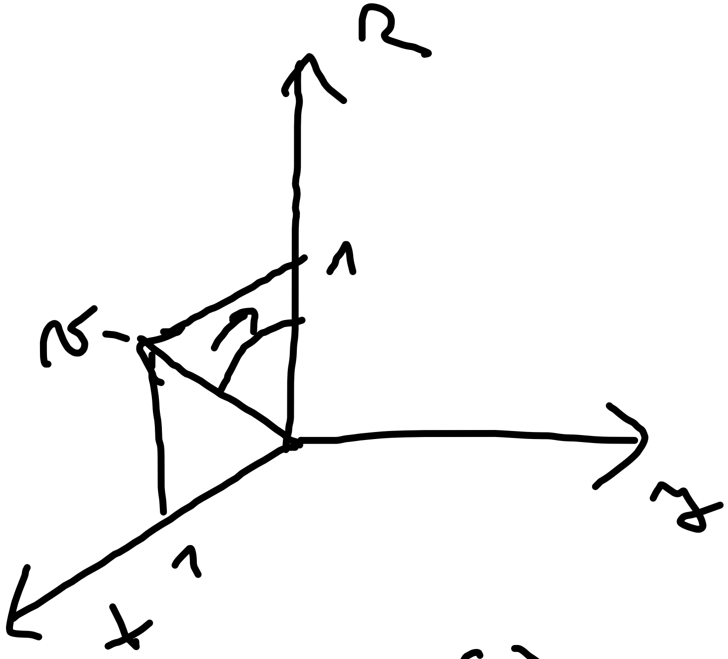


(1) rotace kolem y o $\frac{\pi}{4}$ v záp. sm.

(2) rotace kolem z o π

(3) rotace kolem y o $\frac{\pi}{4}$ v kl. smy

$$\underset{(3)}{\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}} \underset{(2)}{\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \underset{(1)}{\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ -y \\ x \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 1 & 7 & 9 & 8 & 6 & 5 & 3 & 4 \end{pmatrix} \begin{matrix} i \\ \sigma(i) \end{matrix}$$

$1 \mapsto 2$, $2 \mapsto 1$, $3 \mapsto 7$, ...

inverze $i < j$ & $\sigma(i) > \sigma(j)$

$1<2$, $1<3$, $1<4$, $1<5$ ...

$2>1$, $2<7$, $2<9$, $2<8$ ...

inv. ok ok

spočítáme inverze

$2 > 1$

1 OK

$7 > 6, 5, 3, 4$

$9 > 8, 6, 5, 3, 4$

$8 > 6, 5, 3, 4$

$6 > 5, 3, 4$

$5 > 3, 4$

3 OK

4 OK

$1 + 4 + 5 + 4 + 3 + 2 = 19$

$(-1)^{19} = -1$ lichá!

$$\tau = \begin{pmatrix} 1 & 2 & 3 & \dots & 2m-1 & 2m \\ 2m & 2m-1 & 2m-2 & \dots & 2 & 1 \end{pmatrix}$$

$2m$: vpředu je menší, $2m-1$ prvků

$2m-1$: —— " ——, $2m-2$ prvků

$\vdots$

Počet inverzí: $(2m-1) + (2m-2) + (2m-3) + \dots + 2 + 1 =$

$$\sum_{i=1}^{2m-1} i = \frac{2m(2m-1)}{2} = m\cdot(2m-1)$$

$$(-1)^{m\cdot(2m-1)}$$

$n = 6$; $a_{31}\, a_{43}\, a_{14}\, a_{52}\, a_{66}\, a_{25}$

napíšeme si "permutaci"

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 1 & 3 & 2 & 6 \end{pmatrix}$$

$4 > 1, 2, 3$

$5 > 1, 2, 3$

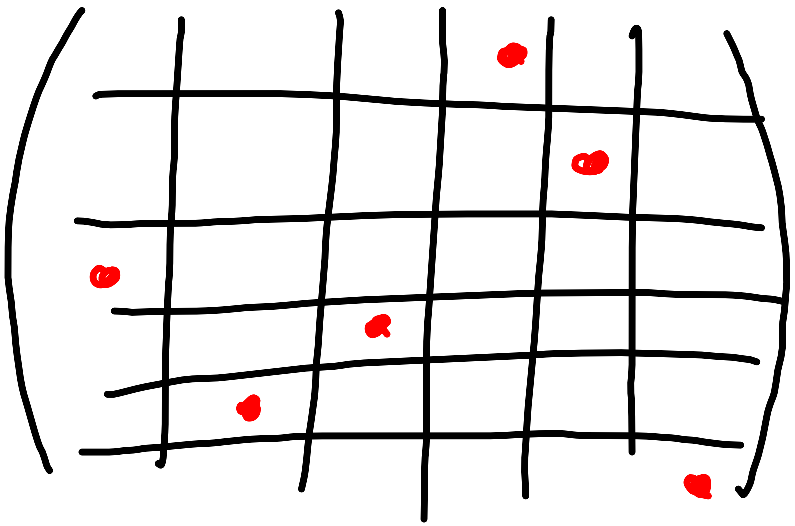
1 OK

$3 > 2$

2 OK

6 OK

lichá



$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & ? \end{pmatrix}$ NE

jen 7 čísel . NE

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$$a_{12} a_{21} a_{34} a_{43} \qquad a_{12} a_{23} a_{34} a_{41}$$

$2 > 1$
1 ok
$4 > 3$ $\Rightarrow +$
3 ok

$2 > 1$
$3 > 1$ $\Rightarrow -$
$4 > 1$
1 ok

$$\begin{vmatrix} 5 & 2 \\ 6 & 4 \end{vmatrix} = 5 \cdot 4 - 2 \cdot 6 = 8$$

$$\begin{vmatrix} 3 & -2 & 5 \\ 1 & 4 & 1 \\ 2 & -3 & 4 \end{vmatrix}$$

$+ \searrow \qquad - \swarrow$

$$\begin{matrix} 3 & -2 & 5 \\ 1 & 4 & 4 \end{matrix}$$

$$= 3 \cdot 4 \cdot 4 + (-2) \cdot 1 \cdot 2 + 5 \cdot 1 \cdot (-3) - 5 \cdot 4 \cdot 2 - (-2) \cdot 1 \cdot 4 - 3 \cdot 1 \cdot (-3) = 6$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & 0 & 0 & 0 \\ a_{41} & a_{42} & 0 & 0 & 0 \\ a_{51} & a_{52} & 0 & 0 & 0 \end{vmatrix} = 0$$

$$a_{51}\, a_{42} \cdot 0 \cdots$$

$$a_{41}\, a_{52} \cdot 0 \cdots$$

$$\begin{vmatrix} 3 & -2 & 1 & -2 \\ -3 & -5 & 2 & 0 \\ 2 & 1 & -2 & -4 \\ -1 & 0 & 3 & 1 \end{vmatrix} = - \begin{vmatrix} -1 & 0 & 3 & 1 \\ -3 & -5 & 2 & 0 \\ 2 & 1 & -2 & -4 \\ 3 & -2 & 1 & -2 \end{vmatrix} \begin{matrix} (-3)\,(2)\,(3) \\ + \\ + \\ + \end{matrix}$$

$$= \begin{vmatrix} -1 & 0 & 3 & 1 \\ 0 & -5 & -7 & -3 \\ 0 & 1 & 4 & -2 \\ 0 & -2 & 10 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 3 & 1 \\ 0 & 1 & 4 & -2 \\ 0 & -5 & -7 & -3 \\ 0 & -2 & 10 & 1 \end{vmatrix} \begin{matrix} (5)\,(2) \\ + \\ + \end{matrix} =$$

$$= \begin{vmatrix} -1 & 0 & 3 & 1 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 13 & -13 \\ 0 & 0 & 18 & -3 \end{vmatrix} \left(\cdot \frac{1}{13}\right) = 13 \cdot \begin{vmatrix} -1 & 0 & 3 & 1 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 18 & -3 \end{vmatrix} = 13 \cdot \begin{vmatrix} -1 & 0 & 3 & 1 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 15 \end{vmatrix}$$

$$= 13 \cdot (-1) \cdot 1 \cdot 1 \cdot 15 = -195$$

$$\begin{vmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix} = 2 \cdot (-1)^{1+1} \cdot \underbrace{\overbrace{\begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}}^{=4}}_{\text{alg. doplněk}}$$

$$+ 1 \cdot (-1)^{1+2} \overbrace{\begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}}^{=3} + 0 \cdot (-1)^{1+3} \cdot$$

$$\cdot \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{vmatrix} + 0 \cdot (-1)^{1+4} \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} =$$

$$= 2 \cdot 4 + (-1) \cdot 3 = 5$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & -2 & 3 & -1 \\ 4 & 1 & 4 & 9 & 1 \\ 8 & 1 & -8 & 27 & -1 \\ 16 & 1 & 16 & 81 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -3 & 2 & -2 \\ 3 & 0 & 3 & 8 & 0 \\ 7 & 0 & -9 & 26 & -2 \\ 15 & 0 & 15 & 80 & 0 \end{vmatrix} = 1\cdot(-1)^{1+2} \begin{vmatrix} 1 & -3 & 2 & -2 \\ 3 & 3 & 8 & 0 \\ 7 & -9 & 26 & -2 \\ 15 & 15 & 80 & 0 \end{vmatrix}$$

Laplace

$$= \begin{vmatrix} 1 & -3 & 2 & -2 \\ 3 & 3 & 8 & 0 \\ 6 & -6 & 24 & 0 \\ 15 & 15 & 80 & 0 \end{vmatrix} = (-1)\cdot(-2)\cdot(-1)^{1+4} \cdot \begin{vmatrix} 3 & 3 & 8 \\ 6 & -6 & 24 \\ 15 & 15 & 80 \end{vmatrix} =$$

$$= (-2)\cdot 6 \cdot 5 \cdot \begin{vmatrix} 3 & 3 & 8 \\ 1 & -1 & 4 \\ 3 & 3 & 16 \end{vmatrix} = -60 \cdot \begin{vmatrix} 3 & 3 & 8 \\ 1 & -1 & 4 \\ 0 & 0 & 8 \end{vmatrix} =$$

$$= (-60)\cdot 8 \cdot (-1)^{3+3} \begin{vmatrix} 3 & 3 \\ 1 & -1 \end{vmatrix} = -480(-3-3) =$$

$$= 2880$$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \qquad A^{-1} = \frac{1}{|A|} \cdot A^{*}$$

$$|A| = 1 + 2 - 1 = 2$$

$$\begin{pmatrix} (-1)^{1+1} \cdot \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \\ -\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 1 & -1 \\ -2 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix} \qquad A^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 & -1 \\ -2 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$$

$$
\begin{vmatrix}
x & y & 0 & \cdots & 0 & 0 \\
0 & x & y & \cdots & 0 & 0 \\
0 & 0 & x & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & \cdots & x & y \\
y & 0 & 0 & \cdots & 0 & x
\end{vmatrix}
= x \cdot (-1)^{1+1}
\underbrace{\begin{vmatrix}
x & y & \cdots & 0 & 0 \\
0 & x & \cdots & 0 & 0 \\
\vdots & & & \vdots & \vdots \\
0 & 0 & \cdots & x & y \\
0 & 0 & \cdots & 0 & x
\end{vmatrix}}_{= x^{n-1}}
$$

$$
+ y \cdot (-1)^{n+1}
\underbrace{\begin{vmatrix}
y & 0 & \cdots & 0 & 0 \\
x & y & \cdots & 0 & 0 \\
0 & x & \cdots & 0 & 0 \\
\vdots & \vdots & & \vdots & \vdots \\
0 & 0 & & x & y
\end{vmatrix}}_{= y^{n-1}}
= x^n + (-1)^{n+1} y^n
$$

$$\begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ a_2 & 1 & 0 & \cdots & 0 & 0 \\ a_3 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_m & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} \stackrel{\cdot}{\lrcorner} = \begin{vmatrix} -a_2 & \boxed{0} & 1 & \cdots & 1 & 1 \\ a_2 & 1 & 0 & \cdots & 0 & 0 \\ a_3 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \\ a_m & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} -a_2-a_3 & 0 & 0 & \cdots & 1 & 1 \\ a_2 & 1 & 0 & \cdots & 0 & 0 \\ a_3 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & & & & \vdots & \vdots \\ a_m & 0 & 0 & \cdots & 0 & 1 \end{vmatrix} = \cdots = \begin{vmatrix} -a_2-a_3-\cdots-a_m & 0 & 0 & \cdots & 0 & 0 \\ a_2 & 1 & 0 & \cdots & 0 & 0 \\ a_3 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_m & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$= -\sum_{i=2}^{m} a_i$$

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ -1 & 0 & 3 & \cdots & n-1 & n \\ -1 & -2 & 0 & \cdots & n-1 & n \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -1 & -2 & -3 & \cdots & -n+1 & 0 \end{vmatrix} \begin{matrix} + \\ + \\ \vdots \end{matrix} = \begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ 0 & 2 & 6 & \cdots & 2n-2 & 2n \\ 0 & 0 & 3 & \cdots & & \\ \vdots & \vdots & \vdots & \ddots & & \\ 0 & 0 & 0 & \cdots & 0 & n \end{vmatrix}$$

[illegible] má $m-1$, co tu je.

$$= 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n = n!$$

$$|A \cdot B| = |A| \cdot |B|$$

$$\begin{pmatrix} x_1 - y_1 & x_1 - y_2 & \cdots & x_1 - y_n \\ x_2 - y_1 & x_2 - y_2 & \cdots & x_2 - y_n \\ \vdots & \vdots & & \vdots \\ x_n - y_1 & x_n - y_2 & \cdots & x_n - y_n \end{pmatrix} =$$

$$= \begin{pmatrix} x_1 & -1 & 0 \cdots 0 \\ x_2 & -1 & 0 \cdots 0 \\ \vdots & \vdots & \vdots \\ x_n & -1 & 0 \cdots 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & \cdots & 1 \\ y_1 & y_2 & \cdots & y_n \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

$n = 2$

$$\begin{vmatrix} x_1 & -1 \\ x_2 & -1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 \\ y_1 & y_2 \end{vmatrix} = (y_2 - y_1) \cdot (x_2 - x_1)$$

$n > 2$

$$|\quad| = 0$$