MUNI ECON

Applied Financial Econometrics

Multivariate volatility models

Lecturer: Axel A. Araneda, Ph.D.

Single asset case

$$r_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1)$$

$$\sigma_t^2 = f\left(\epsilon_{t-1}^2, \sigma_{t-1}^2\right)$$

Multivariate case

$$r_{i,t} = \sigma_{i,t}\epsilon_{i,t}, \quad \epsilon_t \sim MVN(0,1)$$

$$\text{cov}(r_{i,t}, r_{j,t}) = \sigma_{ij,t} = \sigma_{i,t}\sigma_{j,t}\rho_{ij,t}$$

$$\sigma_{i,t}^2 = f\left(\epsilon_{i,t-1}^2, \sigma_{i,t-1}^2\right)$$

Covariance matrix

$$\Sigma_t = \begin{bmatrix} \sigma^2_{1,t} & \sigma_{1,t}\sigma_{2,t}\rho_{12,t} & \cdots & \sigma_{1,t}\sigma_{N,t}\rho_{1N,t} \\ \sigma_{2,t}\sigma_{1,t}\rho_{21,t} & \sigma^2_{2,t} & \cdots & \sigma_{2,t}\sigma_{N,t}\rho_{2N,t} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N,t}\sigma_{1,t}\rho_{N1,t} & \sigma_{N,t}\sigma_{2,t}\rho_{N2,t} & \cdots & \sigma^2_{N,t} \end{bmatrix}$$

– Symmetric and semi-positive definite!

Common Approaches: GARCH-CCC

– Covariance

$$\sigma_{ij,t} = \sigma_{i,t}\sigma_{j,t}\rho_{ij}$$

– Formally (matrix notation):

$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_t + \boldsymbol{\epsilon}_t$$
$$\boldsymbol{\epsilon}_t = \mathbf{H}_t^{1/2}\boldsymbol{\nu}_t$$
$$\mathbf{H}_t = \mathbf{D}_t^{1/2}\mathbf{R}\mathbf{D}_t^{1/2}$$

Common Approaches: GARCH-CCC

$$\mathbf{D}_t = \begin{pmatrix} \sigma^2_{1,t} & 0 & \cdots & 0 \\ 0 & \sigma^2_{2,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2_{m,t} \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1m} \\ \rho_{12} & 1 & \cdots & \rho_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1m} & \rho_{2m} & \cdots & 1 \end{pmatrix}$$

in which each $\sigma^2_{i,t}$ evolves according to a univariate GARCH model of the form

$$\sigma^2_{i,t} = s_i + \sum_{j=1}^{p_i} \alpha_j \epsilon^2_{i,t-j} + \sum_{j=1}^{q_i} \beta_j \sigma^2_{i,t-j}$$

Common Approaches: GARCH-DCC

– Covariance $$\sigma_{ij,t} = \sigma_{i,t}\sigma_{j,t}\rho_{ij,t}$$

– Formally (matrix notation):

$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_t + \boldsymbol{\epsilon}_t$$
$$\boldsymbol{\epsilon}_t = \mathbf{H}_t^{1/2}\boldsymbol{\nu}_t$$
$$\mathbf{H}_t = \mathbf{D}_t^{1/2}\mathbf{R}_t\mathbf{D}_t^{1/2}$$
$$\mathbf{R}_t = \text{diag}(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t\text{diag}(\mathbf{Q}_t)^{-1/2}$$
$$\mathbf{Q}_t = (1-\lambda_1-\lambda_2)\mathbf{R} + \lambda_1\widetilde{\boldsymbol{\epsilon}}_{t-1}\widetilde{\boldsymbol{\epsilon}}'_{t-1} + \lambda_2\mathbf{Q}_{t-1}$$

Common Approaches: GARCH-CCC

$$\mathbf{D}_t = \begin{pmatrix} \sigma^2_{1,t} & 0 & \cdots & 0 \\ 0 & \sigma^2_{2,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2_{m,t} \end{pmatrix} \qquad \mathbf{R}_t = \begin{pmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1m,t} \\ \rho_{12,t} & 1 & \cdots & \rho_{2m,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1m,t} & \rho_{2m,t} & \cdots & 1 \end{pmatrix}$$

$\widetilde{\boldsymbol{\epsilon}}_t$ is an $m \times 1$ vector of standardized residuals, $\mathbf{D}_t^{-1/2}\boldsymbol{\epsilon}_t$; and

λ_1 and λ_2 are parameters that govern the dynamics of conditional quasicorrelations. λ_1 and λ_2 are nonnegative and satisfy $0 \leq \lambda_1 + \lambda_2 < 1$.

Common Approaches: EWMA

– Covariance

$$\sigma_{ij,t+1} = (1-\lambda)\sum_{n=0}^{\infty}\lambda^n r_{i,t-n} r_{j,t-n}$$
$$= (1-\lambda) r_{i,t-n} r_{j,t-n} + \lambda\sigma_{ij,t-n}$$

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Example of DCC-Garch in R

```
rm(list=ls())
#install.packages('rmgarch')
library('rmgarch')
library('rugarch')
getSymbols('AAPL',src='yahoo', from="2020-01-01",periodicity = 'daily')
getSymbols('MSFT',src='yahoo', from="2020-01-01",periodicity = 'daily')
r.AAPL<-diff(log(AAPL[,6]))
r.AAPL<-r.AAPL[2:length(r.AAPL)]
r.MSFT<-diff(log(MSFT[,6]))
r.MSFT<-r.MSFT[2:length(r.MSFT)]
```

Example of DCC-Garch in R

```
# Define univariate Garch model
Garch1.1 <- ugarchspec(variance.model=list(model="sGARCH",garchOrder=c(1,1)),
                       mean.model=list(armaOrder=c(0,0)),
                       distribution.model="norm")
# Define multivariate model
MGARCH<-dccspec(multispec(replicate(2,Garch1.1)),dccOrder = c(1,1),distribution = "mvnorm")
# fit to data
FitMGARCH<-dccfit(MGARCH,data=data.frame(r.AAPL,r.MSFT))
```


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Example of DCC-Garch in R

```
> FitMGARCH

*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  11
[VAR GARCH DCC UncQ] : [0+8+2+1]
No. Series           :  2
No. Obs.             :  1240
Log-Likelihood       :  6898.451
Av.Log-Likelihood    :  5.56
```

```
Optimal Parameters
-----------------------------------
                            Estimate  Std. Error  t value Pr(>|t|)
[AAPL.Adjusted].mu          0.001413    0.000483   2.9252 0.003442
[AAPL.Adjusted].omega       0.000011    0.000003   4.3202 0.000016
[AAPL.Adjusted].alpha1      0.087273    0.011519   7.5762 0.000000
[AAPL.Adjusted].beta1       0.882140    0.012459  70.8018 0.000000
[MSFT.Adjusted].mu          0.001019    0.000482   2.1152 0.034413
[MSFT.Adjusted].omega       0.000013    0.000002   6.3298 0.000000
[MSFT.Adjusted].alpha1      0.094510    0.011758   8.0382 0.000000
[MSFT.Adjusted].beta1       0.867577    0.013422  64.6386 0.000000
[Joint]dcca1                0.076845    0.040745   1.8860 0.059298
[Joint]dccb1                0.841842    0.123231   6.8314 0.000000
```

Example of DCC-Garch in R

```
> FitMGARCH

*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution          :  mvnorm
Model                 :  DCC(1,1)
No. Parameters        :  11
[VAR GARCH DCC UncQ]  : [0+8+2+1]
No. Series            :  2
No. Obs.              :  1240
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[Joint]dccb1                0.841842    0.123231   6.8314 0.000000
```
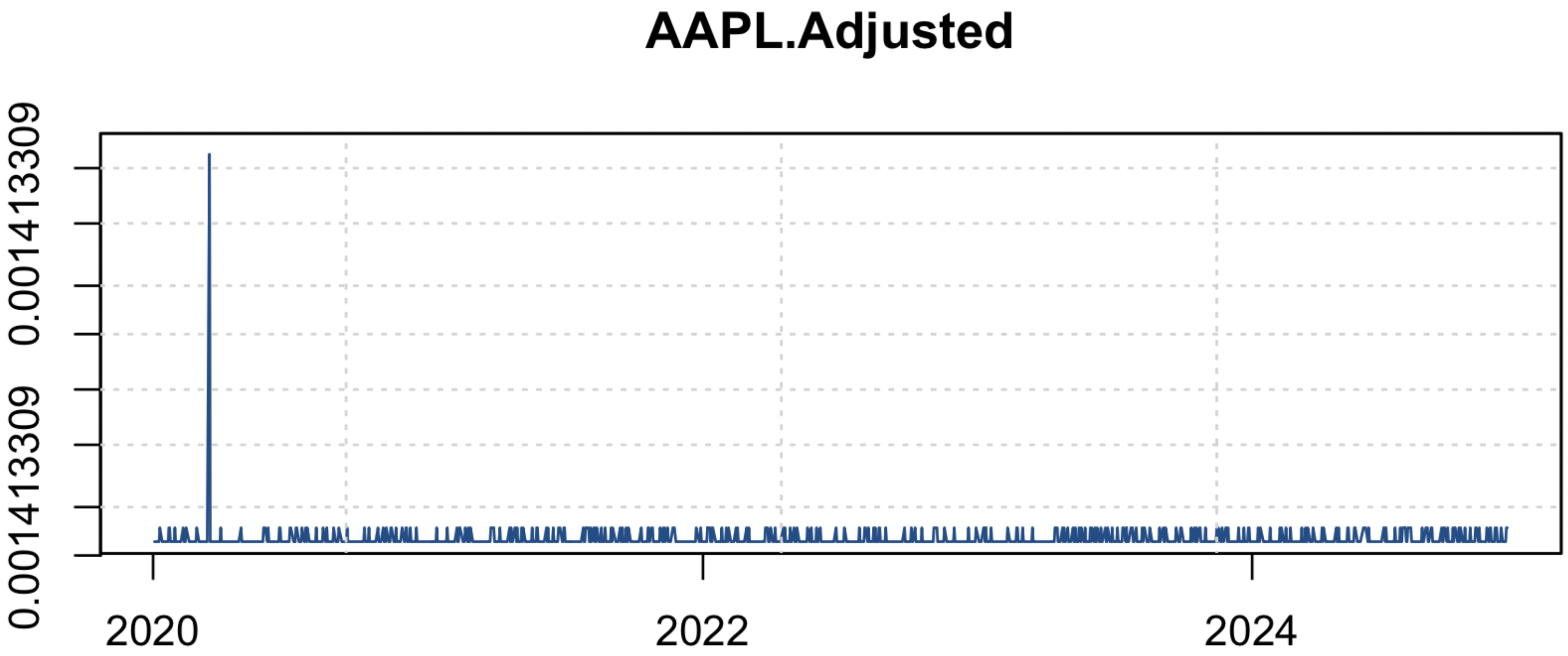

Example of DCC-Garch in R

```
plot(FitMGARCH)
```



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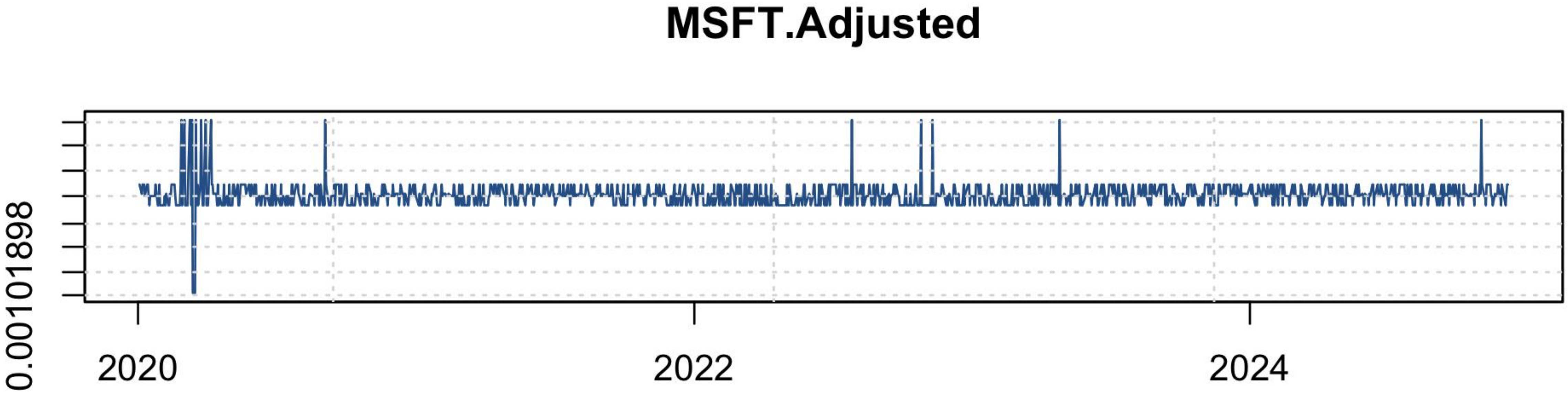
Example of DCC-Garch in R

```
plot(FitMGARCH)
```



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Example of DCC-Garch in R

```
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```

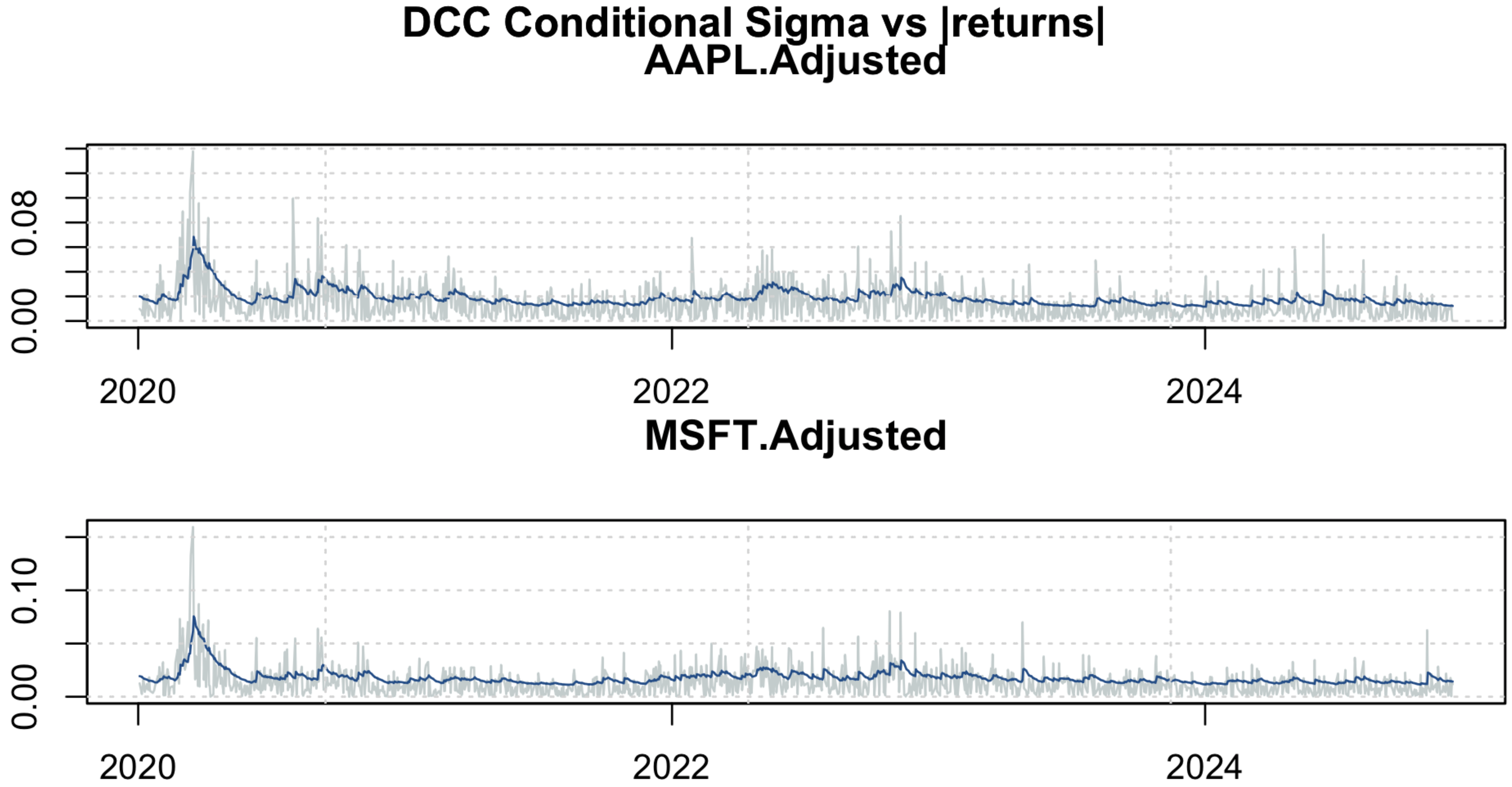

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Example of DCC-Garch in R

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plot(FitMGARCH)
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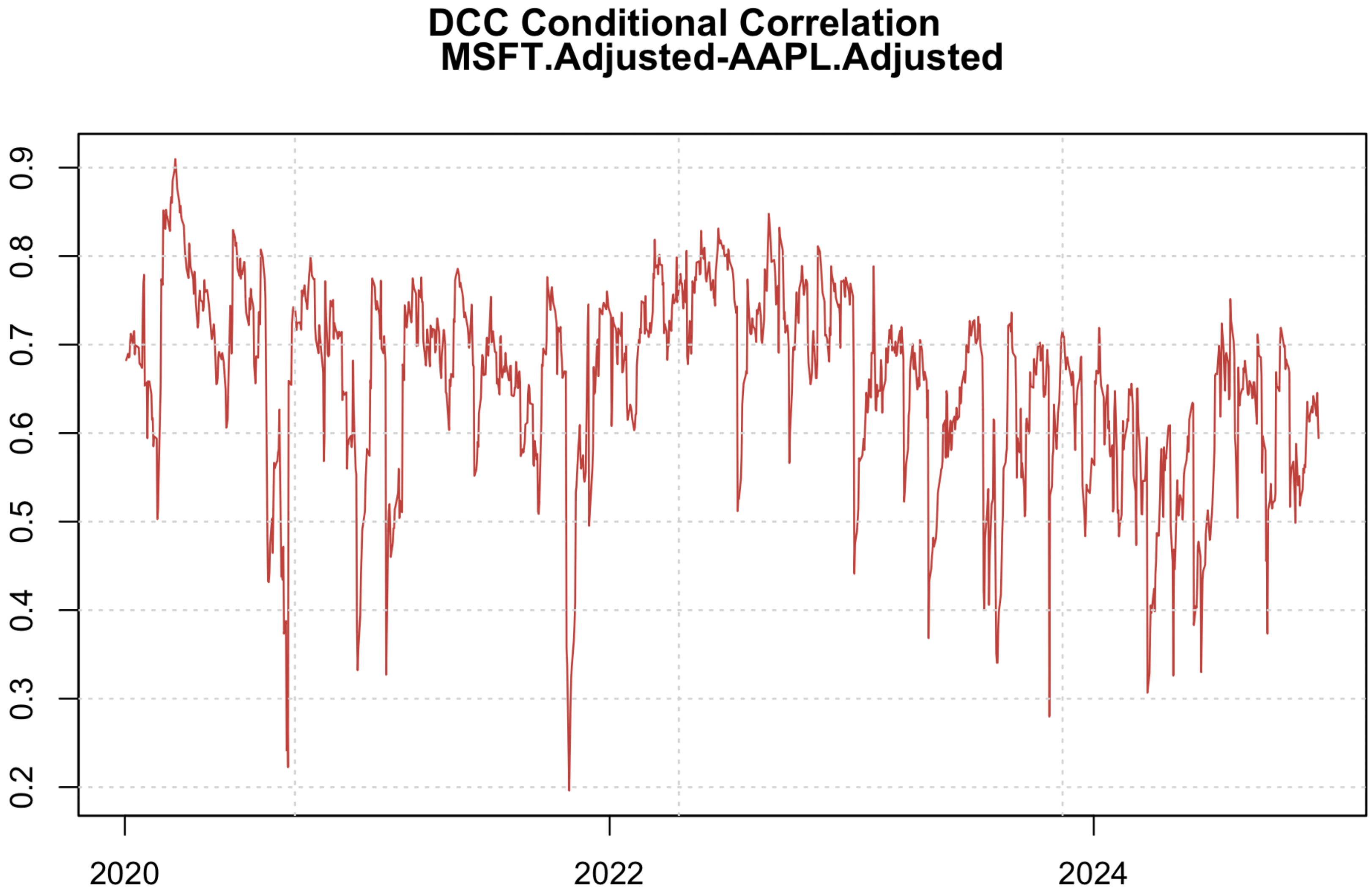

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