ARCH/GARCH models

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Literature

1. Engle, Robert F. (1982). "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation". *Econometrica* **50** (4): 987–1007.

2. Bollerslev, Tim (1986). "Generalized Autoregressive Conditional Heteroskedasticity". *Journal of Econometrics* **31** (3): 307–327.

ARCH and GARCH models

$$\epsilon_t = \sigma_t w_t$$

$$w_t \sim N(0,1)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta \epsilon_{t-1}^2$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-1}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-1}^2$$

Assignment 4

1. Select two assets from two different sectors and download the last-year historical daily prices. Using the first 11 months (train data), calibrate 2-3 ARIMA models and examine the mean forecasting performance for each one of the approaches using the last-month data (out-of-sample or test data) by means of the mean squared error.

2. Consider that the random variable x_t is described by following process: $x_t = \varepsilon_t$ with:

   (a) $\varepsilon_t \sim N(0,1)$

   (b) $\varepsilon_t = \sigma_t w_t, \quad w_t \sim N(0,1), \quad \sigma_t^2 = \alpha + \beta\varepsilon_{t-1}^2$

   (c) $\varepsilon_t = \sigma_t w_t, \quad w_t \sim N(0,1), \quad \sigma_t^2 = \alpha + \beta\varepsilon_{t-1}^2 + \gamma\sigma_{t-1}^2$

   - Comment on each specification (main features).
   - Simulate a path (1000 values) for x_t using each specification and compare the models empirically.

Assignment 4 (1)

```
getSymbols('^AAPL',src='yahoo', from="2021-11-05"
           ,periodicity = 'daily')
AAPL<-AAPL[,6]
L<-length(AAPL)
m<-21
AAPL_train=AAPL[1:(L-m)]
AAPL_test=AAPL[(L-m+1):L]
```

```
m1<-arima(AAPL_train,order = c(1,1,0))
m2<-arima(AAPL_train,order = c(0,1,1))
m3<-arima(AAPL_train,order = c(1,1,1))
m4<-arima(AAPL_train,order = c(0,1,0))
```

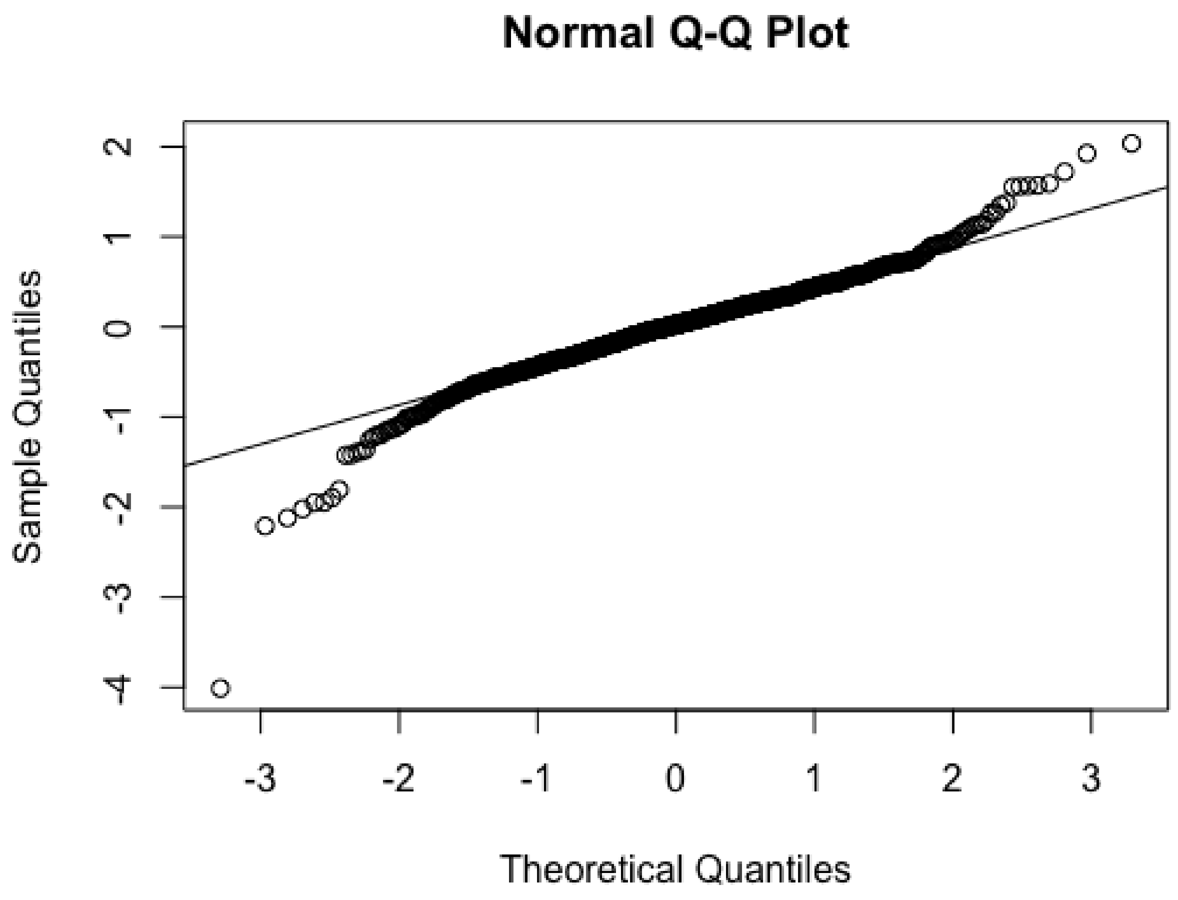
```
m1_test<-arima(AAPL_train, order=c(1,1,0),fixed=m1$coef)
m2_test<-arima(AAPL_train, order=c(0,1,1),fixed=m2$coef)
m3_test<-arima(AAPL_train, order=c(1,1,1),fixed=m3$coef)
m4_test<-arima(AAPL_train, order=c(0,1,0),fixed=m4$coef)
```

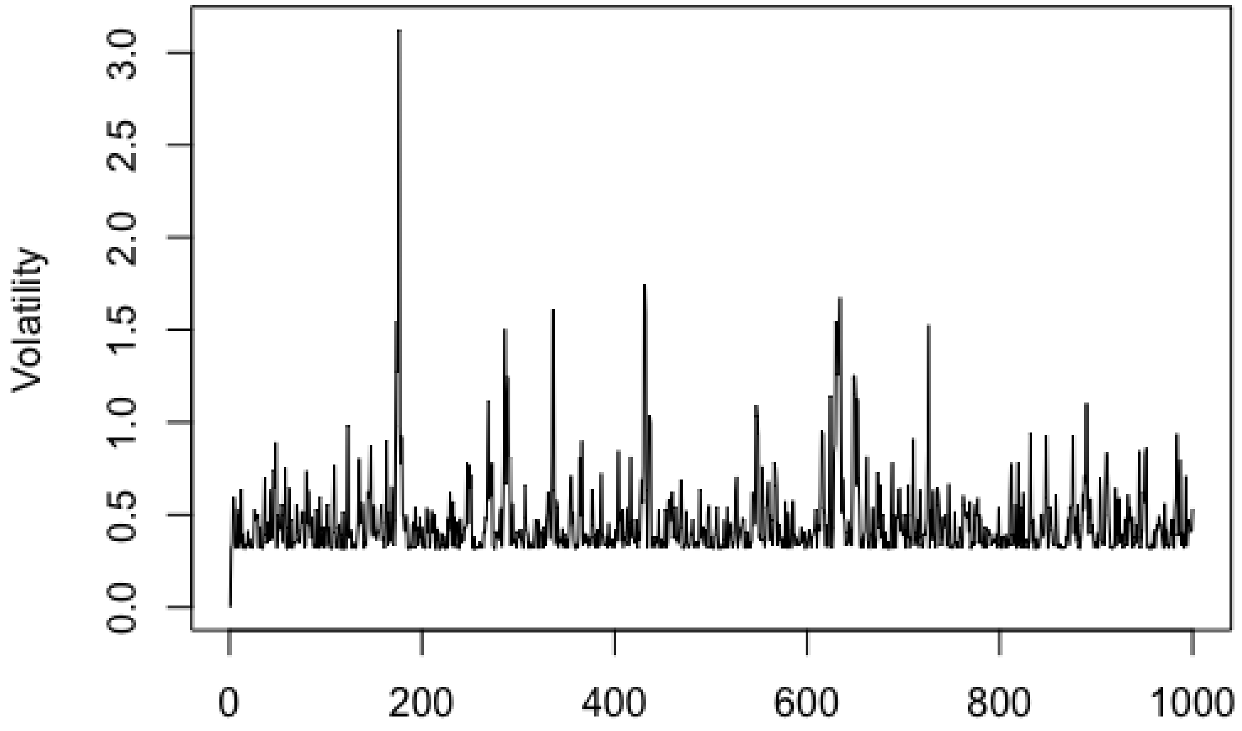
```
A1<-accuracy(m1_test)
A2<-accuracy(m2_test)
A3<-accuracy(m3_test)
A4<-accuracy(m4_test)
RMSE<-c(A1[,2],A2[,2],A3[,2],A4[,2])
```

```
> RMSE
[1] 3.259902 3.259898 3.221729 3.259925
```

Assignment 4 (2b)

```
s2<-rep(0,1000); e<-rep(0,1000)
alpha0<-0.1; alpha1<-0.6
for (i in 2:1000) {
  s2[i]<-alpha0+alpha1*(e[i-1])**2
  e[i]<-sqrt(s2[i])*rnorm(1)
}
plot(sqrt(s2),type='l',ylab = 'Volatility')
qqnorm(e); qqline(e)
```





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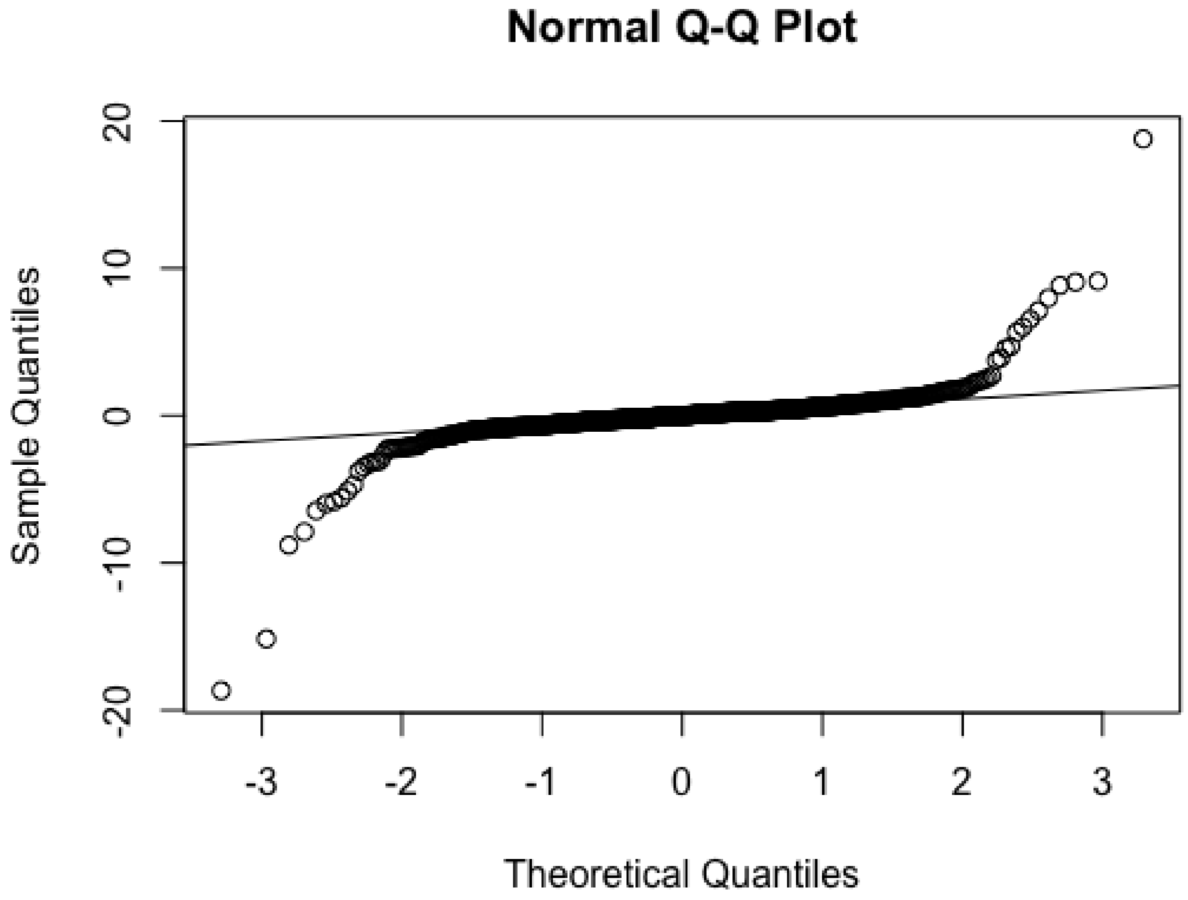
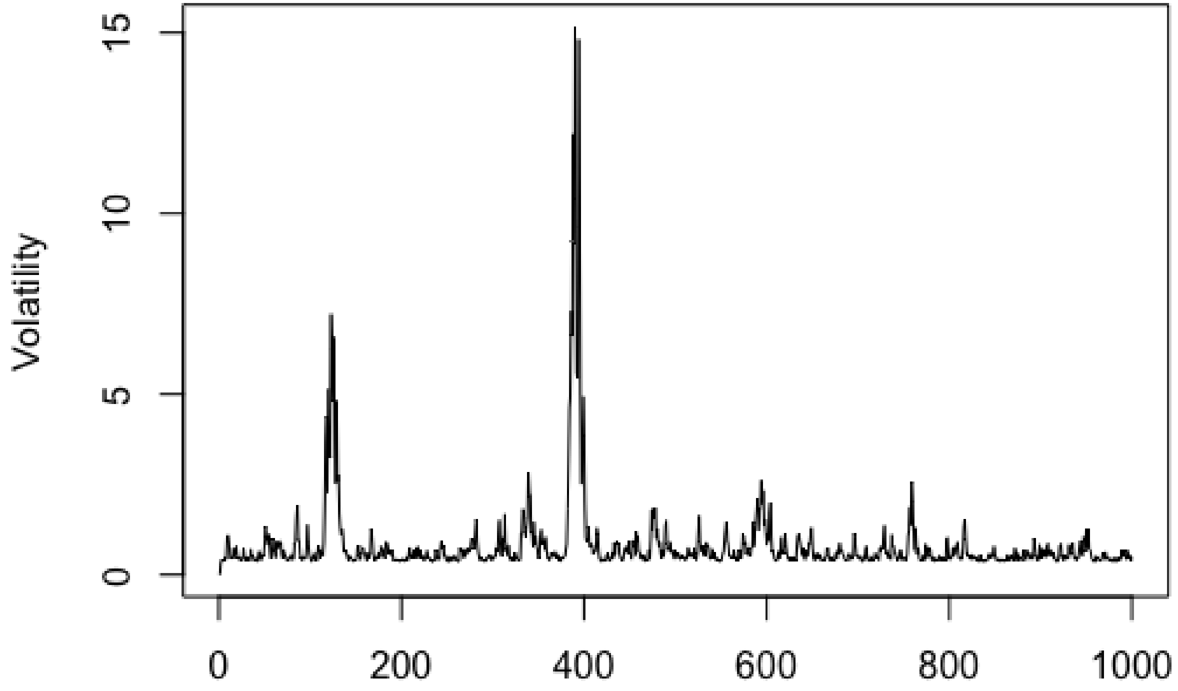
Assignment 4 (2c)

```
s2<-rep(0,1000); e<-rep(0,1000)
alpha0<-0.1; alpha1<-0.6; beta1<-0.25
for (i in 2:1000) {
  s2[i]<-alpha0+alpha1*(e[i-1])**2+beta1*(s2[i-1])
  e[i]<-sqrt(s2[i])*rnorm(1)
}
plot(sqrt(s2),type='l',ylab='Volatility')
qqnorm(e); qqline(e)
```





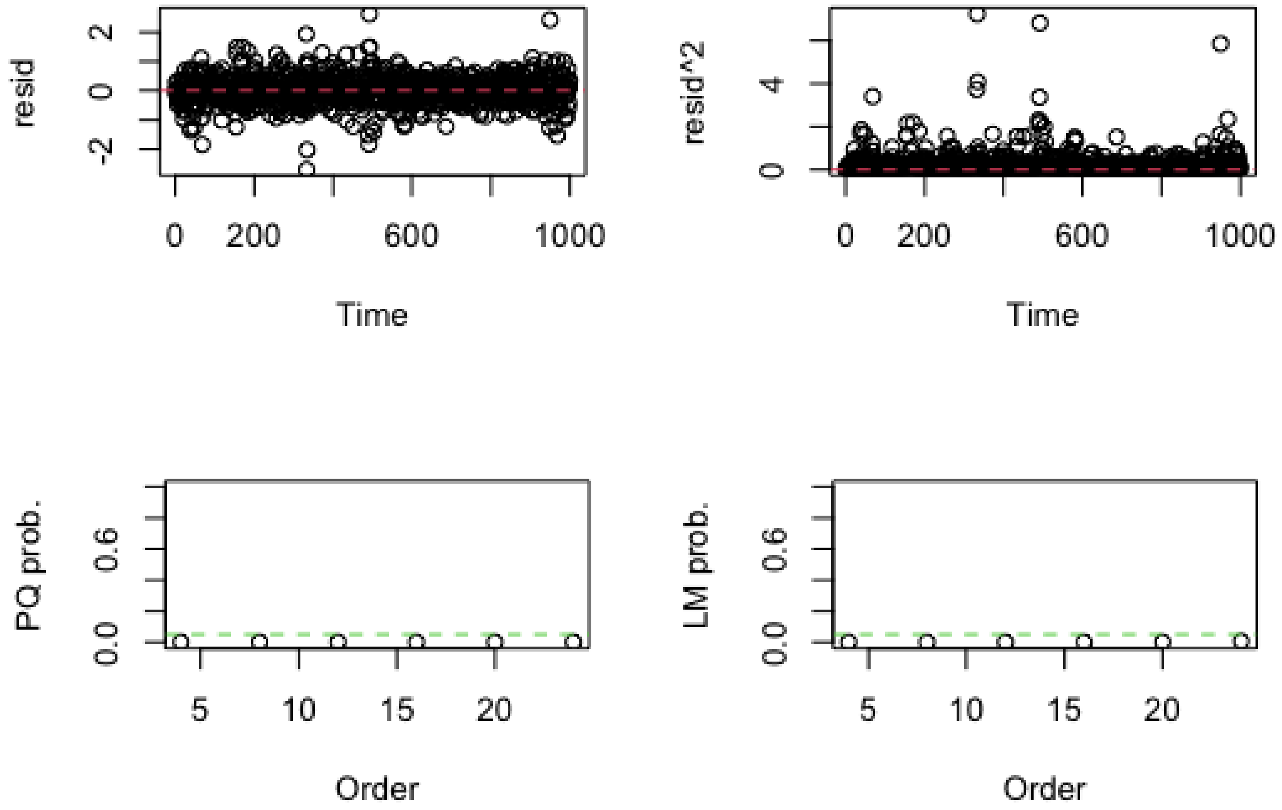
ARCH test

```r
install.packages('aTSA')
library('aTSA')
mod <- arima(e,order = c(0,0,0))
arch.test(mod)
```

```
ARCH heteroscedasticity test for residuals
alternative: heteroscedastic
```

```
Portmanteau-Q test:
     order  PQ p.value
[1,]     4 356       0
[2,]     8 359       0
[3,]    12 360       0
[4,]    16 366       0
[5,]    20 371       0
[6,]    24 374       0
```

```
Lagrange-Multiplier test:
     order    LM  p.value
[1,]     4 290.2 0.00e+00
[2,]     8 143.3 0.00e+00
[3,]    12  94.3 2.44e-15
[4,]    16  68.7 7.53e-09
[5,]    20  53.3 4.28e-05
[6,]    24  44.1 5.15e-03
```

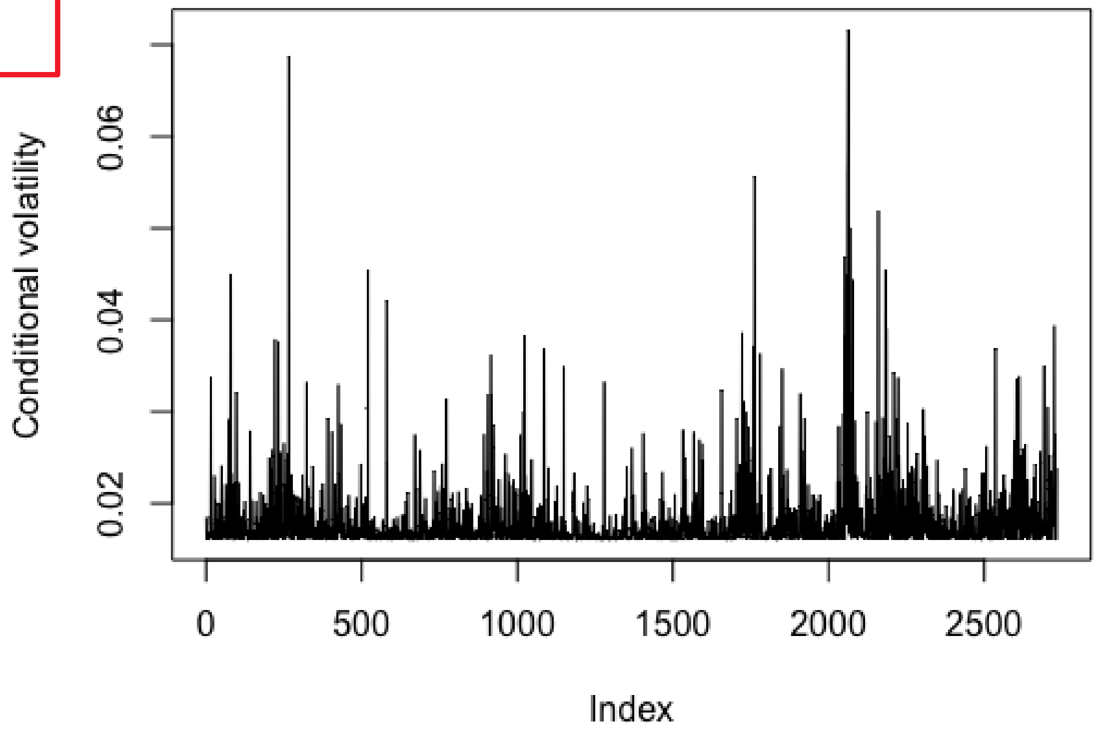


Fitting an ARCH model

```
getSymbols('AAPL',src='yahoo', from="2012-01-01",periodicity = 'daily')
AAPL<-AAPL[,6]
r.AAPL<-diff(log(AAPL))
r.AAPL<-r.AAPL[2:length(AAPL)]
```

```
install.packages('fGarch')
library(fGarch)
arch.fit <- garchFit(~garch(1,0), data = r.AAPL)
plot(arch.fit@sigma.t,type='l',ylab='Conditional volatility')
#plot(arch.fit@h.t) # conditional variance
```

```
#alternatively
garchfit<-garch(r.AAPL, order = c(0, 1), itmax = 200)
plot(garchfit$fitted.values[,1],type='l')
summary(garchfit)
```



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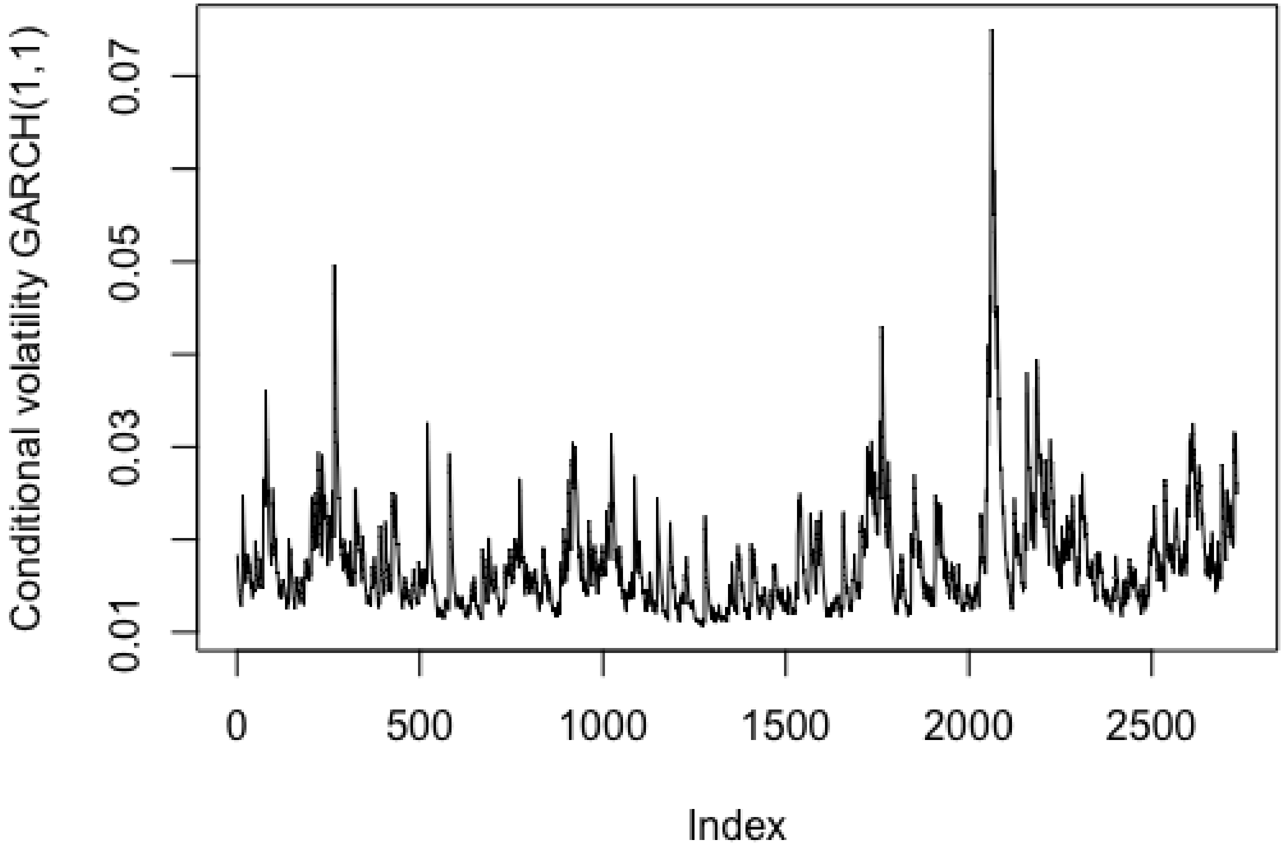
Fitting a GARCH model

```r
Garch.fit <- garchFit(~garch(1,1), data = r.AAPL)
plot(arch.fit@sigma.t,type='l',
     ylab='Conditional volatility GARCH(1,1)')
#plot(arch.fit@h.t) # conditional variance
```

```r
#alternatively
garchfit2<-garch(r.AAPL, order = c(1, 1), itmax = 200)
plot(garchfit$fitted.values[,1],type='l')
summary(garchfit)
```



GARCH extensions

– GARCH-in-Mean
– GJR GARCH
– EGARCH
– And several ones

Fitting GARCH extensions

– Selecting GARCH-type, coefficients and mean-equation:

```
install.packages('rugarch')
library(rugarch)

garchSpec <- ugarchspec(
  variance.model=list(model="sGARCH",
   garchOrder=c(1,1)),
  mean.model=list(armaOrder=c(0,0)),
  distribution.model="std")
```

– Fit the model:

```
garchFit <- ugarchfit(spec=garchSpec, data=r.AAPL)
coef(garchFit)
r_hat <- garchFit@fit$fitted.values
plot.ts(r_hat)
vol_hat <- ts(garchFit@fit$sigma)
plot.ts(hhat)
```

Fitting GARCH extensions

– EGARCH: model=‘eGARCH’

– GJR-GARCH: model=‘’gjrGARCH’

```
# GARCH-in-mean
garchMod <- ugarchspec(
          variance.model=list(model="fGARCH",
                              garchOrder=c(1,1),
                              submodel="APARCH")
           mean.model=list(armaOrder=c(0,0),
                          include.mean=TRUE,
                          archm=TRUE,
                          archpow=2
                          ),
           distribution.model="std"
                          )
```

Assignment

– Considere two assets (one commodity and the S&P500):

1. Assuming an ARIMA(0,0,0) model for the log-returns and determines if there are presence of heteroscedasticyt via ARCH disturbances.
2. Fit the best ARIMA model for the returns and determines if there are ARCH disturbances.
3. For each one of the previous cases, fit an ARCH(1) and GARCH(1,1)
4. In the case of S&P-500, can the GARCH modelling reproduce the VIX shape?