Annuities with focus on future values

Luděk Benada

Department of Finance, office - 402

e-mail: *benada@econ.muni.cz*

Content

Content

1. The basic concept of annuities

2. The magic hidden in the concept of Annuities

3. Some examples

What does annuity mean in the understanding of finance

What does annuity mean in the understanding of finance

In the understanding of finance, in the case of an annuity, it is a regular payment in terms of amount and frequency.

What does annuity mean in the understanding of finance

In the understanding of finance, in the case of an annuity, it is a regular payment in terms of amount and frequency.

Applying annuity calculations following must be respected:

What does annuity mean in the understanding of finance

In the understanding of finance, in the case of an annuity, it is a regular payment in terms of amount and frequency.

Applying annuity calculations following must be respected:

- Time value of money

What does annuity mean in the understanding of finance

In the understanding of finance, in the case of an annuity, it is a regular payment in terms of amount and frequency.

Applying annuity calculations following must be respected:

- Time value of money
- Constant method of interest calculation *(r, IP)*

What does annuity mean in the understanding of finance

In the understanding of finance, in the case of an annuity, it is a regular payment in terms of amount and frequency.

Applying annuity calculations following must be respected:

- Time value of money
- Constant method of interest calculation *(r, IP)*
- Regularity of the payment period

What does annuity mean in the understanding of finance

In the understanding of finance, in the case of an annuity, it is a regular payment in terms of amount and frequency.

Applying annuity calculations following must be respected:

- Time value of money
- Constant method of interest calculation *(r, IP)*
- Regularity of the payment period
- Constant Cash Flow *(…or geometric or arithmetic growth)*

What does annuity mean in the understanding of finance

In the understanding of finance, in the case of an annuity, it is a regular payment in terms of amount and frequency.

Applying annuity calculations following must be respected:

- Time value of money
- Constant method of interest calculation *(r, IP)*
- Regularity of the payment period
- Constant Cash Flow *(... or geometric or arithmetic growth)*
- The calculation can be focused on the **sum** of **future values** at a certain point in time

What does annuity mean in the understanding of finance

In the understanding of finance, in the case of an annuity, it is a regular payment in terms of amount and frequency.

Applying annuity calculations following must be respected:

- Time value of money
- Constant method of interest calculation *(r, IP)*
- Regularity of the payment period
- Constant Cash Flow *(... or geometric or arithmetic growth)*
- The calculation can be focused on the **sum** of **future values** at a certain point in time
- The calculation can be focused on the **sum** of **present values**

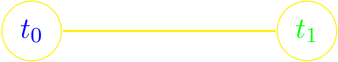
Key elements in the concept

Key elements in the concept

Payment period:

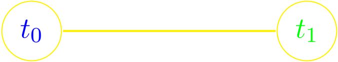
Key elements in the concept

Payment period:



Key elements in the concept

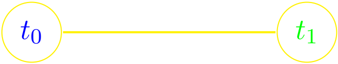
Payment period:



Note: t_0 *...Beginning of the Payment period,* t_1 *...End of the Payment periods*

Key elements in the concept

Payment period:



Note: t_0 *... Beginning of the Payment period,* t_1 *... End of the Payment periods*

Interest period:

Note: t_0 *... Beginning of the Interest period,* t_1 *... End of the Interest periods - when the* **interest** *is* **paid.**

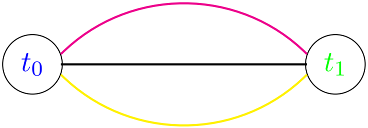
Key elements in the concept

Key elements in the concept

Payment period & Interest period:

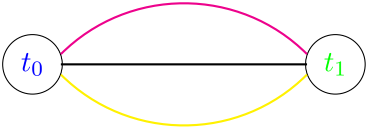
Key elements in the concept

Payment period & Interest period:



Key elements in the concept

Payment period & Interest period:



Note: The simplest case is if the interest and payment period are the same length. However, in practice the relationship may be different.

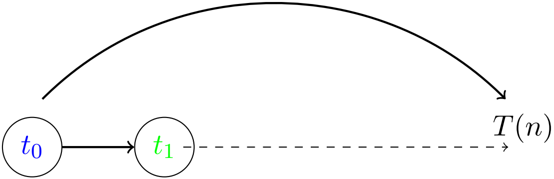
Graphic illustration of the annuity concept - FV´s

Graphic illustration of the annuity concept - FV´s

Annuities over time:

Graphic illustration of the annuity concept - FV´s

Annuities over time:



Sequence of annuities - FV´s

Sequence of annuities - FV´s

t_0 *Time of saving* T

a_1 .... $a_1 * (1+r)^n$ .... FV_{a_1}

$+$

a_2 .... $a_2 * (1+r)^{n-1}$ .... FV_{a_2}

$+$

a_3 .... $a_3 * (1+r)^{n-2}$ .... FV_{a_3}

$+$

$\ddots$ $\vdots$

$+$

FV_{a_n}

Saving: $S = \sum_{i=1}^{n} FV_i = FV_{a_1} + FV_{a_2} + \ldots + FV_{a_n}$

Saving plan

Saving plan

Saving plan represents a practical use of the annuity calculation, where the aggregate sum of partial capitalized annuities is applied.

Content

Understanding the essence of Annuities

Understanding the essence of Annuities

The magic is hidden in:

The magic is hidden in:

G E O M E T R I C S E R I E S

Understanding the essence of Annuities

The magic is hidden in:

G E O M E T R I C S E R I E S

, and its properties . . .

Geometric serie

Geometric serie

An example:

Geometric serie

An example: $2, 4, 8, 16, 32, \ldots$

Geometric serie

An example: *2, 4, 8, 16, 32, . . .*

In general:

a_1

$a_2 = a_1 * q$

$a_3 = a_2 * q$, **where** $q = \frac{a_n}{a_{n-1}}$, **and** $S_n = \frac{q^n - 1}{q - 1}$

Geometric serie

An example: *2, 4, 8, 16, 32, . . .*

In general:

a_1

$a_2 = a_1 * q$

$a_3 = a_2 * q$, **where** $q = \frac{a_n}{a_{n-1}}$, **and** $S_n = \frac{q^n - 1}{q - 1}$

$\vdots$, **or** $S_n = \frac{1 - q^n}{1 - q}$

$a_n = a_{n-1} * q$

Geometric serie

An example: *2, 4, 8, 16, 32, . . .*

In general:

a_1

$a_2 = a_1 * q$

$a_3 = a_2 * q$, **where** $q = \frac{a_n}{a_{n-1}}$, **and** $S_n = \frac{q^n - 1}{q - 1}$

$\vdots$, **or** $S_n = \frac{1 - q^n}{1 - q}$

$a_n = a_{n-1} * q$

Note: A geometric serie can be **finite** *or* **infinite***.*

Application on annuities

Application on annuities

The FV´s of annuities *(PV´s)* are representing a geometric sequence.

Application on annuities

The FV´s of annuities *(PV´s)* are representing a geometric sequence.

Assuming $annuity = 1$ we get:

Application on annuities

The FV´s of annuities *(PV´s)* are representing a geometric sequence.

Assuming $annuity = 1$ we get:

$$\frac{(1+r)^n}{(1+r)^{n-1}} = \frac{(1+r)^{n-1}}{(1+r)^{n-2}} = \cdots = (1+r) = q$$

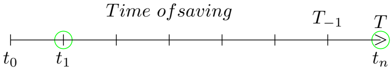
Types of annuities

Types of annuities

The basic criterion in the division of annuities is the moment when the first annuity occurs. According to this concept, we distinguish between ordinary annuity and annuity-due.

Ordinary annuity

Ordinary annuity

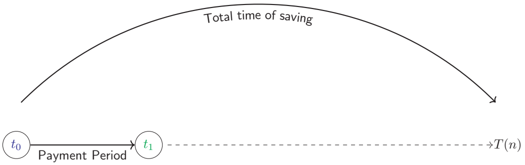


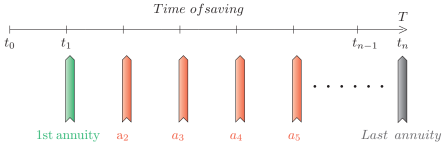
Annuity-ordinary (S^1):

$$S^1 = a * \frac{(1+r)^n - 1}{r}$$

Ordinary annuity - sequence in time

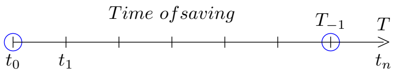
Ordinary annuity - sequence in time





Annuity-due

Annuity-due

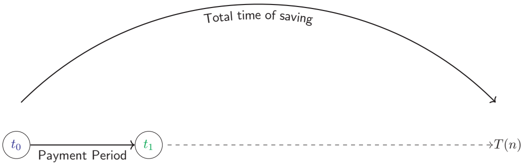


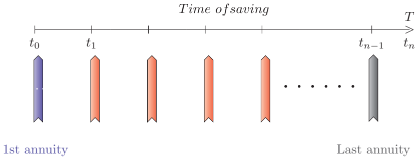
Annuity-due (S^0):

$$S^0 = a * (1 + r) * \frac{(1 + r)^n - 1}{r}$$

Annuity-due - sequence in time

Annuity-due - sequence in time





Content

Saving plans

Saving plans

Every parameter can be calculated in the annuity concept. Most often, it is the value of the amount saved. With the target amount *(the goal of the savings plan)*, it is possible to search for the amount of the annuity under the given conditions.

Or how long it is necessary to save, i.e. how many annuities must be saved. The most **complex** thing is to find out the **interest rate** when the annuity, the target amount and the savings period are known.

Saving plans

Every parameter can be calculated in the annuity concept. Most often, it is the value of the amount saved. With the target amount *(the goal of the savings plan)*, it is possible to search for the amount of the annuity under the given conditions.

Or how long it is necessary to save, i.e. how many annuities must be saved. The most **complex** thing is to find out the **interest rate** when the annuity, the target amount and the savings period are known.

Note: Every parameter except the interest rate can be easily expressed from the basic formula by algebraic modification.

Example 01 - Total amount of saving

Example 01 - Total amount of saving

How much will be the **amount** on the saving account for a client that starts with saving in his age of **22** and will regularly save **until** his **50**. He will **save** regularly **750.00** at the **end** of each **month**. The bank offers an interest rate of **1.6% p.s.** for the entire savings period. The interest is calculated by every deposited annuity. **How much** would be saved if we used the **annuity-due** concept?

Example 01 - Total amount of saving

How much will be the **amount** on the saving account for a client that starts with saving in his age of **22** and will regularly save **until** his **50**. He will **save** regularly **750.00** at the **end** of each **month**. The bank offers an interest rate of **1.6% p.s.** for the entire savings period. The interest is calculated by every deposited annuity. **How much** would be saved if we used the **annuity-due** concept?

First of all we just put all information to the general formula:

Example 01 - Total amount of saving

How much will be the **amount** on the saving account for a client that starts with saving in his age of **22** and will regularly save **until** his **50**. He will **save** regularly **750.00** at the **end** of each **month**. The bank offers an interest rate of **1.6% p.s.** for the entire savings period. The interest is calculated by every deposited annuity. **How much** would be saved if we used the **annuity-due** concept?

First of all we just put all information to the general formula:

$$S^1 = 750 * \frac{(1 + \frac{0.016}{6})^{6*2*28} - 1}{\frac{0.016}{6}}$$

Example 01 - Total amount of saving

How much will be the **amount** on the saving account for a client that starts with saving in his age of **22** and will regularly save **until** his **50**. He will **save** regularly **750.00** at the **end** of each **month**. The bank offers an interest rate of **1.6% p.s.** for the entire savings period. The interest is calculated by every deposited annuity. **How much** would be saved if we used the **annuity-due** concept?

First of all we just put all information to the general formula:

$$S^1 = 750 * \frac{(1 + \frac{0.016}{6})^{6*2*28} - 1}{\frac{0.016}{6}}$$

$$\vdots$$

Example 01 - Total amount of saving

How much will be the **amount** on the saving account for a client that starts with saving in his age of **22** and will regularly save **until** his **50**. He will **save** regularly **750.00** at the **end** of each **month**. The bank offers an interest rate of **1.6% p.s.** for the entire savings period. The interest is calculated by every deposited annuity. **How much** would be saved if we used the **annuity-due** concept?

First of all we just put all information to the general formula:

$$S^1 = 750 * \frac{(1 + \frac{0.016}{6})^{6*2*28} - 1}{\frac{0.016}{6}}$$

$$\vdots$$

$$S^1 = 406,93.7$$

Example 02 - Annuity

Example 02 - Annuity

How much must be the **quarterly** annuity if the target of a saving plan in ten years is **100,000.00**? Further, you know that the financial house offers an interest rate of **4 % p.a** and the interest period corresponds to the payment period. Consider calculation on **annuity-due**.

Example 02 - Annuity

How much must be the **quarterly** annuity if the target of a saving plan in ten years is **100,000.00**? Further, you know that the financial house offers an interest rate of **4 % p.a** and the interest period corresponds to the payment period. Consider calculation on **annuity-due**.

First of all we just put all information to the general formula:

Example 02 - Annuity

How much must be the **quarterly** annuity if the target of a saving plan in ten years is **100,000.00**? Further, you know that the financial house offers an interest rate of **4 % p.a** and the interest period corresponds to the payment period. Consider calculation on **annuity-due**.

First of all we just put all information to the general formula:

$$100000 = a * \frac{(1 + \frac{0.04}{4})^{4*10} - 1}{\frac{0.04}{4}}$$

Example 02 - Annuity

How much must be the **quarterly** annuity if the target of a saving plan in ten years is **100,000.00**? Further, you know that the financial house offers an interest rate of **4 % p.a** and the interest period corresponds to the payment period. Consider calculation on **annuity-due**.

First of all we just put all information to the general formula:

$$100000 = a * \frac{(1 + \frac{0.04}{4})^{4*10} - 1}{\frac{0.04}{4}}$$

$$\vdots$$

Example 02 - Annuity

How much must be the **quarterly** annuity if the target of a saving plan in ten years is **100,000.00**? Further, you know that the financial house offers an interest rate of **4 % p.a** and the interest period corresponds to the payment period. Consider calculation on **annuity-due**.

First of all we just put all information to the general formula:

$$100000 = a * \frac{(1 + \frac{0.04}{4})^{4*10} - 1}{\frac{0.04}{4}}$$

$$\vdots$$

$$a = 2,045.56$$

Example 03 - Time of saving

Example 03 - Time of saving

How long do we have to save to create a budged of **92,476.17** from a regular **annuity** of **5,000.00** deposited every **half year**, always at **the end** of the half year? The interest on the savings account corresponds to **3.8% p.a.**.

Example 03 - Time of saving

How long do we have to save to create a budged of **92,476.17** from a regular **annuity** of **5,000.00** deposited every **half year**, always at **the end** of the half year? The interest on the savings account corresponds to **3.8% p.a.**.

First of all we just put all information to the general formula:

Example 03 - Time of saving

How long do we have to save to create a budged of **92,476.17** from a regular **annuity** of **5,000.00** deposited every **half year**, always at **the end** of the half year? The interest on the savings account corresponds to **3.8% p.a.**.

First of all we just put all information to the general formula:

$$92476.17 = 5000 * \frac{(1 + \frac{0.038}{2})^{2*n} - 1}{\frac{0.038}{2}}$$

Example 03 - Time of saving

How long do we have to save to create a budged of **92,476.17** from a regular **annuity** of **5,000.00** deposited every **half year**, always at **the end** of the half year? The interest on the savings account corresponds to **3.8% p.a.**.

First of all we just put all information to the general formula:

$$92476.17 = 5000 * \frac{(1 + \frac{0.038}{2})^{2*n} - 1}{\frac{0.038}{2}}$$

$$\vdots$$

Example 03 - Time of saving

How long do we have to save to create a budged of **92,476.17** from a regular **annuity** of **5,000.00** deposited every **half year**, always at **the end** of the half year? The interest on the savings account corresponds to **3.8% p.a.**.

First of all we just put all information to the general formula:

$$92476.17 = 5000 * \frac{(1 + \frac{0.038}{2})^{2*n} - 1}{\frac{0.038}{2}}$$

$$\vdots$$

$$0.351409 + 1 = (1 + 0.019)^{2*n}$$

$$n = 8 \;\; \textit{years}$$

Literature:

GUTHRIE, Gary a Larry LEMON. *Mathematics of Interest Rates and Finance.* Pearson New International Edition, 2013. ISBN 978-1-292-03983-1.

DRAKE, P., FABOZZI, F.: Foundations and applications of the time value of money, John Wiley & Sons, 2009. ISBN 978-0-470-40736-3

DAHLQUIST, J., KNIGHT, R. *Principles of Finance.* OpenStax College, 2022.