

MPF_FIFl: Firemní finance

Real options

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- Definition, type of options
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- Option valuation – the Black-Scholes formula

– Real options

- Option to expand
- Options in R&D
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- The abandonment option
- Flexible production and procurement

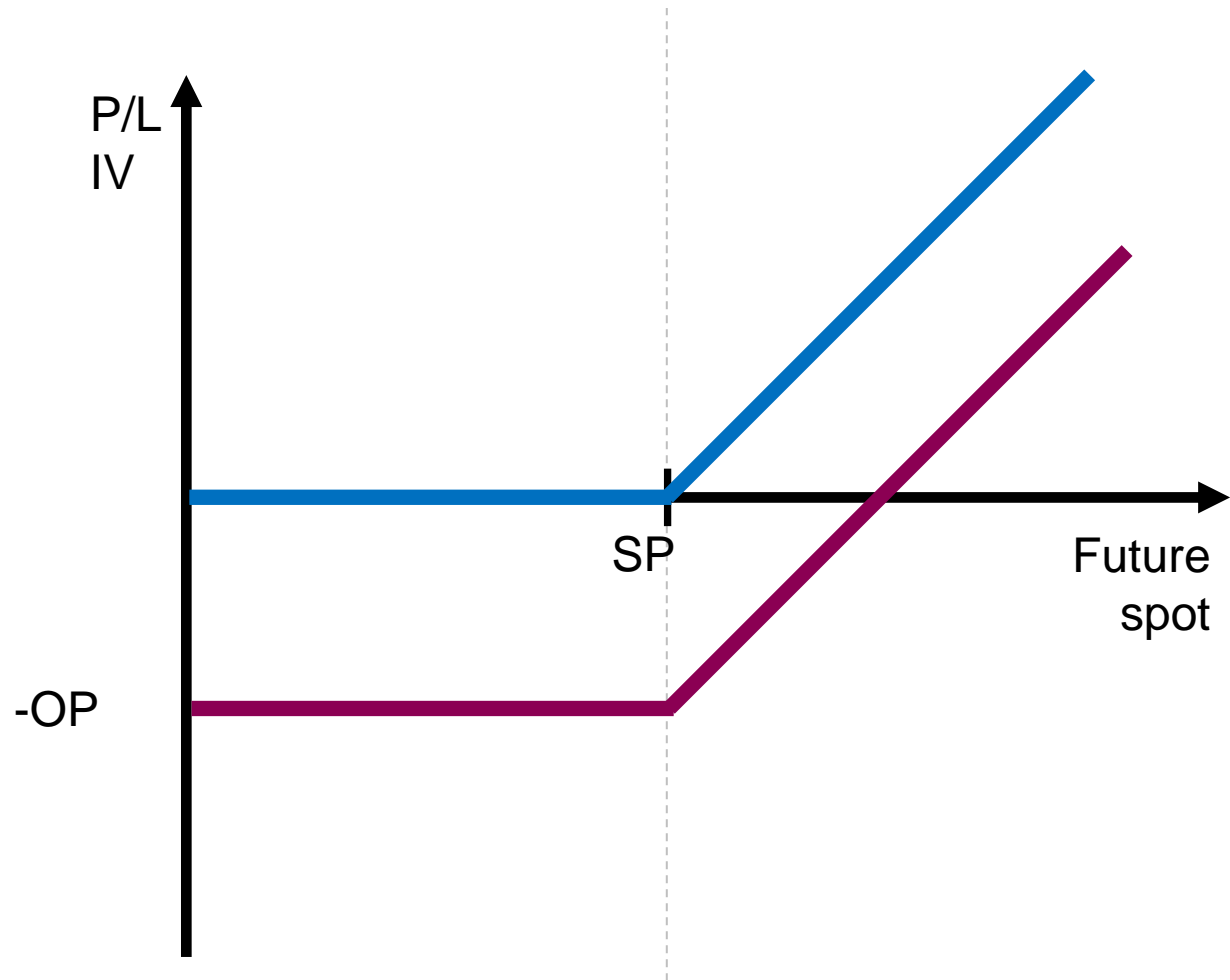
Options - terminology

- **An option is the right** to buy (sell) a certain underlying asset at a set time in the future.
- **The buyer** of the option, the owner, has the right, not the obligation, to enforce the contract.
- **The seller** of the option – the writer, has the obligation to fulfill upon request.
- **Strike price** – agreed in advance.
- **Option expiry** – the time when the option expires (day, month).
- **Option premium** – option price.

Types of options

- **Position** –long, short.
- **Nature of the right** – call, put.

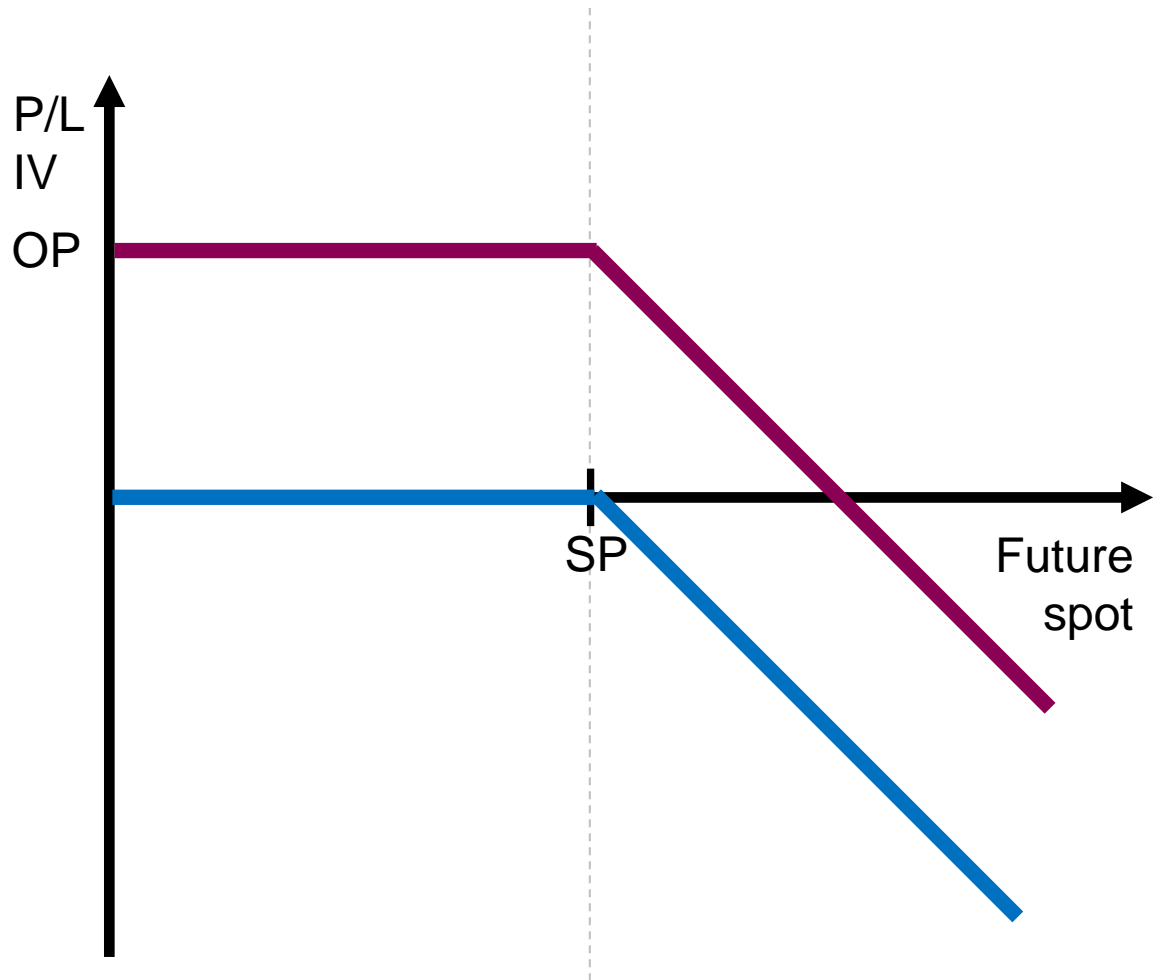
Long call



$$IV = \begin{cases} 0, & S \leq SP \\ S - SP, & S > SP \end{cases}$$

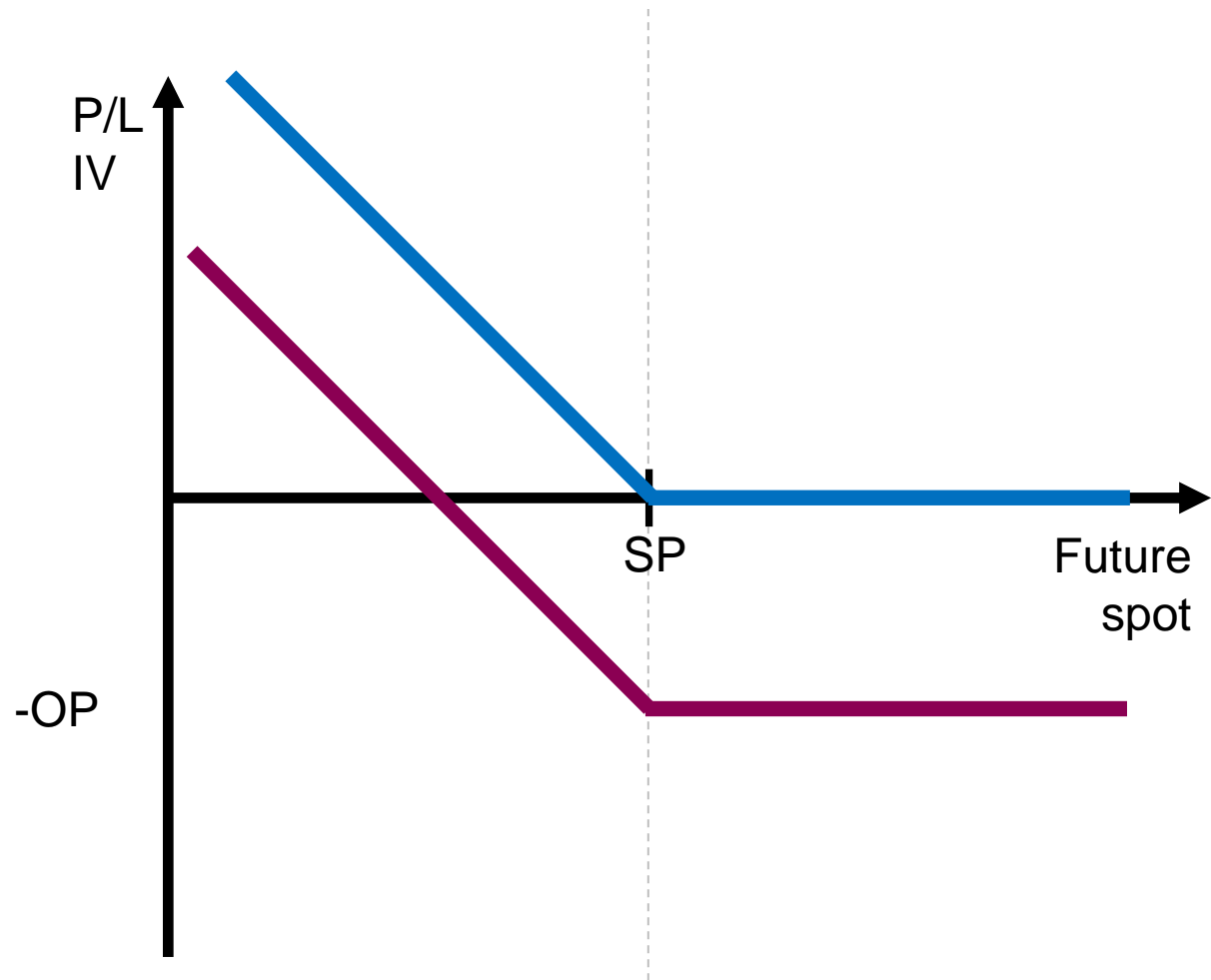
$$P = \begin{cases} -OP, & S \leq SP \\ S - SP - OP, & S > SP \end{cases}$$

Short call



$$P = \begin{cases} OP, & S \leq SP \\ SP + OP - S, & S > SP \end{cases}$$

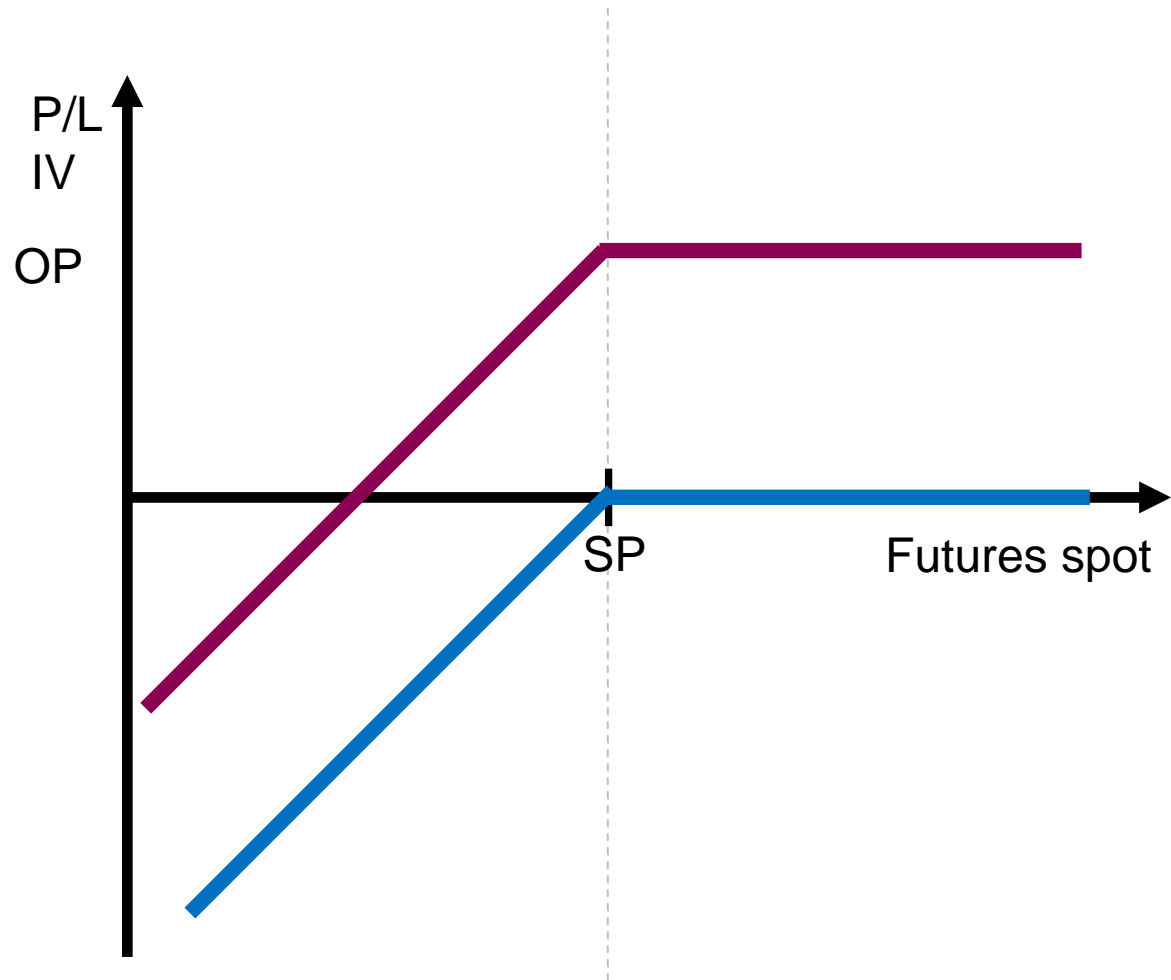
Long put



$$VH = \begin{cases} SP - S, & S \leq SP \\ 0, & S > SP \end{cases}$$

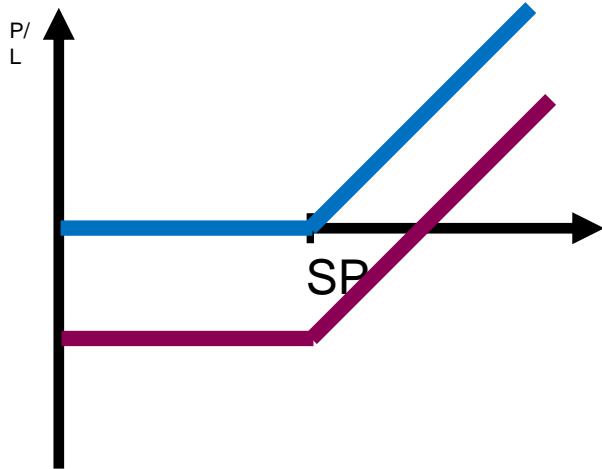
$$P = \begin{cases} SP - S - OP, & S \leq SP \\ -OP, & S > SP \end{cases}$$

Short put

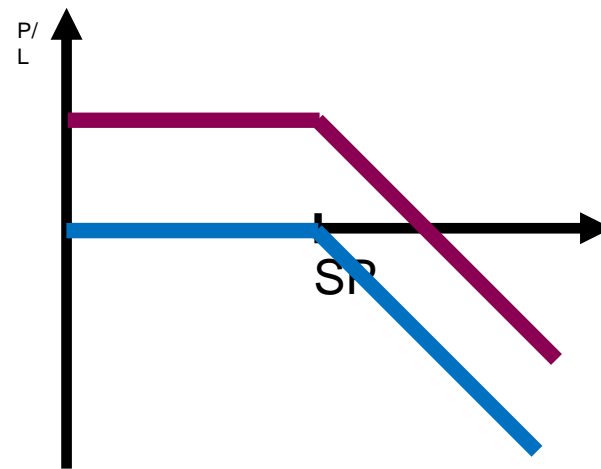


$$P = \begin{cases} OP + S - SP, & S \leq SP \\ OP, & S > SP \end{cases}$$

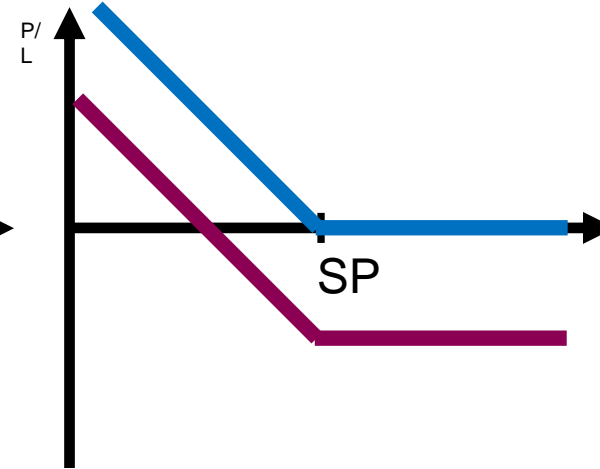
Options



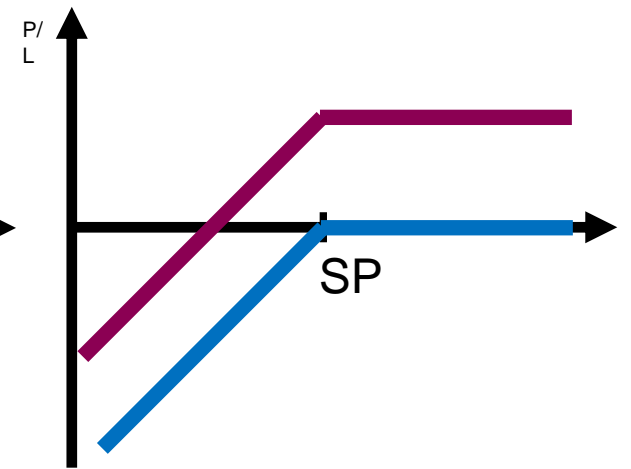
Long call



Short call

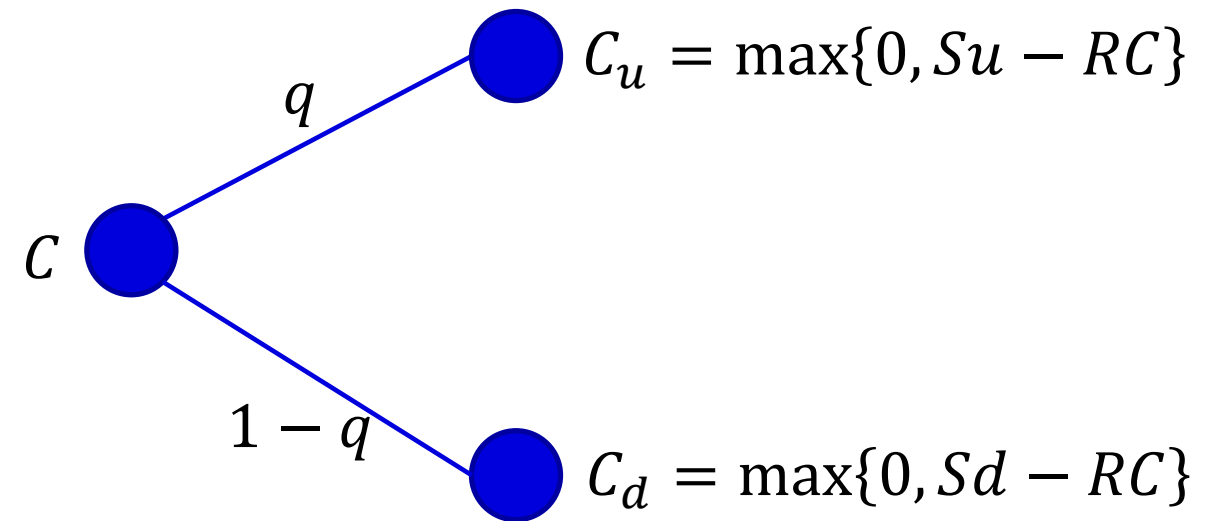
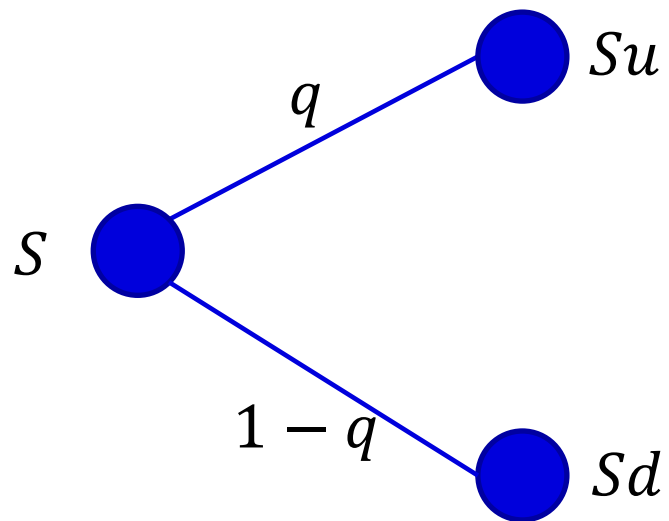


Long put

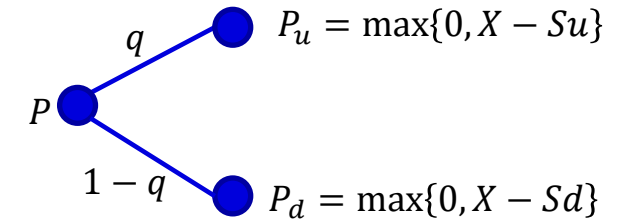


Short put

Binomial option pricing model



Binomial option pricing model



From replication portfolio: $C = hS + B$, $h = \frac{C_u - C_d}{Su - Sd}$, $B = \frac{uC_d - dC_u}{u - d} \frac{1}{1 + r}$

– h – **hedge ratio**, delta.

– The portfolio established a **synthetic call**.

– Replication portfolio buys shares using debt, as

$$uC_d - dC_u = \max\{0, uSd - uRC\} - \max\{0, dSu - dRC\} < 0$$

– For a put option

$$P = hS + B, \quad h = \frac{P_u - P_d}{Su - Sd}, \quad B = \frac{uP_d - dP_u}{u - d} \frac{1}{1 + r}$$

Risk neutral pricing

From replication portfolio

$$C = hS + B = \frac{C_u - C_d}{Su - Sd} S + \frac{uC_d - dC_u}{u - d} \frac{1}{1 + r}$$

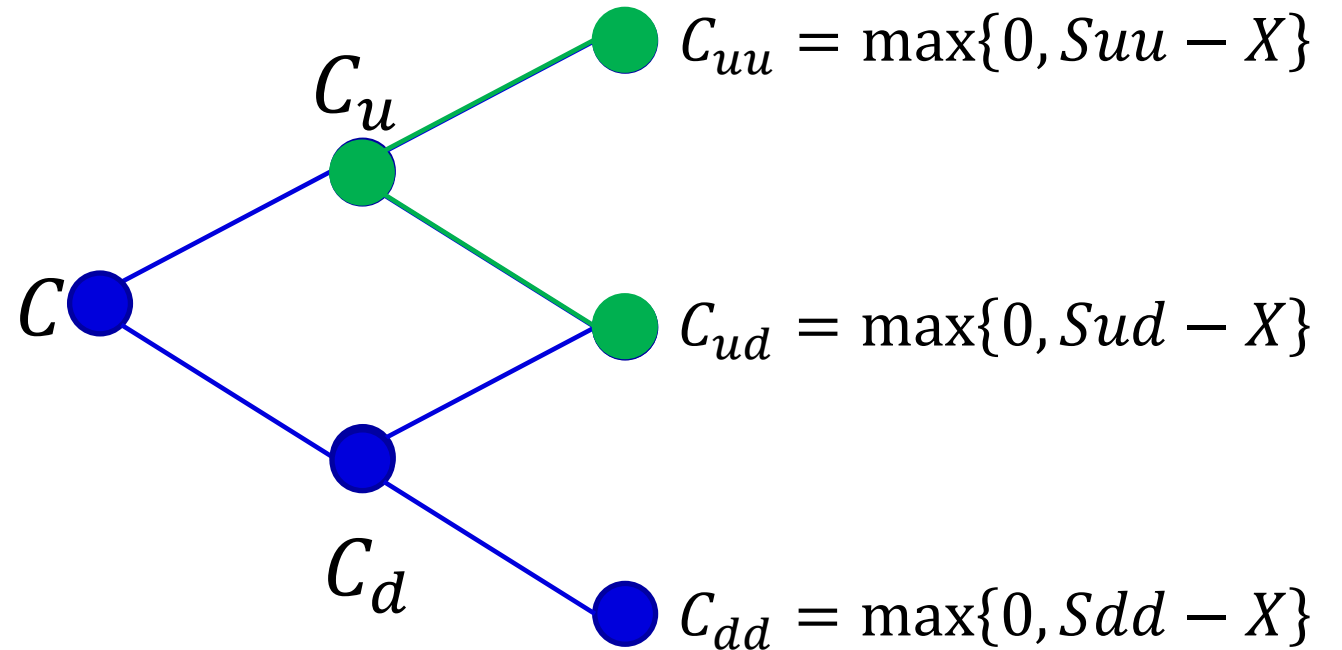
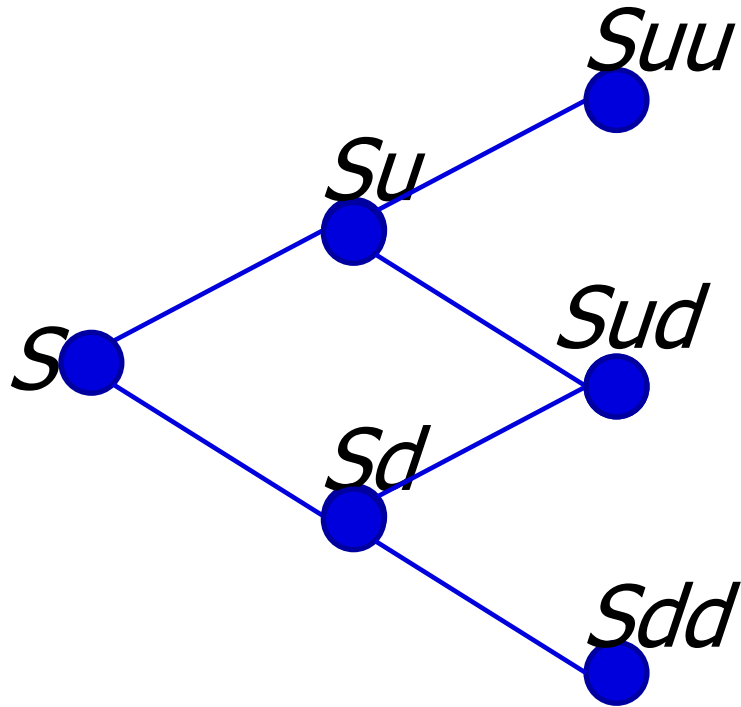
Note that the price does not depend on q , so we define the **risk neutral probability**

$$p = \frac{(1 + r) - d}{u - d}$$

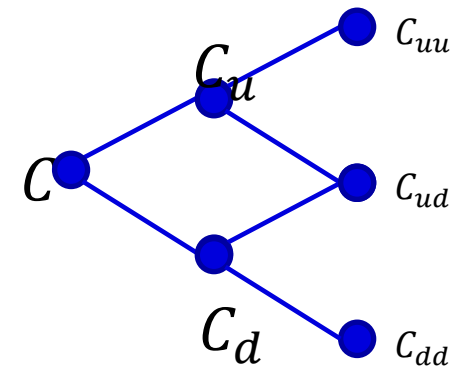
Consequently,

$$C = \frac{pC_u + (1 - p)C_d}{1 + r}$$

Multiperiod binomial pricing model



Multiperiod binomial model



$$C = \frac{\sum_{j=0}^n \left[\binom{n}{j} p^j (1-p)^{n-j} \max\{0, S u^j d^{n-j} - X\} \right]}{(1+r)^n}$$

$$P = \frac{\sum_{j=0}^n \left[\binom{n}{j} p^j (1-p)^{n-j} \max\{0, X - S u^j d^{n-j}\} \right]}{(1+r)^n}$$

Black-Scholes option pricing model

- Spot price:
- Time to expiry (years):
- Strike price:
- Volatility, standard deviation:
- Risk-free return

S

τ

X

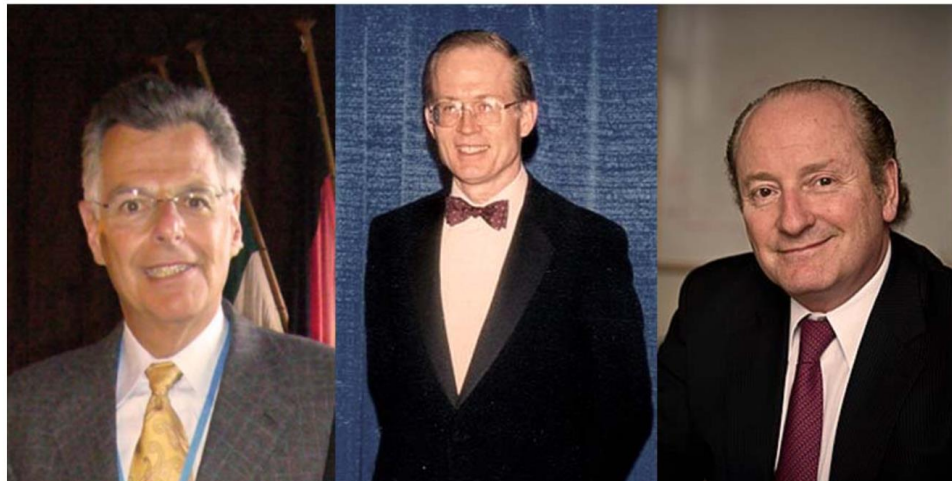
σ

r

Myron S. Scholes

Fischer Black

Robert C. Merton



Black-Scholes model

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}},$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

$$C = S N(d_1) - e^{-r\tau} X N(d_2)$$

$$P = e^{-r\tau} X(-d_2) - S N(-d_1)$$

Real options

- Option to expand, make follow-up investments
- Option to wait, invest later
- The abandonment option, or shrink the project
- Flexible production and procurement

Value of option = NPV with option – NPV without option

Option to expand, follow-up investments

	Year					
	1982	1983	1984	1985	1986	1987
After-tax operating cash flow (1)		+110	+159	+295	+185	0
Capital investment (2)	450	0	0	0	0	0
Increase in working capital (3)	0	50	100	100	-125	-125
Net cash flow (1) – (2) – (3)	-450	+60	+59	+195	+310	+125
NPV at 20% = -\$46.45, or about -\$46 million						

TABLE 23.1
Summary of cash flows and financial analysis of the Mark I microcomputer (\$ millions).

Option to expand, follow-up investments

TABLE 23.3
Cash flows
of the Mark II
microcomputer, as
forecasted from
1982 (\$ millions).

	Year							
	1982		1985	1986	1987	1988	1989	1990
After-tax operating cash flow				+220	+318	+590	+370	0
Increase in working capital				100	200	200	-250	-250
Net cash flow				+120	+118	+390	+620	+250
Present value at 20%	+467	←	+807					
Investment, PV at 10%	676	←	<u>900</u>					
			(PV in 1982)					
Forecasted NPV in 1985								-93

Option to expand, follow-up investments

Assumptions

1. The decision to invest in the Mark II must be made after three years, in 1985.
2. The Mark II investment is double the scale of the Mark I (note the expected rapid growth of the industry). Investment required is \$900 million (the exercise price), which is taken as fixed.
3. Forecasted cash inflows of the Mark II are also double those of the Mark I, with present value of \$807 million in 1985 and $807/(1.2)^3 = \$467$ million in 1982.
4. The future value of the Mark II cash flows is highly uncertain. This value evolves as a stock price does with a standard deviation of 35% per year. (Many high-technology stocks have standard deviations higher than 35%.)
5. The annual interest rate is 10%.

Interpretation

The opportunity to invest in the Mark II is a three-year call option on an asset worth \$467 million with a \$900 million exercise price.

$$C = S N(d_1) - e^{-r\tau} X N(d_2)$$

Option to expand, follow-up investments

Valuation

$$PV(\text{exercise price}) = \frac{900}{(1.1)^3} = 676$$

$$\text{Call value} = [N(d_1) \times S] - [N(d_2) \times PV(EX)]$$

$$\begin{aligned} d_1 &= \log [S/PV(EX)] / \sigma \sqrt{t} + \sigma \sqrt{t} / 2 \\ &= \log [0.691] / 0.606 + 0.606 / 2 = -0.3072 \end{aligned}$$

$$d_2 = d_1 - \sigma \sqrt{t} = -0.3072 - 0.606 = -0.9134$$

$$N(d_1) = 0.3793, N(d_2) = 0.1805$$

$$\text{Call value} = [0.3793 \times 467] - [0.1805 \times 676] = \$55.1 \text{ million}$$

$$C = S N(d_1) - e^{-r\tau} X N(d_2)$$

Option to expand, follow-up investments

Valuation

$$PV(\text{exercise price}) = \frac{900}{(1.1)^3} = 676$$

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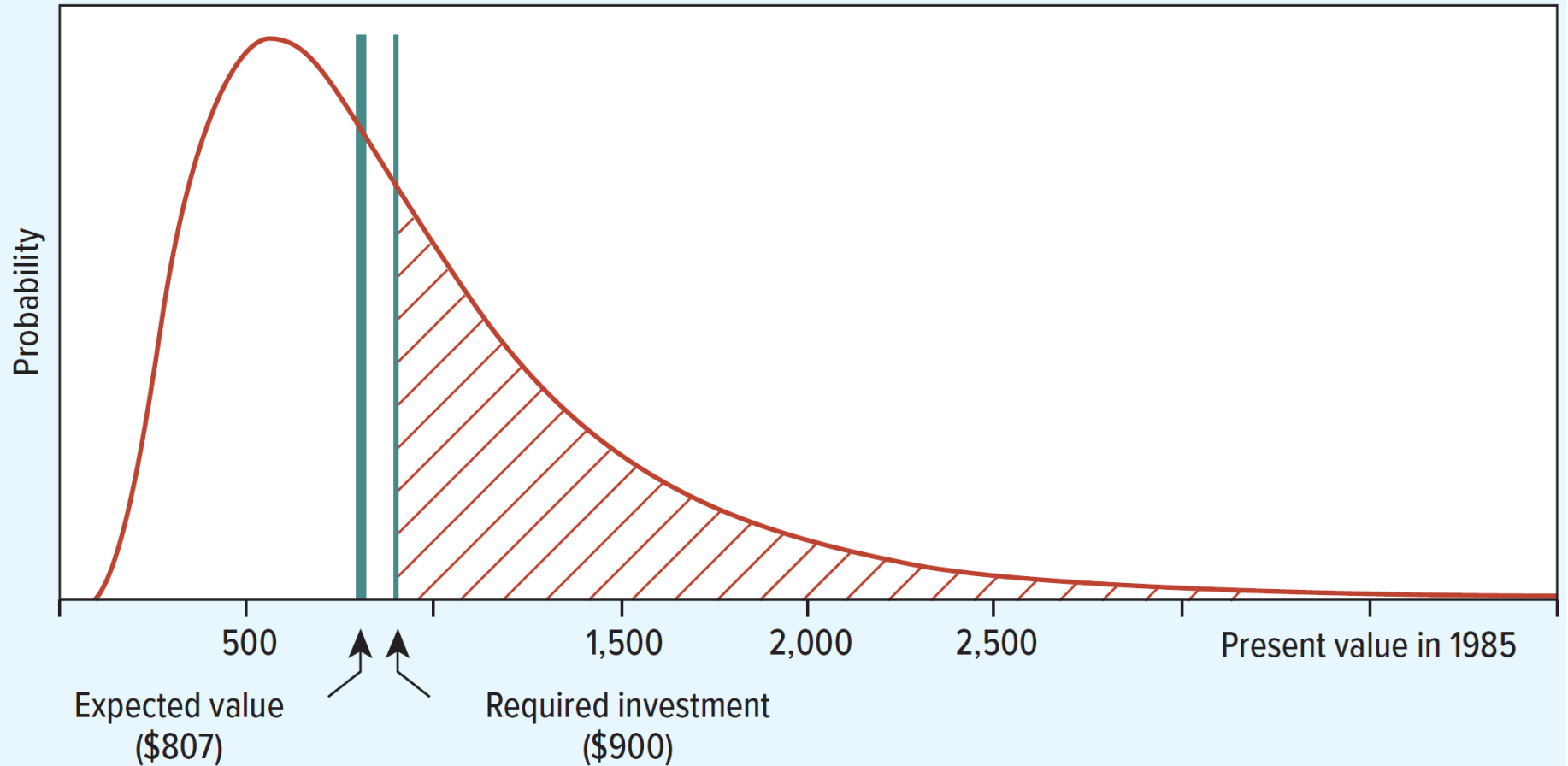
$$\text{Call value} = [0.3793 \times 467] - [0.1805 \times 676] = \$55.1 \text{ million}$$

NPV at 20% = $-\$46.45$, or about $-\$46$ million

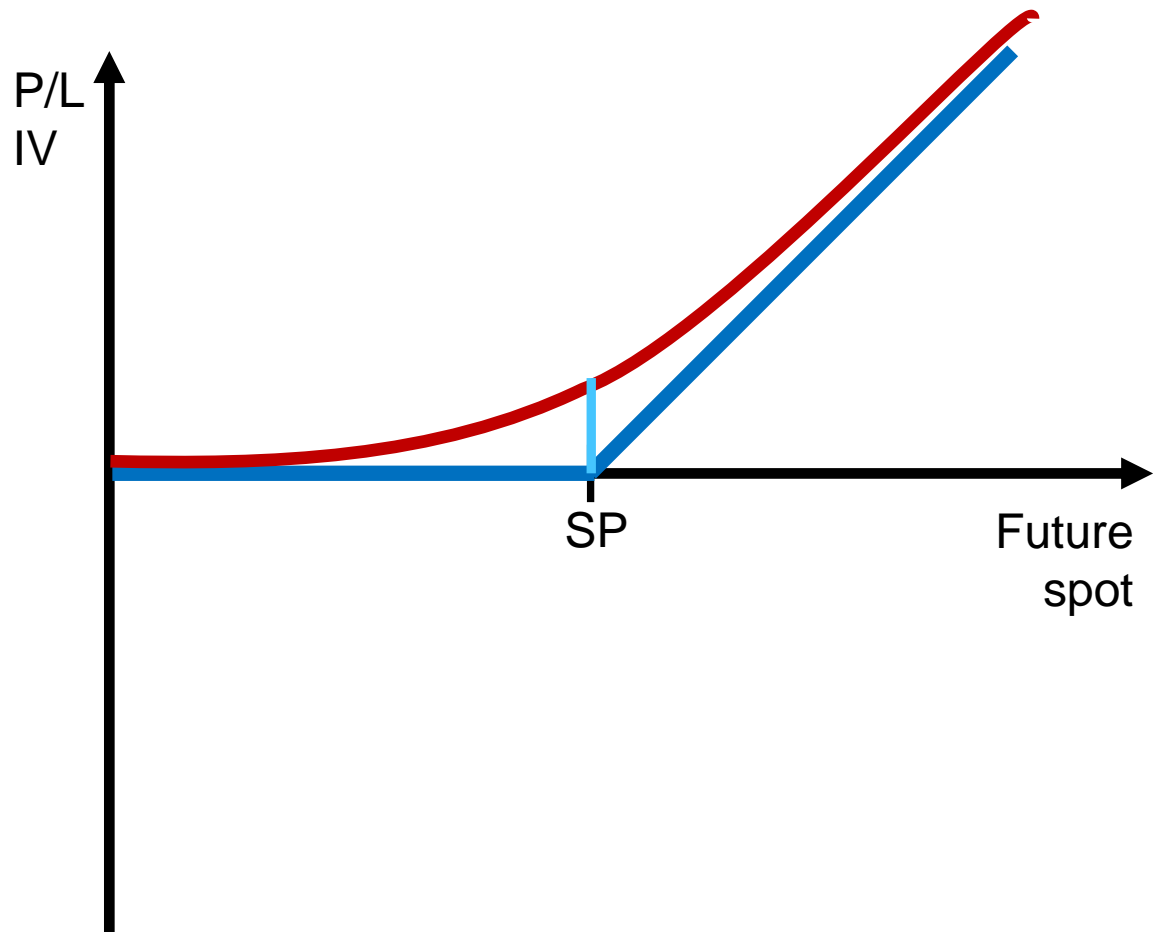
Option to expand, follow-up investments

FIGURE 23.1

This distribution shows the range of possible present values for the Mark II project in 1985. The expected value is about \$800 million, less than the required investment of \$900 million. The option to invest pays off in the shaded area above \$900 million.



Option to wait



- Period to maturity
- Current spot vs. SP
- Volatility of the underlying asset
- Risk-free interest rate
- Dividends

R&D options

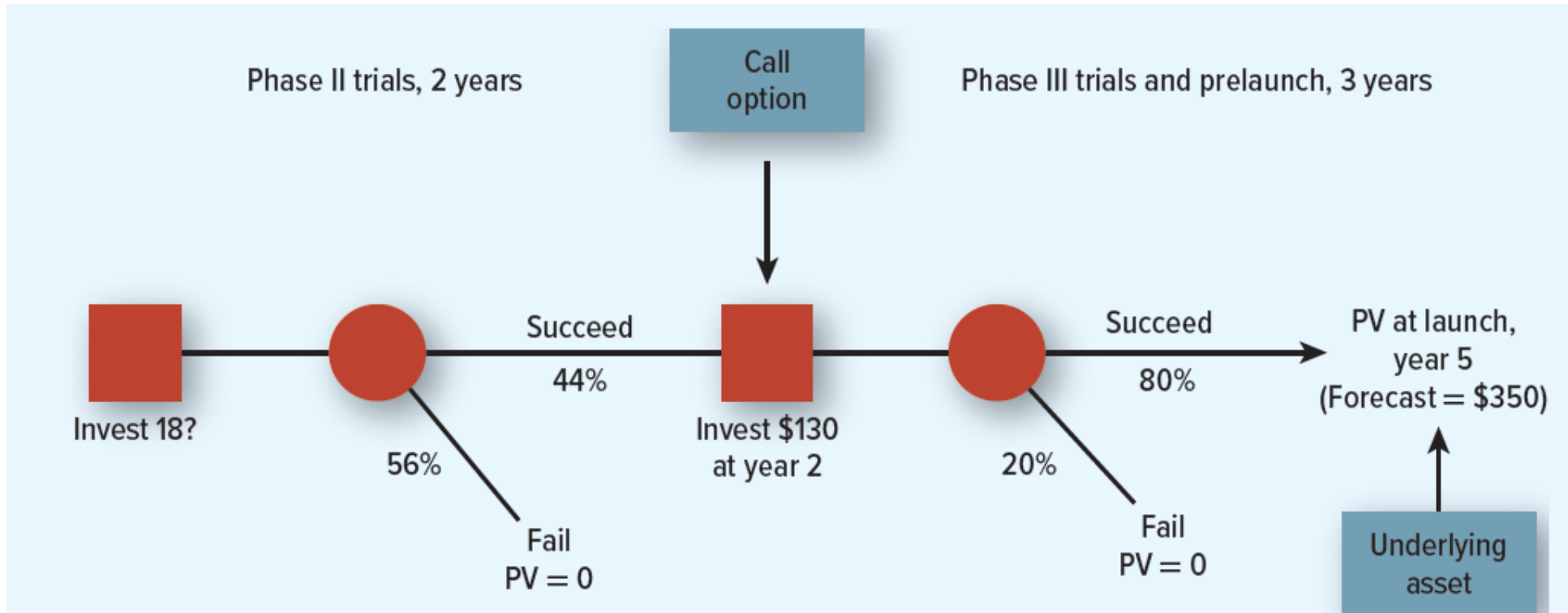


FIGURE 23.2

The decision tree from Figure 10.3 recast as a real option. If phase II trials are successful, the company has a real call option to invest \$130 million. If the option is exercised, the company gets an 80% chance of launching an approved drug. The PV of the drug, which is forecasted at \$350 million in year 5, is the underlying asset for the call option.

Option to wait

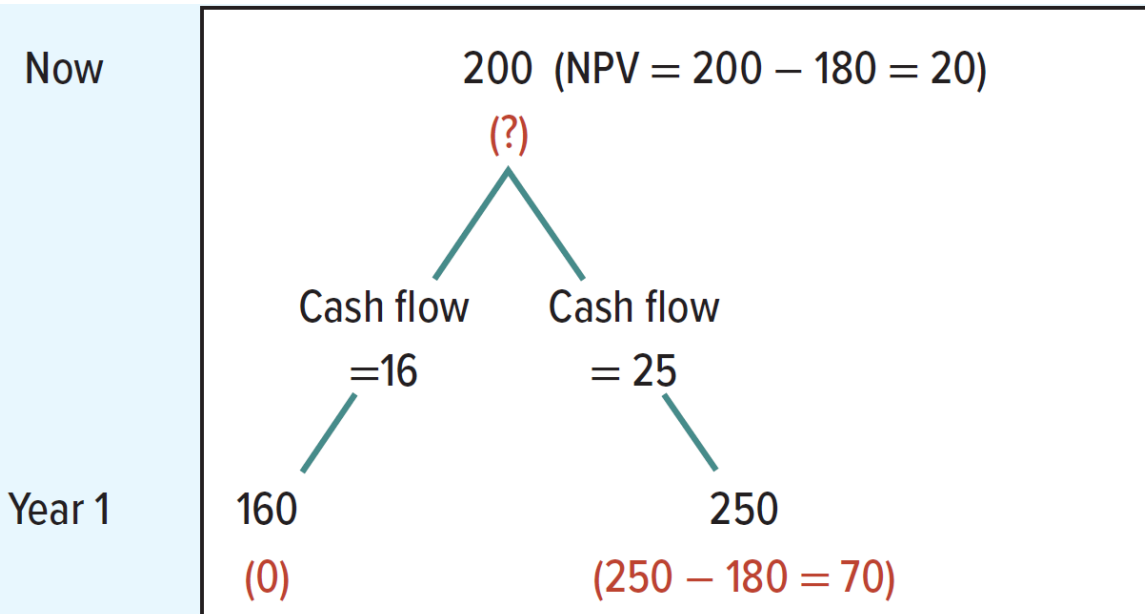
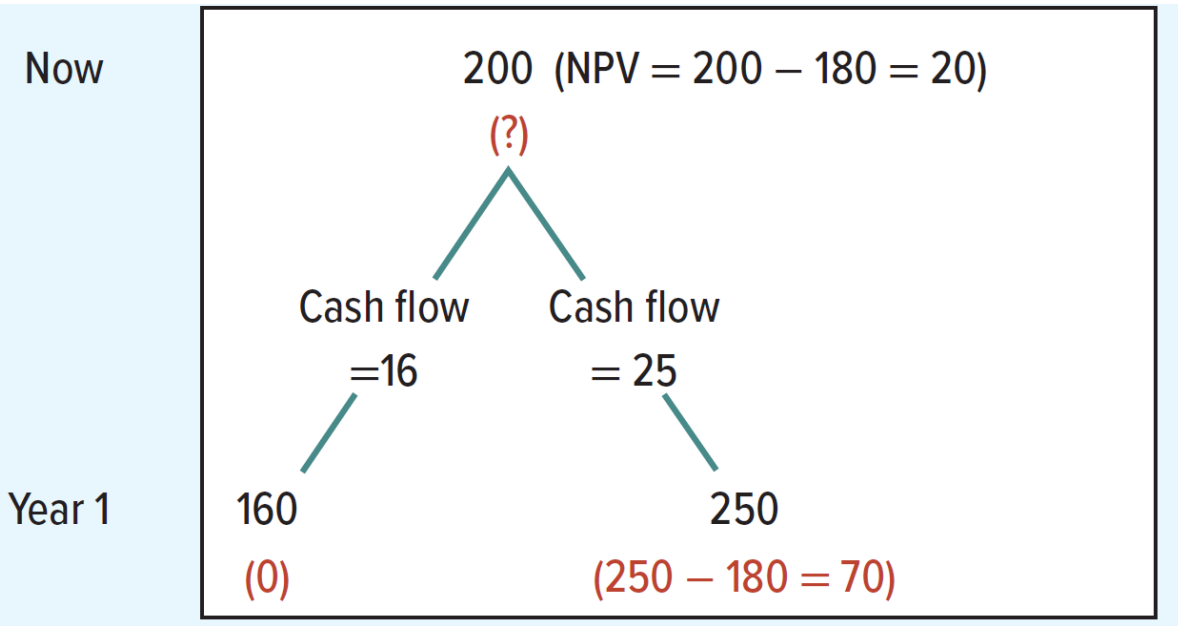


FIGURE 23.3 Possible cash flows and end-of-period values for the malted herring project are shown in black. The project costs \$180 million, either now or later. The red figures in parentheses show payoffs from the option to wait and to invest later if the project is positive NPV at year 1. Waiting means loss of the first year's cash flows. The problem is to figure out the current value of the option.

Option to wait



- High demand generates \$25 million, \$250 million value at end of year
- Low demand generates \$16 million with no value

High Demand

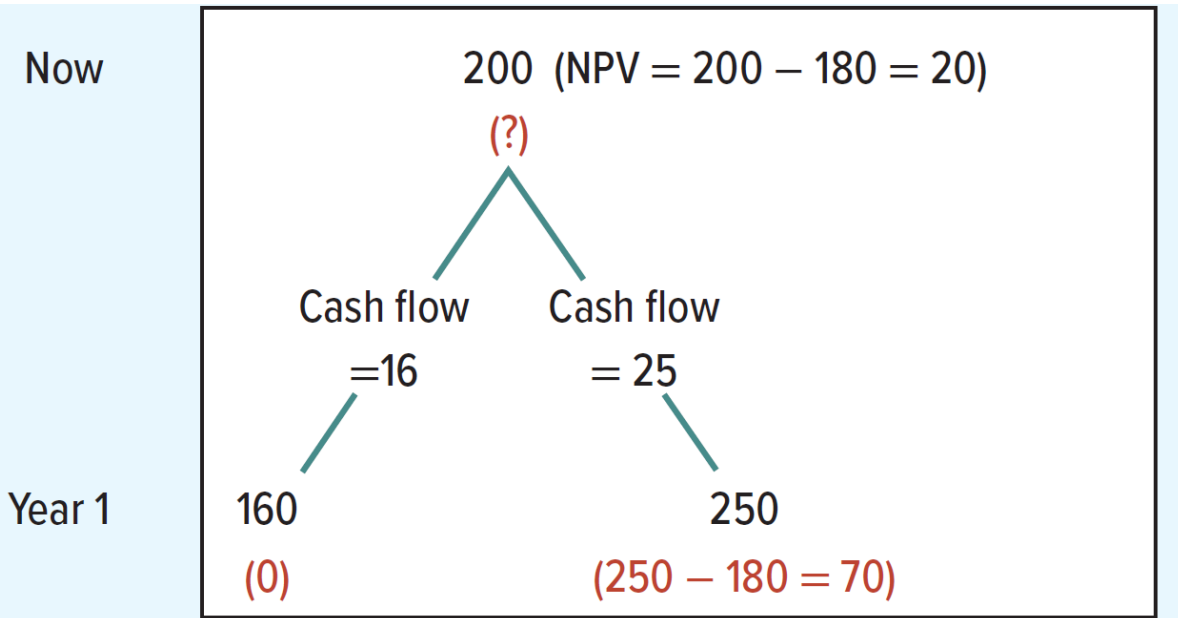
Low Demand

$$\text{Total return} = \frac{(25 + 250)}{200} - 1 = .375$$

$$\text{Total return} = \frac{(16 + 160)}{200} - 1 = -.12$$

Risk neutral return
= 5%

Option to wait



- Calculate probability of high demand for malted herring project

$$\text{Expected return} = (\text{prob of high demand}) \times .375 + (1 - \text{prob of high demand}) \times (-.12)$$

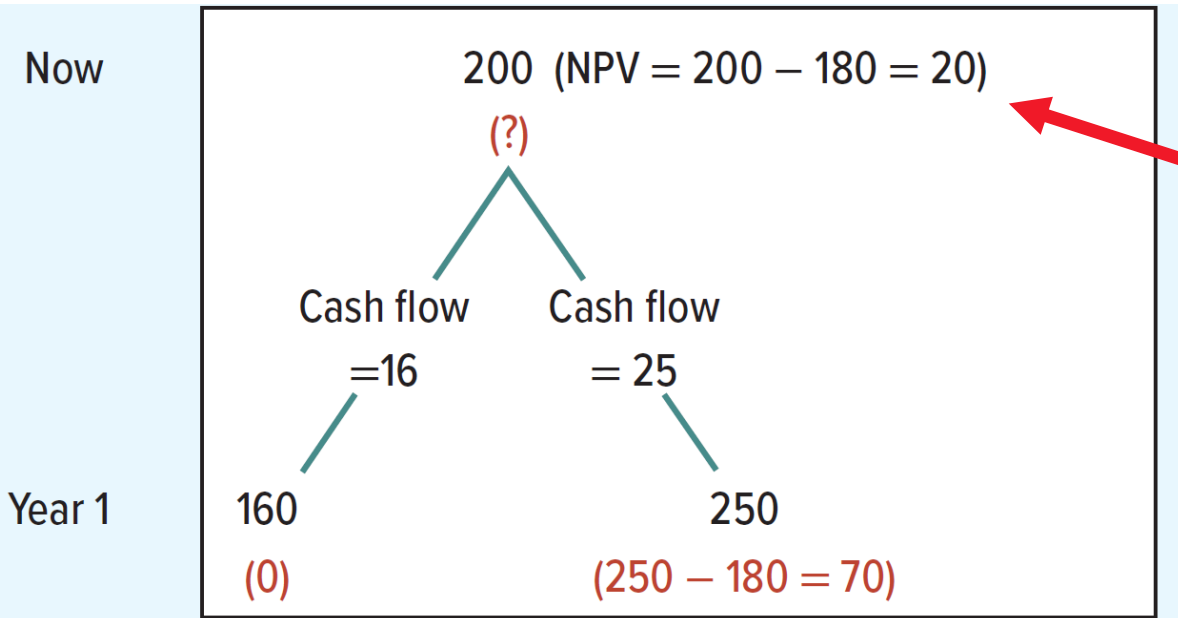
$$\text{Expected return} = .05$$

$$\text{Prob of high demand} = .343$$

- Option value

$$\frac{(.343 \times 70) + (.657 \times 0)}{1.05} = \$22.9 \text{ million}$$

Option to wait



- Calculate probability of high demand for malted herring project

$$\text{Expected return} = (\text{prob of high demand}) \times .375 + (1 - \text{prob of high demand}) \times (-.12)$$

$$\text{Expected return} = .05$$

$$\text{Prob of high demand} = .343$$

- Option value

$$\frac{(.343 \times 70) + (.657 \times 0)}{1.05} = \$22.9 \text{ million}$$

Optimal timing

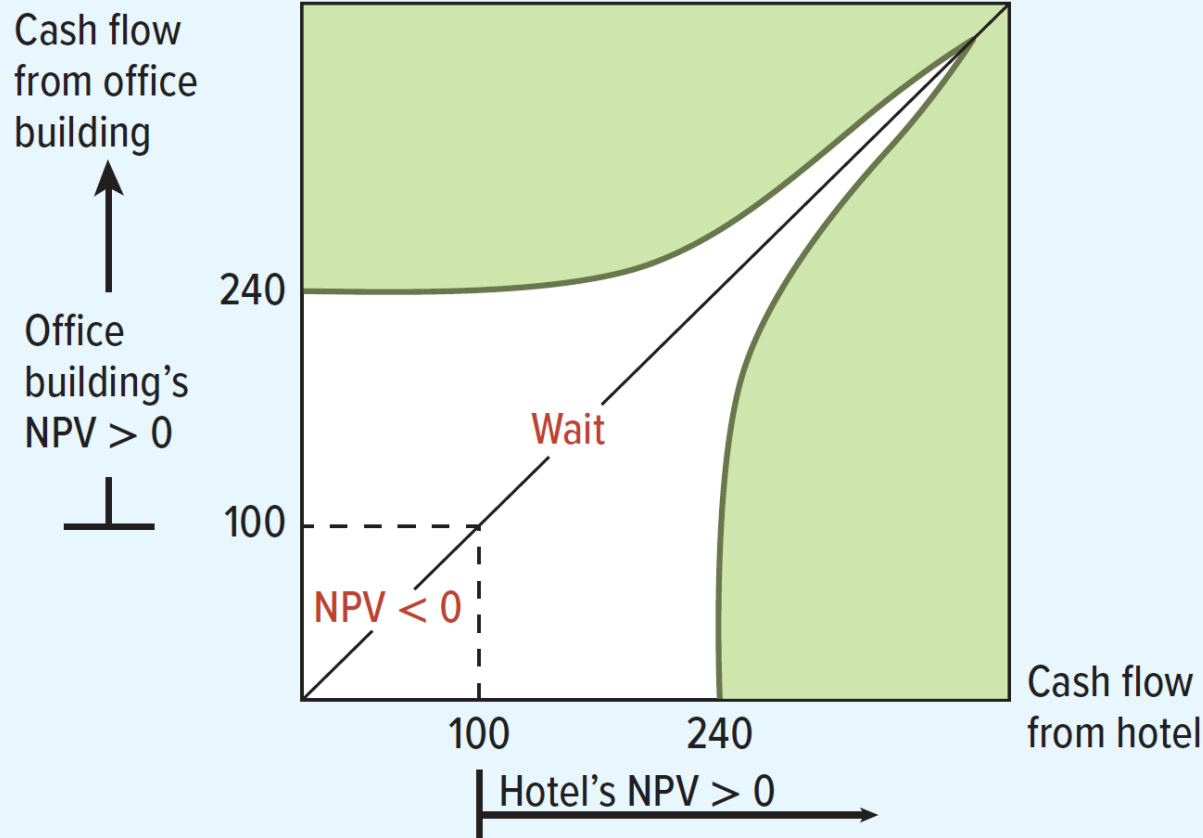


FIGURE 23.4 Development option for vacant land, assuming two mutually exclusive uses, either hotel or office building. The developer should “wait and see” unless the hotel’s or office building’s cash flows end up in one of the shaded areas.

Source: Adapted from Figure 1 in P. D. Childs, T. J. Riddiough, and A. J. Triantis, “Mixed Uses and the Redevelopment Option,” *Real Estate Economics* 24 (Fall 1996), pp. 317–339.

Purchase option – airplane example

- *Commit now.* It can commit now to buy the plane, in exchange for Airbus's offer of locked-in price and delivery date.
- *Acquire option.* It can seek a purchase option from Airbus, allowing the airline to decide later whether to buy. A purchase option fixes the price and delivery date if the option is exercised.
- *Wait and decide later.* Airbus will be happy to sell another A320 at any time in the future if the airline wants to buy one. However, the airline may have to pay a higher price and wait longer for delivery, especially if the demand for air travel is high and many planes from other airlines are on order.

Purchase option

	<u>Year 0</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5 or later</u>
Buy option	Airline and manufacturer set price and delivery date	Exercise? (Yes or no)	Aircraft delivered if option exercised	
Wait	Wait and decide later	Buy now? If yes, negotiate price and wait for delivery.		Aircraft delivered if purchased at year 3

FIGURE 23.7 This aircraft purchase option, if exercised at year 3, guarantees delivery at year 4 at a fixed price. Without the option, the airline can still order the plane at year 3, but the price is uncertain and the wait for delivery is longer.

Source: Adapted from Figure 17–17 in J. Stonier, “What Is an Aircraft Purchase Option Worth? Quantifying Asset Flexibility Created through Manufacturer Lead-Time Reductions and Product Commonality,” in *Handbook of Airline Finance*, G. F. Butler and M. R. Keller, eds.

Purchase option

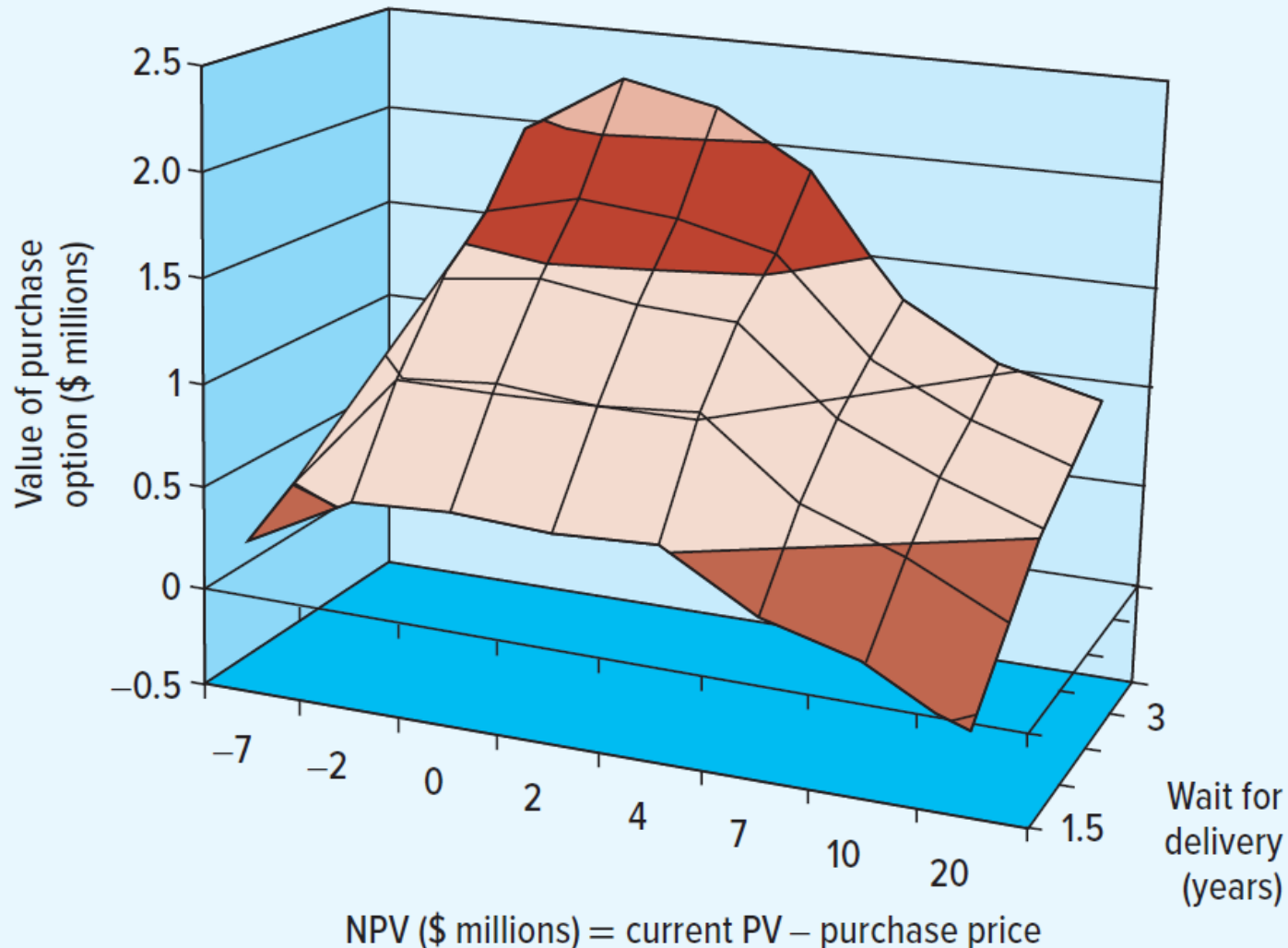


FIGURE 23.8

Value of aircraft purchase option—the extra value of the option versus waiting and possibly negotiating a purchase later. (See Figure 23.7.) The purchase option is worth most when NPV of purchase now is about zero and the forecasted wait for delivery is long.

Source: Adapted from Figure 17–20 in J. Stonier, “What Is an Aircraft Purchase Option Worth? Quantifying Asset Flexibility Created Through Manufacturer Lead-Time Reductions and Product Commonality,” in *Handbook of Aviation Finance*, G. F. Butler and M. R. Keller, eds.

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