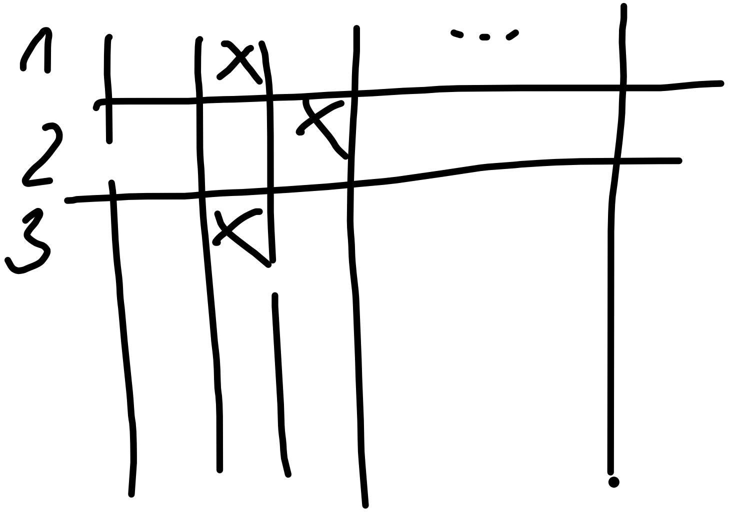
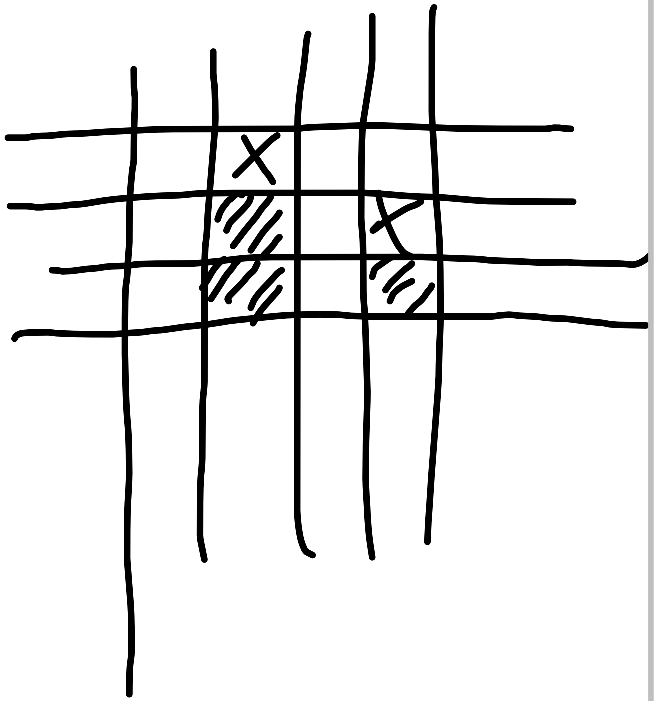
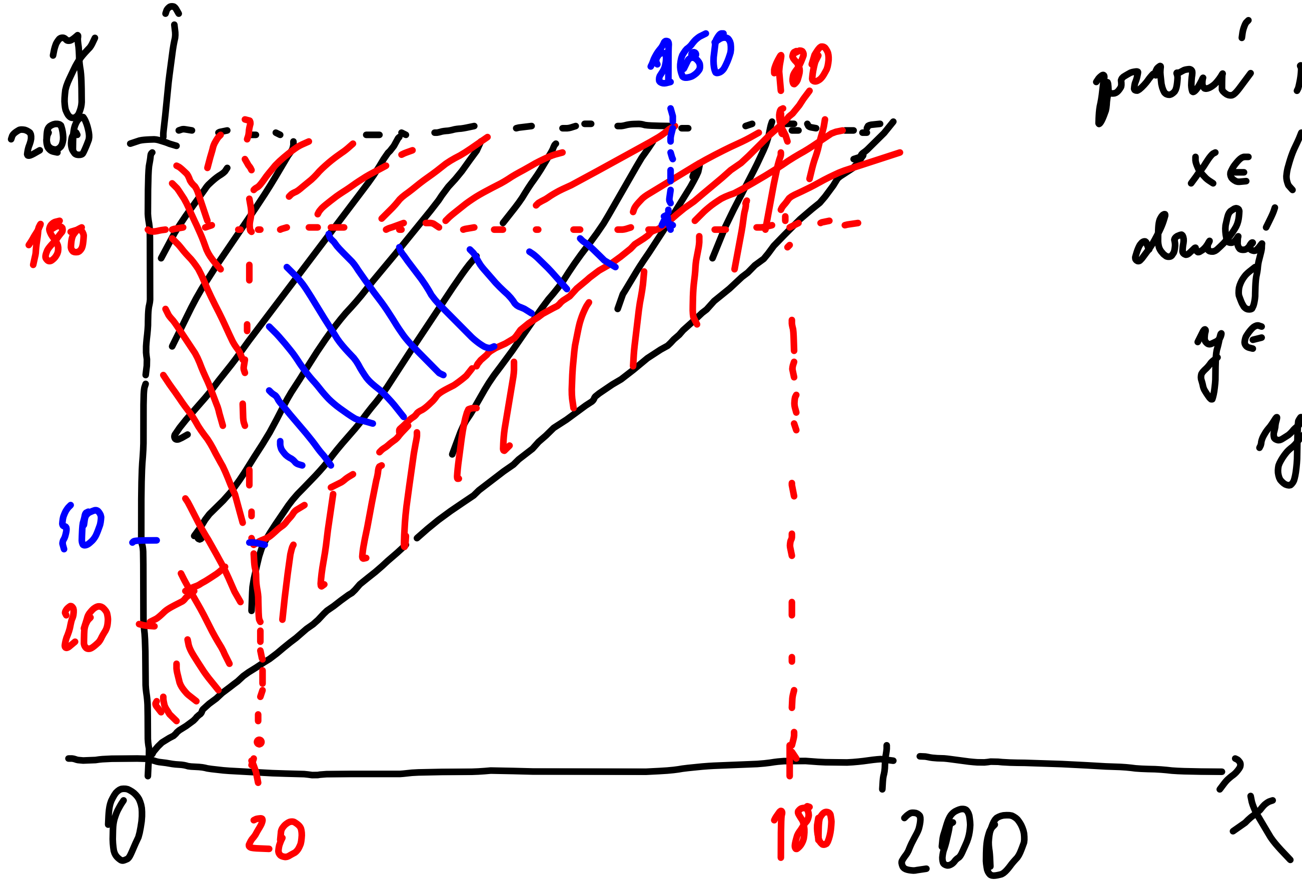

1
2
3





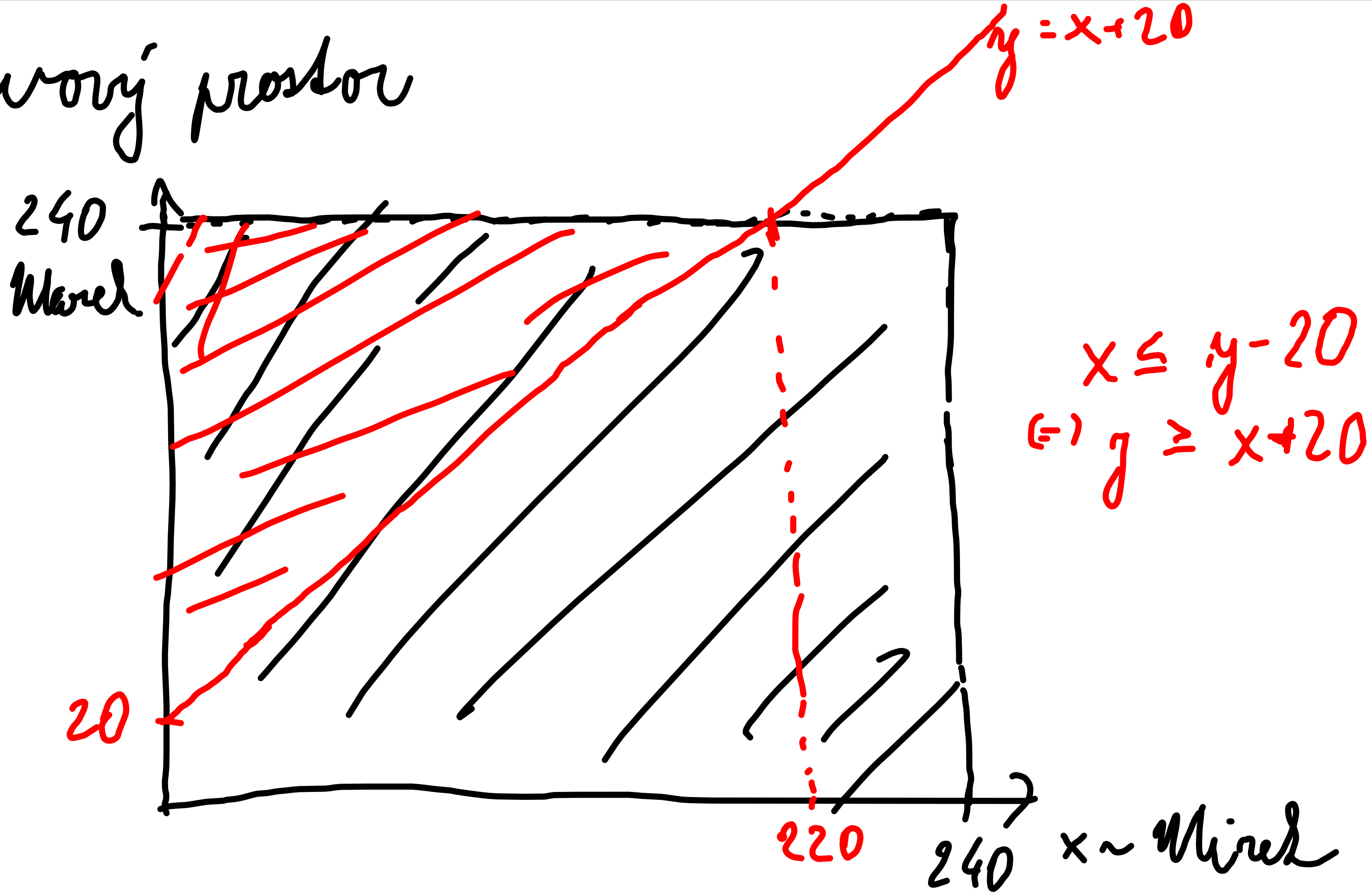
první řez:
$x \in (0, 200)$
druhý řez
$y \in (x, 200)$
$y > x$

příznivé případy:
$x \leq 20$
$y \geq 180$
$y - x \leq 20$

$$p(\bar{A}) = \frac{\frac{(140)^2}{2}}{\frac{(200)^2}{2}} = \left(\frac{7}{10}\right)^2 = \frac{49}{100}$$

$$p(A) = \frac{51}{100}$$

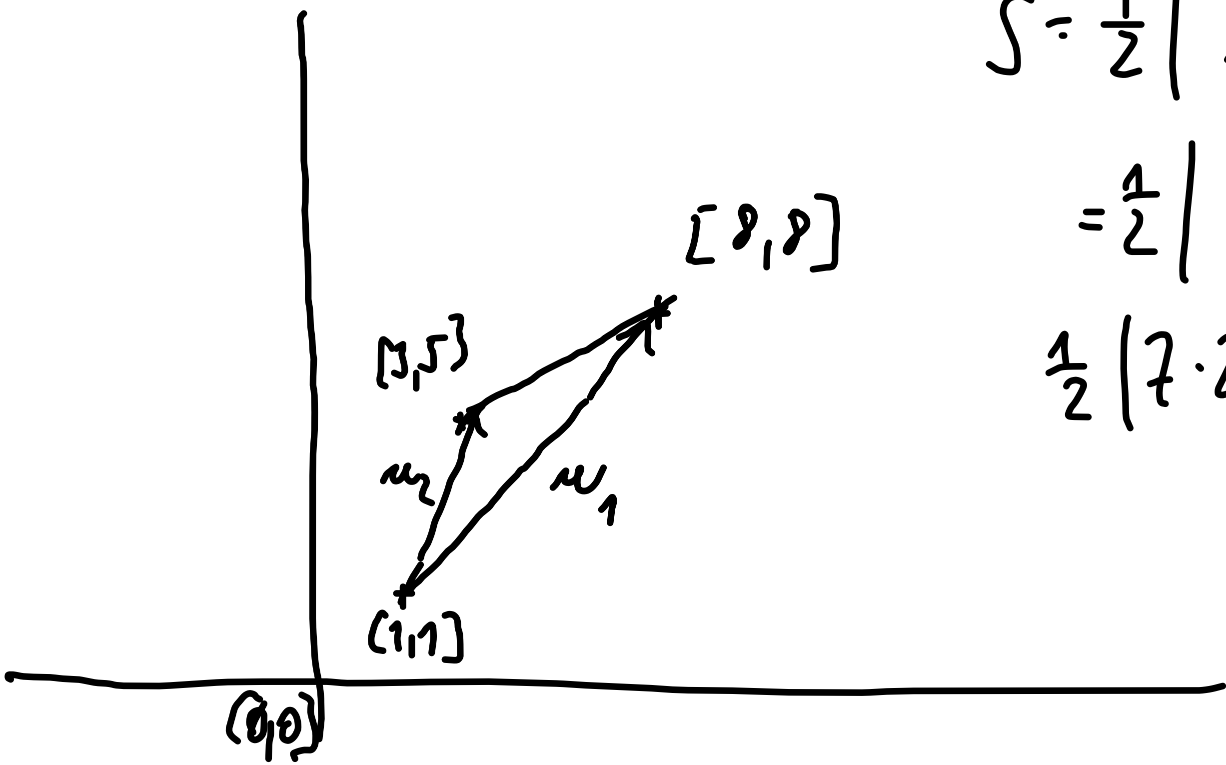
Stavový prostor



Marek jede do Prahy $\frac{200}{120}\,h = 1\frac{2}{3}\,h = 100$ min

Mirek jede do Prahy $\frac{200}{100}\,h = 2\,h = 120$ min

$$p(\text{Mirek přijede dříve}) = \frac{\frac{(220)^2}{2}}{(240)^2} = \frac{11^2}{2 \cdot 12^2} = \frac{121}{288}$$



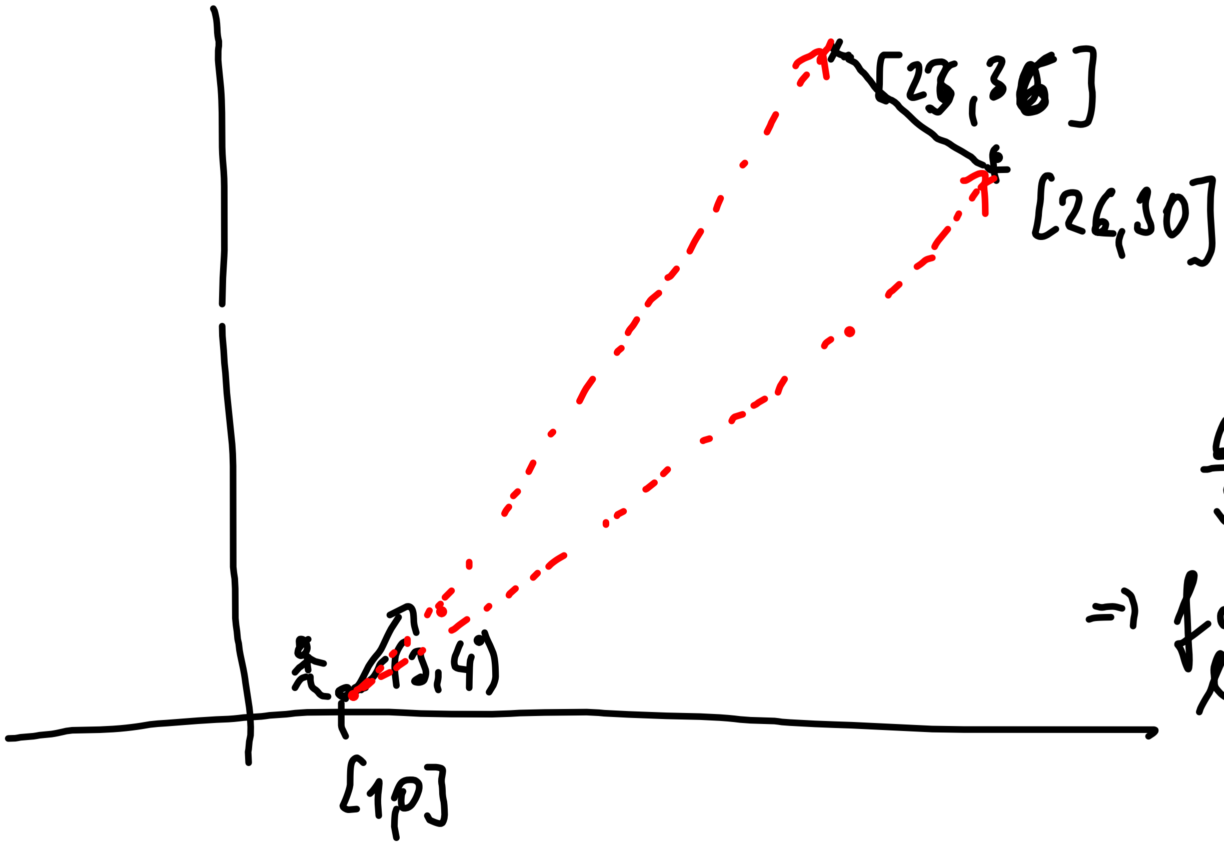
$$S = \frac{1}{2}\begin{vmatrix} u_1^T \\ u_2^T \end{vmatrix} = $$

$$= \frac{1}{2}\begin{vmatrix} 7 & 7 \\ 2 & 4 \end{vmatrix} =$$

$$\frac{1}{2}(7 \cdot 2) = 7$$

$$u_1 = [8,8] - [1,1] = (7,7)$$

$$u_2 = [3,5] - [1,1] = (2,4)$$



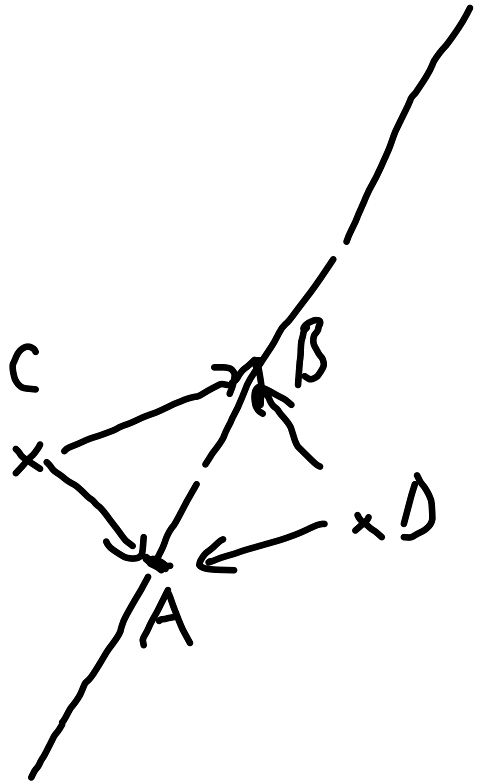
$$\frac{6}{5} < \frac{5}{3} < \frac{18}{11}$$

$\Rightarrow$ fotbalista bránu trefí

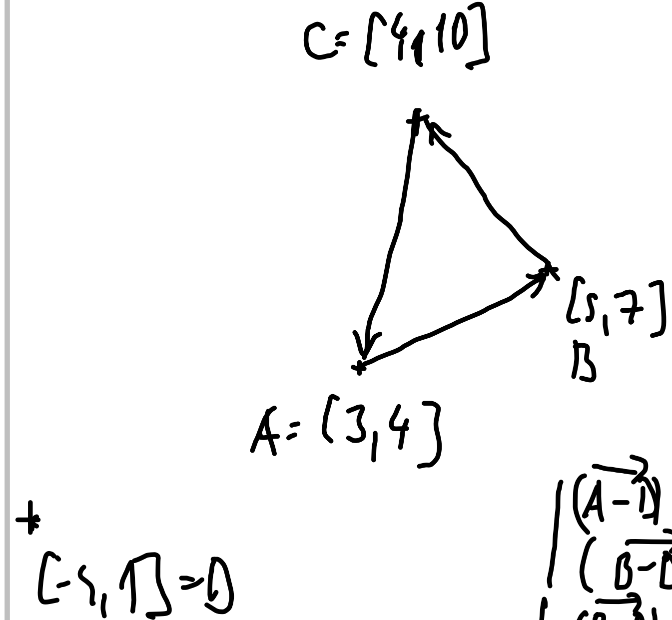
Porovnáme směrnice daných vektorů $\Big(\underset{s_1}{\overrightarrow{F1T}},\ \underset{s_2}{\overrightarrow{F2T}},\ \underset{s_3}{(3,5)}\Big)$:

$$s_1 = \frac{36-0}{23-1} = \frac{36}{22} = \frac{18}{11} \qquad s_3 = \frac{5}{3}$$

$$s_2 = \frac{30-0}{26-1} = \frac{30}{25} = \frac{6}{5}$$



$$\operatorname{sgn}\begin{vmatrix} \overrightarrow{(A-C)} \\ \overrightarrow{(B-C)} \end{vmatrix} = -\operatorname{sgn}\begin{vmatrix} \overrightarrow{(A-D)} \\ \overrightarrow{(B-D)} \end{vmatrix}!$$



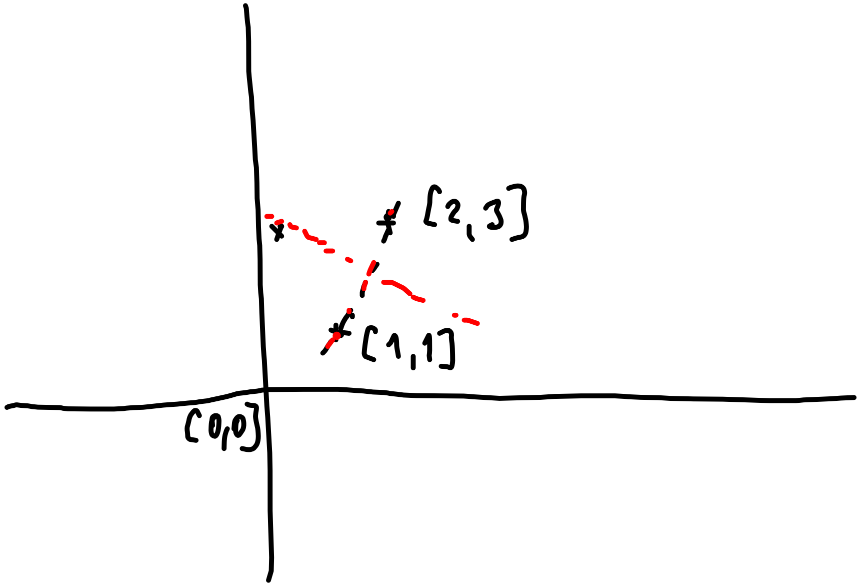
Orientujeme hranici trojúhelníka. Zjistíme na kterou stranu od vektorů v orientované hranici leží bod [-4,1]. To zjistíme pomocí determinantů:

$$\begin{vmatrix} (\overrightarrow{A-D}) \\ (\overrightarrow{B-D}) \end{vmatrix} = \begin{vmatrix} 7 & 3 \\ 9 & 6 \end{vmatrix} = 42-27>0$$

$$\begin{vmatrix} (\overrightarrow{B-D}) \\ (\overrightarrow{C-D}) \end{vmatrix} = \begin{vmatrix} 9 & 6 \\ 8 & 9 \end{vmatrix} = 81-48>0$$

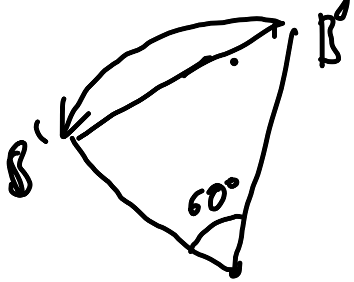
$$\begin{vmatrix} (\overrightarrow{C-D}) \\ (A-D) \end{vmatrix} = \begin{vmatrix} 8 & 9 \\ 7 & 3 \end{vmatrix} = 8\cdot 3-9\cdot 7<0$$

$\Rightarrow$ bod D leží napravo od úseček $\vec{AB}$, $\vec{BC}$, nalevo od $\vec{CA}$. Z D je tedy vidět pouze strana AC.



Nejprve zjistíme třetí bod $\triangle$:

Ten nalezneme jako obraz bodu [2,3] v rotaci v kladném smyslu kolem bodu [1,1] o $60°$.



Matice zobrazení rotace o úhel α v kladném smyslu je:

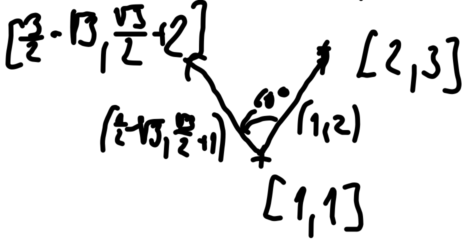
$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

V našem případě $\alpha = 60^\circ$, tedy matice je

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}.$$

Touto maticí zobrazíme vektor $([2,3]-[1,1]) = (1,2)$:

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} - \sqrt{3} \\ \frac{\sqrt{3}}{2} + 1 \end{pmatrix}$$



Nyní aplikujeme rotaci o 60° kolem $[0,0]$ na tři vrcholy:

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} - \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} + \frac{1}{2} \end{pmatrix} =: A.$$

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 - \frac{3\sqrt{3}}{2} \\ \sqrt{3} + \frac{3}{2} \end{pmatrix} =: B' - A' = \begin{pmatrix} \frac{1}{2} - \sqrt{3} \\ \frac{\sqrt{3}}{2} + 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{3}{2} - \sqrt{3} \\ \frac{\sqrt{3}}{2} + 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{4} - \frac{\sqrt{3}}{2} - \frac{3}{4} - \sqrt{3} \\ \frac{3\sqrt{3}}{4} - \frac{3}{2} + \frac{\sqrt{3}}{4} + 1 \end{pmatrix} = \Bigg($$

$$= \begin{pmatrix} -\frac{3\sqrt{3}}{2} \\ \sqrt{3} - \frac{1}{2} \end{pmatrix} =: C'$$

Ověření správnosti:

$$\|C' - B'\| = \left\| \begin{matrix} -1 \\ -2 \end{matrix} \right\| = \underline{\underline{\sqrt{5}}} = \sqrt{1^2 + 2^2}$$

$$\|A - B\| = \left\| \begin{pmatrix} \frac{1}{2} - \sqrt{3} \\ \frac{\sqrt{3}}{2} + 1 \end{pmatrix} \right\| = \sqrt{\left(\frac{1}{2} - \sqrt{3}\right)^2 + \left(\frac{\sqrt{3}}{2} + 1\right)^2} =$$

$$= \sqrt{\frac{1}{4} \cancel{- \sqrt{3}} + 3 + \frac{3}{4} \cancel{+ \sqrt{3}} + 1} = \sqrt{5}$$