EXERCISES

EXERCISE 1

A coupon bond will pay a coupon of 50 euro 3 months from now, another coupon of 50 euro 9 months from now, and final coupon of 50 euro plus the face value of 1000 euro 15 months from now. The yield to maturity is 2% and the current price of the bond is 1123.34 euro. Compute the Macaulay duration.

The cash flows are 50 euro in $3/12 = 0.25$ years, 50 euro in $9/12 = 0.75$ years, and 1050 euro in $15/12 = 1.25$ years.

The duration is therefore:

$$MacaulayD = \sum_{i=1}^{3} \frac{PV(C_i)}{P} t_i = \frac{\frac{50}{1.02^{0.25}}}{1123.34} 0.25 + \frac{\frac{50}{1.02^{0.75}}}{1123.34} 0.75 + \frac{\frac{1050}{1.02^{1.25}}}{1123.34} 1.25 \approx 1.184$$

EXERCISE 2

The log-return of the four assets included in an equally weighted portfolio is:

$r_1 = 0.1,\ r_2 = -0.06,\ r_3 = 0.07,\ r_4 = 0.05$

What is the return of the portfolio?

Log-returns are not asset additive. We first need to convert them to simple returns:

$$R_1 = \exp(r_1) - 1 = \exp(0.1) - 1 = 0.1052$$

$$R_2 = \exp(r_2) - 1 = \exp(-0.06) - 1 = -0.0582$$

$$R_3 = \exp(r_3) - 1 = \exp(0.07) - 1 = 0.0725$$

$$R_4 = \exp(r_4) - 1 = \exp(0.05) - 1 = 0.0513$$

We can now compute the return of the portfolio:

$$R_p = w_1 R_1 + w_2 R_2 + w_3 R_3 + w_4 R_4$$

$$= 0.25 * 0.1052 + 0.25 * (-0.0582) + 0.25 * 0.0725 + 0.25 * 0.0513 = 0.0427$$

EXERCISE 3

The returns of a security over four periods are:

$R_{t=1} = 0.2,\ \ R_{t=2} = -0.1,\ \ R_{t=3} = 0.08,\ \ R_{t=4} = 0.04$

If we invested 1000 euro in this asset at t=0, how much is our investment worth at t=4?

The value of the investment is:

$$V_4 = V_0 + V_0\left[\prod_{t=1}^{4}(1+R_t) - 1\right] = 1000 + 1000[(1+0.2)(1-0.1)(1+0.08)(1+0.04) - 1]$$

$$\approx 1000 + 1000 * [1.213 - 1] = 1213$$

Alternatively, we can transform the returns in log-returns, which are time-additive:

$$r_{t=1} = \ln(R_{t=1} + 1) = \ln(0.2 + 1) \approx 0.1823$$

$$r_{t=2} = \ln(R_{t=2} + 1) = \ln(-0.1 + 1) \approx -0.1054$$

$$r_{t=3} = \ln(R_{t=3} + 1) = \ln(0.08 + 1) \approx 0.0770$$

$$r_{t=4} = \ln(R_{t=4} + 1) = \ln(0.04 + 1) \approx 0.0392$$

The cumulative log-return from $t = 1$ to $t = 4$ is:

$$cumret_{1-4} = 0.1823 - 0.1054 + 0.0770 + 0.0392 = 0.1931$$

We need to convert this into a simple return:

$$cumRet_{1-4} = \exp(cumret_{1-4}) - 1 = \exp(0.1931) - 1 \approx 0.213$$

And the value of the investment at $t = 4$ is therefore:

$$V_4 = V_0 + V_0 * cumRet_{1-4} = 1000 + 1000 * 0.231 = 1231$$

EXERCISE 4

The vector of weights and the covariance matrix of a portfolio with three assets are:

$$\boldsymbol{w} = \begin{bmatrix} 0.5 \\ 0.7 \\ -0.2 \end{bmatrix} \qquad \boldsymbol{\Sigma} = \begin{bmatrix} 0.004 & 0.006 & 0.003 \\ 0.006 & 0.008 & 0.007 \\ 0.003 & 0.007 & 0.005 \end{bmatrix}$$

Compute, using matrix form, the variance of the portfolio.

We just need to apply the formula:

$$Var(R_P) = \boldsymbol{w}'\Sigma\boldsymbol{w} = \begin{bmatrix} 0.5 & 0.7 & -0.2 \end{bmatrix} \begin{bmatrix} 0.004 & 0.006 & 0.003 \\ 0.006 & 0.008 & 0.007 \\ 0.003 & 0.007 & 0.005 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.7 \\ -0.2 \end{bmatrix} =$$

$$\begin{bmatrix} 0.5 * 0.004 + 0.7 * 0.006 - 0.2 * 0.003 & 0.5 * 0.006 + 0.7 * 0.008 - 0.2 * 0.007 & 0.5 * 0.003 + 0.7 * 0.007 - 0.2 * 0.005 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.7 \\ -0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0056 & 0.0072 & 0.0054 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.7 \\ -0.2 \end{bmatrix} = 0.0056 * 0.5 + 0.0072 * 0.7 + 0.0054 * (-0.2) = 0.00676$$