Financial Management

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Lecture 3

Content:

- General evaluation principles
- Present value of some common investments
- Amortized loans
- Investment decision rules

General evaluation principles

Given the interest rate R and the number of periods n, the value of an investment:

- in terms of dollars today is the **present value (PV)**:

$$PV = \frac{FV}{(1+R)^n}$$

- in terms of dollars in the future is the **future value (FV):**

$$FV = PV * (1+R)^n$$

(Risky cash flows must be discounted at a rate equal to the risk-free rate plus an appropriate risk premium.)

General evaluation principles

General process for pricing securities (in a market without arbitrage opportunities):

1. Identify the cash flows that will be paid by the security
2. Determine the PV of the security's cash flows

Unless the price of the security equals this present value, there is an arbitrage opportunity:

No-Arbitrage Price = PV(All cash flows paid by the security)

General evaluation principles

It is only possible to compare or combine values at the same point in time.

To move a cash flow *C* forward in time you must compound it (compute the future value):

$$FV_n = C * (1+R)^n$$

To find the equivalent value today of a future cash flow C you must discount it (compute the present value):

$$PV = \frac{C}{(1+R)^n}$$

General evaluation principles

Present Value of a Cash Flow Stream:

$$PV = \sum_{n=0}^{N} PV(C_n) = \sum_{n=0}^{N} \frac{C_n}{(1+R)^n}$$

Future Value of a Cash Flow Stream with Present Value PV:

$$FV_n = PV * (1+R)^n$$

Present value of some common investments

A perpetuity is a stream of equal cash flows that occur at regular intervals and last forever.

The present value of a perpetuity with payment *C* and interest rate *R* is given by

$$PV = \sum_{n=1}^{\infty} \frac{C}{(1+R)^n}$$

Because the first cash flow is in one period, $C_0 = 0$.

It can be shown that:

$$PV(C\ in\ perpetuity) = \frac{C}{R}$$

Present value of some common investments

An **annuity** is a stream of N equal cash flows paid at regular intervals (the first one at date 1). At the end of it, you get your initial investment back.

The present value of an *N*-period annuity with payment *C* and interest rate *R* is $PV = \sum_{n=1}^{N} \frac{C}{(1+R)^n}$

It can be shown that the **PV of an annuity** is:

$$PV = C\frac{1}{R}\left(1 - \frac{1}{(1+R)^N}\right)$$

The **Future Value of an Annuity** is:

$$FV = C\frac{1}{R}((1+R)^N - 1)$$

Present value of some common investments

A **growing perpetuity** is a stream of cash flows that occur at regular intervals and grow at a constant rate forever. A growing perpetuity with a first payment *C* and a growth rate *g* has present value

$$PV = \sum_{n=1}^{\infty} \frac{C(1+g)^{n-1}}{(1+R)^n}$$

It must be $g < R$, so that each successive term in the sum is less than the previous term and the overall sum is finite.

It can be shown that:

$$PV(growing\ perpetuity) = \frac{C}{R-g}$$

Present value of some common investments

A **growing annuity** is a stream of *N* growing cash flows, paid at regular intervals. The first cash flow arrives at the end of the first period, and the first cash flow does not grow. The present value of an *N*-period growing annuity with initial cash flow *C*, growth rate *g*, and interest rate r is given by

$$PV = C\frac{1}{R-g}\left(1-\left(\frac{1+g}{1+R}\right)^{N}\right)$$

Because the annuity has only a finite number of terms, we can have $g > R$. The formula does not work for $g = R$, but in that case growth and discounting cancel out: $PV = C\frac{N}{1+R}$

Amortized loans

An important application of compound interests are **amortized loans**. Given a loan of amount L that has to be repaid in N periods, with interest R, the payments P are determined from

$$L = \sum_{n=1}^{N} \frac{P}{(1+R)^n}$$

Notice that the longer the amortization period, the higher the amount to be paid in form of interests.

Amortized loans are often use in automobile, home mortgage, and business loans.

Amortized loans

In a standard amortized loan, fixed payments are made at regular intervals, with a fixed interest rate. This qualifies it **as an annuity**, and we can compute the amount of the periodic payments using the annuity formula:

$$L = P\,\frac{1}{R}\left(1 - \frac{1}{(1+R)^N}\right)$$

Given the amount of the loan L, the number of periods N and the interest rate R, one simply needs to solve for P:

$$P = \frac{L}{\frac{1}{R} - \frac{1}{R(1+R)^N}}$$

Investment decision rules

Net present value (NPV): *NPV = PV(benefits) – PV(costs)*

If *NPV > 0* the investment is profitable.

Internal rate of return (IRR): the interest rate that sets the net present value of the cash flows equal to zero.

To compute it you just need to invert the formulas. This is only easy with one or two periods with cash flows.

Abel–Ruffini theorem (Abel's impossibility theorem): there is no general algebraic solution to polynomial equations of degree five or higher!

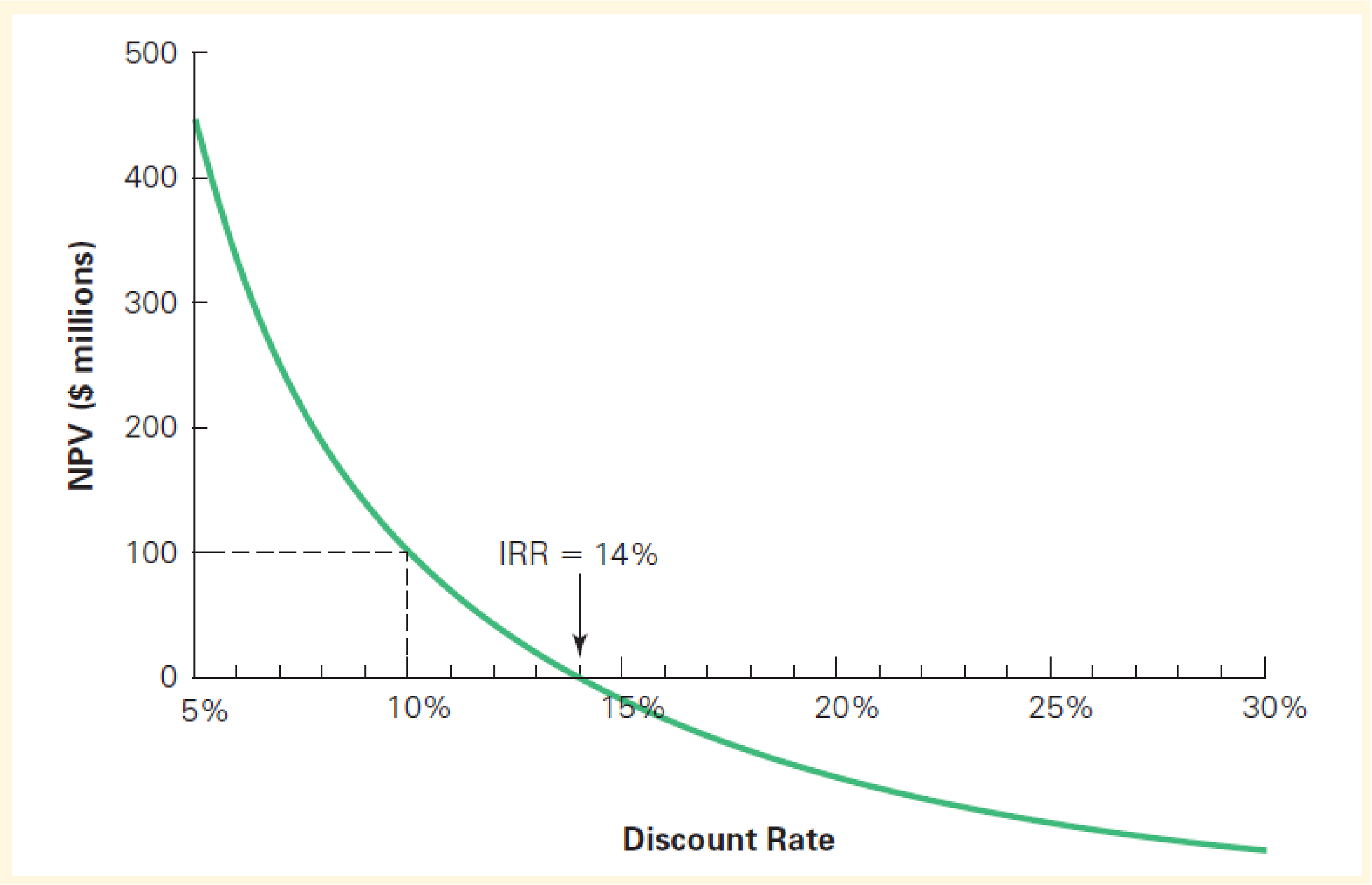
Investment decision rules

NPV Investment Rule: take the investment with the highest NPV. This is equivalent to receiving its NPV in cash today.

NPV profile: a graph of the project's NPV over a range of discount rates. Useful when there is some uncertainty regarding the project's cost of capital.

The difference between the cost of capital and the IRR is the maximum estimation error in the cost of capital that can exist without altering the original decision.

Investment decision rules



In this example, the cost of capital r is estimated = 10%

The investment remains profitable as long as the real r < 14%

Investment decision rules

The NPV rule is the most accurate and reliable, but other tools are applied by practitioners. They often give the same indication; if they don’t, they should be disregarded.

The internal rate of return (IRR) investment rule suggests that if the average return on the investment opportunity (i.e. the IRR) is greater than the return on other alternatives in the market with equivalent risk and maturity (i.e. the project’s cost of capital), you should undertake the investment.

IRR Investment Rule: Take any investment opportunity where the IRR exceeds the opportunity cost of capital. Turn down any opportunity whose IRR is less than the opportunity cost of capital.

Investment decision rules

The IRR rule is only guaranteed to work for a stand-alone project if all of the project’s negative cash flows precede its positive cash flows. When the benefits of an investment occur before the costs, the NPV is an increasing function of the discount rate, and the IRR rule fails.

Another problematic situation occurs with multiple IRRs.

Finally, the IRR might be non-existent because the NPV is positive for every discount rate.

Investment decision rules

Payback investment rule: you should only accept a project if its cash flows pay back its initial investment within a prespecified period, called payback period.

Problems with this rule:

1. It ignores the project's cost of capital and the time value of money;
2. it ignores cash flows after the payback period;
3. it relies on an ad hoc decision criterion (the choice of the length of the payback period is arbitrary).

Investment decision rules

Sometimes a firm must choose just one among several **mutually exclusive** possible projects.

The correct choice is to pick the project with the highest **NPV**.

Picking the project with the larger **IRR** can lead to mistakes: when projects differ in their scale of investment, the timing of their cash flows, or their riskiness, their IRRs cannot be meaningfully compared.

A modified version called incremental IRR has been proposed but it also can be problematic, and it is also more difficult to use than the NPV, so it should not be used.