① $R: \forall x \in \mathbb{N} \quad x = x$

$S: NSD(x,y) = 1 \Rightarrow NSD(y,x) = 1$

$x = y \Rightarrow y = x$

$T: NSD(2,3) = 1 \wedge NSD(3,4) = 1$

$\not\Rightarrow NSD(2,4) = 1 \wedge 2 = 4$

② $5 \mid x - y \Leftrightarrow \exists k \in \mathbb{Z} : x - y = k \cdot 5$

$R: x - x = 0 = 5 \cdot 0$

$5 \mid x - y \Leftrightarrow x - y = k \cdot 5$ ($k \in \mathbb{Z}$) neb $y - x = -k \cdot 5$ ($-k \in \mathbb{Z}$)

$T: x \sim y \wedge y \sim z \Leftrightarrow \exists k, l \in \mathbb{Z}$

$x - y = k \cdot 5 \wedge y - z = l \cdot 5$

neb $x - z = x - y + y - z = k \cdot 5 + l \cdot 5$

$= (k + l) \cdot 5$

$\{1, 6, 11, \dots\}, \{2, 7, 12, \dots\}$

$\{3, 8, 13, \dots\}, \dots$

$\{\{i + 5k \mid k \in \mathbb{N}\}, i = 1, 2, 3, 4, 5\}$

③ $x - x = 0 \in \mathbb{Q}$

$x - y = \frac{p}{q} \in \mathbb{Q} \Rightarrow y - x = -\frac{p}{q} \in \mathbb{Q}$

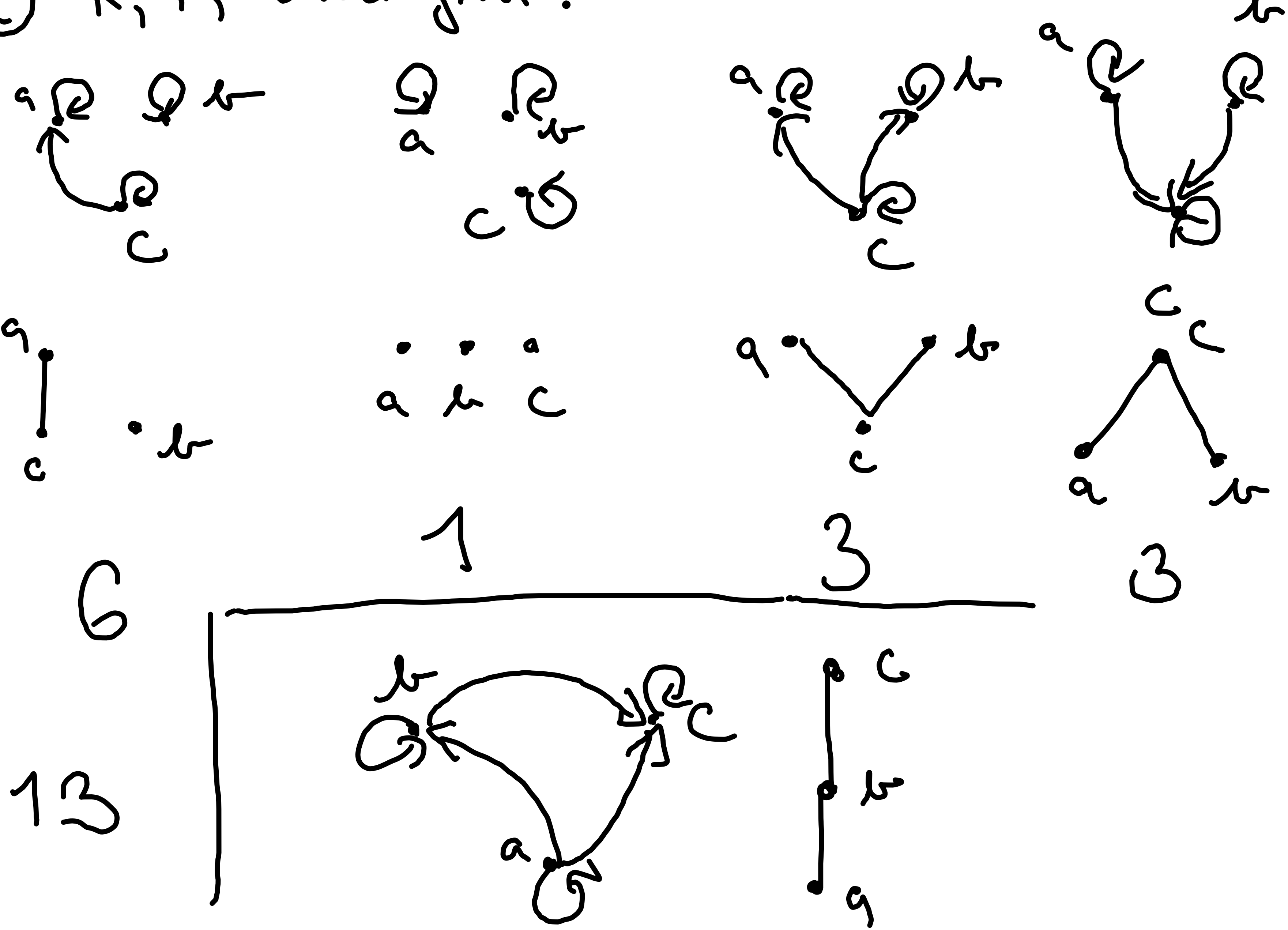
$x - y \in \mathbb{Q} \wedge y - z \in \mathbb{Q}$

$\frac{p}{q}$ $\frac{r}{s}$

$x - y + y - z = \frac{p}{q} + \frac{r}{s} \in \mathbb{Q}$

$\mathbb{Q}, \{i + \pi \mid \pi \in \mathbb{Q}\}$

② R, T, antisym.



$$2(A-C)+3B$$

$$A-C=\begin{pmatrix}0 & 1+i & -1\\ 2-i & 1 & -2+2i\end{pmatrix}-\begin{pmatrix}2-i & 5 & -3\\ 1 & 1-2i & -2\end{pmatrix}=$$

$$=\begin{pmatrix}-2+i & -4+i & 2\\ 1-i & 2i & 2i\end{pmatrix}$$

$$2(A-C)+3B=$$

$$\begin{pmatrix}-4+2i & -8+2i & 4\\ 2-2i & 4i & 4i\end{pmatrix}+\begin{pmatrix}3 & 6-3i & -6\\ 9 & 3+9i & 12\end{pmatrix}=$$

$$=\begin{pmatrix}-1+2i & -2-i & -2\\ 11-2i & 3+13i & 12+4i\end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 0\cdot4+1\cdot7 & 0\cdot5+1\cdot8 & 0\cdot6+1\cdot9 \\ 2\cdot4+3\cdot7 & 2\cdot5+3\cdot8 & 2\cdot6+3\cdot9 \end{pmatrix}$$

2/2 2/3

2/3

$$= \begin{pmatrix} 7 & 8 & 9 \\ 29 & 34 & 39 \end{pmatrix}$$

$B \cdot A$ = nejde

2/3 2/2

s čím se nejde!

$$\begin{pmatrix} 0 & 0 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}\begin{pmatrix} 0 & \cdot \\ 0 & \cdot \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 12 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 6 & 11 \\ 4 & 5 \end{pmatrix}$$

$$\underset{1/3}{\begin{pmatrix} -1 & 1 & 2 \end{pmatrix}} \underset{3/1}{\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}} = -1 \cdot 2 + 1 \cdot 0 + 2 \cdot 3 = 4$$

$$\underset{3/1}{\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}} \underset{1/3}{\begin{pmatrix} -1 & 1 & 2 \end{pmatrix}} = \begin{pmatrix} -2 & 2 & 4 \\ 0 & 0 & 0 \\ -3 & 3 & 6 \end{pmatrix}$$

① $f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} kx \\ ky \\ kz \end{pmatrix}$

$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

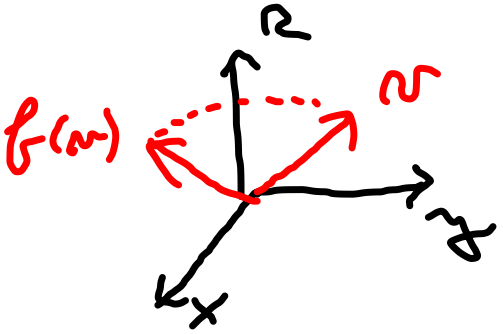
$A \cdot E = E \cdot A = A$

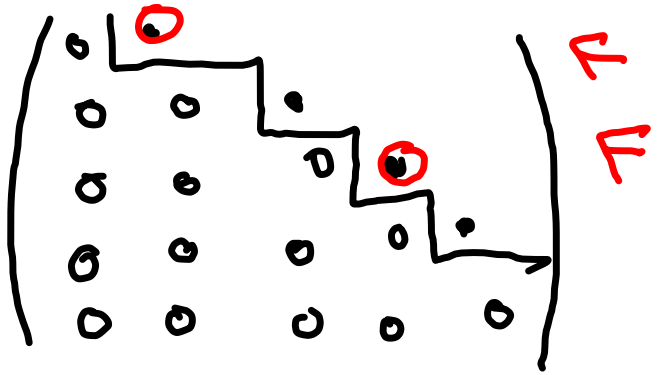
$$\begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} kx \\ ky \\ kz \end{pmatrix}$$

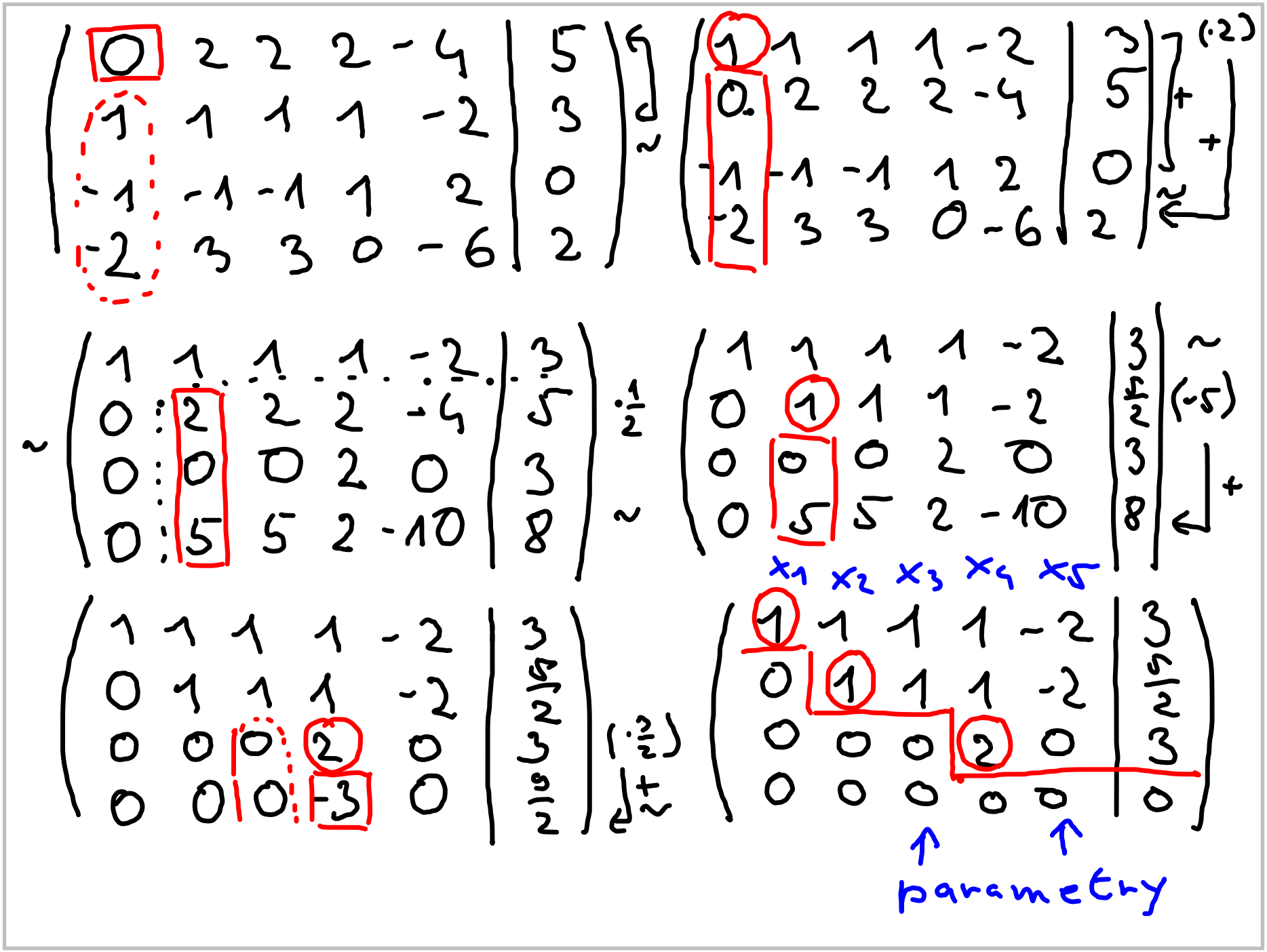
3/3 3/1 3/1

② $f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$

③ $f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} x \\ -y \\ z \end{pmatrix}$ $\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$






x1 x2 x3 x4 x5
parametry

$$x_1 + x_2 + x_3 + x_4 - 2x_5 = 3$$
$$x_2 + x_3 + x_4 - 2x_5 = \frac{5}{2}$$
$$2x_4 = 3$$

$x_5 = t$

$2x_4 = 3 \Rightarrow x_4 = \frac{3}{2}$

$x_3 = s$

$x_2 + s + \frac{3}{2} - 2t = \frac{5}{2}$

$\Rightarrow x_2 = 1 - s + 2t$

$x_1 + 1 - s + 2t + s + \frac{3}{2} - 2t = 3$

$\Rightarrow x_1 = \frac{1}{2}$

$$\left\{ \begin{pmatrix} \frac{1}{2} \\ 1 - s + 2t \\ s \\ \frac{3}{2} \\ t \end{pmatrix} : s, t \in \mathbb{R} \right\}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 2 & -1 & -3 & 5 \\ 1 & 1 & 1 & 3 \\ 1 & 2 & -3 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & -3 & 3 & 7 \\ 0 & 0 & 4 & 4 \\ 0 & 1 & 0 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & -3 & 3 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 13 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 10 \end{array}\right)$$

$0x_1 + 0x_2 + 0x_3 = 10$

NŘ nad $\mathbb{R}$

$\mathbb{Z}_5$

$$\left(\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \leftarrow$$

$x_3 = 1$

$x_2 = 2$

$x_1 + \underbrace{2 - 3}_{-1 = 4} = 4$

$x_1 = 0$

mod $\mathbb{Z}_5$: $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$

A $\quad A^{-1}$; $A \cdot A^{-1} = E = A^{-1} \cdot A$

m/m

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 1 & -1 & 6 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right) \sim$$

řádky chceme vynulovat

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -4 & 2 & 1 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right) \sim$$

řádky jsme hotové

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -17 & 7 & 4 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -17 & 7 & 4 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right)$$

A^{-1}

$$A^{-1} = \begin{pmatrix} -17 & 7 & 4 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ 1 & -1 & 6 \end{pmatrix} \cdot \begin{pmatrix} -17 & 7 & 4 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \cdots & \cdots & \cdots \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A $\quad$ A^{-1}

$$\begin{pmatrix} 3 & 2 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ 4 & 1 & 1 \end{pmatrix} \begin{matrix} (-3) & (-4) \end{matrix} \sim$$

A

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 5 & -5 \\ 0 & 5 & -7 \end{pmatrix} (-1) \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 5 & -5 \\ 0 & 0 & -2 \end{pmatrix} = B$$

$$\underbrace{\begin{pmatrix} 1 & -1 & 2 \\ 0 & 5 & -5 \\ 0 & 0 & -2 \end{pmatrix}}_{B} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}}_{P_3} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{pmatrix}}_{P_2} \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{P_1} \underbrace{\begin{pmatrix} 3 & 2 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 1 \end{pmatrix}}_{A}$$

$$P = P_3 \cdot P_2 \cdot P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -3 & 0 \\ -1 & -1 & 1 \end{pmatrix}$$

~~$P_2 \cdot P_3 \cdot P_1$~~

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -3 & 0 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 3 & 2 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 5 & -5 \\ 0 & 0 & -2 \end{pmatrix}$$