

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & -1 & 1 & -1 & 5 \\ 2 & 3 & 2 & 3 & 3 & 5 \\ 2 & 2 & 3 & 2 & 3 & 1 \end{array} \right) \sim \dots \sim \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 & -1 & 5 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\left\{ \begin{pmatrix} -6 \\ 5-\lambda \\ 1 \\ \lambda \\ 0 \end{pmatrix} : \lambda \in \mathbb{R} \right\}$$

$$\begin{pmatrix} \lambda + \frac{9}{5}\lambda & -\frac{3}{5}\lambda \\ \lambda & \lambda \end{pmatrix}$$

$$X = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} \quad A \cdot X = X \cdot A$$

$$5x_2 + 3x_3 = 0$$

$$3x_1 + 9x_2 - 3x_4 = 0$$

$$5x_1 - 9x_3 - 5x_4 = 0$$

$$\begin{pmatrix} 0 & 5 & 3 & 0 \\ 3 & 9 & 0 & -3 \\ 5 & 0 & -9 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 0 & -1 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4 = 0$$

$$a_1 \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} + a_2 \begin{pmatrix} 2 \\ -2 \\ 1 \\ 3 \end{pmatrix} + a_3 \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix} + a_4 \begin{pmatrix} 3 \\ 4 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$a_1 + 2a_2 + 3a_4 = 0$$

$$-a_1 + 2a_2 + a_3 + 2a_4 = 0$$

$$-a_2 + a_3 = 0$$

$$2a_1 + 3a_2 + 5a_4 = 0$$

$$\begin{pmatrix} 1 & 2 & 0 & 3 \\ -1 & 2 & 1 & 2 \\ 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

$\sim \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$

 \Rightarrow lin. závisle

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 2 \\ 2 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 2 \\ 0 & -3 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix}$$

\Rightarrow lim. rozávině

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

\Rightarrow lim. rozávině

$$h \leq \min\{3, 4\} = 3 \quad \uparrow \text{rozávině}$$

$$\begin{pmatrix} 2 & -4 & 8 & 0 & 4 \\ 3 & -6 & 1 & 4 & -3 \\ -4 & 2 & 5 & -1 & 7 \\ 5 & -4 & -12 & 5 & -14 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 4 & 0 & 2 \\ 0 & 0 & -11 & 4 & -9 \\ 0 & -6 & 21 & -1 & 15 \\ 0 & 6 & -32 & 5 & -24 \end{pmatrix} \sim \\
 \sim \begin{pmatrix} 1 & -2 & 4 & 0 & 2 \\ 0 & -6 & 21 & -1 & 15 \\ 0 & 0 & -11 & 4 & -9 \\ 0 & 0 & -11 & 4 & -9 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 4 & 0 & 2 \\ 0 & -6 & 21 & -1 & 15 \\ 0 & 0 & -11 & 4 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

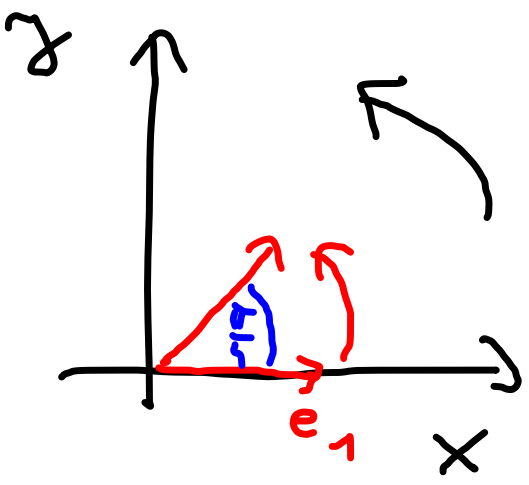
$$h = 3$$

$$A = (a_{ij}) \quad A^T = (a_{ji})$$

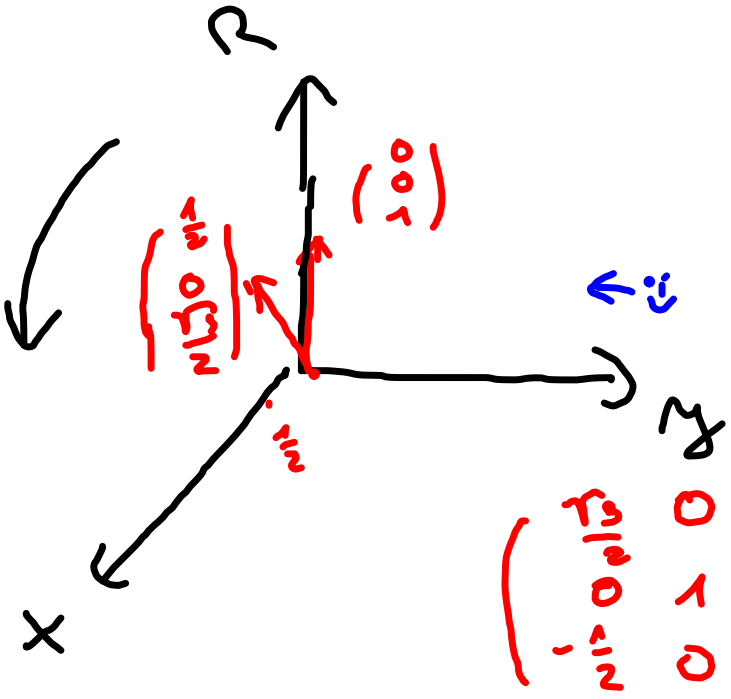
$$\begin{pmatrix} 3 & 4 & 1 & -2 & 5 & 1 \\ -1 & 1 & 2 & 3 & 3 & -5 \\ 2 & 1 & -1 & -3 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & 2 & 3 & 3 & -5 \\ 0 & 7 & 7 & 7 & 14 & -14 \\ 0 & 3 & 3 & 3 & 6 & -6 \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & 1 & 2 & 3 & 3 & -5 \\ 0 & 7 & 7 & 7 & 14 & -14 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h = 2$$



$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\
 \text{||} \\
 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 \text{||} \\
 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

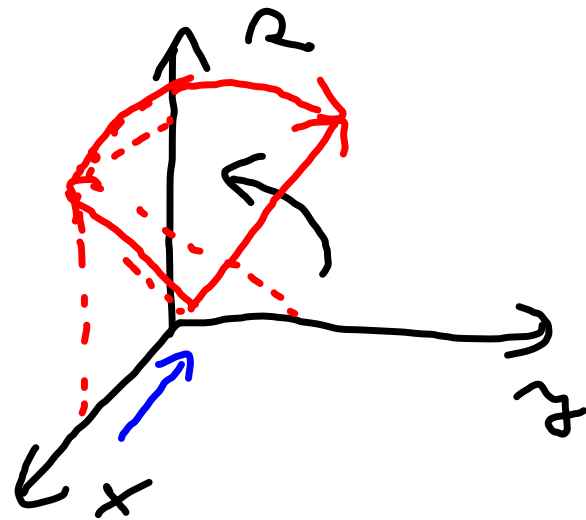


$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 \text{||} \\
 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 \text{||} \\
 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4 je 2 vlni:

- ① stlačenie dĺžky λ o $\frac{\lambda}{2\sqrt{2}}$ v kladnom s.
- ② roztiahnutie podľa $\sqrt{2}$

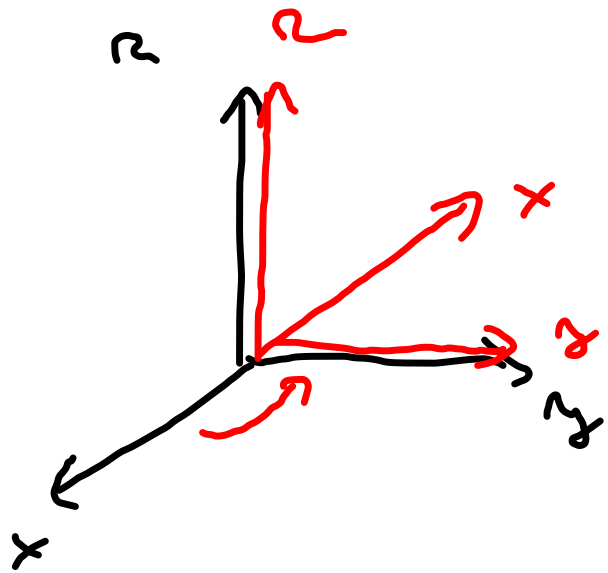
$$\varphi \begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} 2 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix}$$

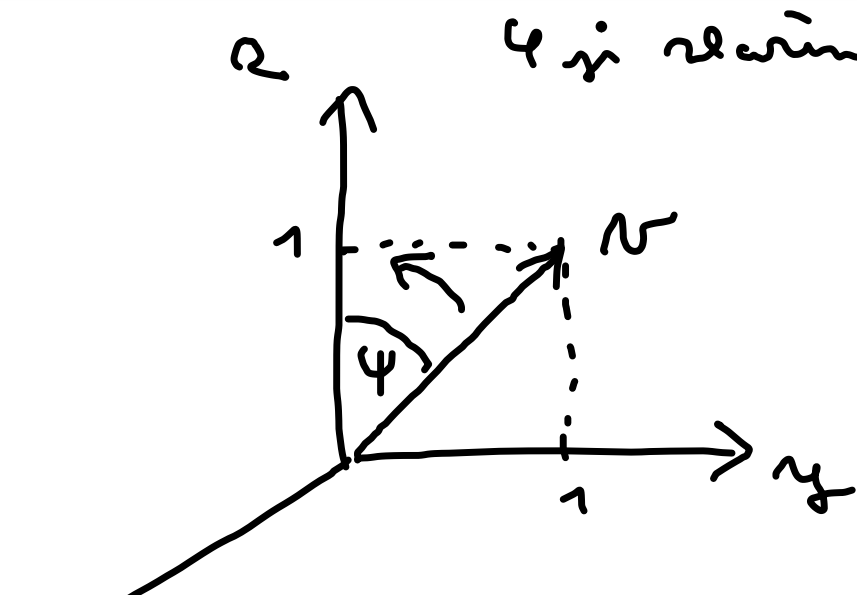


$$\textcircled{1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\lambda}{2\sqrt{2}} & 0 \\ 0 & \frac{\lambda}{2\sqrt{2}} & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\textcircled{2} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} -x \\ z \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$





4. je rovina: (1) rovine kolem x
 σ_{x1} v desce
 myslu

(2) rovine kolem z
 σ_{z1}

(3) rovine kolem x
 σ_{x2} v desce
 myslu

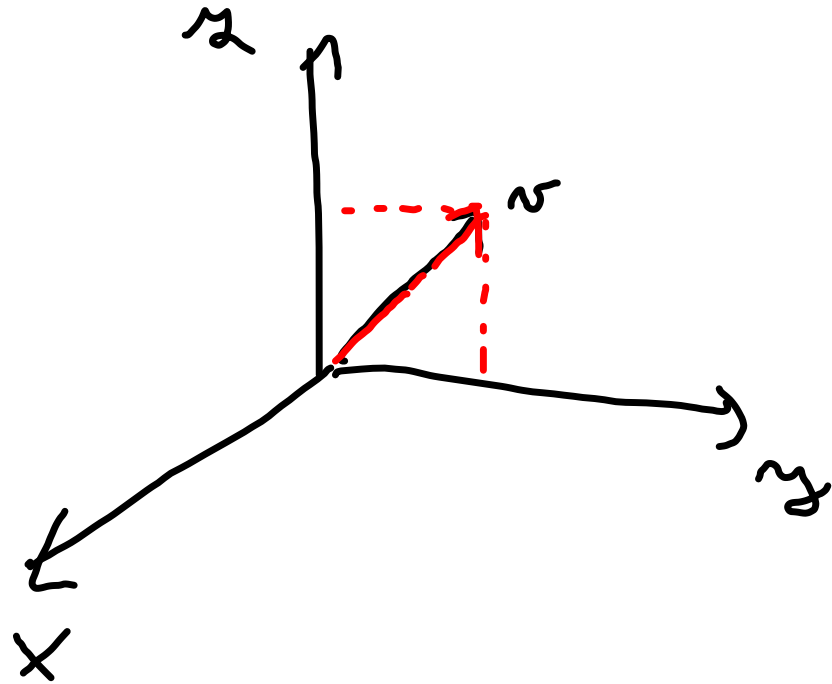
$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \cos \varphi & \sin \varphi \\ \sin \varphi & \cos \varphi \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \cos \varphi & \sin \varphi \\ \sin \varphi & \cos \varphi \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

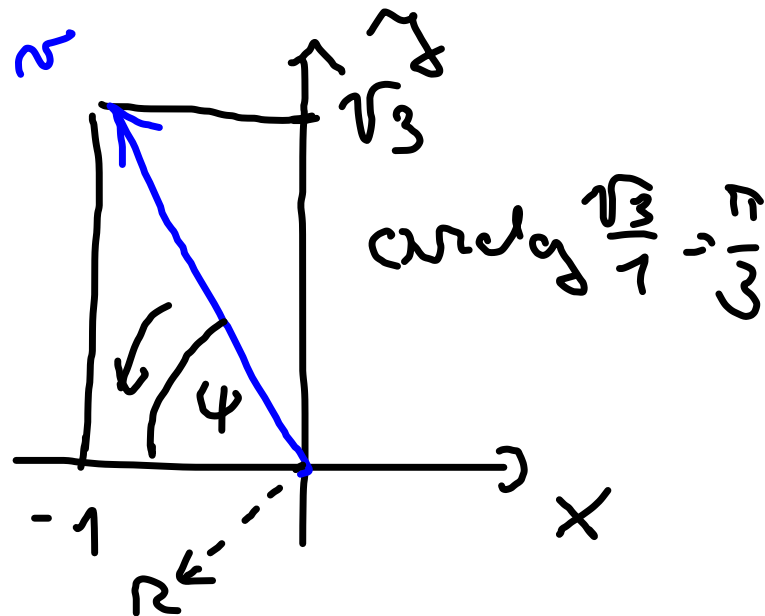
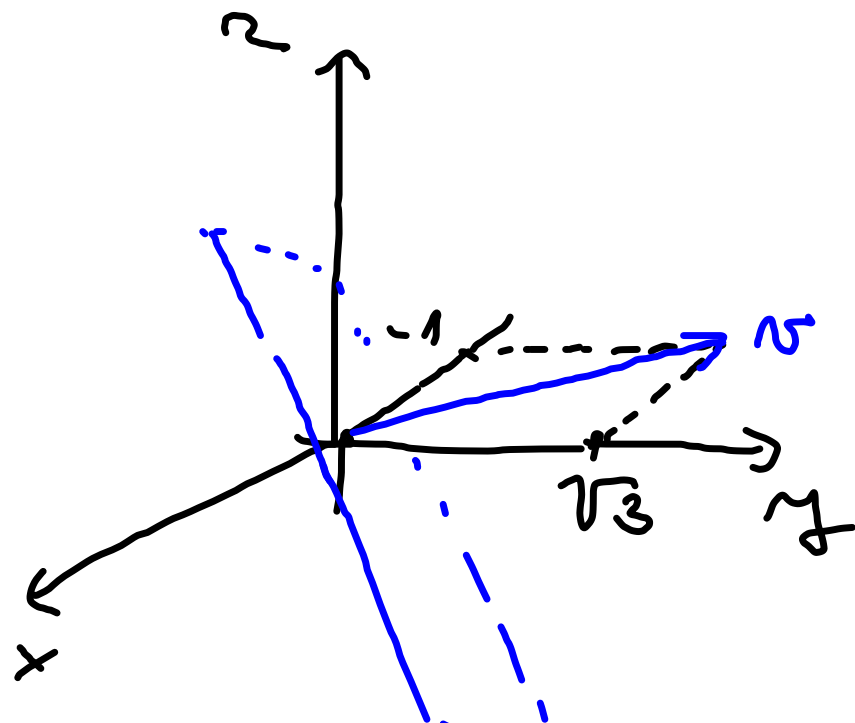
$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} \cos \varphi & \sin \varphi \\ \sin \varphi & \cos \varphi \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \cos \varphi \\ 0 & \sin \varphi \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} \cos \varphi & \sin \varphi \\ \sin \varphi & \cos \varphi \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \cos \varphi & \sin \varphi \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned}
 & \text{6} \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} \right) = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\
 & \begin{pmatrix} 0 & 0 & 1 \\ \text{నిచ్చే నిచ్చే} & 0 & 0 \\ \text{నిచ్చే నిచ్చే} & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ \text{నిచ్చే నిచ్చే} & 0 \\ \text{నిచ్చే నిచ్చే} & 0 \end{pmatrix} =
 \end{aligned}$$

$$= \left(\begin{array}{ccc|cc} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 & 1 \end{array} \right)$$





ψ je sklon :

(1) sklaci o $\frac{\pi}{3}$ kolem osy z v
hlad. systemu

(2) rotacemi podle yz

(3) sklaci o $\frac{\pi}{2}$ kolem z v rot. systemu

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

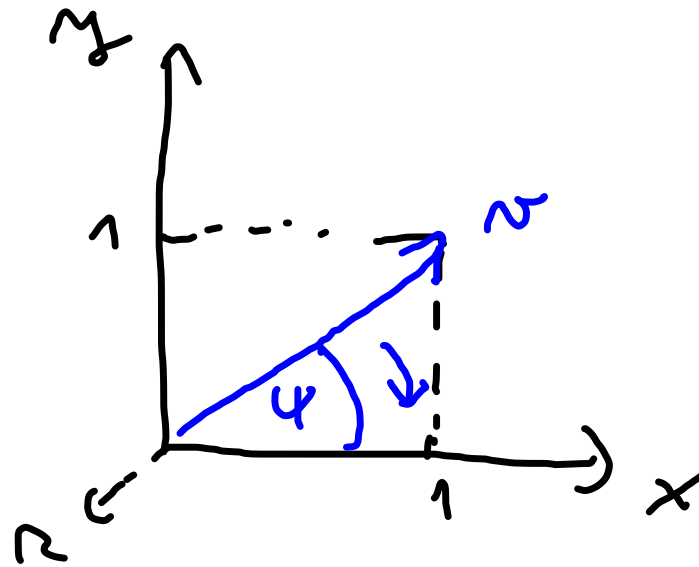
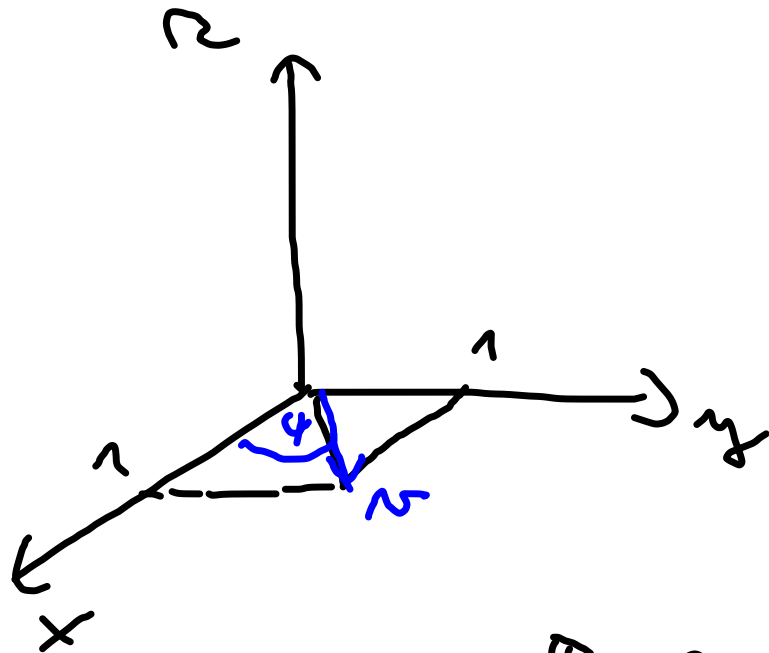
(1)
$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(2)
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(3)
$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

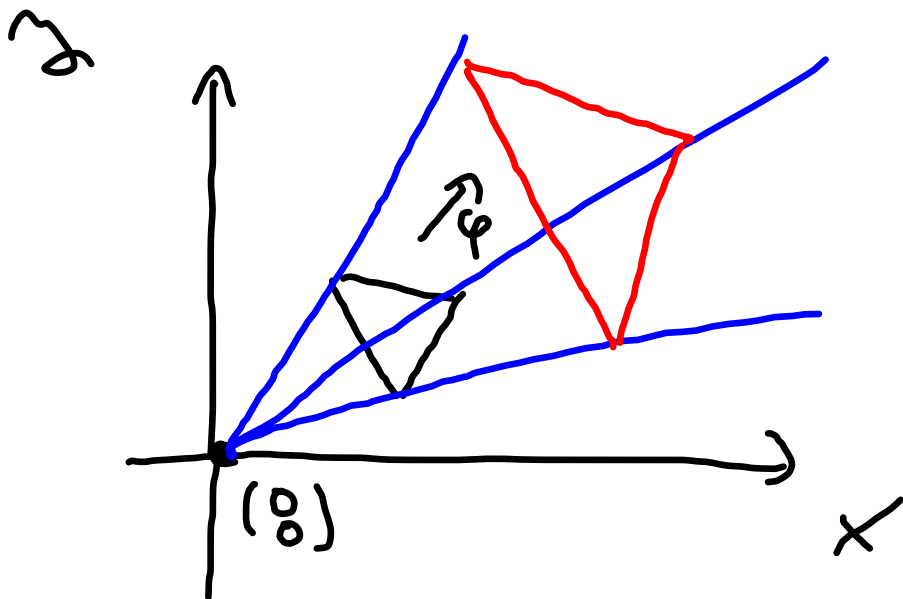


- | | | | |
|-----|----------|-----------------|--------------------------|
| (1) | rotace o | $\frac{\pi}{5}$ | kolem z v rovině xOy |
| (2) | rotace o | $\frac{\pi}{2}$ | kolem x v rovině yOz |
| (3) | rotace o | $\frac{\pi}{4}$ | kolem z v rovině xOy |

$$\begin{aligned}
 (1) \quad & \begin{pmatrix} 0 & \sin(\frac{\pi}{2}) & -\cos(\frac{\pi}{2}) \\ \cos(\frac{\pi}{2}) & \sin(\frac{\pi}{2}) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 (2) \quad & \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\frac{\pi}{2}) & \sin(\frac{\pi}{2}) \\ 0 & -\sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\
 (3) \quad & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 & (3) (2) (1) =
 \end{aligned}$$

$$(f \circ g)(x) = f(g(x))$$

$$x \mapsto g(x) \mapsto f(g(x))$$



$$\varphi\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x \\ 3y \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$