EXERCISES

EXERCISE 1

In a perfectly competitive market where the interest rate r remains constant over time, security A pays 1200 euro after three years and it costs 1000 euro.

Security B pays 1100 euro after one year. If there is a 20% tax on financial profits, how much does the investor who buys B earn after one year?

The interest rate is:

$$1000 = \frac{1200}{(1+r)^3}$$

$$(1+r)^3 = \frac{1200}{1000}$$

$$1 + r = 1.2^{1/3}$$

$$r = 1.2^{1/3} - 1 \approx 0.0627 = 6.27\%$$

Since the interest rate is 6.27% and we are in a perfect market, the price of B has to be:

$$P_B = \frac{1100}{1+\mathrm{r}} = \frac{1100}{1+0.0627} \approx 1035.1$$

The investor earned $1100 - 1035.1 = 64.9$ euro, but has to pay a 20% tax on it, so after taxes he actually earns:

$$64.9 - 64.9 * 0.2 = 51.92$$

EXERCISE 2

A project requires an initial investment of 10000 euro and is expected to generate a positive cash flow of 5000 euro after one year and one of 8000 euro after two years. What is the maximum cost of capital with which the project is profitable?

The project is profitable as long as the cost of capital is lower than the IRR, i.e., the discount rate that sets the NPV equal to zero. Therefore, we compute the IRR.

$$0 = -10000 + \frac{5000}{1+IRR} + \frac{8000}{(1+IRR)^2}$$

$$10000 = \frac{5000(1+IRR)+8000}{(1+IRR)^2}$$

$$10000(1+2IRR+IRR^2) = 5000 + 5000IRR + 8000$$

$$10000 + 20000IRR + 10000IRR^2 - 5000IRR - 13000 = 0$$

$$10000IRR^2 + 15000IRR - 3000 = 0$$

$$10IRR^2 + 15IRR - 3 = 0$$

$$IRR = \frac{-15 \pm \sqrt{225+120}}{20} \approx \frac{-15 \pm 18.57}{20}$$

The first solution is 0.1785; the second is -1.6785. The second one is clearly not meaningful, so we consider the first. The project is profitable if the cost of capital is lower than 17.85%.

EXERCISE 3

A three-year 1000-euro coupon bond pays a 200-euro coupon each year.

Knowing that a bond that pays 100 euro after one-year costs 97 euro, one that pays 100 euro after two years costs 95 euro, and one that pays 100 euro after three years costs 90 euro, what is the price at which the coupon bond should trade in a market in which the law of one price holds?

Due to the law of one price, equal cash flows should have the same price. The coupon bond pays 200 euro after one year, 200 euro after two years and 1200 (the 1000 euro plus the 200 coupon) after three years.

Therefore, we can replicate its cash flows with two one-year zero-coupon bonds, two two-year zero-coupon bonds and twelve three-year zero-coupon bonds. The price of this portfolio of zero-coupon bonds, which must be equal to the price of the coupon bond, is:

$$97 * 2 + 95 * 2 + 90 * 12 = 1464$$

EXERCISE 4

The security S pays 990 euro after one year and it costs 900 euro.

If there is a 20% tax on financial profits and the inflation rate is 2% per year, what is the real return paid by security S after taxes?

After one year we earn $990 - 900 = 90$ euro.

On this we have to pay a 20% tax, so we actually earn $90 - 90 * 0.2 = 72$ euro

The nominal return net of taxes is therefore:

$$R = \frac{972 - 900}{900} = 8\%$$

We now need to account for the inflation. The real return is:

$$R_r = \frac{1 + R}{1 + i} - 1 = \frac{1 + 0.08}{1 + 0.02} - 1 \approx 0.0588$$

EXERCISE 5

An amortized loan of 100000 euro is paid back in five payments done at regular intervals. If the interest rate is 5%, what is the total amount paid in interests?

An amortized loan of this type is an annuity. Therefore, we have:

$$L = P\frac{1}{R}\left(1 - \frac{1}{(1+R)^N}\right)$$

The amount paid at each individual payment is:

$$P = \frac{L}{\frac{1}{R} - \frac{1}{R(1+R)^N}} = \frac{100000}{\frac{1}{0.05} - \frac{1}{0.05(1+0.05)^5}}$$

$$\approx \frac{100000}{20 - 15.67} = 23094.69$$

So the total payment is $23094.69 * 5 = 115473.4$ euro, and the total amount paid in interests is $115473.4 - 100000 = 15473.4$ euro.

EXERCISE 6

A zero-coupon bond with a face value of 10000 euro expires 3 months from now and costs 9800 euro.

Compute the Modified duration for this bond.

The formula for the Modified duration is:

$$ModifiedD = \frac{MacaulayD}{1 + \frac{YTM}{k}}$$

Since this is a zero-coupon bond, the Macaulay duration is simply equal to its maturity expressed in years, i.e.:

$$MacaulayD = \frac{3}{12} = 0.25$$

The yield to maturity is:

$$YTM = \left(\frac{FV}{P}\right)^{1/n} - 1 = \left(\frac{10000}{9800}\right)^{\frac{1}{0.25}} - 1 \approx 0.0842$$

We also have $k = 1$ because it is a zero-coupon bond, so the Modified duration is:

$$ModifiedD = \frac{MacaulayD}{1 + \frac{YTM}{k}} = \frac{0.25}{1 + \frac{0.0842}{1}} = \frac{0.25}{1.0842} \approx 0.23$$