8 COST–VOLUME–PROFIT ANALYSIS

LEARNING OBJECTIVES After studying this chapter, you should be able to:

- justify the use of linear cost and revenue functions;
- apply the numerical approach to answer questions similar to those listed in Example 8.1;
- construct break-even, contribution and profit–volume graphs;
- apply cost–volume–profit analysis in a multi-product setting;
- explain the meaning of operating leverage and describe how it influences profits;
- identify and explain the assumptions on which cost–volume–profit analysis is based.

You will remember from Chapter 1 that the decision-making process involves selecting from a range of possible courses of action. Before they make their choice, managers need to compare the likely effects of the options they are considering. This chapter looks at one technique that allows them to consider the consequences of particular courses of action. It provides answers to questions such as:

- How many units must be sold to break even?
- What would be the effect on profits if we reduce our selling price and sell more units?
- What sales volume is required to meet the additional fixed charges arising from an advertising campaign?
- Should we pay our sales people on the basis of a salary only, on the basis of a commission only or by a combination of the two?

These and other questions can be answered using cost–volume–profit (CVP) analysis.

CVP analysis examines the relationship between changes in activity (i.e. output) and changes in total sales revenue, costs and net profit. It allows us to predict what will happen to the financial results if a specified level of activity or volume fluctuates. This information is vital to management, since one of the most important variables influencing total sales revenue, total costs and profits is output or volume. Knowledge of this relationship enables management to identify critical output levels, such as the level at which neither a profit nor a loss will occur (i.e. the **break-even point**).

CVP analysis is based on the relationship between volume and sales revenue, costs and profit in the *short run*. This is normally a period of one year, or less, a time in which the output of a firm is likely to be restricted to that available from the current operating capacity. In the short run some inputs can be increased, but others cannot. Additional supplies of materials and unskilled labour may be obtained at short notice, but operating capacity cannot be significantly changed. For example, it is not possible for a hospital to expand its facilities in the short run in order to increase the number of beds. Similarly, a hotel cannot increase the number of rooms in the short run to increase the number of guests. It is also important to remember that most of the costs and prices of a firm's products or services will already have been predetermined over a short-run period, and the major area of uncertainty will be sales volume. Short-run profitability will, therefore, be most sensitive to sales volume. CVP analysis thus highlights the effects of changes in sales volume on the level of profits in the short run.

The term 'volume' is used within CVP analysis but this has multiple meanings. Different measures can be used to represent the term. For example, sales revenue is a generic term that can be used by most organizations. However, units of output, or activity, tend to be the most widely used terms. This raises the question of what constitutes a unit of output or activity. For a manufacturing organization, such as a car manufacturer, determining units of output is straightforward. It is the number of cars produced. For a computer manufacturer, it is the number of computers produced. Service organizations face a more difficult choice. Hotels may define units as the number of guest nights, leisure centres may use the number of visitors as a measure of output/activity and airlines might use the number of passenger miles.

CVP analysis is dependent on the ability to estimate costs at different activity levels and to do this requires that costs are analysed into their fixed and variable elements. Cost estimation techniques are explained in Chapter 24 in Part Six of this book which focuses on the application of quantitative methods to management accounting. Alternatively, if you require a knowledge of cost estimation techniques now you may prefer to read Chapter 24 immediately after you have read this chapter.

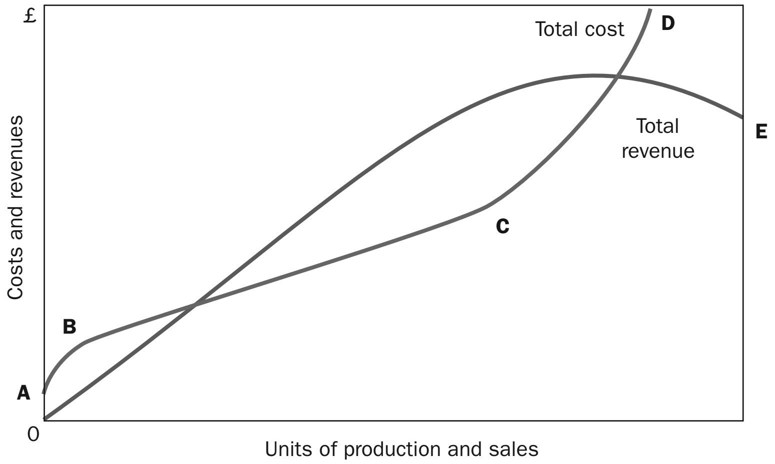
CURVILINEAR CVP RELATIONSHIPS

A diagram showing CVP behaviour is presented in Figure 8.1. You will see that the total revenue and total cost lines are curvilinear. The total revenue line (0–E) initially resembles a straight line but then begins to rise less steeply and eventually starts to decline. This arises because the firm is only able to sell increasing quantities of output by reducing the selling price per unit; thus the total revenue line does not increase proportionately with output. To increase the quantity of sales, it is necessary to reduce the unit selling price, which results in the total revenue line rising less steeply, and eventually beginning to decline. The decline occurs because the adverse effect of price reductions outweighs the benefits of increased sales volume.

The total cost line (A–D) illustrates cost behaviour in a manufacturing firm but similar cost behaviour also applies in non-manufacturing firms. Between points A and B, total costs rise steeply at first as the firm operates at the lower levels of the volume range. This reflects the difficulties of efficiently using manufacturing facilities designed for much larger volume levels. Between points B and C, the total cost line begins to level out and rise less steeply because the firm is now able to operate its manufacturing facilities within the efficient operating range and can take advantage of economies of scale (e.g. specialization of labour, smooth production schedules and discounts from bulk purchases). Economists describe this situation as **increasing returns to scale**. In the upper portion of the volume range, the total cost line between points C and D rises more steeply as the cost per unit increases. This is because manufacturing facilities are being operated beyond their capacity. Bottlenecks develop, production schedules become more complex and equipment breakdowns begin to occur. The overall effect is that the cost per unit of output increases and causes the total cost line to rise steeply. Economists describe this situation as **decreasing returns to scale**.

It is also clear from Figure 8.1 that the shape of the total revenue line is such that it crosses the total cost line at two points. In other words, there are two output levels at which the total costs are equal to the total revenues; or, more simply, there are two break-even points.

FIGURE 8.1
Curvilinear CVP relationships



LINEAR CVP RELATIONSHIPS

In Figure 8.2, the total cost line X–Y and the total revenue line 0–V assume that variable cost and selling price are constant per unit of output. This results in a linear relationship (i.e. a straight line) for total revenue and total cost as output/volume changes. If you look at these two lines, you will see that a linear relationship results in only one break-even point. You can also see that the profit area (i.e. the difference between the total revenue line O–V and the total cost line X–Y) widens as volume increases. For comparative purposes, the curvilinear relationships shown in Figure 8.1 are also reproduced in Figure 8.2 (with line A–D and line 0–E showing, respectively, curvilinear total cost and total revenue relationships).

Management accounting assumes linear CVP relationships when applying CVP analysis to short-run business problems. Curvilinear relationships appear to be more realistic of cost and revenue behaviour, so how can we justify CVP analysis based on the assumption of linear relationships? The answers are provided in the following sections.

Relevant range

Linear relationships are not intended to provide an accurate representation of total cost and total revenue throughout all ranges of output. The objective is to represent the behaviour of total cost and revenue over the range of output at which a firm expects to be operating within a short-term planning horizon.

FIGURE 8.2
Linear CVP relationships

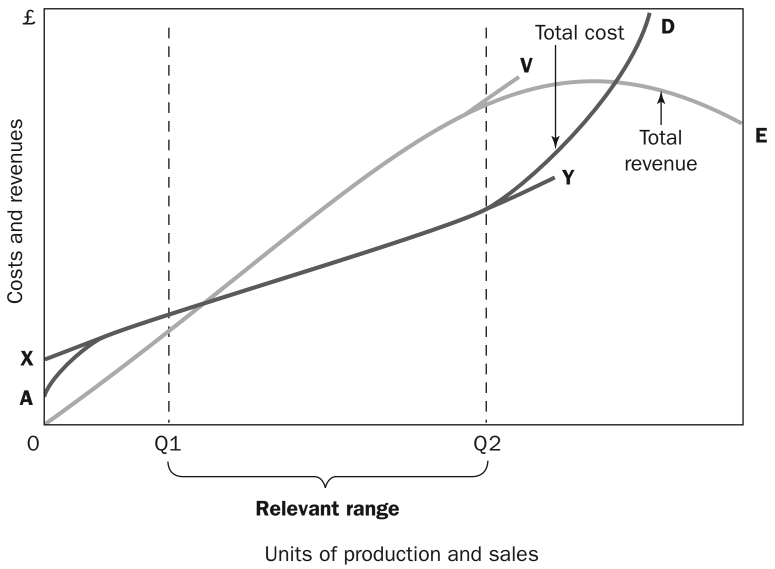


FIGURE 8.3
Fixed costs applicable within the relevant range

This range of output is represented by the output range between points Q1 and Q2 in Figure 8.2. The term **relevant range** is used to refer to the output range at which the firm expects to be operating within a short-term planning horizon. This relevant range also broadly represents the output levels that the firm has had experience of operating in the past and for which cost information is available.

It is clear from Figure 8.2 that, between points Q1 and Q2, the cost and revenue relationships are more or less linear. It would be unwise, however, to make this assumption for output levels outside the relevant range. CVP analysis should therefore only be applied within the relevant range. If the relevant range changes, different fixed and variable costs and selling prices must be used.

Fixed cost function

Figure 8.2 indicates that at zero output level fixed costs equivalent to 0X would be incurred. This fixed cost level of 0X is assumed to be applicable to activity level Q1 to Q2, shown in Figure 8.3. If there were to be a prolonged economic recession then output might fall below Q1, and this could result in redundancies and shutdowns. Therefore, fixed costs may be reduced to 0B if there is a prolonged and a significant decline in sales demand. Alternatively, additional fixed costs will be incurred if long-term sales volume is expected to be greater than Q2. Over a longer-term time horizon, the fixed cost line will consist of a series of step functions as shown in Figure 8.3. However, since within its short-term planning horizon the firm expects to be operating between output levels Q1 and Q2 (i.e. the relevant range), it will be committed, in the short term, to fixed costs of 0X. Thus the fixed cost of 0X shown in Figures 8.2 and 8.3 represent the fixed costs that would be incurred only for the relevant range.

Total revenue function

Linear CVP relationships assume that selling price is constant over the relevant range of output, and therefore the total revenue line is a straight line. This is a realistic assumption in those firms that operate in industries where selling prices tend to be fixed in the short term. Beyond the relevant range, increases in output may only be possible by offering substantial reductions in price. As it is not the intention of firms to operate outside the relevant range, it is appropriate to assume constant selling prices.

A NUMERICAL APPROACH TO COST–VOLUME–PROFIT ANALYSIS

As an alternative to using diagrams for CVP analysis we can also use a numerical approach. Diagrams are useful for presenting the outcomes in a more visual form to non-accounting managers, but the numerical approach is often a quicker and more flexible method for producing the appropriate information. Indeed, it is possible to express CVP relationships in a simple mathematical equation format so that they can form an input for computer financial models. To keep things simple we shall avoid mathematical formulae and use a simple numerical approach.

In the previous sections, we pointed out that CVP analysis is based on the assumption that selling price and variable cost are constant per unit of output. In contrast, you will remember from Chapter 2 that over a short-run period fixed costs are a constant total amount whereas unit cost changes with output levels. As a result, profit per unit also changes with volume. For example, if fixed costs are £10 000 for a period and output is 10 000 units, the fixed cost will be £1 per unit. Alternatively, if output is 5000 units, the fixed cost will be £2 per unit. Profit per unit will not therefore be constant over varying output levels and it is incorrect to unitize fixed costs for CVP decisions.

Instead of using profit per unit, we shall use contribution margins to apply the numerical approach. **Contribution margin** is equal to sales revenue minus variable costs. Because the variable cost per unit and the selling price per unit are assumed to be constant the contribution margin per unit is also assumed to be constant. We will use Example 8.1 to illustrate the application of the numerical approach to CVP analysis.

Example 8.1 calculations

1 Break-even point in units (i.e. number of tickets sold)

You will see from Example 8.1 that each ticket sold generates a contribution of £10 (£20 selling price – £10 variable cost), which is available to cover fixed costs and, after they are covered, to contribute to profit. When we have obtained sufficient total contribution to cover fixed costs, the break-even point is achieved, and so:

$$\text{Break-even point in units} = \frac{\text{Fixed costs (£60 000)}}{\text{Contribution per unit (£10)}}$$
$$= 6000 \text{ tickets}$$

2 Units to be sold to obtain a £30 000 profit

To achieve a profit of any size we must first obtain sufficient contribution to cover the fixed costs (i.e. the break-even point). If the total contribution is not sufficient to cover the fixed costs then a loss will occur. Once a sufficient total contribution has been achieved any excess contribution represents profit.

EXAMPLE 8.1

Lee Enterprises operates in the leisure and entertainment industry and one of its activities is to promote concerts at locations throughout the world. The company is examining the viability of a concert in Singapore. Estimated fixed costs are £60 000. These include the fees paid to performers, the hire of the venue and advertising costs. Variable costs consist of the cost of a pre-packed buffet that will be provided by a firm of caterers at a price, which is currently being negotiated, but it is likely to be in the region of £10 per ticket sold. The proposed price for the sale of a ticket is £20.

The management of Lee has requested the following information:

1. The number of tickets that must be sold to break even (that is, the point at which there is neither a profit nor a loss).
2. How many tickets must be sold to earn £30 000 target profit?
3. What profit would result if 8000 tickets were sold?
4. What selling price would have to be charged to give a profit of £30 000 on sales of 8000 tickets, fixed costs of £60 000 and variable costs of £10 per ticket?
5. How many additional tickets must be sold to cover the extra cost of television advertising of £8000?

REAL WORLD VIEWS 8.1

Airbus A380 likely to break even in 2015

The Airbus A380 was the world's first double-decker aircraft. It can accommodate from 555 to 853 passengers depending on the class configuration. Long haul airlines such as Singapore Airlines were early adopters of the aircraft back in 2007.

The 2016 list price of an A380 is approximately $433 million. According to the company website, there are over 120 A380 aircraft in service with 13 airlines. Each aircraft is built to order and airlines often place orders years in advance. According to the *FlightGlobal* website, the Airbus A380 will breakeven in 2015. The site quotes Chief executive, Tom Enders, who says 'Most importantly, we confirm the A380 break-even for 2015.' As of early 2015, €42 billion in revenue has been raised from A380 sales. It also has an order book for 319 A380 aircraft as of the end of March 2016.

Questions

1. Is it true to say that any A380 aircraft sold before break-even has been achieved is making a loss?
2. Can you think of some major fixed costs likely to be incurred on the A380 by Airbus?

References

www.flightglobal.com/news/articles/airbus-a380-aircraft-profile-205274/
www.airbus.com/presscentre/pressreleases/press-release-detail/detail/new-airbus-aircraft-list-prices-for-2016/
www.flightglobal.com/news/articles/airbus-assures-on-a380-break-even-this-year-409534/

Thus to determine the total contribution to obtain a target profit we simply add the target profit to the fixed costs and divide by the contribution per unit, so that:

$$\text{Units sold for the target profit} = \frac{\text{Fixed costs (£60 000)} + \text{Target profit (£30 000)}}{\text{Contribution per unit (£10)}}$$
$$= 9000 \text{ tickets}$$

3 Profit from the sale of 8000 tickets

The total contribution from the sale of 8000 tickets is £80 000 (8000 × £10). To ascertain the profit, we deduct the fixed costs of £60 000, giving a net profit of £20 000. Let us now assume that we wish to ascertain the impact on profit if a further 1000 tickets are sold so that sales volume increases from 8000 to 9000 tickets. Assuming that fixed costs remain unchanged, the impact on a firm's profits resulting from a change in the number of units sold can be determined by multiplying the unit contribution margin by the change in units sold. Therefore the increase in profits will be £10 000 (1000 units times a unit contribution margin of £10).

4 Selling price to be charged to show a profit of £30 000 on sales of 8000 tickets

First, we must determine the total required revenue to obtain a profit of £30 000. This is £170 000, which is derived from the sum of the fixed costs (£60 000), variable costs (8000 × £10) and the target profit (£30 000). Dividing the required sales revenues of £170 000 by the sales volume (8000 tickets) gives a selling price of £21.25.

5 Additional sales volume to meet £8000 additional fixed advertisement charges

The contribution per unit is £10 and fixed costs will increase by £8000. Therefore an extra 800 tickets must be sold to cover the additional fixed costs of £8000.

THE PROFIT–VOLUME RATIO

The **profit–volume ratio** (also known as the **contribution margin ratio**) is the contribution divided by sales. It represents the proportion of each £1 of sales available to cover fixed costs and provide for profit. In Example 8.1, the contribution is £10 per unit and the selling price is £20 per unit; the profit–volume ratio is 0.5. This means that for each £1 sale a contribution of £0.50 is earned. Because we assume that selling price and contribution per unit are constant, the profit–volume ratio is also assumed to be constant. This means that the profit–volume ratio can be computed using either unit figures or total figures. Given an estimate of total sales revenue, it is possible to use the profit–volume ratio to estimate total contribution. For example, if total sales revenue is estimated to be £200 000, the total contribution will be £100 000 (£200 000 × 0.5). To calculate the profit, we deduct fixed costs of £60 000; thus a profit of £40 000 will be obtained from total sales revenue of £200 000.

This computation can be expressed in equation form:

$$\text{Profit} = (\text{Sales revenue} \times \text{PV ratio}) - \text{Fixed costs}$$

REAL WORLD VIEWS 8.2

Why is the break-even price of crude oil so important?

The break-even price of crude oil includes production costs, exploring or finding costs, oil well development costs, transportation costs, and selling and general administration expenses. A survey published in 2015 showed some interesting insights into the break-even price for producing crude oil. Petroleum extraction in the Arctic region shows the highest break-even price of $75 per barrel. On the other hand, Middle Eastern countries have the lowest price at $27 per barrel. US shale oil producers have a break-even price of $65 per barrel. These estimates are average break-even prices. The costs may vary depending on the oil well and its location. The chart below describes the break-even price for crude oil.

According to a publication in *Market Realist* by Gordon Kristopher WTI (West Texas Intermediate) crude oil was currently trading at $45 per barrel at the time of the publication and Brent crude oil was trading at $46.4 per barrel. This massive price decline in the last six months will impact oil producers with high break-even prices. The margins of high break-even-price US shale oil producers will be impacted the most. As long as crude oil prices are around the break-even range of US shale oil, then US oil production growth will be slow. Production will likely decline over the long term, which in turn will have a positive impact on oil prices.

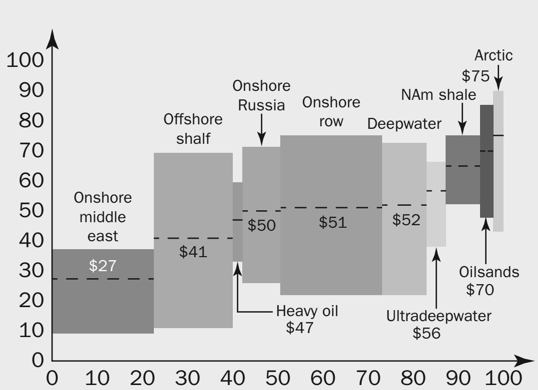
Questions

1. Why will a decline in production have a positive impact on oil prices?
2. Why does the break-even price in the above chart differ according to the location of the oil wells?
3. Is the break-even price of crude oil more important than the break-even volume?

Reference

Kristopher, G. (2015) A key investor's guide to the crude oil market (Part 4 of 1 of 15 *Market Monitor*). Available at marketrealist.com/2015/01/crude-oil-market-key-overview/

Breakeven Price of Crude Oil



Note: *Market Realist*
Source: Seadrill, Morgan Stanley Equity Research, International Energy Agency

We can rearrange this equation:

$$\text{Profit} + \text{Fixed costs} = \text{Sales revenue} \times \text{PV ratio}$$

Therefore the break-even sales revenue (where profit = 0) = Fixed costs/PV ratio.

If we apply this approach to Example 8.1, the break-even sales revenue is £120 000 (£60 000 fixed costs/ 0.5 PV ratio).

RELEVANT RANGE

It is vital to remember that CVP analysis can only be used for decisions that result in outcomes within the relevant range. Outside this range, the unit selling price and the variable cost are no longer deemed to be constant per unit and any results obtained from the formulae that fall outside the relevant range will be incorrect. The concept of the relevant range is more appropriate for production settings but it can apply within non-production settings. Returning to Lee Enterprises in Example 8.1, we shall now assume that the caterers' charges will be higher per ticket if ticket sales are below 4000 but lower if sales exceed 12 000 tickets. Thus, the £10 variable cost relates only to a sales volume within a range of 4000 to 12 000 tickets. Outside this range, other costs apply. In other words, we will assume that the relevant range is a sales volume of 4000 to 12 000 tickets and outside this range the results of our CVP analysis do not apply.

MARGIN OF SAFETY

The **margin of safety** indicates by how much sales may decrease before a loss occurs. Using Example 8.1, where unit selling price and variable cost were £20 and £10 respectively and fixed costs were £60 000, we noted that the break-even point was 6000 tickets or £120 000 sales value. If sales are expected to

REAL WORLD VIEWS 8.3

Airlines struggling to break even will make 'less than £4 profit per passenger'

According to the International Air and Transport Association (IATA) conference airlines were expected to make around £3.18 profit from each passenger in 2014. Although carriers were expecting net profits of £11 billion, margins were so thin the air industry was expected to make less money than the oil industry makes from selling the fuel it consumes. Tony Tyler, the director general of IATA, said the headline figures masked 'a daily struggle for airlines to break even. The brutal economic reality is that on revenues of $746 billion (£445bn), we will earn an average net margin of 2.4 per cent.'

IATA research revealed that carriers would spend an estimated $212 billion (£126bn) on jet fuel over the next 12 months, representing almost 30 per cent of their total operating costs. Intense competition from low-cost carriers has seen air fares fall in real terms by 3.5 per cent this year, with the number of passengers worldwide reaching 3.3 billion. Planes are flying fuller than ever before but lower fares mean that a higher percentage of occupied seats is needed to break even. IATA's chief economist, Brian Pearce, said, 'It's remarkable that the industry is generating any profit at all.'

Questions

1. Is break even a good performance monitor over the longer term?
2. How do decreasing margins affect the break-even point and margin of safety?

Reference

Catherine Eade (2014) 'Airlines struggling to break even will make "less than £4 profit per passenger" this year'. *Daily Mail*, 3 June. Available at www.dailymail.co.uk/travel/article-2647105/Airlines-struggling-break-make-4-profit-passenger-year.html#ixzz3UjUsCZm4

be 8000 tickets or £160 000, the margin of safety will be 2000 tickets or £40 000. Alternatively, we can express the margin of safety in a percentage form based on the following ratio:

$$\text{Percentage margin of safety} = \frac{\text{Expected sales} - \text{Break-even sales}}{\text{Expected sales}}$$

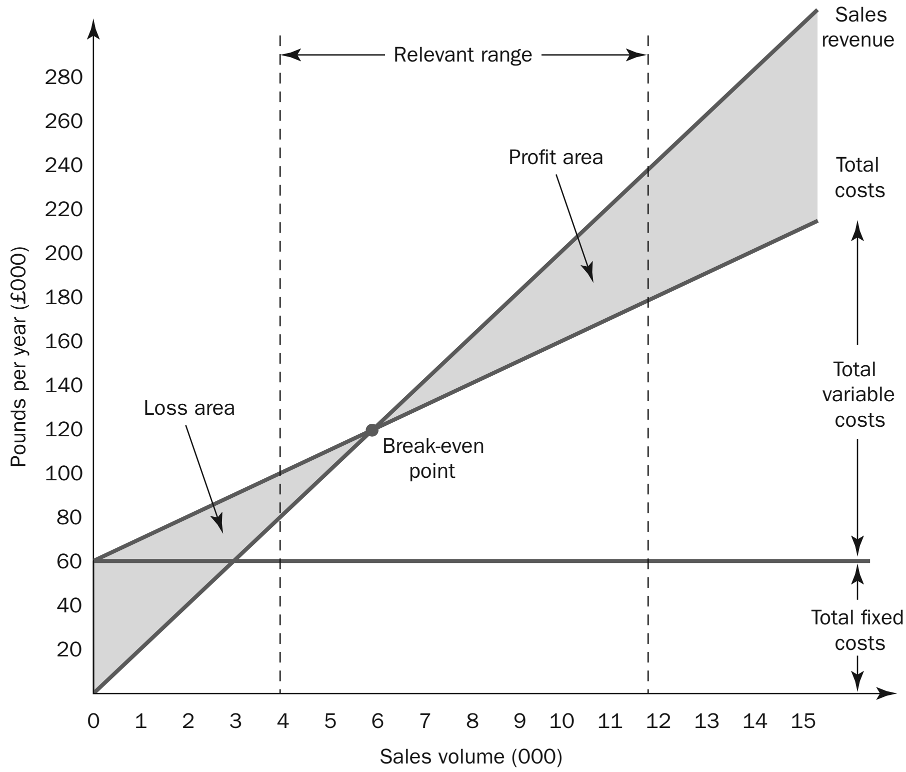
$$= \frac{£160\,000 - £120\,000}{£160\,000} = 25\%$$

Note that higher margins of safety are associated with less risky activities.

CONSTRUCTING THE BREAK-EVEN CHART

Managers may obtain a clearer understanding of CVP behaviour if the information is presented in graphical format. Using the data in Example 8.1, we can construct the **break-even chart** for Lee Enterprises (Figure 8.4). Note that activity/output is plotted on the horizontal axis and monetary amounts for total costs, total revenues and total profits (or loss) are recorded on the vertical axis. In constructing the graph, the fixed costs are plotted as a single horizontal line at the £60 000 level. Variable costs at the rate of £10 per unit of volume are added to the fixed costs to enable the total cost line to be plotted. Two points are required to insert the total cost line. At zero sales, volume total cost will be equal to the fixed costs of £60 000. At 12 000 units, sales volume total costs will be £180 000 consisting of £120 000 variable costs plus £60 000 fixed costs. The total revenue line is plotted at the rate of £20 per unit of volume. At zero output, total sales are zero and at 12 000 units, total sales revenue is £240 000. The total revenues for these two points are plotted on the graph and a straight line is drawn that joins these points. The constraints of the relevant range consisting of two vertical lines are then added to the graph; beyond these lines, we have little assurance that the CVP relationships are valid.

FIGURE 8.4
Break-even chart for Example 8.1



The point at which the total sales revenue line cuts the total cost line is the point where the concert makes neither a profit nor a loss. This is the break-even point and is 6000 tickets or £120 000 total sales revenue. The distance between the total sales revenue line and the total cost line at a volume below the break-even point represents losses that will occur for sales levels below 6000 tickets. Similarly, if the company operates at a sales volume above the break-even point, the difference between the total revenue and the total cost lines represents the profit that results from sales levels above 6000 tickets.

ALTERNATIVE PRESENTATION OF COST–VOLUME–PROFIT ANALYSIS

Contribution graph

In Figure 8.4, the fixed cost line is drawn parallel to the horizontal axis, and the variable cost is the difference between the total cost line and the fixed cost line. An alternative to Figure 8.4 for the data contained in Example 8.1 is illustrated in Figure 8.5. This alternative presentation is called a **contribution graph**. In Figure 8.5, the variable cost line is drawn first at £10 per unit of volume. The fixed costs are represented by the difference between the total cost line and the variable cost line. Because fixed costs are assumed to be a constant sum throughout the entire output range, a constant sum of £60 000 for fixed costs is added to the variable cost line, which results in the total cost line being drawn parallel to the variable cost line. The advantage of this form of presentation is that it emphasizes the total contribution, which is represented by the difference between the total sales revenue line and the total variable cost line.

Profit–volume graph

Neither the break-even nor the contribution graphs highlight the profit or loss at different volume levels. To ascertain the profit or loss figures from a break-even graph, it is necessary to determine

FIGURE 8.5
Contribution chart for Example 8.1

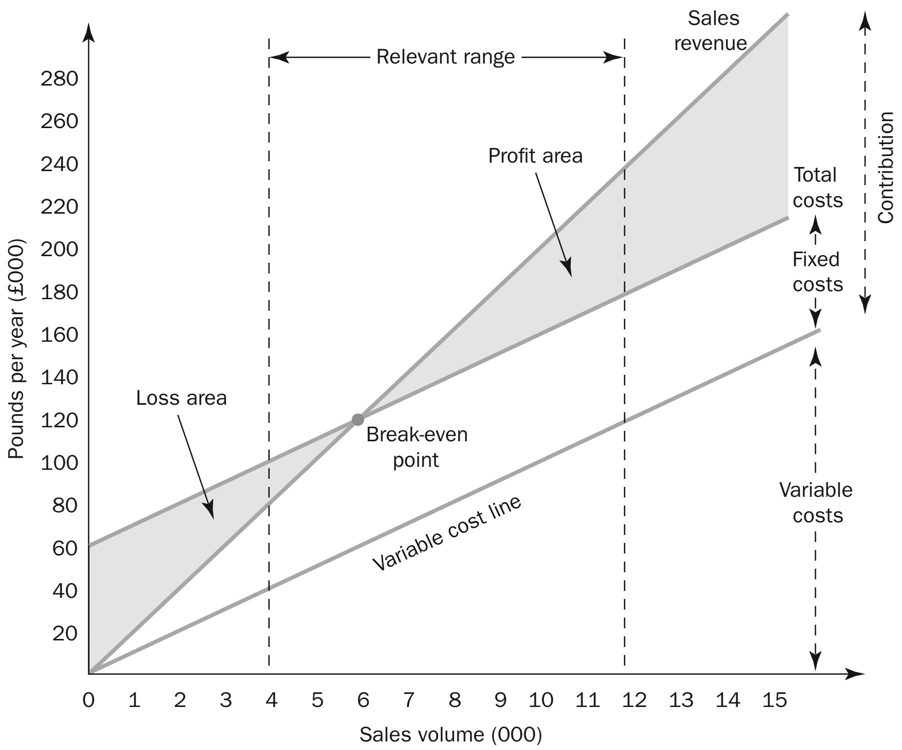
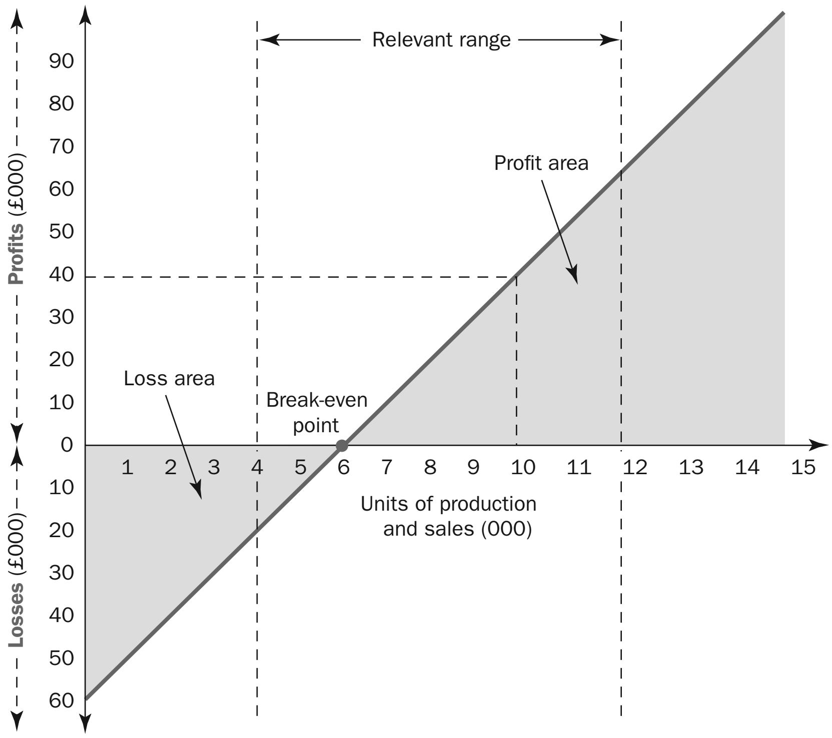


FIGURE 8.6
Profit–volume graph for Example 8.1



the difference between the total cost and total revenue lines. The **profit–volume graph** is a more convenient method of showing the impact of changes in volume on profit. Such a graph is illustrated in Figure 8.6. The horizontal axis represents the various levels of sales volume, and the profits and losses for the period are recorded on the vertical scale. You will see from Figure 8.6 that profits or losses are plotted for each of the various sales levels and these points are connected by a profit line. Two points are required to plot the profit line. When units sold are zero, a loss equal to the amount of fixed costs (£60 000) will be reported. At the break-even point (zero profits) sales volume is 6000 units. This is plotted at the point where the profit line intersects the horizontal line at a sales volume of 6000 tickets. The profit line is drawn between the two points. With each unit sold, a contribution of £10 is obtained towards the fixed costs, and the break-even point is at 6000 tickets, when the total contribution exactly equals the total of the fixed costs. With each additional unit sold beyond 6000 tickets, a surplus of £10 per ticket is obtained. If 10 000 tickets are sold, the profit will be £40 000 (4000 tickets at £10 contribution). You can see this relationship between sales and profit at 10 000 tickets from the dotted lines in Figure 8.6.

MULTI-PRODUCT COST–VOLUME–PROFIT ANALYSIS

Our analysis so far has assumed a single product setting. However, most firms produce and sell many products or services. In this section, we shall consider how we can adapt CVP analysis to a multi-product setting. Consider the situation described in Example 8.2. You will see that there are two types of fixed costs. Direct avoidable fixed costs can be specifically identified with each product and would not be incurred if the product was not made. For example, the deluxe and standard machines might be produced in different departments and the departmental supervisors fixed salaries would represent fixed costs directly attributable to each machine. The common fixed costs relate to the costs of common facilities (e.g. factory rent) that cannot be specifically identified with either of the products since they can only be avoided if *both* products are not sold.

EXAMPLE 8.2

The Super Bright Company sells two types of washing machine – a deluxe model and a standard model. The financial controller has prepared the following information based on the sales forecast for the period:

Sales volume (units)	*Deluxe machine 1200 (£)*	*Standard machine 600 (£)*	*Total (£)*
Unit selling price	300	200	
Unit variable cost	150	110	
Unit contribution	150	90	
Total sales revenues	360000	120000	480000
Less: Total variable cost	180000	66000	246000
Contribution to direct and common fixed costs[a]	180000	54000	234000
Less: Direct avoidable fixed costs	90000	27000	117000
Contribution to common fixed costs[a]	90000	27000	117000
Less common (indirect) fixed costs			39000
Operating profit			78000

The common fixed costs relate to the costs of common facilities and can only be avoided if neither of the products is sold. The managing director is concerned that sales may be less than forecast and has requested information relating to the break-even point for the activities for the period.

Note

[a]Contribution was defined earlier in this chapter as sales less variable costs. Where fixed costs are divided into direct and common (indirect) fixed costs it is possible to identify two separate contribution categories. The first is described as contribution to direct and common fixed costs and this is identical to the conventional definition, being equivalent to sales less variable costs. The second is after a further deduction of direct fixed costs and is described as 'contribution to common or indirect fixed costs'.

You might think that the break-even point for the firm as a whole can be derived if we allocate the common fixed costs to each individual product. However, this approach is inappropriate because the allocation will be arbitrary. The common fixed costs cannot be specifically identified with either of the products and can only be avoided if *both* products are not sold. The solution to our problem is to convert the sales volume measure of the individual products into standard batches of products based on the planned sales mix. You will see from Example 8.2 that Super Bright plans to sell 1200 deluxe and 600 standard machines giving a sales mix of 1200:600. Reducing this sales mix to the smallest whole number gives a mix of 2:1. In other words, for the sale of every two deluxe machines one standard machine is expected to be sold. We therefore define our standard batch of products as comprising two deluxe and one standard machine giving a contribution of £390 per batch (two deluxe machines at a contribution of £150 per unit sold plus one standard machine at a contribution of £90).

The break-even point in standard batches can be calculated by using the same break-even equation that we used for a single product, so that:

$$\text{Break-even number of batches} = \frac{\text{Total fixed costs (£156 000)}}{\text{Contribution margin per batch (£390)}}$$
$$= 400 \text{ batches}$$

The sales mix used to define a standard batch (2:1) can now be used to convert the break-even point (measured in standard batches) into a break-even point expressed in terms of the required combination of individual products sold. Thus, 800 deluxe machines (2 × 400) and 400 (1 × 400) standard machines must be sold to break even. The following profit statement verifies this outcome:

Units sold	*Deluxe machine 800 (£)*	*Standard machine 400 (£)*	*Total (£)*
Unit contribution margin	150	90	
Contribution to direct and common fixed costs	120 000	36 000	156 000
Less: Direct fixed costs	90 000	27 000	117 000
Contribution to common fixed costs	30 000	9 000	39 000
Less: Common fixed costs			39 000
Operating profit			0

Let us now assume that the actual sales volume for the period was 1200 units, the same total volume as the break-even volume, but consisting of a sales mix of 600 units of each machine. Thus, the actual sales mix is 1:1 compared with a planned sales mix of 2:1. The total contribution to direct and common fixed costs will be £144 000 ([£150 × 600 for deluxe] + [£90 × 600 for standard]) and a loss of £12 000 (£144 000 contribution − £156 000 total fixed costs) will occur. It should now be apparent to you that *the break-even point (or the sales volumes required to achieve a target profit) is not a unique number: it varies depending on the composition of the sales mix*. Because the actual sales mix differs from the planned sales mix, the sales mix used to define a standard batch has changed from 2:1 to 1:1 and the contribution per batch changes from £390 to £240 ([1 × £150] + [1 × £90]). This means that the revised break-even point will be 650 batches (£156 000 total fixed costs/£240 contribution per batch), which converts to a sales volume of 650 units of each machine based on a 1:1 sales mix. Generally, an increase in the proportion of sales of higher contribution margin products will decrease the break-even point whereas increases in sales of the lower margin products will increase the break-even point.

OPERATING LEVERAGE

Companies can sometimes influence the proportion of fixed and variable expenses in their cost structures. For example, they may choose to rely heavily either on automated facilities (involving high fixed and low variable costs) or on manual systems (involving high variable costs and low fixed costs). The chosen cost structure can have a significant impact on profits. Consider the situation presented in Exhibit 8.1 where the managers of an airline company are considering an investment in automated ticketing equipment.

You will see from Exhibit 8.1 that it is unclear which system should be chosen. If periodic sales exceed £960 000, the automated system will result in higher profits. Automation enables the company to lower its variable costs by increasing fixed costs. This cost structure results in greater increases in profits as sales increase compared with the manual system. Unfortunately, it is also true that a high fixed cost and lower variable cost structure will result in a greater reduction in profits as sales decrease. The term **operating leverage** is used as a measure of the sensitivity of profits to changes in sales. The greater the degree of operating leverage, the more that changes in sales activity will affect profits. The **degree of operating leverage** can be measured for a given level of sales by the following formula:

$$\text{Degree of operating leverage} = \text{Contribution margin}/\text{Profit}$$

REAL WORLD VIEWS 8.4

Operating leverage captures relationships

Operating leverage can tell investors a lot about a company's risk profile, and although high operating leverage can often benefit companies, firms with high operating leverage are also vulnerable to sharp economic and business cycle swings. In good times, high operating leverage can supercharge profit. But companies with a lot of costs tied up in machinery, plants, real estate and distribution networks cannot easily cut expenses to adjust to a change in demand. So, if there is a downturn in the economy, earnings do not just fall, they can plummet.

Consider the software developer Inktomi. During the 1990s investors marvelled at the nature of its software business. The company spent tens of millions of dollars to develop each of its digital delivery and storage software programs. But thanks to the internet, Inktomi's software could be distributed to customers at almost no cost. In other words, the company had close to zero cost of goods sold. After its fixed development costs were recovered, each additional sale was almost pure profit.

After the collapse of dotcom technology market demand in 2000, Inktomi suffered the dark side of operating leverage. As sales took a nosedive, profits swung dramatically to a staggering $58 million loss in a single quarter – plunging down from the $1 million profit the company had enjoyed in Q1 of the previous year. The high leverage involved in counting on sales to repay fixed costs can put companies and their shareholders at risk. High operating leverage during a downturn (such as the recession following the 2008 financial crisis) can be an Achilles heel, putting pressure on profit margins and making a contraction in earnings unavoidable.

Indeed, companies such as Inktomi with high operating leverage typically have larger volatility in their operating earnings and share prices. As a result, investors need to treat these companies with caution.

Question

1 Provide examples of other companies that have high and low degrees of operating leverage.

Reference

www.investopedia.com/articles/stocks/06/opleverage.asp

The degree of operating leverage in Exhibit 8.1 for sales of £1 million is 7 (£700 000/£100 000) for the automated system and 2.5 (£200 000/£80 000) for the manual system. This means that profits change by seven times more than the change in sales for the automated system and 2.5 times for the manual system. Thus, for a 10 per cent increase in sales from £1 million to £1.1 million, profits increase by 70 per cent for the automated system (from £100 000 to £170 000) and by 25 per cent for the manual system (from £80 000 to £100 000). In contrast, you will see in Exhibit 8.1 that if sales decline by 10 per cent from £1 million to £0.9 million, profits decrease by 70 per cent (from £100 000 to £30 000) for the automated system and by 25 per cent from (£80 000 to £60 000) for the manual system.

The degree of operating leverage provides useful information for the airline company in choosing between the two systems. Higher degrees of operating leverage can provide significantly greater profits when sales are increasing but higher percentage decreases will also occur when sales are declining. Higher operating leverage also results in a greater volatility in profits. The manual system has a break-even point of £600 000 sales (£120 000 fixed expenses/PV ratio of 0.2) whereas the break-even point for the automated system is £857 143 (£600 000 fixed expenses/PV ratio of 0.7). Thus, the automated system has a lower margin of safety. High operating leverage leads to higher risk arising from the greater volatility of profits and higher break-even point. Contrariwise, the increase in risk provides the potential for higher profit levels (as long as sales exceed £960 000). We can conclude that if management is confident that sales will exceed £960 000, the automated system is preferable.

It is apparent from the above discussion that labour intensive organizations, such as McDonald's and Pizza Hut have high variable costs and low fixed costs, and thus have low operating leverage. These companies can continue to report profits even when they experience wide fluctuations in sales levels.

EXHIBIT 8.1 Sensitivity of profits arising from changes in sales for an automated and manual system

An airline company is considering investing in automated ticketing equipment. The estimated sales revenues and costs for the current manual system and the proposed automated system for a typical period are as follows:

	Automated system (£)		*Manual system (£)*	
Sales revenue	1 000 000		1 000 000	
Less: Variable expenses	300 000		800 000	
Contribution	700 000	(70%)	200 000	(20%)
Less: Fixed expenses	600 000		120 000	
Profit	100 000		80 000	

The above cost structure suggests that the automated system yields the higher profits. However, if sales decline by 10 per cent the following calculations show that the manual system will result in the higher profits:

	Automated system (£)		*Manual system (£)*	
Sales revenue	900 000		900 000	
Less: Variable expenses	270 000		720 000	
Contribution	630 000	(70%)	180 000	(20%)
Less: Fixed expenses	600 000		120 000	
Profit	30 000		60 000	

What will happen if sales are 10 per cent higher than the predicted sales for the period?

	Automated system (£)		*Manual system (£)*	
Sales revenue	1 100 000		1 100 000	
Less: Variable expenses	330 000		880 000	
Contribution	770 000	(70%)	220 000	(20%)
Less: Fixed expenses	600 000		120 000	
Profit	170 000		100 000	

The sales revenue where both systems result in the same profits is £960 000. The automated system yields higher profits when periodic sales revenue exceeds £960 000 whereas the manual system gives higher profits when sales revenue is below £960 000.[a]

	Automated system (£)		*Manual system (£)*	
Sales revenue	960 000		960 000	
Less: Variable expenses	288 000		768 000	
Contribution	672 000	(70%)	192 000	(20%)
Less: Fixed expenses	600 000		120 000	
Profit	72 000		72 000	

Note

[a]The profit–volume ratio is 0.7 for the automated system and 0.2 for the manual system. Let x = periodic sales revenue: the indifference point is where $0.7x - £600\,000 = 0.2x - £120\,000$, so $x = £960\,000$.

Conversely, organizations that are highly capital intensive, such as easyJet and Volkswagen, have high operating leverage. These companies must generate high sales volumes to cover fixed costs, but sales above the break-even point produce high profits. In general, these companies tend to be more vulnerable to sharp economic and business cycle swings.

COST–VOLUME–PROFIT ANALYSIS ASSUMPTIONS

It is essential that anyone preparing or interpreting CVP information is aware of the underlying assumptions on which the information has been prepared. If these assumptions are not recognized, or the analysis is modified, errors may result and incorrect conclusions may be drawn from the analysis. We shall now consider these important assumptions. They are as follows:

1. All other variables remain constant.
2. A single product or constant sales mix.
3. Total costs and total revenue are linear functions of output.
4. Profits are calculated on a variable costing basis.
5. Costs can be accurately divided into their fixed and variable elements.
6. The analysis applies only to the relevant range.
7. The analysis applies only to a short-term time horizon.

1 All other variables remain constant

It has been assumed that all variables other than the particular one under consideration have remained constant throughout the analysis. In other words, it is assumed that volume is the only factor that will cause costs and revenues to change. However, changes in other variables such as production efficiency, sales mix and price levels can have an important influence on sales revenue and costs. If significant changes in these other variables occur, the CVP analysis presentation will be incorrect and it will be necessary to revise the CVP calculations based on the projected changes to the other variables.

2 Single product or constant sales mix

CVP analysis assumes that either a single product is sold or, if a range of products is sold, that sales will be in accordance with a predetermined sales mix. When a predetermined sales mix is used, it can be depicted in the CVP analysis by measuring sales volume using standard batch sizes based on a planned sales mix. As we have discussed, any CVP analysis must be interpreted carefully if the initial product mix assumptions do not hold.

3 Total costs and total revenue are linear functions of output

The analysis assumes that unit variable cost and selling price are constant. This assumption is only likely to be valid within the relevant range of production described earlier in this chapter.

4 Profits are calculated on a variable costing basis

The analysis assumes that the fixed costs incurred during the period are charged as an expense for that period. Therefore variable-costing profit calculations are assumed. If absorption-costing profit calculations are used, it is necessary to assume that production is equal to sales for the analysis to predict absorption costing profits. For the application of CVP analysis with an absorption costing system, you should refer to Learning Note 8.1 on the dedicated digital support resources (see Preface for details).

5 Costs can be accurately divided into their fixed and variable elements

CVP analysis assumes that costs can be accurately analysed into their fixed and variable elements. In practice, you will see in Chapter 24 that the separation of semi-variable costs into their fixed and variable elements is extremely difficult. Nevertheless, a reasonably accurate analysis is necessary if CVP analysis is to provide relevant information for decision-making.

6 Analysis applies only to the relevant range

Earlier in this chapter we noted that CVP analysis is appropriate only for decisions taken within the relevant production range and that it is incorrect to project cost and revenue figures beyond the relevant range.

7 Analysis applies only to a short-term time horizon

CVP analysis is based on the relationship between volume and sales revenue, costs and profit in the short run, typically a period of one year, in which the output of a firm is likely to be restricted to that available from the current operating capacity. During this period significant changes cannot be made to selling prices and fixed and variable costs. CVP analysis thus examines the effects of changes in sales volume on the level of profits in the short run. It is inappropriate to extend the analysis to long-term decision-making.

THE IMPACT OF INFORMATION TECHNOLOGY

The output from a CVP model is only as good as the input. The analysis will include assumptions about sales mix, production efficiency, price levels, total fixed costs, variable costs and selling price per unit. In practice, estimates regarding these variables will be subject to varying degrees of uncertainty.

Sensitivity analysis is one approach for coping with changes in the values of the variables. Sensitivity analysis focuses on how a result will be changed if the original estimates or the underlying assumptions change. With regard to CVP analysis, sensitivity analysis answers questions such as the following:

1. What will the profit be if the sales mix changes from that originally predicted?
2. What will the profit be if fixed costs increase by 10 per cent and variable costs decline by 5 per cent?

Today's information technology enables management accountants to build CVP computerized models and consider alternative plans by keying the information into a computer, which can quickly show changes both graphically and numerically. Thus, managers can study various combinations of change in selling prices, fixed costs, variable costs and product mix and can react quickly without waiting for formal reports from the management accountant.

SUMMARY

The following items relate to the learning objectives listed at the beginning of the chapter.

- **Justify the use of linear cost and revenue functions.**

Within the relevant range, it is generally assumed that cost and revenue functions are approximately linear. Outside the relevant range linearity is unlikely to apply. Care is therefore required in interpreting CVP relationships outside the relevant range.

- **Apply the numerical approach to answer questions similar to those listed in Example 8.1.**

In Example 8.1, the break-even point was derived by dividing fixed costs by the contribution per unit. To ascertain the number of units sold to achieve a target profit, the sum of the fixed costs and the target profit is divided by the contribution per unit.

- **Construct break-even, contribution and profit–volume graphs.**

Managers may obtain a clearer understanding of CVP behaviour if the information is presented in graphical format. With the break-even chart, the fixed costs are plotted as a single horizontal line. The total cost line is plotted by adding variable costs to fixed costs. The reverse situation applies with a contribution graph. The variable costs are plotted first and the fixed costs are added to variable costs to plot the total cost line. Because fixed costs are assumed to be a constant sum throughout the output range, the total cost line is drawn parallel to the variable cost line. The break-even and contribution graphs do not highlight the profit or loss at different output levels and must be ascertained by comparing the differences between the total cost and total revenue lines. The profit–volume graph shows the impact of changes in volume on profits. The profits and losses are plotted for each of the various sales levels and these are connected by a profit line. You should refer to Figures 8.4–8.6 for an illustration of the graphs.

- **Apply cost–volume–profit analysis in a multi-product setting.**

Multi-product CVP analysis requires that an assumption is made concerning the expected sales mix. The approach that is used is to convert the multi-product CVP analysis into a single product analysis based on the assumption that output consists of standard batches of the multiple products based on the expected sales mix. However, you should note that the answers change as the sales mix changes.

- **Explain the meaning of operating leverage and describe how it influences profits.**

Operating leverage measures the sensitivity of profits in relation to fluctuations in sales. It is measured by dividing total contribution by total profit. An operating leverage of four indicates that profits change by four times more than the change in sales. Therefore, if sales increase/decrease by 10 per cent, profits will increase/decrease by 40 per cent. High levels of operating leverage lead to higher risk arising from highly volatile profits but the increase in risk also provides the potential for higher profit levels when sales are expanding.

- **Identify and explain the assumptions on which cost–volume–profit analysis is based.**

Cost–volume–profit analysis is based on the following assumptions: (a) all variables, other than volume, remain constant; (b) the sales mix remains constant; (c) total costs and revenues are linear functions of output; (d) profits are calculated on a variable costing basis; (e) the analysis applies only to the relevant range; (f) the analysis applies only to a short-term horizon; and (g) costs can be accurately divided into their fixed and variable elements. The techniques that can be used to divide costs into their fixed and variable elements are explained in Chapter 24 in Part Six of this book which focuses on the application of quantitative methods to management accounting. Alternatively, if you require a knowledge of cost estimation techniques now you may prefer to read Chapter 24 immediately after you have completed this chapter.

KEY TERMS AND CONCEPTS

Break-even chart A chart that plots total costs and total revenues against sales volume and indicates the break-even point.

Break-even point The level of output at which costs are balanced by sales revenue and neither a profit nor a loss will occur.

Contribution graph A graph that plots variable costs and total costs against sales volume and fixed costs represent the difference between the total cost line and the variable cost line.

Contribution margin The margin calculated by deducting variable expenses from sales revenue.

Contribution margin ratio The proportion of sales available to cover fixed costs and provide for profit, calculated by dividing the contribution margin by the sales revenue, also known as profit–volume ratio.

Decreasing returns to scale A situation that arises when unit costs rise as volume increases.

Degree of operating leverage The contribution margin divided by the profit for a given level of sales.

Increasing returns to scale A situation that arises when unit costs fall as volume increases.

Margin of safety The amount by which sales may decrease before a loss occurs.

Operating leverage A measure of the sensitivity of profits to changes in sales.

Profit–volume graph A graph that plots profit/losses against volume.

Profit–volume ratio The proportion of sales available to cover fixed costs and provide for profit, calculated by dividing the contribution margin by the sales revenue, also known as contribution margin ratio.

Relevant range The output range at which an organization expects to be operating with a short-term planning horizon.

Sensitivity analysis Analysis that shows how a result will be changed if the original estimates or underlying assumption changes.

RECOMMENDED READING

For additional reading relating to CVP analysis, you should refer to an article that can be accessed from the ACCA Student Accountant www.accaglobal.com/gb/en/student/exam-support-resources/fundamentals-exams-study-resources/f2/technical-articles.html

KEY EXAMINATION POINTS

Students tend to experience little difficulty in preparing break-even charts, but many cannot construct profit–volume charts. Remember that the horizontal axis represents the level of activity, while profit/losses are shown on the vertical axis. The maximum loss is at zero activity, and is equal to fixed costs. For practice on preparing a profit–volume chart, you should attempt Review problem 8.16 and compare your answer with the solution. Students also experience difficulty with the following:

1. coping with multi-product situations;
2. calculating the break-even point when total sales and costs are given but no information is given on the unit costs;
3. explaining the assumptions of CVP analysis.

For multi-product situations you should base your answer on the average contribution per unit, using the approach shown in Example 8.2. Review problem 8.19 requires the computation of a break-even point in a multi-product setting. When unit costs are not given, the break-even point in sales value can be calculated as follows:

$$\text{Fixed costs} \times \frac{\text{total estimated sales}}{\text{total estimated contribution}}$$

or

$$\frac{\text{Fixed costs}}{\text{Profit/volume ratio}}$$

You should refer to the solutions to Review problem 8.17 for an illustration of the application of the above approach. Sometimes questions will give details of costs but not the split into the fixed and variable elements. You can separate the total costs into their fixed and variable elements using the high–low method described in Chapter 24. This approach is required for Review problem 8.17.