Exchange and production

Varian: Intermediate Microeconomics, chapters 31, 32.7–9, 32.11–14

In this lecture you will learn

- what general equilibrium is
- what general equilibrium in pure exchange looks like
- what general equilibrium in an economy with production is
- what Walras law is
- what the first and second welfare theorems are

Partial and general equilibrium

In the previous lectures we ignored effects of prices of other goods on market equilibrium.

Partial equilibrium analysis examines how the price of the good influences the quantities demanded and supplied.

In this lecture we will study general equilibrium.

General equilibrium analysis examines how the interaction of demand and supply in more markets influences prices of many goods.

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We simplify our analysis by using the following assumptions:

- competitive markets (= everyone is the price taker)
- we have only 2 goods and 2 consumers

Pure exchange

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We have two consumers A and B and two goods 1 and 2:

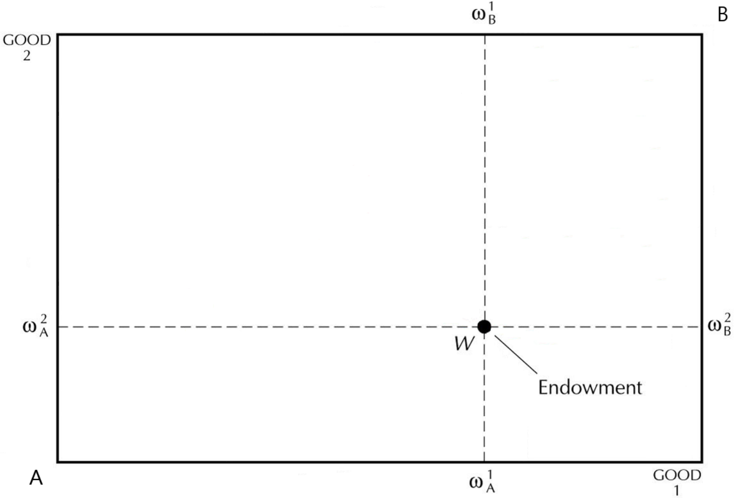
- $W_A = (\omega_A^1, \omega_A^2)$ is the endowment of consumer A
- $W_B = (\omega_B^1, \omega_B^2)$ is the endowment of consumer B
- $X_A = (x_A^1, x_A^2)$ is the bundle of consumer A
- $X_B = (x_B^1, x_B^2)$ is the bundle of consumer B

The *pair* of consumption bundles X_A and X_B is an **allocation**.
In a **feasible allocation** consumption equals quantities available:

$$x_A^1 + x_B^1 = \omega_A^1 + \omega_B^1$$

$$x_A^2 + x_B^2 = \omega_A^2 + \omega_B^2$$

Pure exchange – the Edgeworth box



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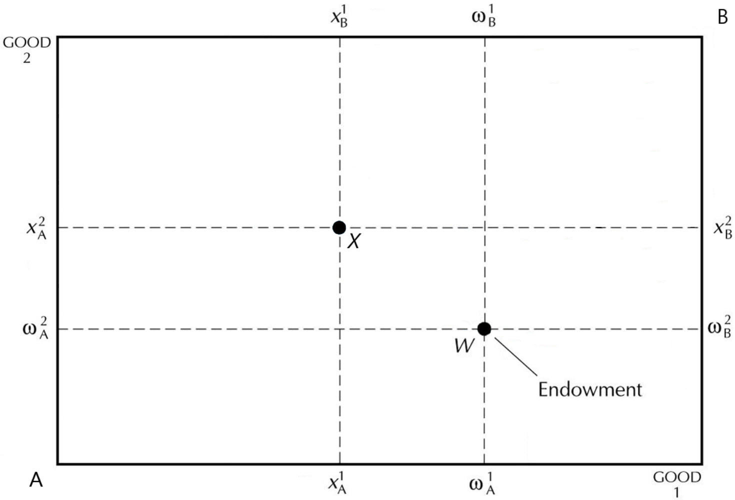

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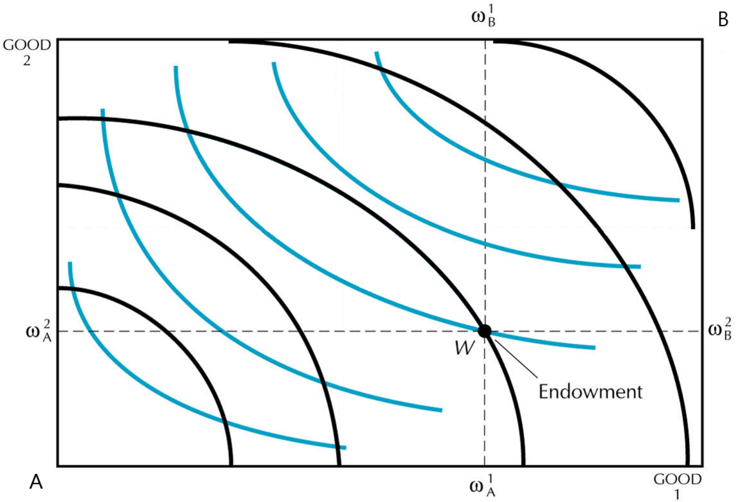
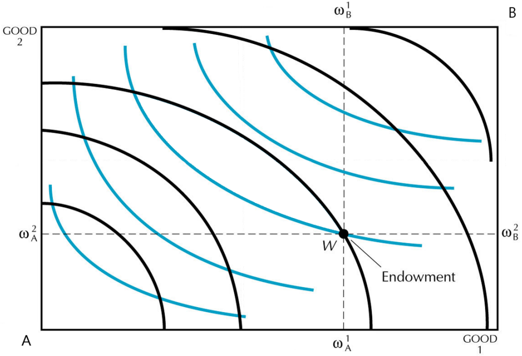


Trade

Trade begins in the **endowment**, point W.



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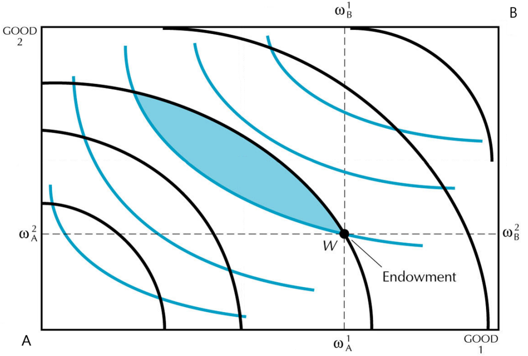
Consumers are better off if they shift to the blue area.



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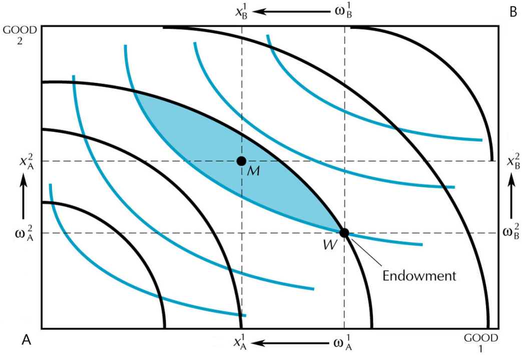
When moving to point M:

- consumer A exchanges $|x_A^1 - \omega_A^1|$ for $|x_A^2 - \omega_A^2|$
- consumer B exchanges $|x_B^2 - \omega_B^2|$ for $|x_B^1 - \omega_B^1|$



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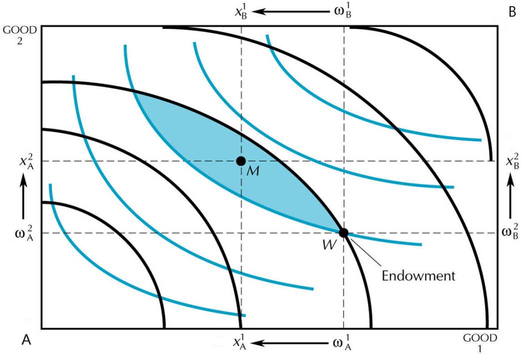
We can draw ICs though point M again and look for allocation in which the consumers are better off...

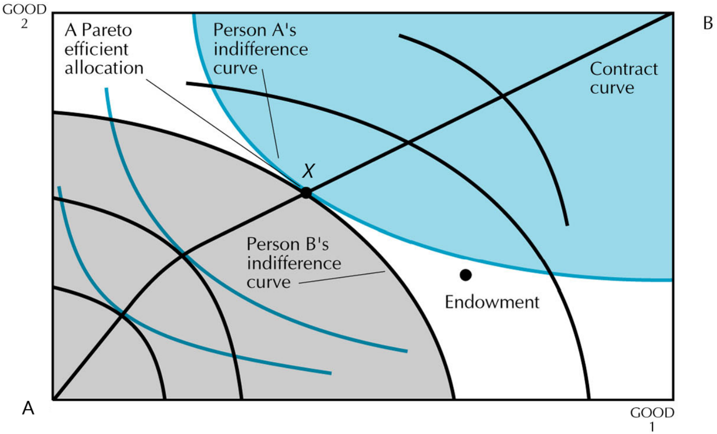


Finally we get the Pareto efficient allocation X.

Pareto efficient allocation

In a **Pareto efficient** allocation one consumer cannot better off without making anyone worse off (the point where the ICs touch).

Contract curve – all Pareto efficient points in the Edgeworth box.



Trade in the market

If the consumers can trade in any way, they can end up in any point of the contract curve that was in the blue area initially.

If the consumers have

- given relative prices (price takers + auctioneer assumption)
- same MRS where the ICs touch (smooth, convex IC, inner solution)

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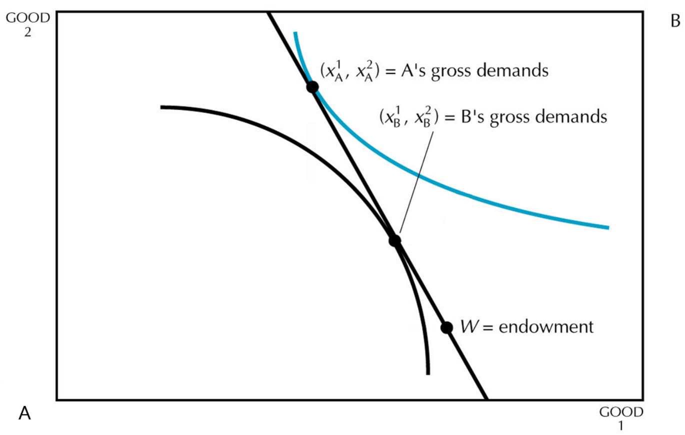
The auctineer set prices (p_1, p_2) we have

- gross demands (x),
- nett demands $(x - \omega)$.

Trade in the market (cont'd)

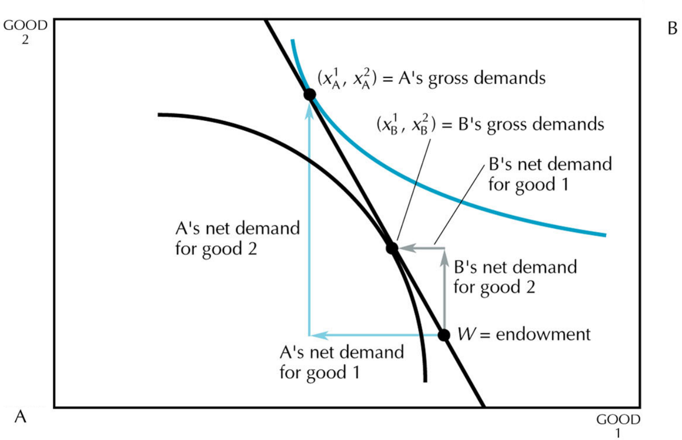
Gross demand of consumer A for good 1 is x_A^1 and for good 2 is x_A^2.



Trade in the market (cont'd)

Gross demand of consumer A for good 1 is x_A^1 and for good 2 is x_A^2.

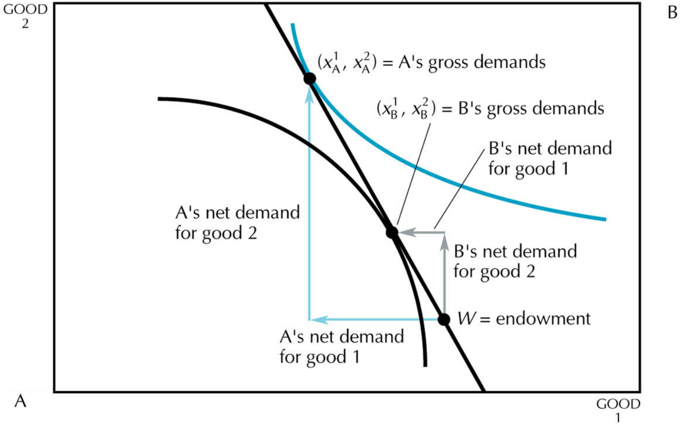
Net demand or **excess demand** of consumer A for good 1 is $e_A^1 = x_A^1 - \omega_A^1$ and for good 2 is $e_A^2 = x_A^2 - \omega_A^2$.



Trade in the market (cont'd)

For (p_1, p_2) the market in the graph is in disequilibrium:

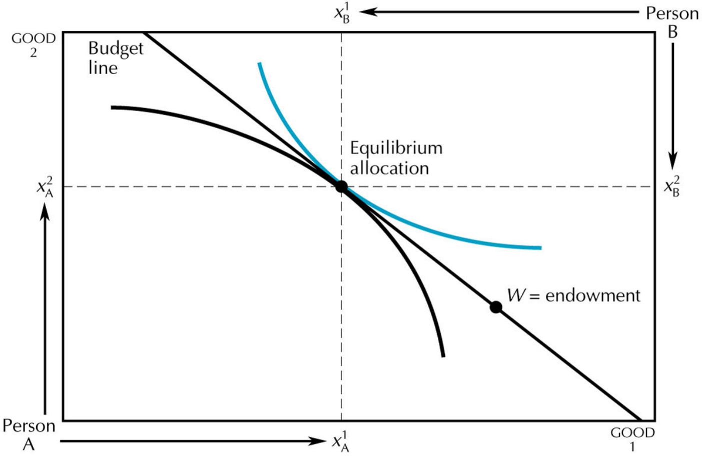
- net demands differ – q demanded $\neq$ q supplied
- gross demands differ – consumption $\neq$ endowment



Trade in the market (cont'd)

If the auctioneer changes the price so that there is no excess demand (supply), we get a **general** or **Walrasian equilibrium**.

Equilibrium: slope of BL = slope of ICs: $\text{MRS}_A = -p_1/p_2 = \text{MRS}_B$.



General equilibrium

For equilibrium prices (p_1^*, p_2^*) holds that demand equals supply:

$$x_A^1(p_1^*, p_2^*) + x_B^1(p_1^*, p_2^*) = \omega_A^1 + \omega_B^1,$$

$$x_A^2(p_1^*, p_2^*) + x_B^2(p_1^*, p_2^*) = \omega_A^2 + \omega_B^2.$$

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For equilibrium prices (p_1^*, p_2^*) holds that **aggregate excess demands** for goods 1 and 2 are zero:

$$z_1(p_1^*, p_2^*) = x_A^1(p_1^*, p_2^*) + x_B^1(p_1^*, p_2^*) - \omega_A^1 - \omega_B^1 = 0,$$

$$z_2(p_1^*, p_2^*) = x_A^2(p_1^*, p_2^*) + x_B^2(p_1^*, p_2^*) - \omega_A^2 - \omega_B^2 = 0.$$

Walras' law

Walras' law: If the entire endowment is demanded, the sum of excess demands for goods 1 and 2 at any prices (p_1, p_2) must be equal to 0:

$$p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) \equiv 0$$

Example – Walras' law

Let us assume that at prices $p_1 = 1$ and $p_2 = 2$

- consumer A whats to buy 20 goods 1, i.e. $x_A^1(1,2) - \omega_A^1 = 20$,
- consumer B whats to sell 10 goods 1, i.e. $x_B^1(1,2) - \omega_B^1 = -10$.

What are their net demands for good 2? What are their aggregate excess demands for both goods? Does Walras' law hold?

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Net demand of A for good 2 is $x_A^2(1,2) - \omega_A^2 = -10$.
Net demand of B for good 2 is $x_B^2(1,2) - \omega_B^2 = 5$.

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Aggregate excess demand for good 1 $z_1(1,2) = 20 - 10 = 10$.
Aggregate excess demand for good 2 $z_2(1,2) = -10 + 5 = -5$.

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Walras' law holds:

$$p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) = 1 \cdot 10 + 2 \cdot (-5) = 0$$

Consequences of Walras' law

It follows from Walras' law that if one market is in equilibrium, the other market has to be in equilibrium too.

If it is true that

$$p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) \equiv 0 \text{ and } z_1(p_1, p_2) = 0,$$

then at $p_2 > 0$ it must also be true that

$$z_2(p_1, p_2) = 0.$$

It is sufficient to find equilibrium in one market and the other market will be automatically in equilibrium too.

Consequences of Walras' law (cont'd)

In order to have equilibrium in k markets, it is sufficient to find such prices that ensure equilibrium in $k-1$ markets.

Hence we have only $k-1$ independent equations (supply = demand), but we have k markets with k prices.

Can this equation be solved?

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Can this equation be solved?

Yes. In equilibrium we can set one price equal to any number. Typically we set $p_1 = 1$ (numeraire).

It means that we have $k-1$ independent prices.

Example – finding the equilibrium

Utility functions of A and B: $u_A = \sqrt{x_A^1}\sqrt{x_A^2}$ and $u_B = \sqrt{x_B^1}\sqrt{x_B^2}$

Endowment of A and B: $(\omega_A^1, \omega_A^2) = (15, 4)$ and $(\omega_B^1, \omega_B^2) = (5, 11)$

What are the equilibrium prices of goods 1 and 2?

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We want to find the equilibrium price p_1^*, at which the sum of demands equals to the quantities of the good in the endowments:

$$x_A^1(p_1^*) + x_B^1(p_1^*) = \omega_A^1 + \omega_B^1$$
$$x_A^2(p_1^*) + x_B^2(p_1^*) = \omega_A^2 + \omega_B^2$$

Substituting for the endowments, we get

$$x_B^1(p_1^*) = 20 - x_A^1(p_1^*),$$
$$x_B^2(p_1^*) = 15 - x_A^2(p_1^*).$$

Example – finding the equilibrium (cont'd)

Cobb–Douglas preferences – smooth and convex ICs + inner solution $\implies$ for allocations $(x_A^1(p_1^*), x_A^2(p_1^*))$ and $(x_B^1(p_1^*), x_B^2(p_1^*))$ holds:

$$\text{MRS}_A = \text{MRS}_B$$

$$-\frac{x_A^2(p_1^*)}{x_A^1(p_1^*)} = -\frac{x_B^2(p_1^*)}{x_B^1(p_1^*)}$$

$$\frac{x_A^2(p_1^*)}{x_A^1(p_1^*)} = \frac{15 - x_A^2(p_1^*)}{20 - x_A^1(p_1^*)}$$

$$x_A^2(p_1^*) = \frac{3}{4} x_A^1(p_1^*)$$

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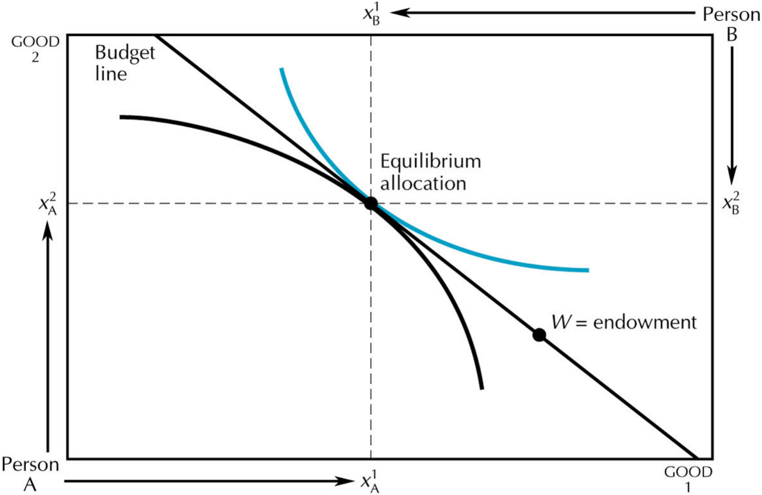
Because for the equilibrium allocation holds that the slope of IC equals the sole of BL:

$$\text{MRS}_A = -p_1^*$$
$$p_1^* = \frac{3}{4}$$

Example – finding the equilibrium (graph)



The existence of equilibrium

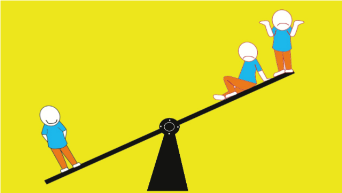
Is there always an equilibrium (for all preferences and endowments)?

Yes, if the aggregate excess demand is continuous.

It means that small changes in relative prices lead to small changes in demand and supply surpluses.

It is true if

- individual demands are continuous, which requires convex preferences,
- individual demands are not continuous, but we have a large quantity of consumers (competitive markets).



Equilibrium and efficiency

We know what are the conditions for the existence of equilibrium and can find equilibrium prices for all endowments and prefereces.

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But is this equilibrium Pareto efficient? Yes. The ICs touch in equilibrium. Hence a Pareto improvement is not possible.

The first theorem of welfare economics – all competitive markets are Pareto efficient.

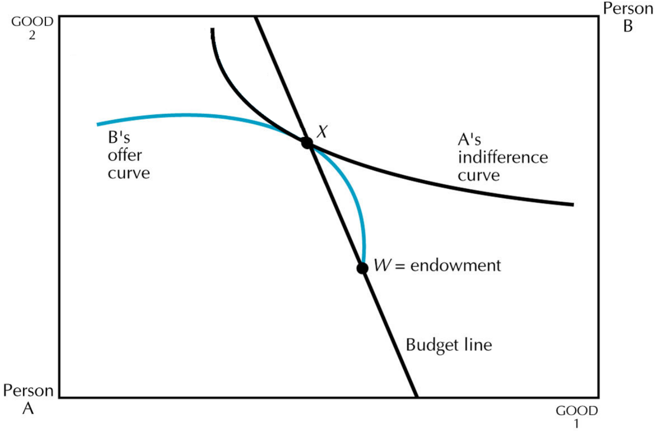
The first welfare theorem holds if

- there is not a consumption externality (each consumer cares only about her consumption),
- consumers behave competitively and there is a competitive equilibrium.

Example – inefficiency of monopoly

Monopoly – consumer A sets prices and consumer B accepts them.

Consumer A (monopoly) chooses a point on the price consumption (offer) curve of consumer B that brings her the highest utility.



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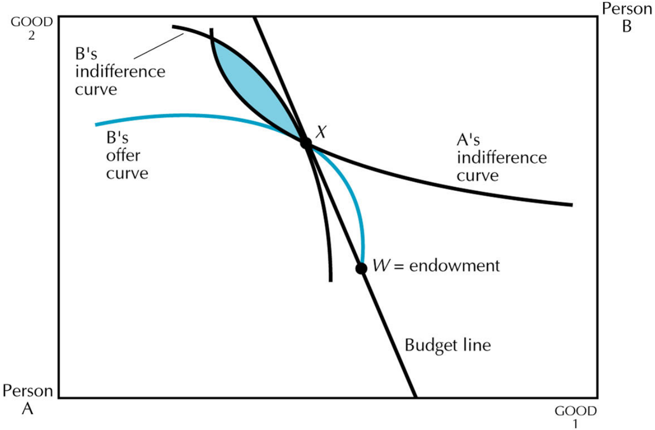
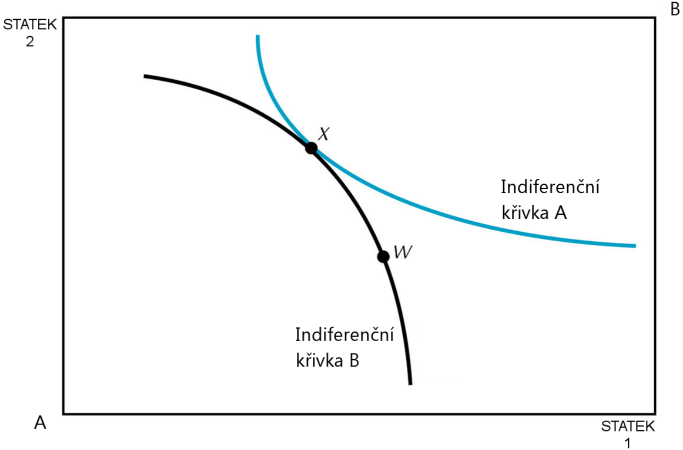


Example – efficiency of perfectly discriminating monopoly

Perfectly discriminating monopoly (consumer A) sells each unit of product for the willingness to pay.

A chooses the utility-maximizing point on IC_B going though W.



Efficiency and equilibrium

First welfare theorem = competitive equilibrium is Pareto efficient.

Does it work the other way round? Is each Pareto-efficient allocation an equilibrium? No, if consumers have non-convex preferences.

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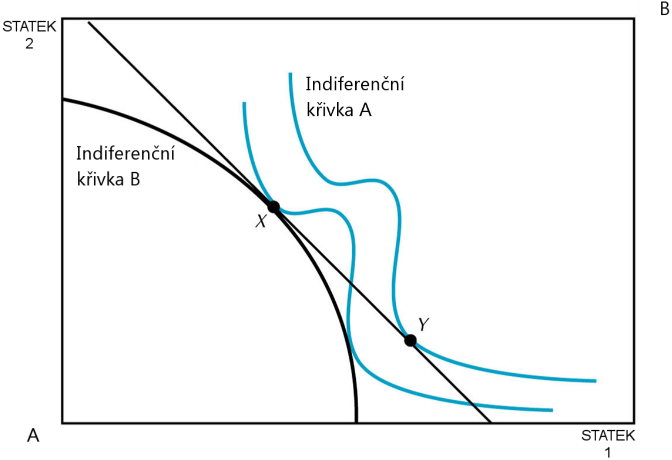
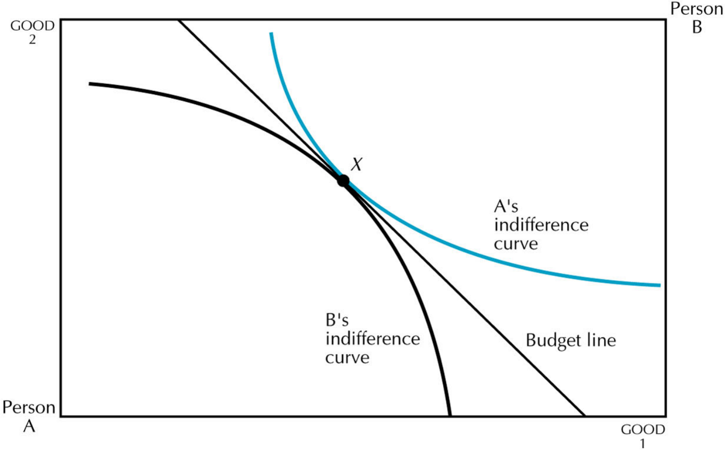


Efficiency and equilibrium (cont'd)

Second welfare theorem – if all agents have convex preferences, then there will always be a set of prices and endowments such that each Pareto efficient allocation is a market equilibrium.



Implications of the first welfare theorem

First welfare theorem – perfectly competitive market with utility-maximizing consumers generates Pareto-efficient allocation.

If we are dealing with a resource problem involving many people, the use of competitive markets economizes on the information that any one agent needs to possess.

If consumers know prices and market finds competitive equilibrium, the allocation is Pareto efficient.

Implications of the second welfare theorem

Second theorem – under some conditions any Pareto-efficient allocation may arise as a competitive equilibrium.

We can separate the allocative and distributive role of prices. We can choose the consumption distribution and while allocating efficiently.

How?

Implications of the second welfare theorem

Second theorem – under some conditions any Pareto-efficient allocation may arise as a competitive equilibrium.

We can separate the allocative and distributive role of prices. We can choose the consumption distribution and while allocating efficiently.

How? Using competitive market, we can get from given endowments to any Pareto-efficient allocation.

If redistribution creates the required endowment and we let the market find competitive prices, consumers reach a Pareto-efficient allocation.

Implications of the second welfare theorem (cont'd)

A practical problem: How to set a tax system to move the endowments efficiently in the required direction?

An ideal solution: a lump sum tax, but it is difficult to implement since consumers' endowments hinge on their labor productivity.

If we tax their labor, we have a distortionary tax = the tax that changes the quantity of labor supplied.

If we want efficient resource allocation, prices should reflect relative scarcity of resources and not be used for redistribution.

Production

Up to now we assumed that the quantities of goods for consumption are given and these are only distribution between consumers.

Now the quantity of goods for consumption depends on production decisions of firms.

Both welfare theorems hold in the production economy if

- there are no production externalities,
- competitive equilibrium exists,
- there are not increasing returns to scale for a significant range of output.

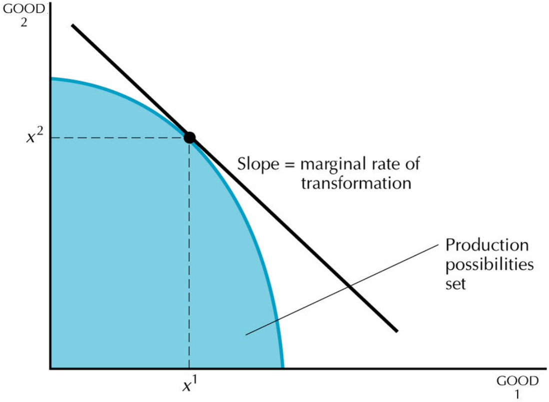
Production (cont'd)

We have firms that produce goods 1 and 2. If they use all available inputs they can produce different combinations of these goods.

Production possibility set – good combinations that a firm can produce using given inputs.

Frontier of this set is **production possibility frontier** (PPF).

The slope of PPF is **marginal rate of transformation** (MRT).



Equilibrium in exchange and production

Two ways how to exchange one good for the other:

- consumers can exchange in the ratio given by the prices,
- producers may exchange goods at the ratio given by MRT.

For allocations on the CC holds that $\text{MRS}_A = -p_1^*/p_2^* = \text{MRS}_B$.
What if it was true that $\text{MRS}_A = \text{MRS}_B \neq \text{MRT}$?

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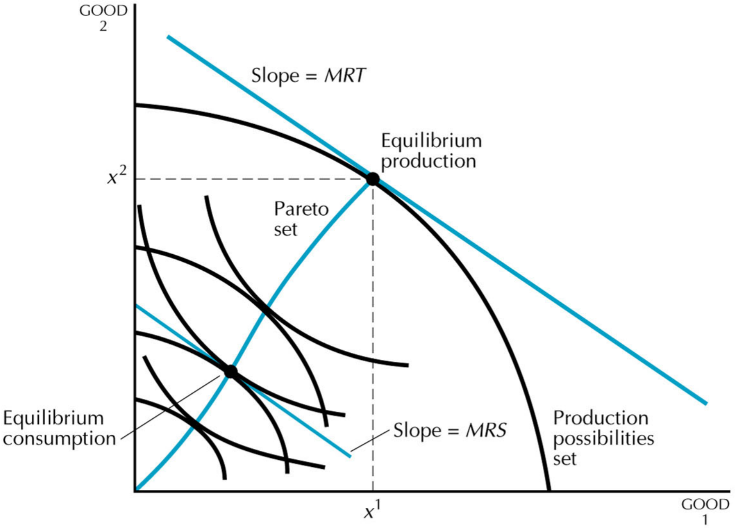
What if it was true that $\text{MRS}_A = \text{MRS}_B \neq \text{MRT}$?

This equilibrium would not be Pareto efficient –both consumers could increase their utility by changing the structure of production.

The allocation in a production economy is Pareto efficient iff

$$\text{MRS}_A = -\frac{p_1^*}{p_2^*} = \text{MRS}_B = \text{MRT}.$$

Production and the Edgeworth box



Example – Robinson, Friday and Castaways, Inc.

This example contains

- 2 consumers – Robinson Crusoe and Friday,
- 2 inputs – Robinson's labor L_R and Friday's labor L_F,
- 2 goods – coconuts C and fish F,
- 1 firm, Castaways, Inc. that produces both goods.

Robinson and Friday work in Castaways and are also the only shareholders and customers of this firm.

Example – Robinson, Friday and Castaways, Inc. (cont'd)

Castaways maximizes profit

$$\max_{C,F,L_C,L_D} p_C^* C + p_F^* F - w_R^* L_R - w_F^* L_F$$

subject to the production possibility set.

For simplicity, we assume that it is optimal for firm to hire L_R^* and L_F^* and the cost of labor at given wages is L^*. The profit of Castaways is

$$\pi = p_C^* C + p_F^* F - L^*.$$

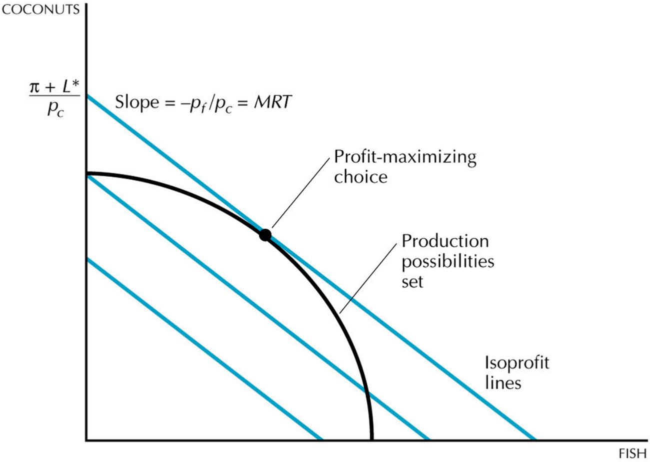
From the profit function we can derive the isoprofit lines

$$C = \frac{\pi + L^*}{p_C^*} - \frac{p_F^* F}{p_C^*}.$$

Example – Robinson, Friday and Castaway, Inc. (cont'd)

Castaway chooses the point from the production possibility frontier that touches the highest isoprofit line.

Isoprofit line has to be tangent to PPF, hence MRT $= -p_F^*/p_C^*$.



Example – Robinson, Friday and Castaway, Inc. (cont'd)

Robinson and Friday as consumers receive wages and profits. They can buy the entire profit of Castaway (a variation of Walras' law).

In equilibrium each buys the most preferred combination of C and F. If they have convex preferences, then $\text{MRS}_R = \text{MRS}_F = -p_F^*/p_C^*$.

In equilibrium: $\text{MRS}_R = \text{MRS}_F = \text{MRT}$.

Example – Robinson, Friday and Castaway, Inc. (cont'd)

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Conclusion:

The problem of efficient use of resources in society can be solved at an individual level.

If individual firms maximize profits and individual consumers maximize utility, and the market sets competitive prices, the resulting equilibrium will be Pareto efficient.

What should you know?

- Under specific conditions, the competitive general equilibrium exists for all preferences and endowments.
- It holds under given conditions that each competitive equilibrium is Pareto efficient (1st theorem) and that each Pareto-efficient allocation can be achieved in a competitive market (2nd welfare theorem).
- In order for an allocation to be Pareto efficient, marginal rates of substitution must be equal to the slopes of BL and PPF.
- The advantage of competitive markets is that decentralized decisions of consumers and firms allocate resources efficiently.