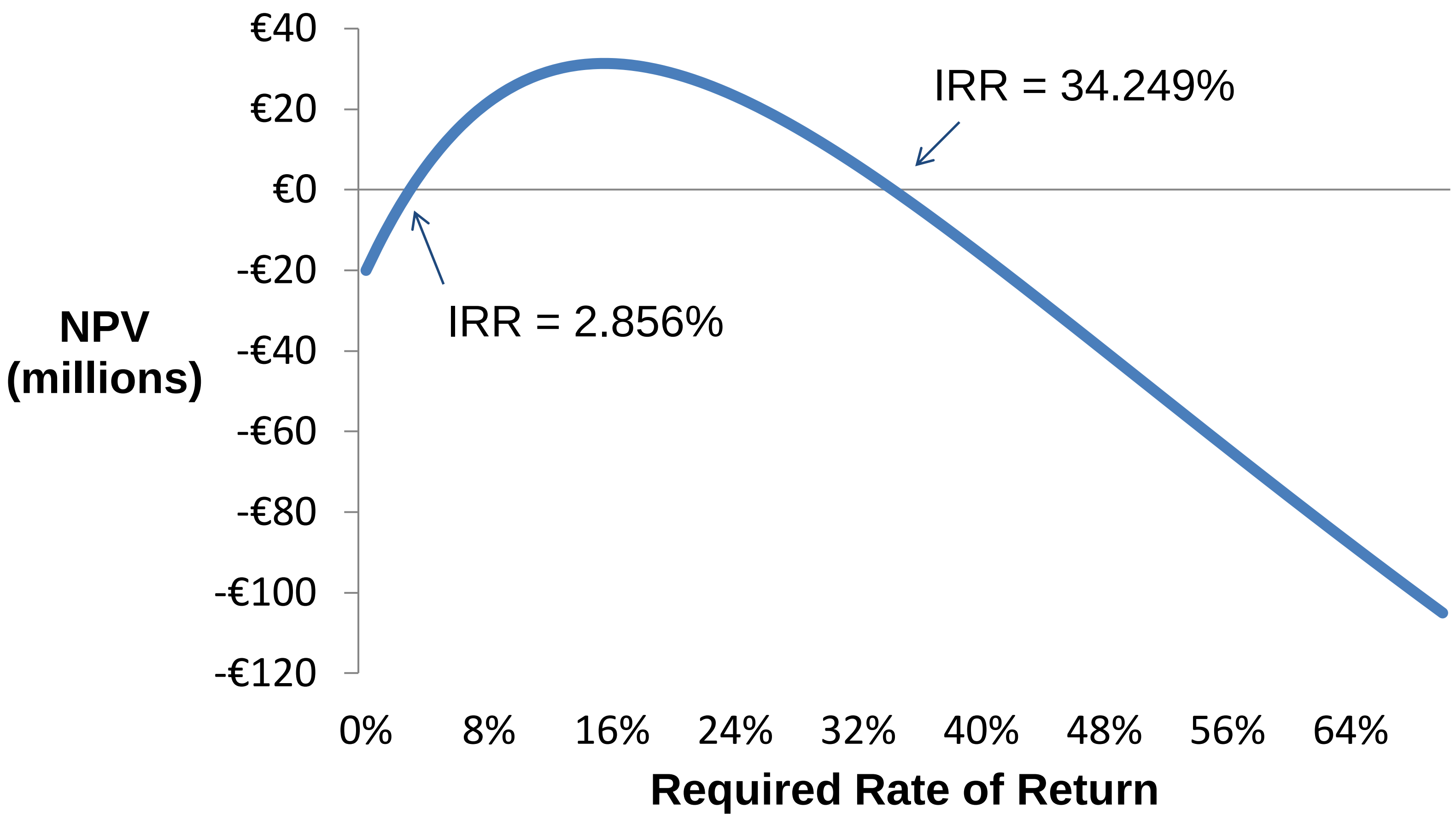
Example: The multiple IRR problem

Consider the fluctuating capital project with the following end of year cash flows, in millions:

Year	Cash Flow
0	–€550
1	€490
2	€490
3	€490
4	–€940

What is the IRR of this project?

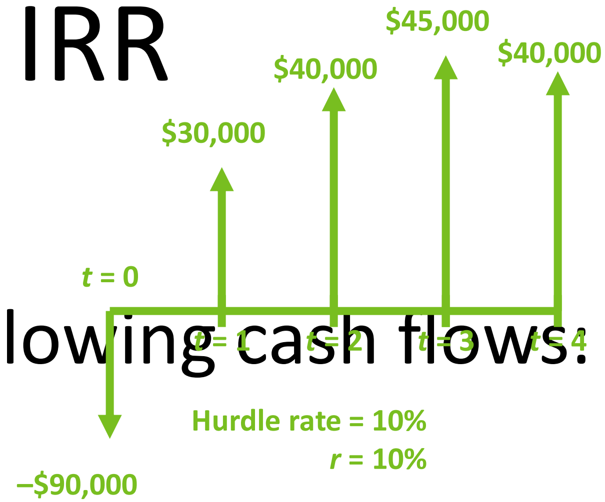
Example: The Multiple IRR Problem



NPV vs. IRR

- If projects are **independent**, the decision to invest in one does not preclude investment in the other.
  - NPV and IRR will yield the same investment decisions.
- Projects are **mutually exclusive** if the selection of one project precludes the selection of another project → project selection is determined by rank.
  - NPV and IRR may give different ranks when
    - The projects have different scales (sizes)
    - The timing of the cash flows differs
  - If projects have different ranks → use NPV.

NPV vs. IRR



Focus On: Calculations

- Consider Project C with the following cash flows:
  - The NPV is $28,600.26.
  - The IRR is 24.42%.

	Project A	Project B	Project C
NPV	$29,872.52	$27,783.12	$28,600.26
IRR	22.84%	25.62%	24.42%
Decision	Accept	Accept	Accept

  - If the projects are independent, you accept all three.
  - If the projects are mutually exclusive, you accept Project A even though it has the smallest IRR.
  - If Projects B and C are mutually exclusive, you accept Project C.

Example NPV and IRR

5. Westcott–Smith is a privately held investment management company. Two other investment counseling companies, which want to be acquired, have contacted Westcott–Smith about purchasing their business. Company A's price is £2 million. Company B's price is £3 million. After analysis, Westcott–Smith estimates that Company A's profitability is consistent with a perpetuity of £300,000 a year. Company B's prospects are consistent with a perpetuity of £435,000 a year. Westcott–Smith has a budget that limits acquisitions to a maximum purchase cost of £4 million. Its opportunity cost of capital relative to undertaking either project is 12 percent.

A. Determine which company or companies (if any) Westcott–Smith should purchase according to the NPV rule.

B. Determine which company or companies (if any) Westcott–Smith should purchase according to the IRR rule.

C. State which company or companies (if any) Westcott–Smith should purchase. Justify your answer.

Payback Period

- The **payback period** is the length of time it takes to recover the initial cash outlay of a project from future incremental cash flows.
- In the Hoofdstad Project example, the payback occurs in the last year, Year 4:

Period	Cash Flow (millions)	Accumulated Cash flows
0	–$1,000	–$1,000
1	200	–$800
2	300	–$500
3	400	–$100
4	500	+400

Discounted Payback Period

- The **discounted payback period** is the length of time it takes for the cumulative discounted cash flows to equal the initial outlay.
  - In other words, it is the length of time for the project to reach NPV = 0.

Example: Discounted Payback Period

Consider the example of Projects X and Y. Both projects have a discounted payback period close to three years. Project X actually adds more value but is not distinguished from Project Y using this approach.

	Cash Flows		Discounted Cash Flows		Accumulated Discounted Cash Flows	
Year	Project X	Project Y	Project X	Project Y	Project X	Project Y
0	–£100.00	–£100.00	–£100.00	–£100.00	–£100.00	–£100.00
1	20.00	20.00	19.05	19.05	–80.95	–80.95
2	50.00	50.00	45.35	45.35	–35.60	–35.60
3	45.00	45.00	38.87	38.87	3.27	3.27
4	60.00	0.00	49.36	0.00	52.63	3.27

Summary

- The quantitative processes underlying the time value of money and its associated calculations are central to the investment process.
  - Using the time value of money concepts, we can
    - Determine the value of a series of future cash flows today (present value)
    - Determine the value of a series of cash flows in the future (future value)
    - Determine the value of a regular series of cash flows, known as an annuity, that is equivalent to a specific value today or in the future (annuitizing)

- The process of evaluating projects incorporates two primary decision criteria known as net present value (NPV) and internal rate of return (IRR).
  - When projects are independent, NPV and IRR can be used interchangeably.
  - When projects are mutually exclusive, analysts should use the NPV criteria.