

Little's law basics

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Different times used in Little's law

(used in different resources)

- Lead time (LT)
- Flow time (FT)
- **Cycle time (CT) we will use only this time**

It is **essential** to define precisely all mentioned times to better understand the principles of Little's law- see time variables explanation in the following slides

$$WIP=TH \times CT$$

Definitions – time variables

- **CT**=average time from when the job is released into the station (machine or line) to when it exits

Only explanations if you will use different literature

- **LT**=management **constant** indicating the time allotted (assigned) for production of a part on a given routing – used more often in planning
- **CT =FT** (in different publications they use **FT** instead of **CT**), where FT stands for=Flow Time
- **CT=Throughput Time** (in different publications they use Throughput Time instead of CT)

Other two variables of Little's law

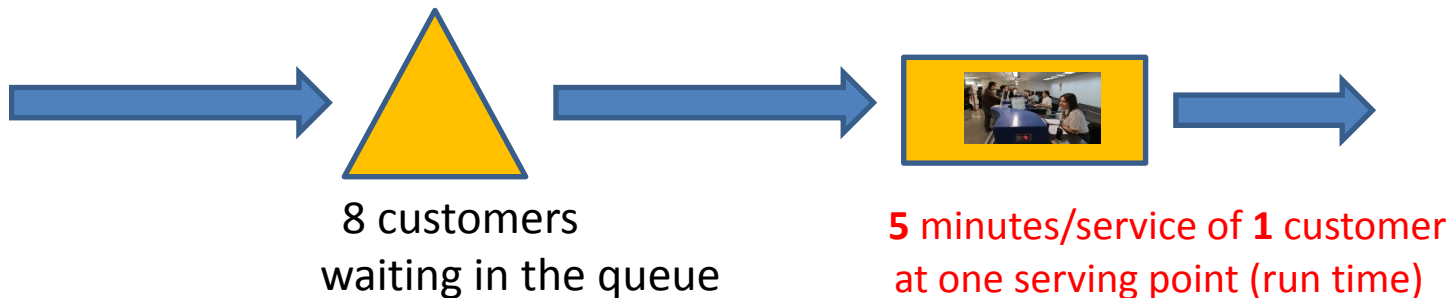
- **WIP**= Work in Process (Work in Progress)
- **TH**=Throughput=Throughput Rate = average output of the production process (machine, workstations) per unit time

Facilities -suitable for the application of the law



One example to be solved

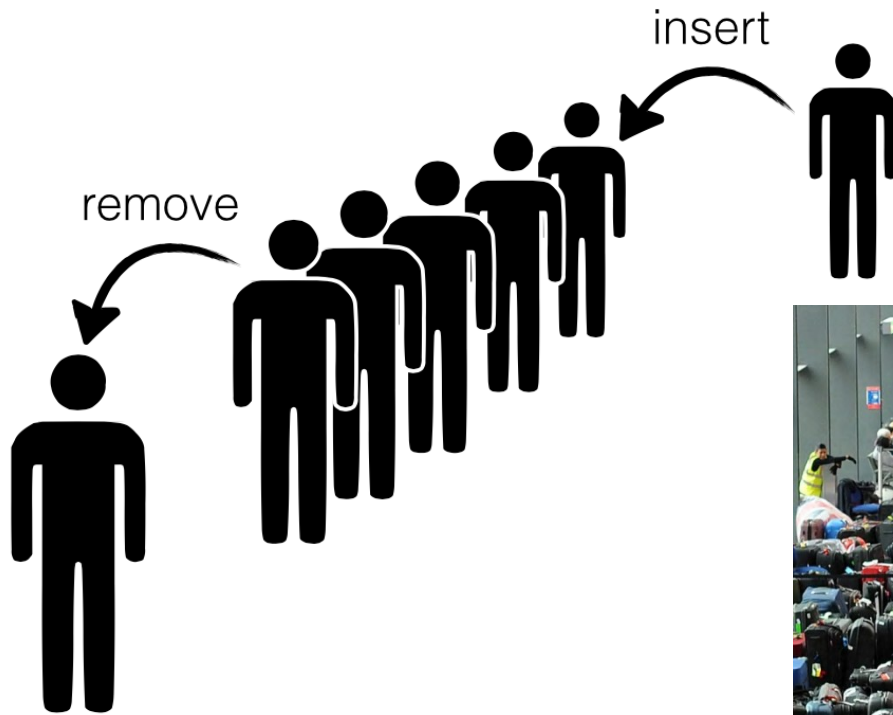
- **30** customers/hour – (max capacity of the facility). The **Facility** may be a hairdresser, fast food, bank counter, airport checking, massages and so on.
- 8 customers waiting in the queue (buffer) – **see next slide**
- **5 minutes** take service of one customer



- **You need to remove all times that do not add value to the process**
- **1 serving point** = **12** customers per hour -> (**60** minutes/**5** = **12**), for **30** customers/hour the needed **2.5** = **30/12** of serving points

$$WIP = TH \times CT$$





Buffering

A [buffer](#) is used to temporarily keep data (customers in our model) while it is being moved from one place to another. A buffer often adjusts timing by implementing a queue or **FIFO** algorithm in memory, simultaneously writing data into the queue at one rate and reading it at another rate.

Questions to be answered

- How long does the customer wait in the queue?
- How many people on an average can be served at once?
- How many customers are in the facility just in time (both pending and those served by facility staff)?
- What is the average time of the "flow" of the customer by the facility (wait and service)

Simplifying conditions :

"Input flow" (average) = "Output flow" (average)- **steady flow** (see video at the end of this PWP)

We do not consider fluctuation due to averaging (see flip a coin situation)

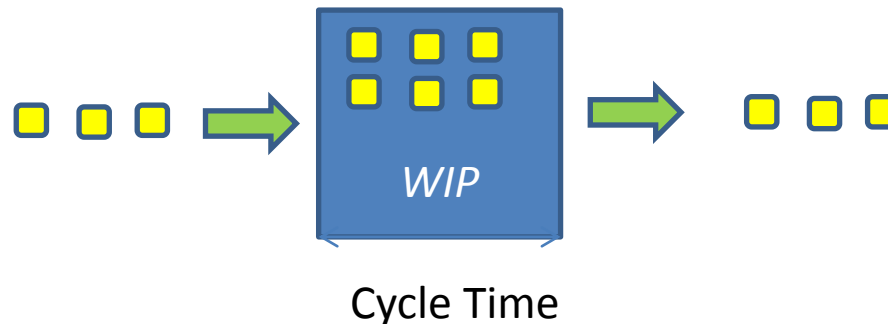
Key metrics and variables (completion of definitions)

- **CT = Cycle Time** (how long takes the whole process) = **5 minutes take the service of one customer**
- **Work In Progress = WIP** (how many units are in process at any moment = **Work In Progress**)
- **Throughput = TH** (how many customers/time unit) – e.g. In our case 30/hour = 30/60

• These metrics are tied together by Little's law **WIP = TH x CT**

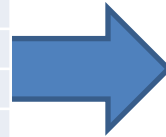
- Our example : **TH = 30 customers/hour** , service of one takes **5 minutes**, **WIP = (30/60) * (5/1) = (1/2) * 5 = 2,5**

↑
TH in minutes



Solution (home study, C=Customer)

Proces	WIP	TH (Cust/hour)	CT
Buffer	8	30	
Service		30	5
Total		30	



Proces	WIP	TH (Cust/hour)	CT (min/Cust)	Time
Buffer	8	30	5	
Service		30	5	5
Total		30	5	

$$\text{WIP} = \text{TH} \times \text{CT}$$



Proces	WIP	TH (C/h)	CT (min/C)	Time
Buffer	8	30	5	
Service	2,5	30	5	5
Total	10,5	30	5	

$$\text{CT} = \text{WIP} / \text{TH} \text{ (third column is only see the units)}$$



Proces	WIP	TH (C/h)	CT (/min/C)	Time
Buffer	8	30	5	16
Service	2,5	30	5	5
Total	10,5	30	5	21

Input data (from previous slides)

30 customers/hour -> (max Capacity of the facility) = Throughput = TH

8 customers are waiting in the queue = WIP2 = buffer

5 minutes per customer service = CT

$\text{WIP1} = \text{TH} \times \text{CT} = ((30/60) * 5) = (3 * 5) / 6 = 2,5$, so how many customers can be served at the same time and total quantity of customers is after that $10.5 = 8.0$ (queue) + 2.5 (WIP1+WIP2) and then:

$\text{CT} = \text{WIP2} / \text{TH} = 8 / (30/60) = (8 * 6) / 3 = 48/3 = 16$ (as long as the customer waits in the queue=buffer) and finally for control $\text{CT} = 5 = (2.5 / (3/6)) = 2.5 * 6/3 = 15/3$ is the service time (already entered). So total time is $16+5=21$

Questions

- How long does the customer wait in the queue? -> **16 minutes**
- How many average people can be served at once? -> **>2,5 customers**
- How many customers are in the facility just in time (both pending and those just served by facility staff)? **10,5 customers**
- What is the average time of the "flow" of the customer by the facility (wait and service) -> **21 minutes**

Little's law-2nd part

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Based on resource : Factory Physics (Hopp and Spearman)

Little's law - definition (formula)

- Fundamental relationships among :
 - WIP (Work In Process)
 - Cycle Time (CT)
 - Throughput (T or sometimes TH)

- Formula

$$WIP = TH \times CT$$

- Can be applied to :
 - Single machine station
 - Complex production line
 - Entire plant

Relationships among these variables will serve to se clearly precise (quantitative) description of behaviour of the single production line . It helps user to use a given scale to benchmark actual production systems

I finally figured it out !!!!

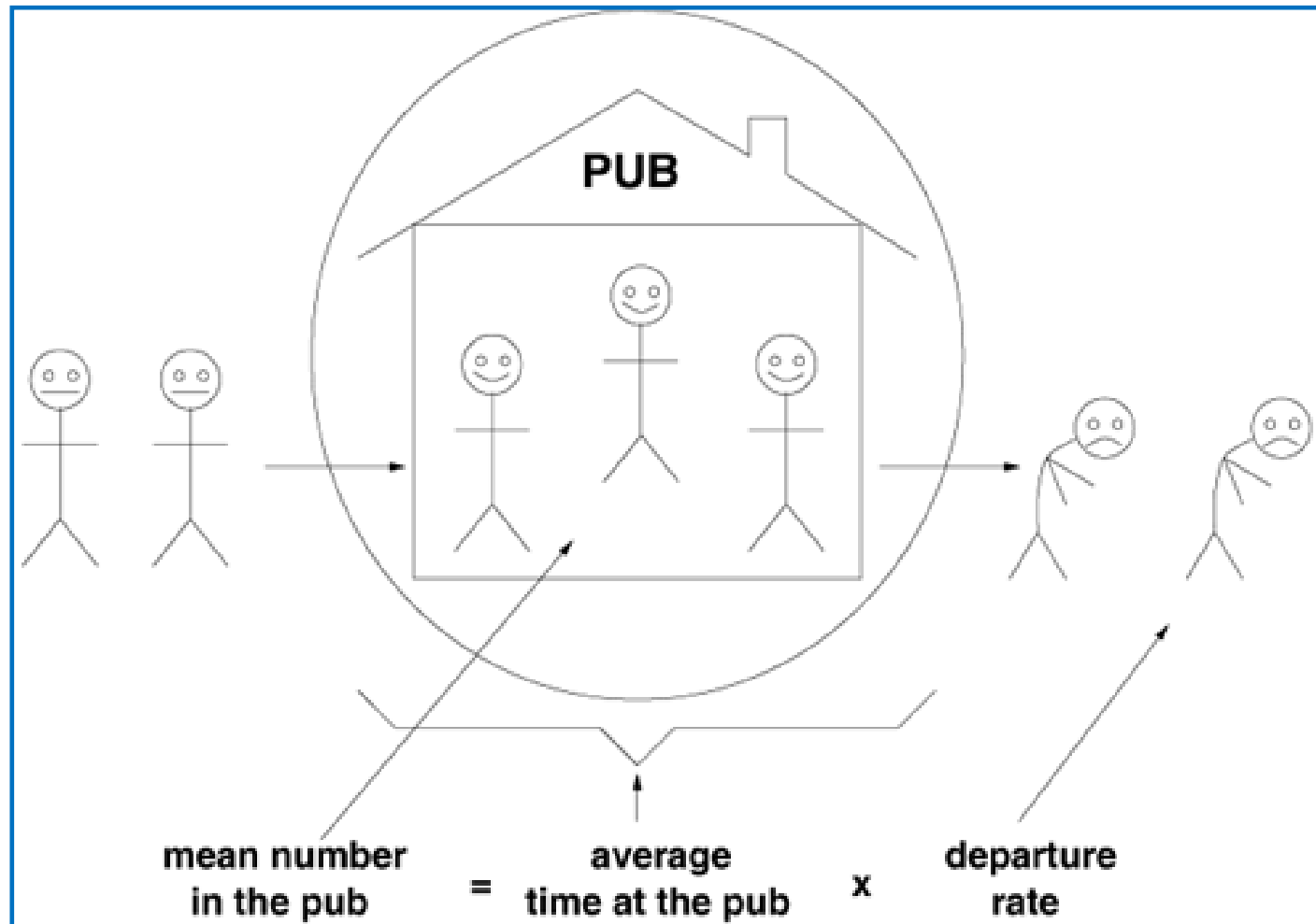
Little's Law

$$\text{avg. Lead Time} = \frac{\text{avg. Work in Progress}}{\text{avg. Throughput}}$$

Avg. Lead Time = Cycle Time !!



Daily application of the law....



Definition of basic parameters (supplements)

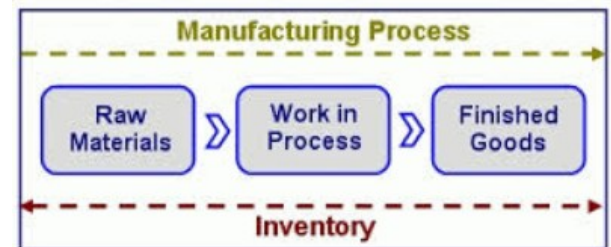
- **Throughput (Throughput rate, TH)** : production per unit time that is sold (see TOC definition)
- If **TH** is measured in **cost dollars (not in customers/time)** , it is typically called :

Cost of good sold (COGS)

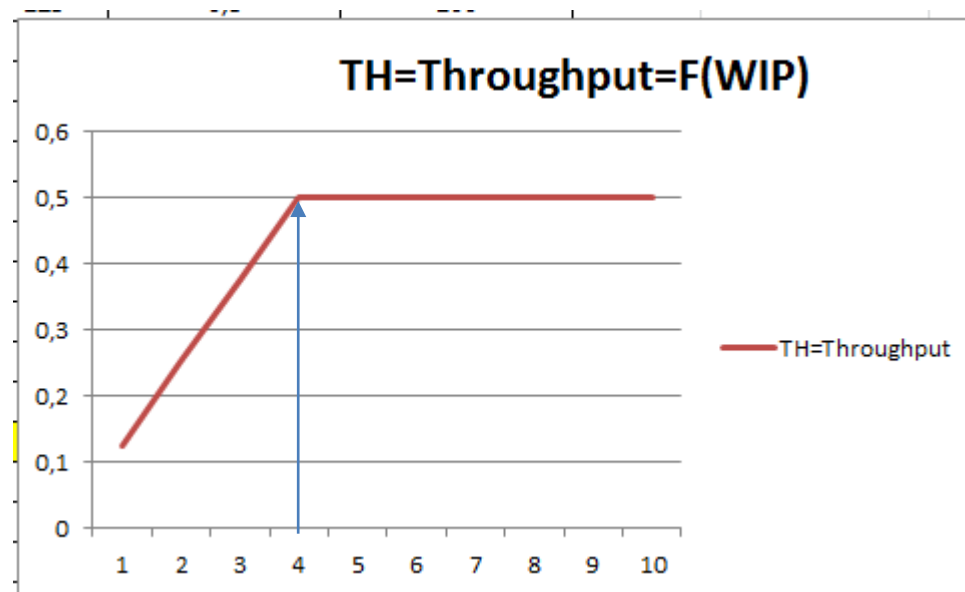
- **The upper limit of TH** in the **production process** is the **capacity**
- If you release more raw material above the capacity of the line (machine), **the system becomes unstable → WIP goes up !! See later ...**

Definition of basic parameters (supplements)

- **WIP (Work In Process)** : inventory between start and end points of the production routing
- **WIP** can be used as one parameter to calculate (measure) an **efficiency**
- **Efficiency** can be defined as **Turnover Ratio** = TH/FGI for warehouses or $\text{TH}/(\text{FGI}+\text{WIP})$ for production plants where **FGI**=Finished goods inventory
- **WIP** : inventory still in line
- **FGI** : inventory waiting for dispatch (shipping)



Best case performance



Resources

• **WIP=TH * CT**

- *Source : Factory Physics, Wallace J Hopp and Mark L. Spearman ; ISBN 13: 978-1-57766-739-1 or ISBN 10 :1-57766-739-5*

<http://www.factoryphysics.com/principle/littleslaw.htm>

Example 1 (home study)

- **Estimating Waiting Times:** If are in a grocery queue behind 10 persons and estimate that the clerk is taking around 5 minutes/per customer, we can calculate that it will take us 50 minutes (10 persons x 5 minutes/person) to start service.
- This is essentially **Little's law**. We take the number of persons in the **queue** (10) as the "**inventory**".
- The inverse of the average time per customer (1/5 customers/minute) provides us the rate of service or the Throughput.
- Finally, we obtain the waiting time as equal to number of persons in the queue divided by the processing rate $10/(1/5) = 50$ minutes).

Example 2 (home study)

- **Planned Inventory Time:** Suppose a product is scheduled so that we expect it to wait for 2 days in finished goods inventory before shipping to the customer. This two days is called **planned inventory time** and is sometimes used as protection against system variability to ensure high delivery service. Using Little's law the total size of inventory in finished goods can be computed as :
- **FGI = throughput × planned inventory time**

Youtube examples (6 minutes)

- <http://www.youtube.com/watch?v=VU8TUSnQ-vw>
- <http://www.youtube.com/watch?v=rtGihR-bm-U>