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Applied Financial Econometrics

Value at Risk (VaR)

Lecturer: Axel A. Araneda, Ph.D.

Risk measures: Value at Risk (VaR)

- Probability of (potential) losses given some probability.

- Downside risk based on current levels and normal market conditions.

- For a computed T-horizon p-VaR, a loss (L) will not exceed p-VaR with p probability T-periods ahead. $Prob(L > \text{VaR}) \leq 1-p$.

- Example: One-day 95% VaR of 100.
    - 5% of losses falls over 100.
    - Expected losses greater than 100 for 1 day over 20 days.
    - 95% of confidence of don't have losses greater than 100.

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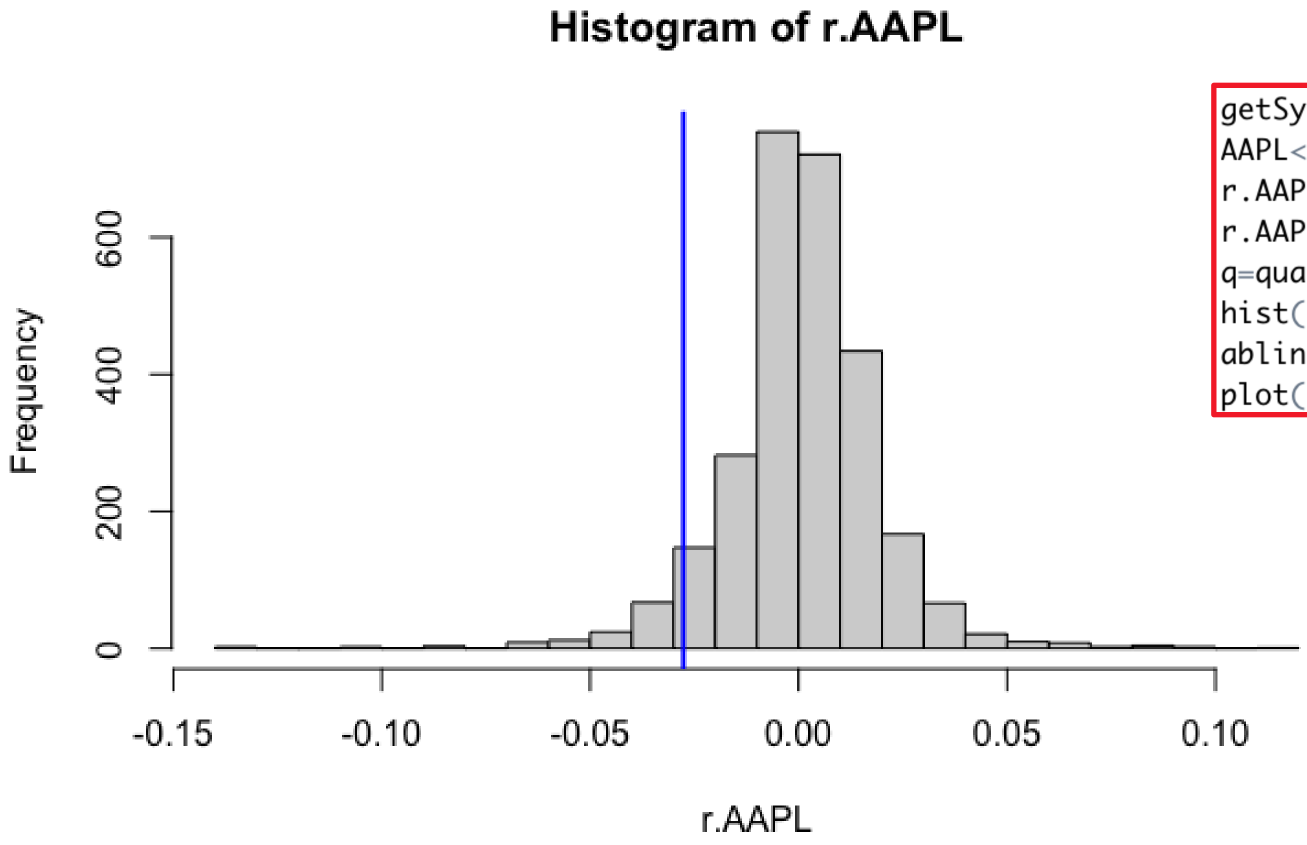
Computing VaR




```
getSymbols('AAPL',src='yahoo', from="2012-01-01",periodicity = 'daily')
AAPL<-AAPL[,6]
r.AAPL<-diff(log(AAPL))
r.AAPL<-r.AAPL[2:length(AAPL)]
q=quantile(r.AAPL,0.05)
hist(r.AAPL,breaks = 20)
abline(v=q,col="blue",lwd=2)
plot(density(r.AAPL))
```

```
> quantile(r.AAPL,0.05)
        5%
-0.02761346
```

– 1-day 95% VaR: 2.76 %

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Estimating VaR (in-sample)

– Assuming normality:

$$VaR(a) = \bar{x} + \sigma * N^{-1}(1 - a)$$

$$VaR(95) = \bar{x} - \sigma * 1.644854$$

Example

– Estimating 97.5-VaR for AAPL, assuming the asset follows an ARMA(0,0) process; i.e., constant drift plus constant volatility, with normal disturbances.

$$r.AAPL_t = \mu + \sigma * \epsilon_t$$

$$\epsilon_t \sim N(0,1)$$

$$VaR(97.5) = \mu - \sigma * 1.96$$

Example: 97.5-VaR for AAPL

```r
#Downloading data
#library('quantmod')
getSymbols('AAPL',src='yahoo',
       from="2015-01-01",periodicity = 'daily')

#log-returns
r.AAPL<-diff(log(AAPL[,6]))
r.AAPL<-r.AAPL[2:length(r.AAPL)]
# or equivalently:
#r.APPL<-na.omit(r.AAPL)
```

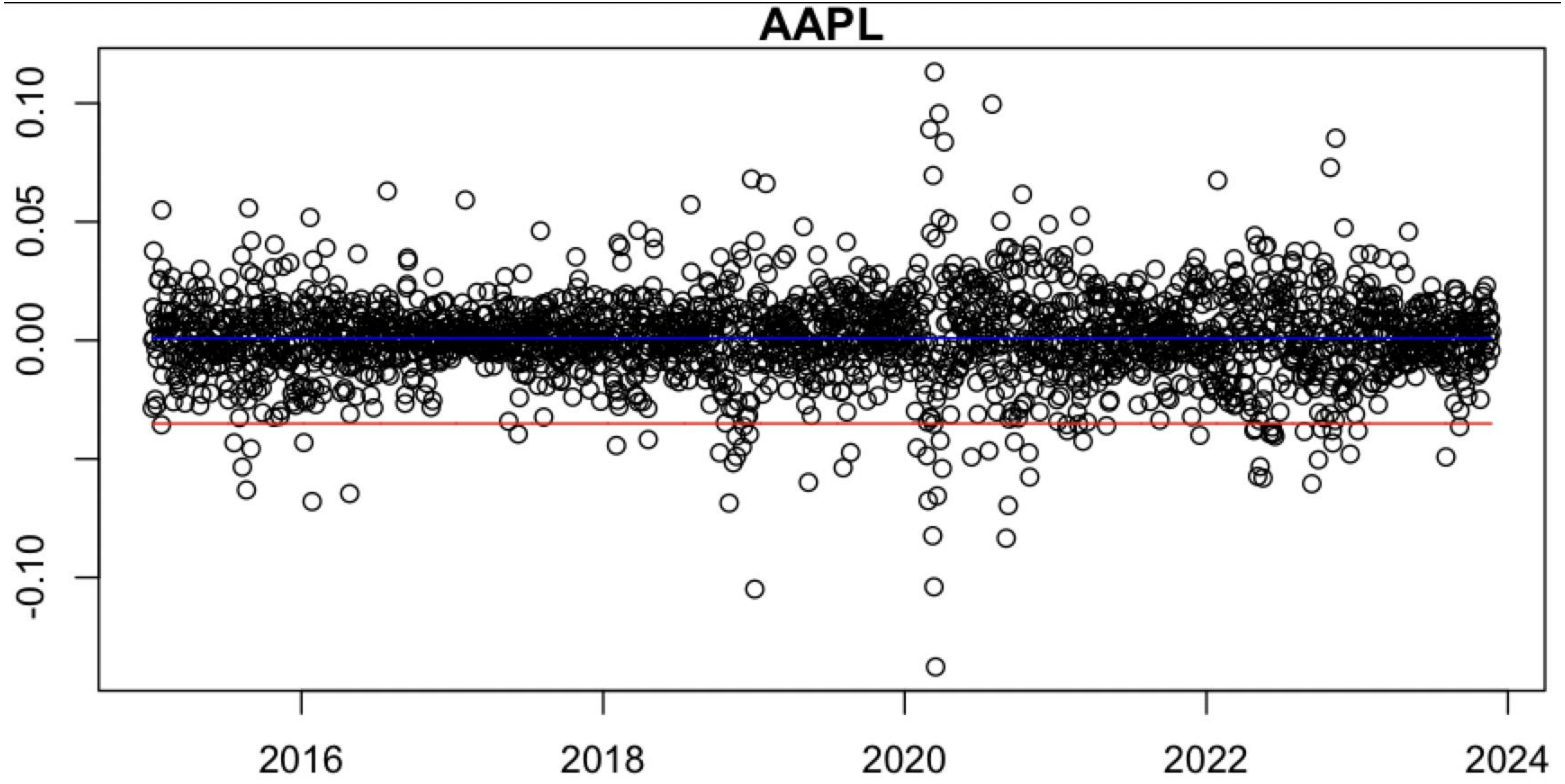
```r
### The model #####
arima0<-arima(r.AAPL,order=c(0,0,0))
mu<-arima0$coef
sigma<-sqrt(arima0$sigma2)
## or equivalently
#mu<mean(r.AAPL)
#sigma<-sqrt(var(r.AAPL))
```

```r
# 97.5-VAR under normality
VaR_AAPL<-mu-1.96*sigma
```

```
VaR_AAPL: 0.03510028
```

```r
par(mfrow=c(2,1),mar=c(2.5,2.5,1,1))
plot(time(r.AAPL),r.AAPL,title('AAPL'))
lines(time(r.AAPL),rep(mu,
      length(r.AAPL)),col="blue")
lines(time(r.AAPL),rep(VaR_AAPL,
      length(r.AAPL)),col="red")
```

Example: 97.5-VaR for AAPL



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Assignment

1. For the S&P500 and another asset of your choice, estimate the in-sample 1-day 95%VaR assuming normality and:
   a) Constant volatility and ARIMA(0,0,0) for the returns
   b) Constant volatility, finding the best ARIMA model possible for the returns, if any (auto.arima)
   c) GARCH(1,1) volatility.

2. Compare the results.

Assignment

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   c) GARCH(1,1) volatility.

2. Compare the results.

Review of the assignment (1.a)

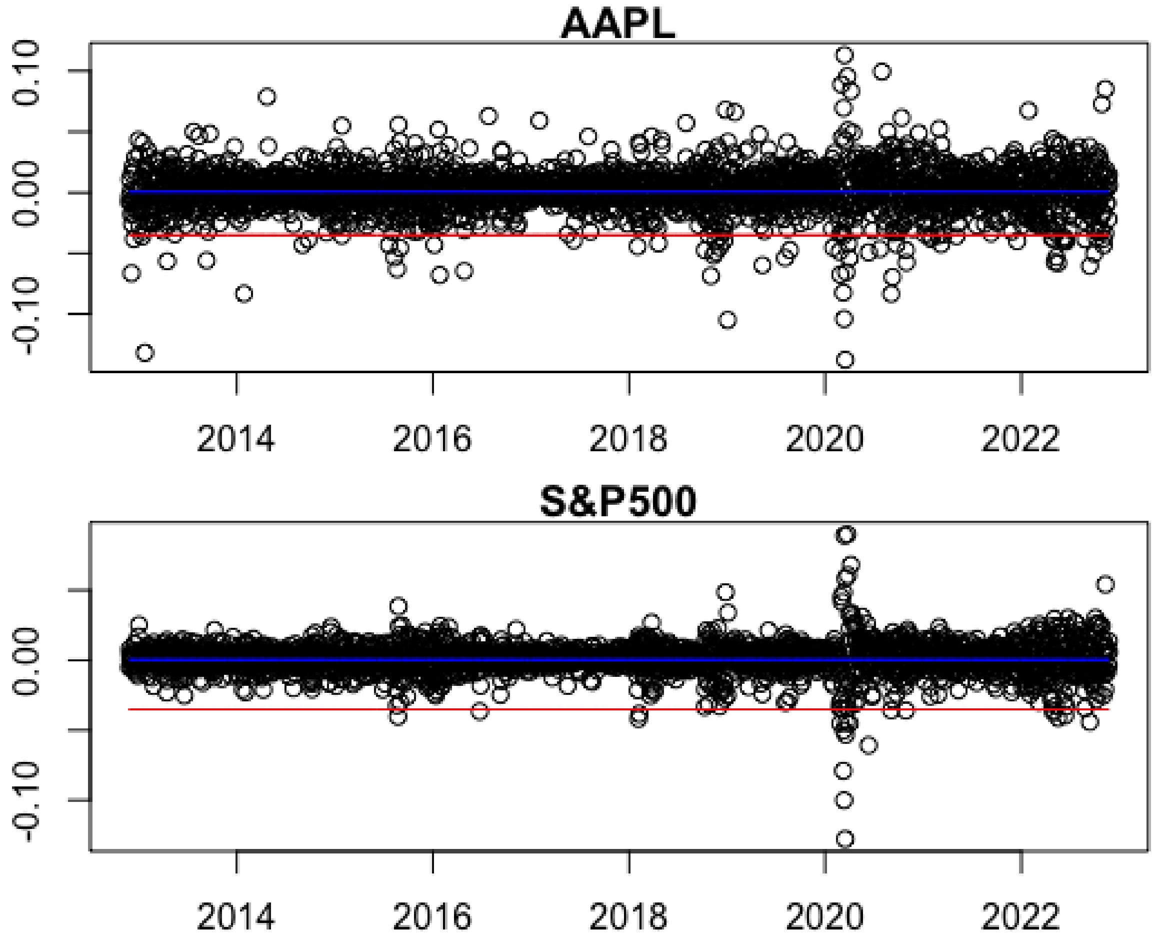
```
library('quantmod')
getSymbols(c('AAPL','^GSPC'),src='yahoo',
           from="2012-11-25",periodicity = 'daily')
r.AAPL<-diff(log(AAPL[,6]))
r.SP500<-diff(log(GSPC[,6]))
r.AAPL<-r.AAPL[2:length(r.AAPL)]
r.SP500<-na.omit(r.SP500)
```

```
VAR_AAPL<-arima0$coef-1.96*vol0AAPL
VAR_SP500<-mean(r.SP500)-1.96*vol0SP500
```

```
arima0<-arima(r.AAPL,order=c(0,0,0))
res0AAPL<-arima0$residuals
arima0SP500<-mean(r.SP500)
res0SP500<r.SP500-mean(r.SP500)
vol0AAPL<-sqrt(arima0$sigma2)
vol0SP500<-sqrt(var(r.SP500))
```

Review of the assignment 7 (1.a)

```
par(mfrow=c(2,1),mar=c(2.5,2.5,1,1))
plot(time(r.AAPL),r.AAPL,title('AAPL'))
lines(time(r.AAPL),rep(arima0$coef,
      length(r.AAPL)),col="blue")
lines(time(r.AAPL),rep(VAR_AAPL,
      length(r.AAPL)),col="red")
plot(time(r.SP500),r.SP500,title('S&P500'))
lines(time(r.SP500),rep(mean(r.SP500),
      length(r.SP500)),col="blue")
lines(time(r.SP500),rep(VAR_AAPL,
      length(r.AAPL)),col="red")
```



Review of the assignment (1.b)

```r
library(forecast)
arimaAAPL<-auto.arima(r.AAPL)
arimaSP500<-auto.arima(r.SP500)
arimaAAPL; arimaSP500
```

```
Series: r.AAPL
ARIMA(0,0,1) with non-zero mean

Coefficients:
          ma1   mean
      -0.0637  8e-04
s.e.   0.0200  3e-04
```

```
Series: r.SP500
ARIMA(3,0,3) with non-zero mean

Coefficients:
          ar1     ar2     ar3     ma1      ma2      ma3   mean
      -0.7649  0.8182  0.8577  0.6572  -0.8582  -0.7461  4e-04
s.e.   0.0317  0.0344  0.0276  0.0373   0.0273   0.0327  1e-04
```

```r
vol2AAPL<-sqrt(arimaAAPL$sigma2)
vol2SP500<-sqrt(arimaSP500$sigma2)
model2_AAPL<-arimaAAPL$fitted
model2_SP500<-arimaSP500$fitted
```

```r
VAR_AAPL_2<-model2_AAPL-1.96*vol2AAPL
VAR_SP500_2<-model2_SP500-1.96*vol2SP500
```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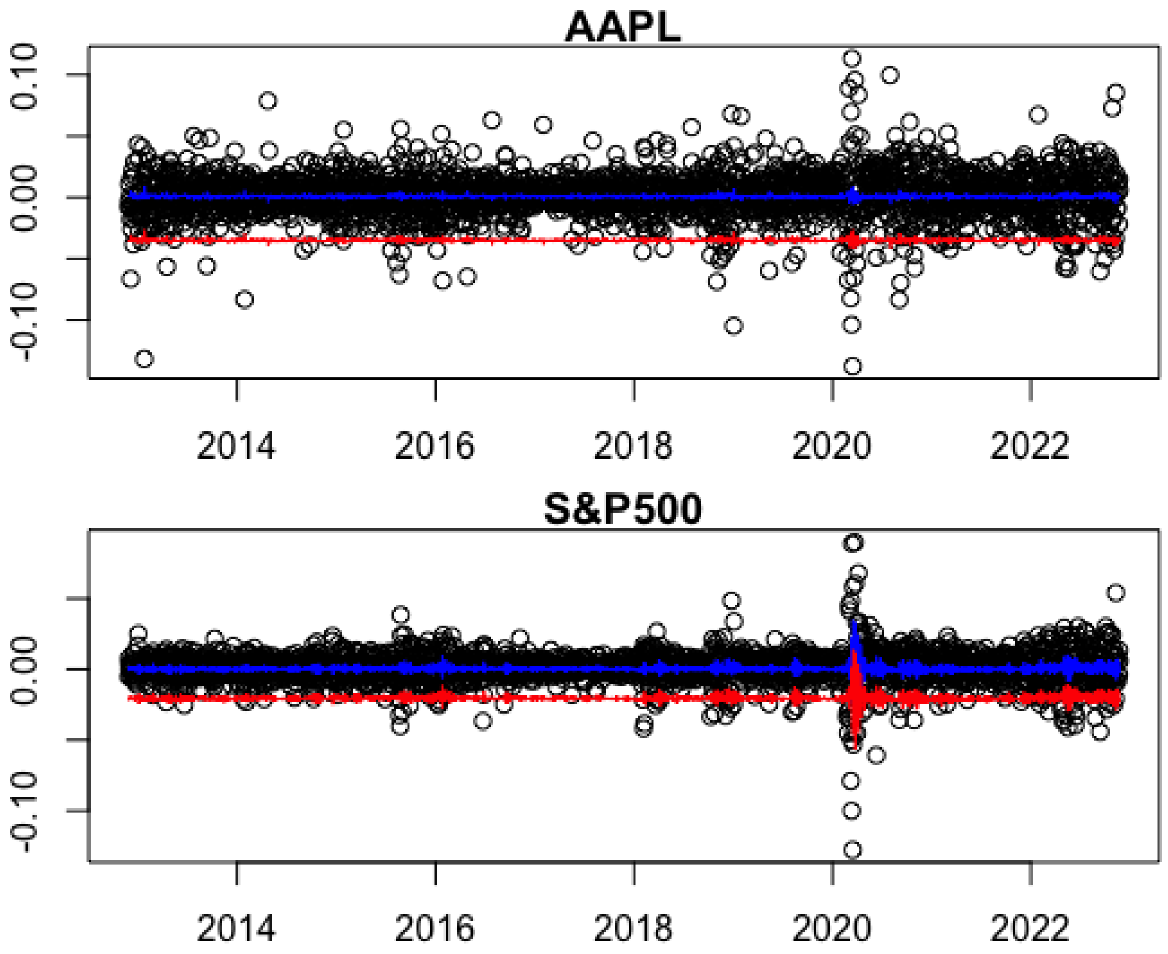
Review of the assignment (1.b)

```
plot(time(r.AAPL),r.AAPL,title('AAPL'))
lines(time(r.AAPL),model2_AAPL,col="blue")
lines(time(r.AAPL),VAR_AAPL_2,col="red")
plot(time(r.SP500),r.SP500,title('S&P500'))
lines(time(r.SP500),model2_SP500,col="blue")
lines(time(r.SP500),VAR_SP500_2,col="red")
```

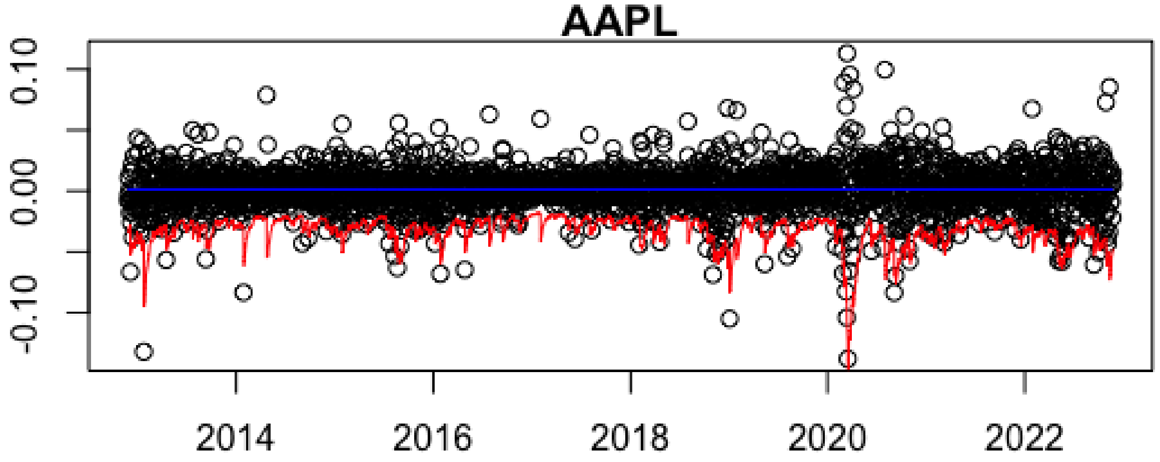


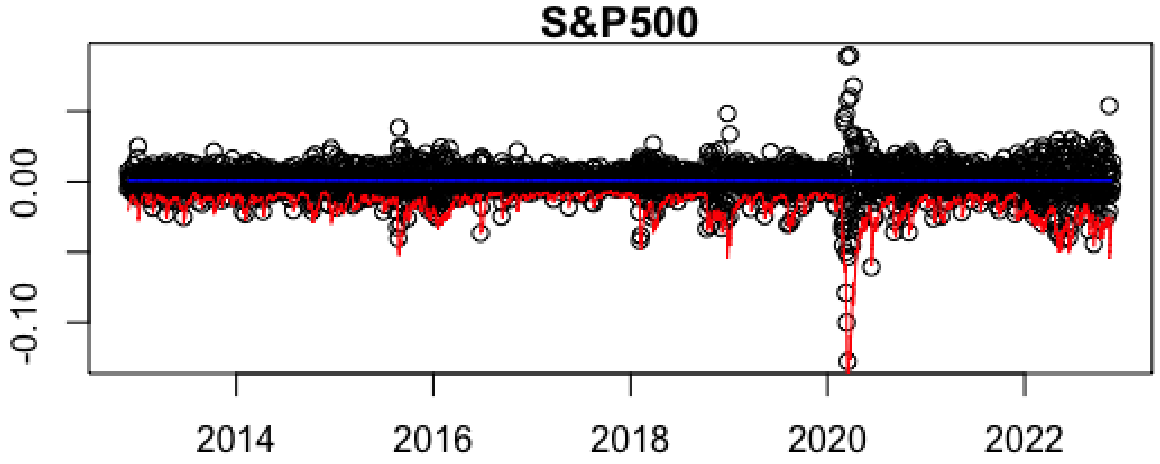
Review of the assignment (1.c)

```
library('rugarch')
garchSpec <- ugarchspec(
  variance.model=list(model="sGARCH",
                      garchOrder=c(1,1)),
  mean.model=list(armaOrder=c(0,0)),
  distribution.model="norm")
garchFit <- ugarchfit(spec=garchSpec, data=r.AAPL)
AAPL_hat <- garchFit@fit$fitted.values
AAPL_vol_hat <- ts(garchFit@fit$sigma)
garchFit2 <- ugarchfit(spec=garchSpec, data=r.SP500)
SP_hat <- garchFit2@fit$fitted.values
SP_vol_hat <- ts(garchFit2@fit$sigma)
```

```
VAR_AAPL_3<-AAPL_hat-1.96*AAPL_vol_hat
VAR_SP500_3<-SP_hat-1.96*SP_vol_hat
```

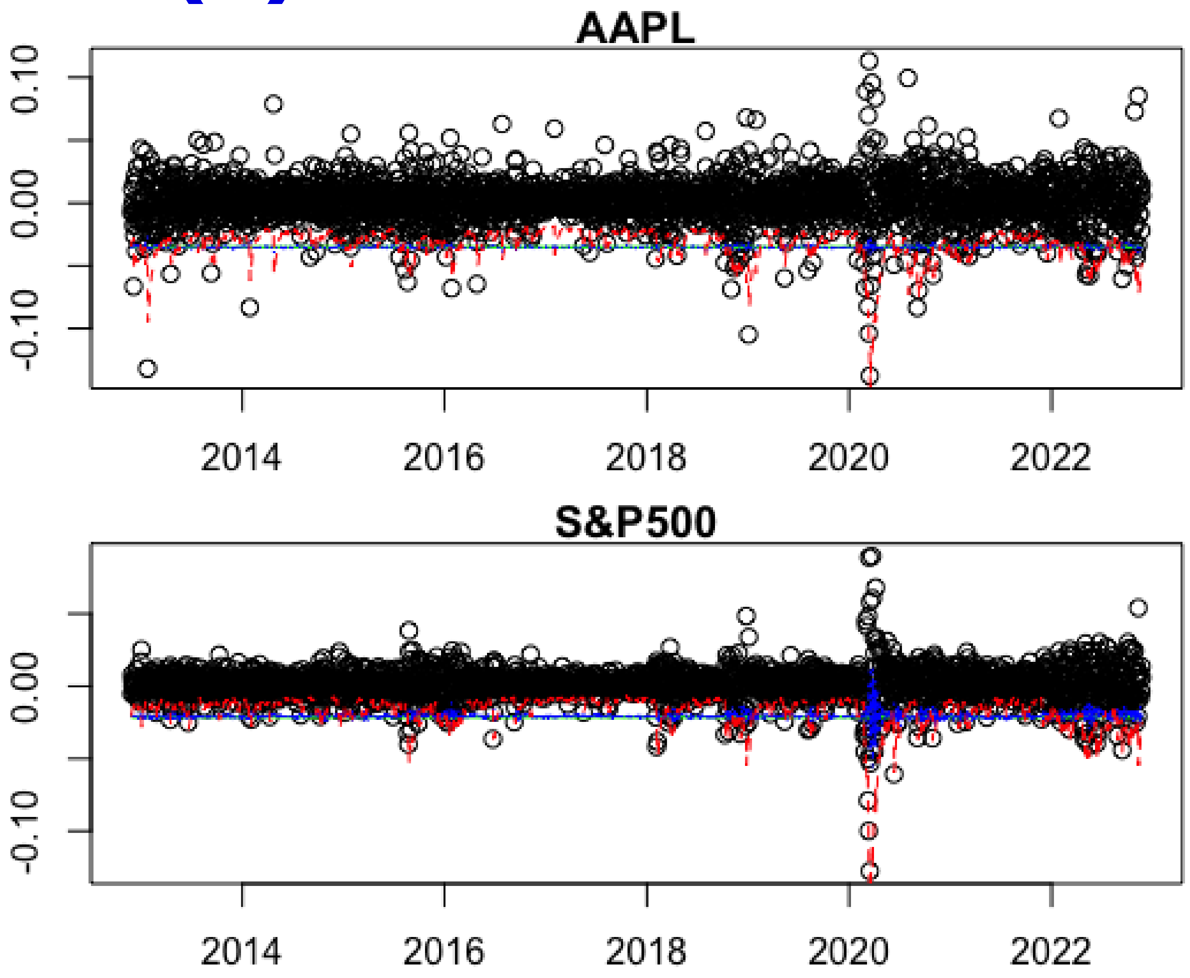
```
plot(time(r.AAPL),r.AAPL,title('AAPL'))
lines(time(r.AAPL),AAPL_hat,col="blue")
lines(time(r.AAPL),VAR_AAPL_3,col="red")
plot(time(r.SP500),r.SP500,title('S&P500'))
lines(time(r.SP500),SP_hat,col="blue")
lines(time(r.SP500),VAR_SP500_3,col="red")
```





Review of the assignment (2)

```
#Comparison
plot(time(r.AAPL),r.AAPL,title('AAPL'))
lines(time(r.AAPL),rep(VAR_AAPL,
      length(r.AAPL)),col="green")
lines(time(r.AAPL),VAR_AAPL_2,col="blue",lty='dotted')
lines(time(r.AAPL),VAR_AAPL_3,col="red",lty='dashed')
plot(time(r.SP500),r.SP500,title('S&P500'))
lines(time(r.SP500),rep(VAR_SP500,
      length(r.AAPL)),col="green")
lines(time(r.SP500),VAR_SP500_2,col="blue",lty='dotted')
lines(time(r.SP500),VAR_SP500_3,col="red",lty='dashed')
```



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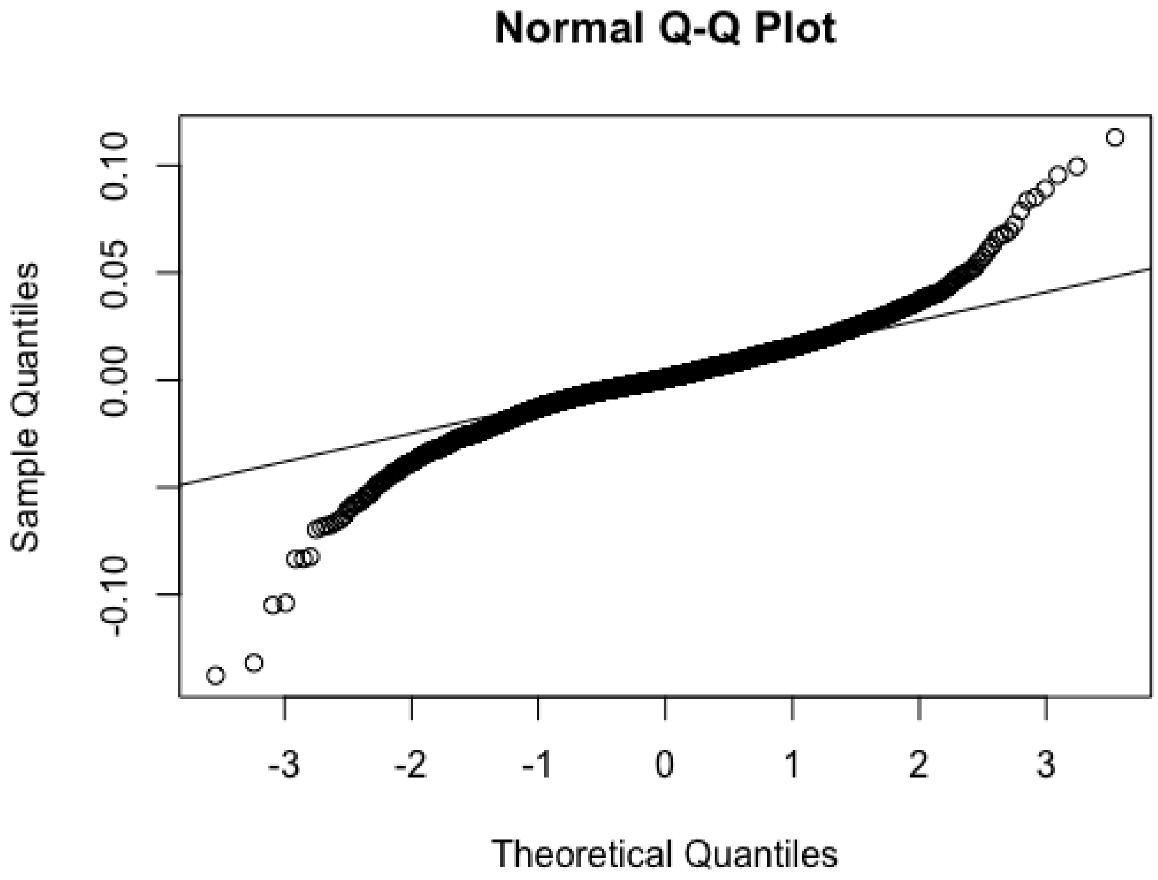
VaR computation

- Returns exhibits heavy tails than normal distribution.

- t-student distribution: Similar shape than normal distribution with heavy tails.



VaR computation

```
pars<-fitdist(distribution = 'std' , x = r.AAPL)$pars
```

```
> pars
         mu       sigma       shape
0.001187087 0.019318014 3.284576519
```

```
pars<-fitdist(distribution = 'std' , x = r.AAPL)$pars
quantile(r.AAPL,0.05)
qnorm(p = 0.05,mean=mean(r.AAPL), sd=sqrt(var(r.AAPL)))
qdist(distribution = 'std' ,mu=pars[1], sigma=pars[2],
      shape = pars[3] , p = 0.05)
```

```
quantile(r.AAPL,0.01)
qnorm(p = 0.01,mean=mean(r.AAPL), sd=sqrt(var(r.AAPL)))
qdist(distribution = 'std' ,mu=pars[1], sigma=pars[2],
      shape = pars[3] , p = 0.01)
```

Uncond.	Normal	t-student
5% -0.027507	-0.02934347	-0.02626725
1% -0.05298592	-0.04185024	-0.0500444

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VaR computation

– Delta normal VaR

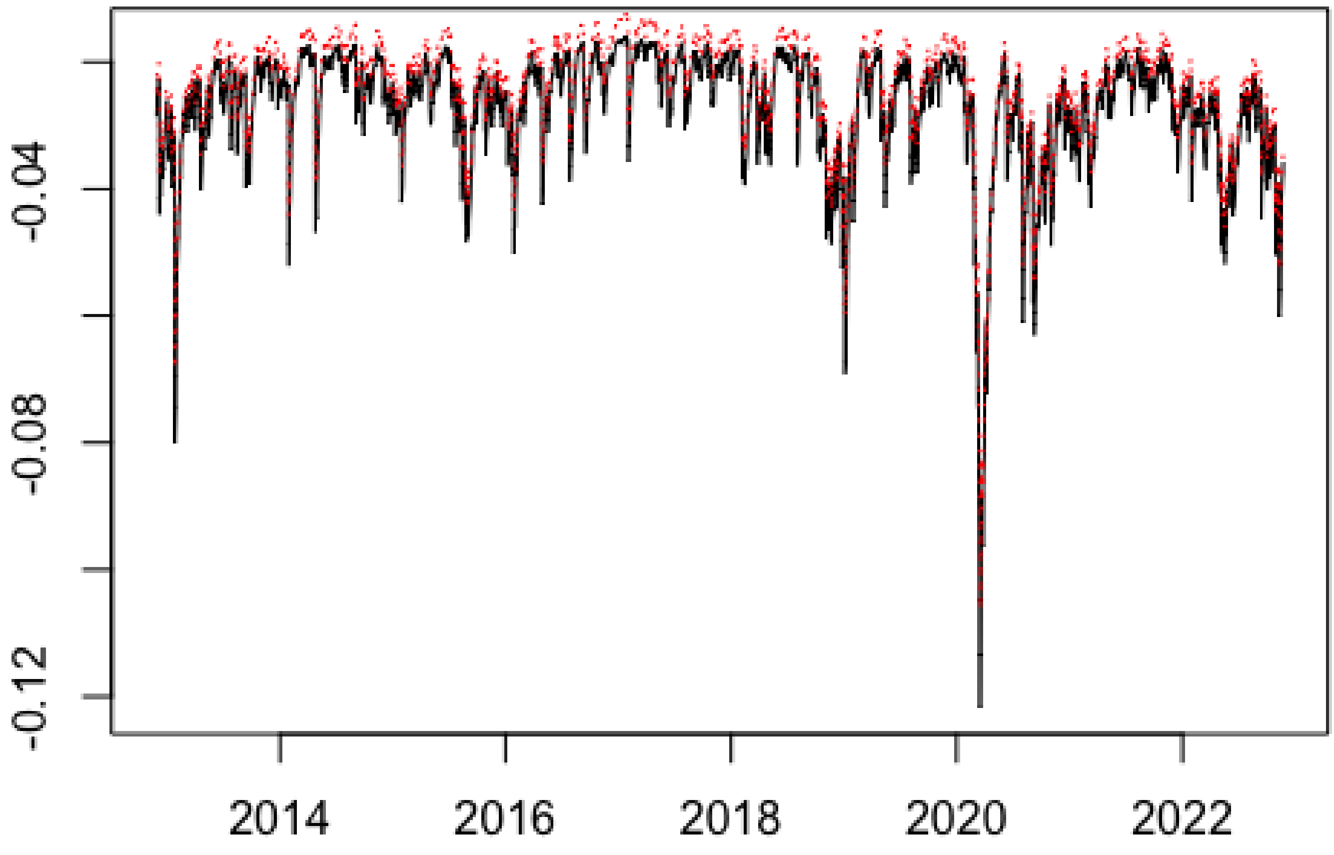
$$\begin{aligned}\mathrm{VaR}(a) &= N^{-1}(a,\mu,\sigma) \\ &= \mu + \sigma * N^{-1}(a)\end{aligned}$$

– GARCH(1,1) VaR with student's t-distribution in the underlying:

$$\mathrm{VaR}(a)_t = \mu + \sigma_{t|t-1} * F^{-1}(a)$$

VaR computation

```
garchSpec2 <- ugarchspec(
  variance.model=list(model="sGARCH",
                      garchOrder=c(1,1)),
  mean.model=list(armaOrder=c(0,0)),
  distribution.model="std")
garchFit2<- ugarchfit(spec=garchSpec2, data=r.AAPL)
AAPL_hat2 <- garchFit2@fit$fitted.values
AAPL_vol_hat2 <- ts(garchFit2@fit$sigma)
# 95-VAR under normality
VAR_AAPL_N<-AAPL_hat+AAPL_vol_hat*qnorm(p=.05)
# 95-VAR under t-student
VAR_AAPL_t<-AAPL_hat2+AAPL_vol_hat2*qdist(
  distribution = 'std', p = 0.05,shape = pars[3])
#plot
plot(time(r.AAPL),VAR_AAPL_N,type='line')
lines(time(r.AAPL),VAR_AAPL_t,col='red',lty='dotted')
```



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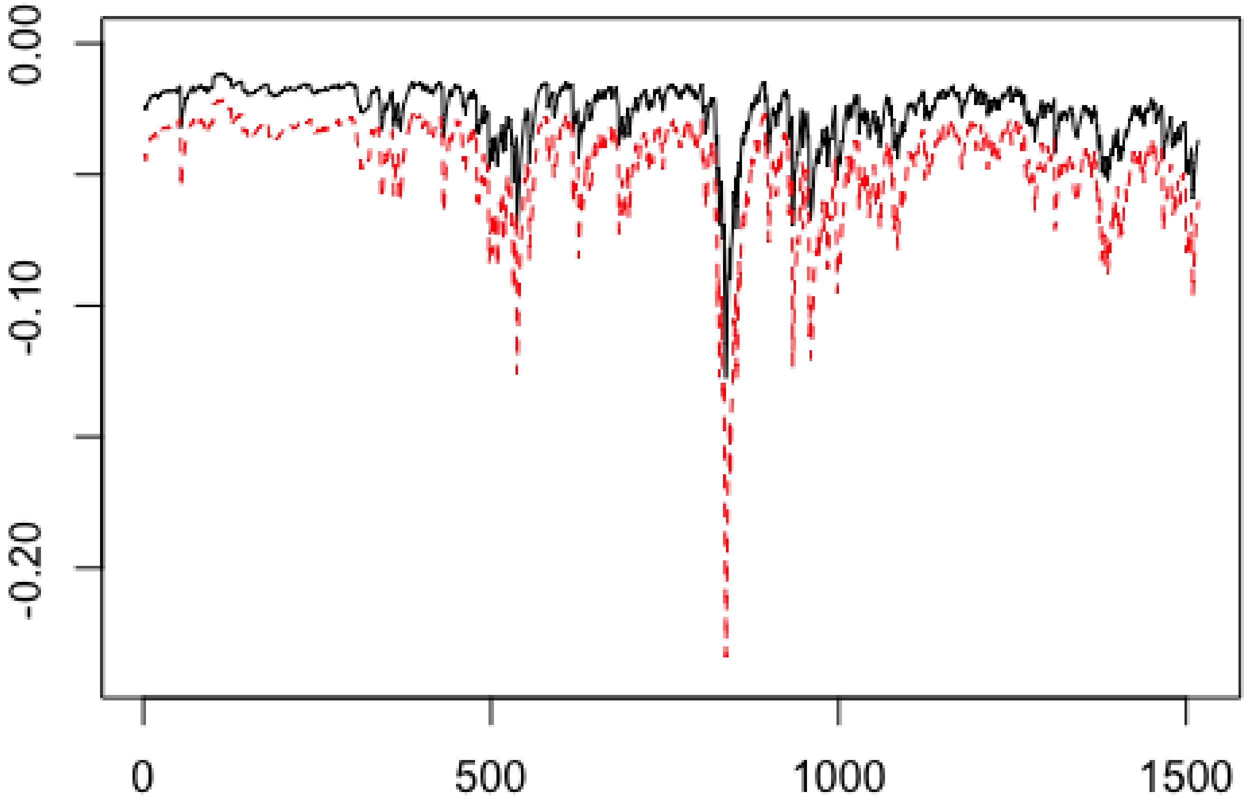
Out of sample GARCH/VaR forecasting

– ugarchroll: Rolling estimation and forecasting via GARCH family.

```
rolling_est<- ugarchroll(garchSpec2,
            r.AAPL, n.start=1000,
            refit.every = 25,
            refit.window = "moving",
            VaR.alpha = c(0.01, 0.05))
Var95<-rolling_est@forecast$VaR[,2]
Var99<-rolling_est@forecast$VaR[,1]
plot(Var95,type='l',ylim=c(-0.24,0))
lines(Var99,lty='dashed',col='red')
```



Out of sample GARCH/VaR performance: Backtesting

- How good/bad was our volatility forecasting?
  - Compare the forecasted value with the model realized value

- How good/bad is the forecasted VaR?
  - Unconditional and conditional Coverage tests

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Realized vs forecasted volatility

```
prediction<-as.data.frame(rolling_est)
head(prediction)
#Prediction error for the mean
error_mean<-prediction$Realized-prediction$Mu
#Prediction error for the variance
error_var<-error_mean^2-prediction$Sigma^2
mean(error_mean^2)
mean(error_var^2)
```

	Mu	Sigma	Skew	Shape	Shape(GIG)	Realized
2016-11-15	0.000747833	0.01650391	0	4.214219	0	0.0131567844
2016-11-16	0.000747833	0.01623713	0	4.214219	0	0.0265334138
2016-11-17	0.000747833	0.01720848	0	4.214219	0	-0.0003638326
2016-11-18	0.000747833	0.01648021	0	4.214219	0	0.0009998037
2016-11-21	0.000747833	0.01584315	0	4.214219	0	0.0150597476
2016-11-22	0.000747833	0.01580330	0	4.214219	0	0.0006262995

```
> mean(error_mean^2)
[1] 0.00038391
> mean(error_var^2)
[1] 0.0003852829
```


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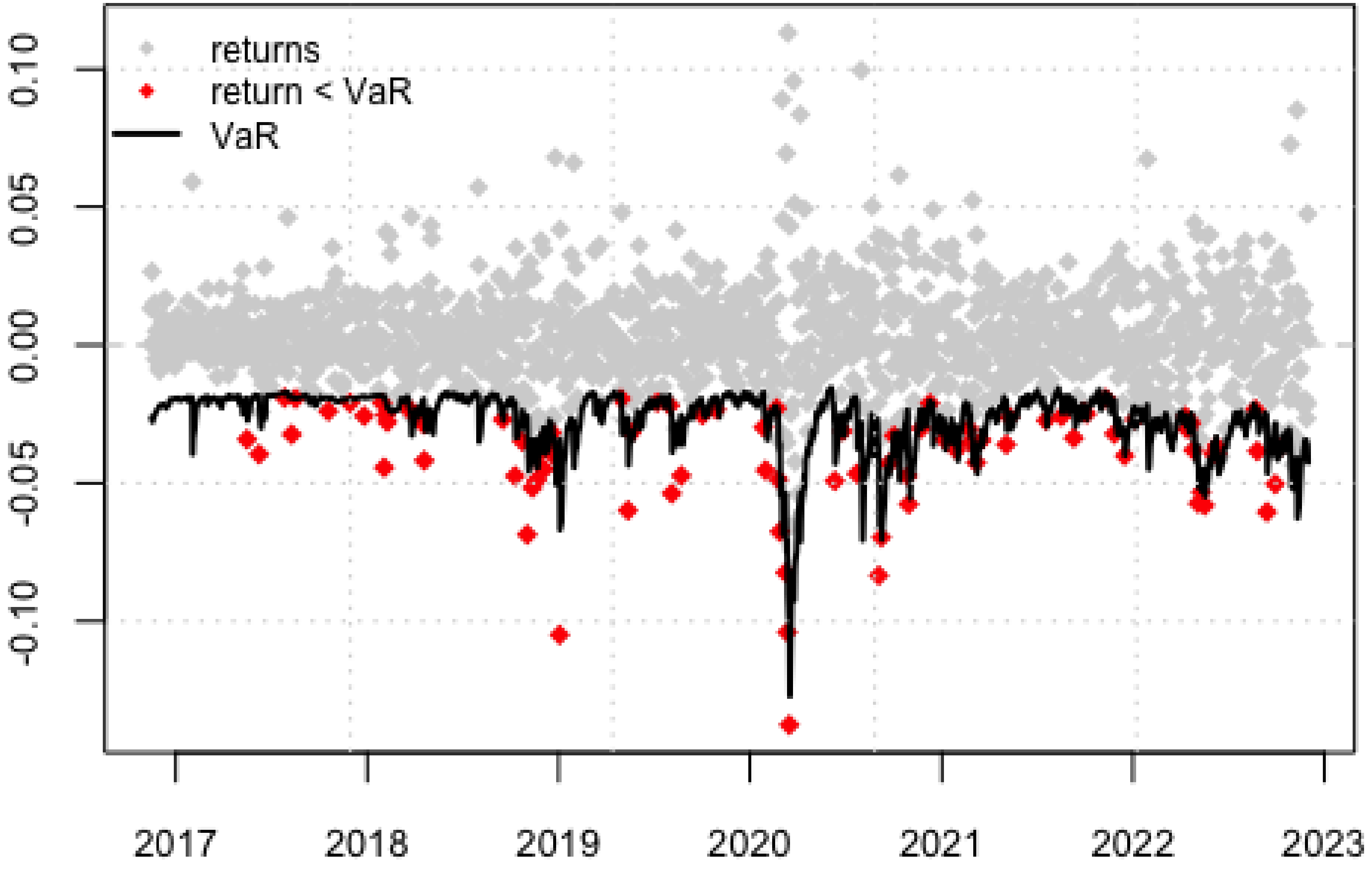
Backtesting VaR models

```
plot(rolling_est,which=4,VaR.alpha=0.05)
# or equivalently
VaRplot(0.05, r.AAPL[1001:length(r.AAPL)], Var95)
```

Forecasted VaR



Backtesting VaR models

```
report(rolling_est,VaR.alpha=0.05)
#analogously
VaRTest(0.05, r.AAPL[1001:length(r.AAPL)], Var95,0.95)
```

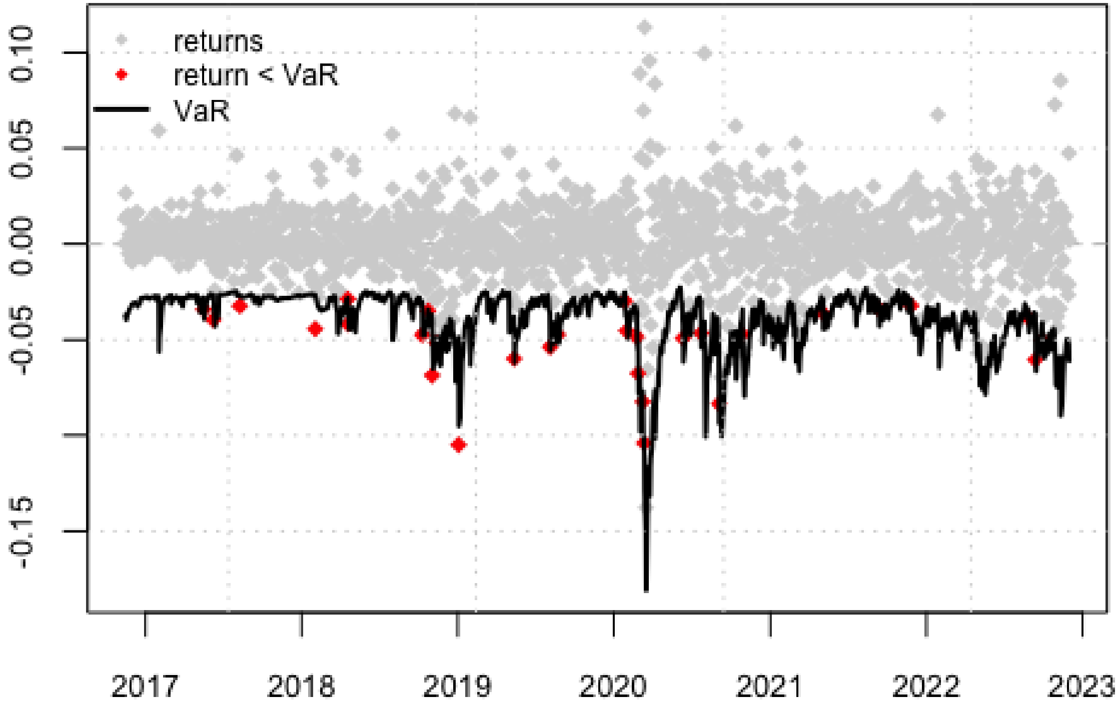
```
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1522
Data:

==========================================
alpha:                          5%
Expected Exceed:        76.1
Actual VaR Exceed:      91
Actual %:                       6%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        2.898
LR.uc Critical:         3.841
LR.uc p-value:          0.089
Reject Null:            NO

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        2.96
LR.cc Critical:         5.991
LR.cc p-value:          0.228
Reject Null:            NO
```

Backtesting VaR models




```
plot(rolling_est,which=4,VaR.alpha=0.01)
```

```
report(rolling_est,VaR.alpha=0.01)
```

```
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1522
Data:

==========================================
alpha:                          1%
Expected Exceed:        15.2
Actual VaR Exceed:      30
Actual %:                       2%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        11.301
LR.uc Critical:         3.841
LR.uc p-value:          0.001
Reject Null:            YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                      Independence of Failures
LR.cc Statistic:        11.545
LR.cc Critical:         5.991
LR.cc p-value:          0.003
Reject Null:            YES
```