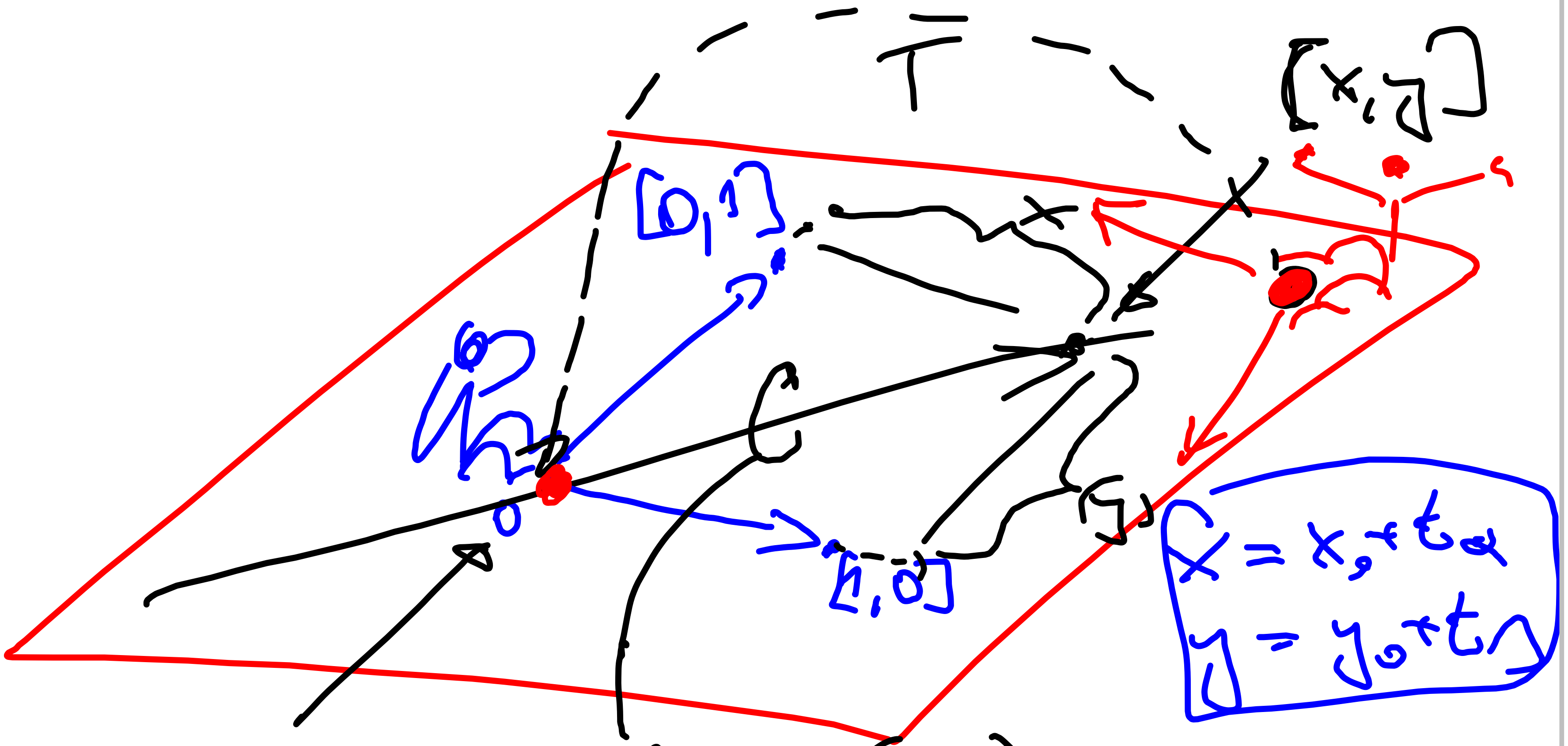


$$x = x_0 + t\alpha$$
$$y = y_0 + t\beta$$

$O = [x_0, y_0]$ vektor $(x,y) = v$

$$[x,y] = O + v$$

$$p: \quad O + u = [x_0, y_0] + t\cdot(\alpha, \beta)$$

$$\underset{\mathbb{R}^2}{\underset{\Uparrow}{\begin{pmatrix}x\\y\end{pmatrix}}} \longmapsto \underset{\mathbb{R}}{\underset{\Uparrow}{F\begin{pmatrix}x\\y\end{pmatrix}}} \qquad \text{p:}\ F\begin{pmatrix}x\\y\end{pmatrix} = \underset{\text{pevné}}{\underset{\Uparrow}{c}}$$

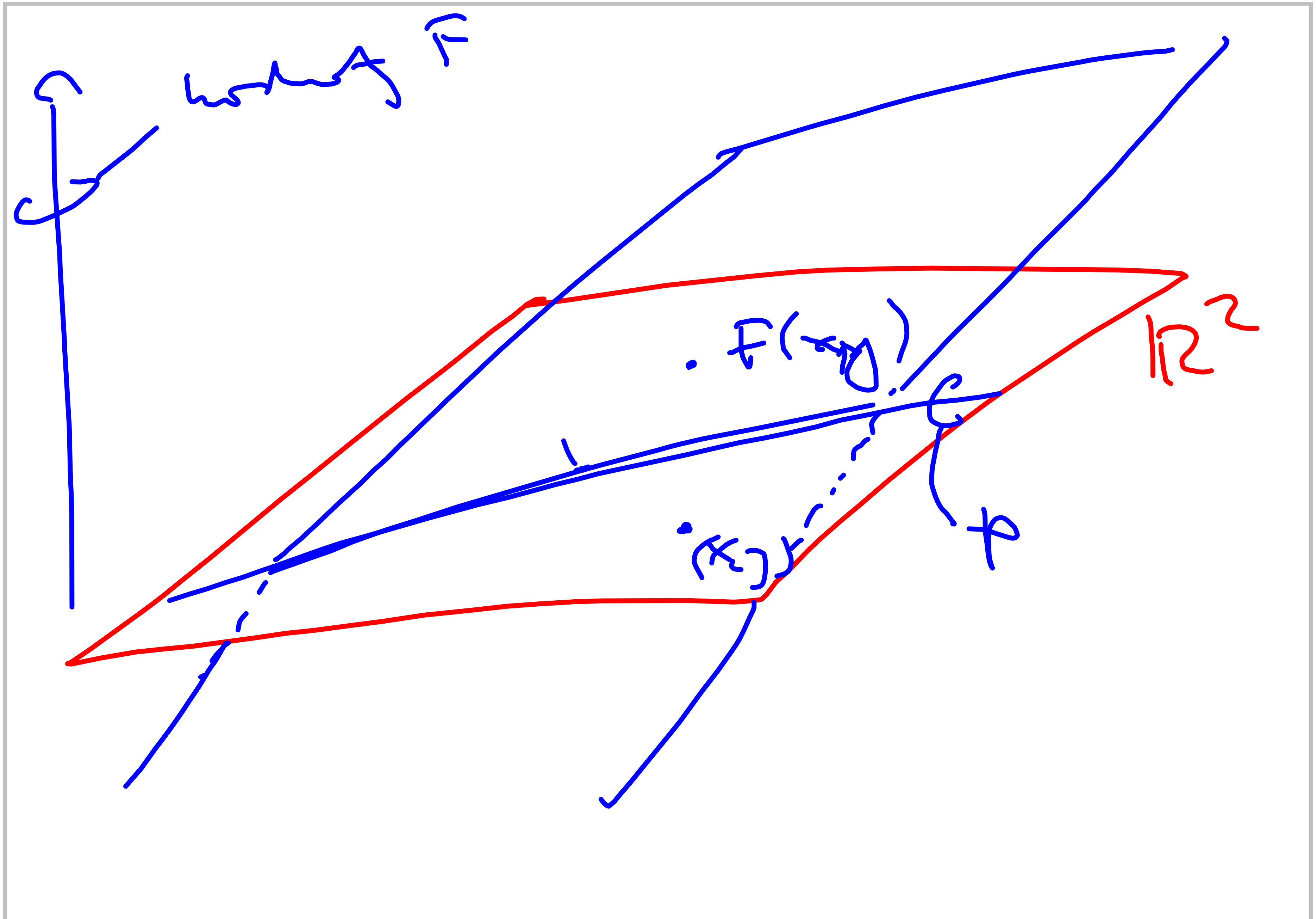
$$F\begin{pmatrix}x+x'\\y+y'\end{pmatrix} = a(x+x') + b(y+y')$$

$$= (ax+by) + (ax'+by')$$

$$= F\begin{pmatrix}x\\y\end{pmatrix} + F\begin{pmatrix}x'\\y'\end{pmatrix}$$

lineární

$$F\begin{pmatrix}\alpha x\\ \alpha y\end{pmatrix} = \alpha F\begin{pmatrix}x\\y\end{pmatrix}$$



graf F
F(x,y)
(x,y)
P
$\mathbb{R}^2$

$F_{1,2} : \mathbb{R}^2 \to \mathbb{R}$

$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} : \mathbb{R}^2 \to \mathbb{R}^2$

$$F(v) = w$$

$v = (x, y)$ $\qquad$ $v + w = (x + x', y + y')$

$w = (x', y')$ $\qquad$ $a \cdot v = (ax, ay)$

$$\begin{pmatrix}0&1\\1&0\end{pmatrix}\cdot\begin{pmatrix}1&2\\1&3\end{pmatrix}=\begin{pmatrix}0\cdot1+1\cdot1 & 0\cdot2+1\cdot3\\1\cdot1+0\cdot1 & 1\cdot2+0\cdot3\end{pmatrix}$$

$$=\begin{pmatrix}1&3\\1&2\end{pmatrix}$$

$$\left[\begin{pmatrix}a&b\\c&d\end{pmatrix}\cdot\begin{pmatrix}x&y\\z&w\end{pmatrix}\right]\cdot\begin{pmatrix}k&l\\m&n\end{pmatrix}=$$

asociativní

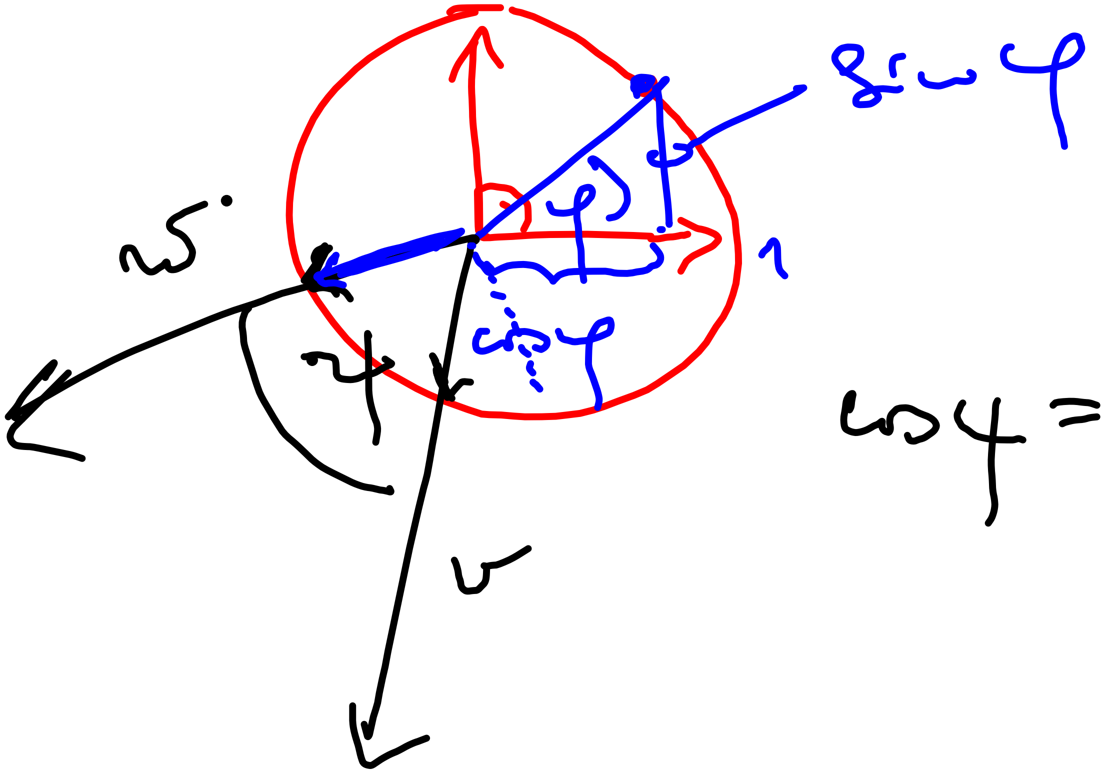
$$\begin{pmatrix}a&b\\c&d\end{pmatrix}+\begin{pmatrix}x&y\\z&w\end{pmatrix}=\begin{pmatrix}a+x&b+y\\c+z&d+w\end{pmatrix}$$

$$ax + by =$$
$$cx + dy =$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\frac{a}{b} = \frac{c}{d} \iff \underbrace{ad - bc = 0}$$

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

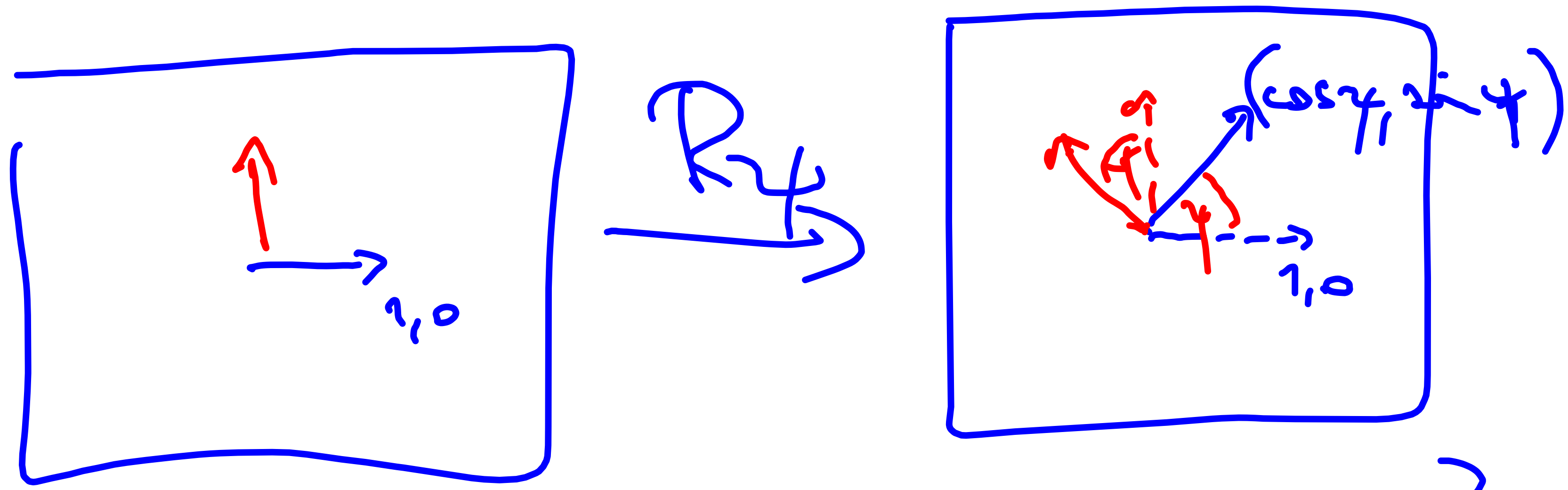


$v = (1, 0)$

$w = (\cos\varphi, \sin\varphi)$

$$\frac{\cos\varphi + 0}{1} = \cos\varphi$$

$$\cos\varphi = \frac{\overbrace{x_v \cdot x_w + y_v \cdot y_w}^{\varphi}}{\|v\| \cdot \|w\|}$$

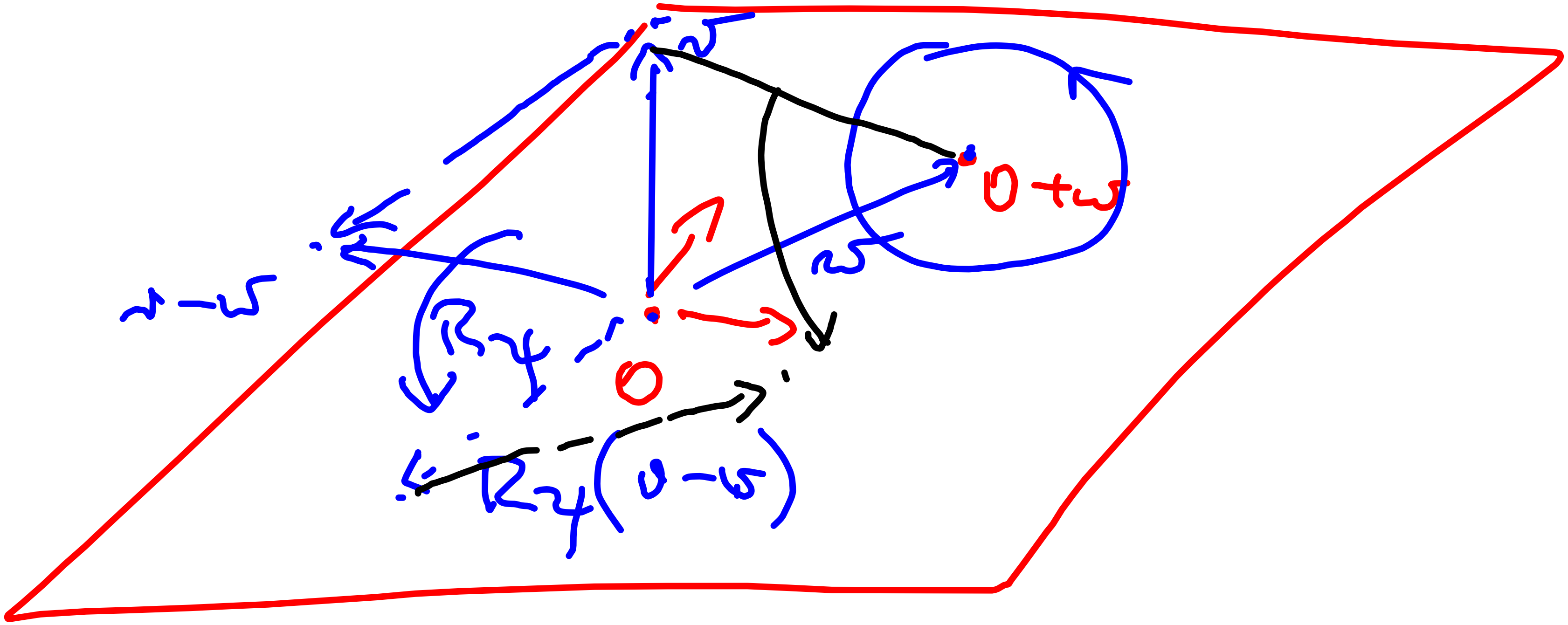


$$(\cos\varphi\cdot x - \sin\varphi\cdot y)^2 + (\sin\varphi\cdot x + \cos\varphi\cdot y)^2$$

$$= \cos^2\varphi\cdot x^2 + \sin^2\varphi\cdot y^2 - 2\,\cancel{\cos\varphi\sin\varphi\cdot xy}$$

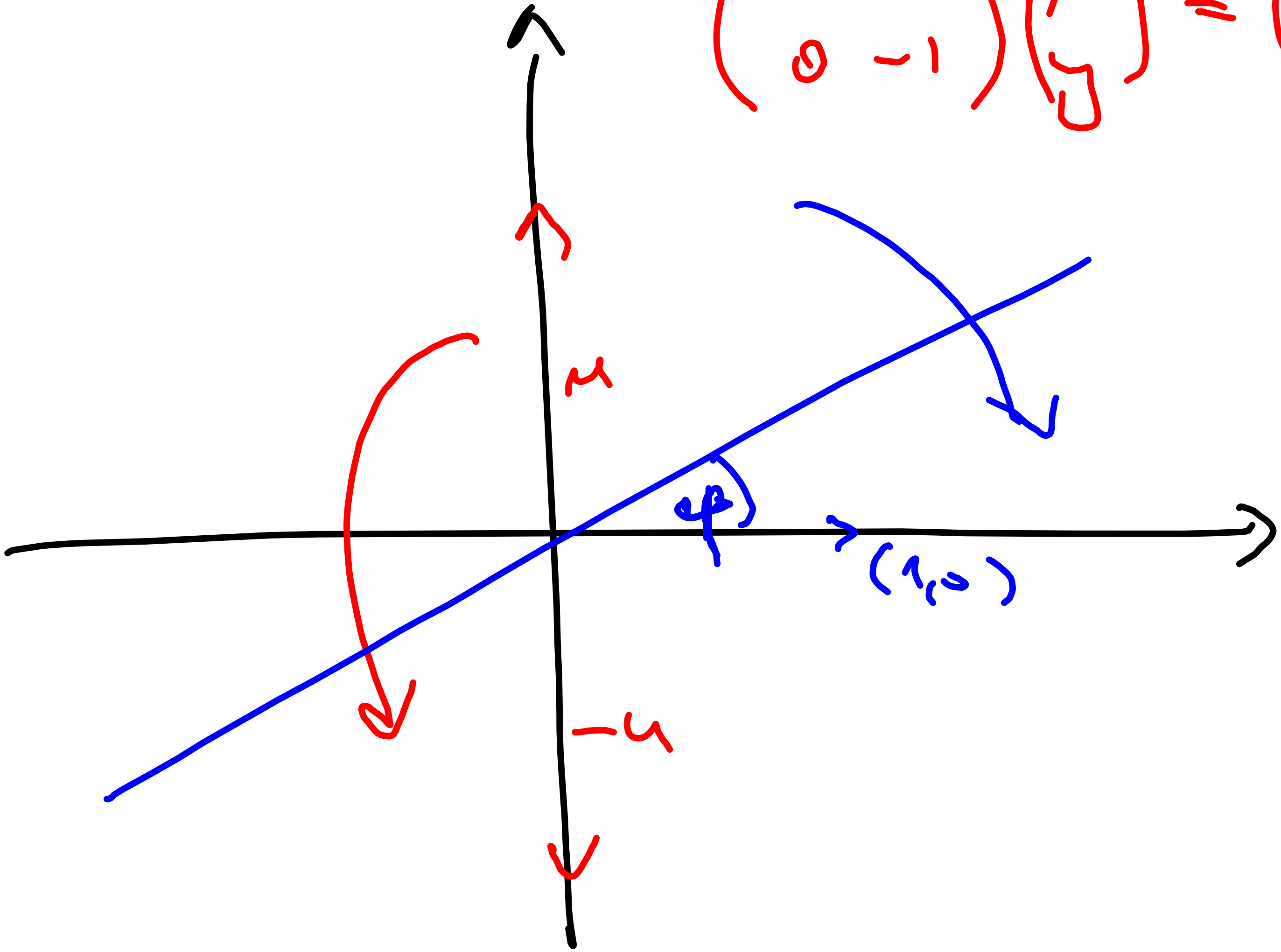
$$\sin^2\varphi\cdot x^2 + \cos^2\varphi\cdot y^2 + 2\,\cancel{\cos\varphi\sin\varphi\cdot x}$$

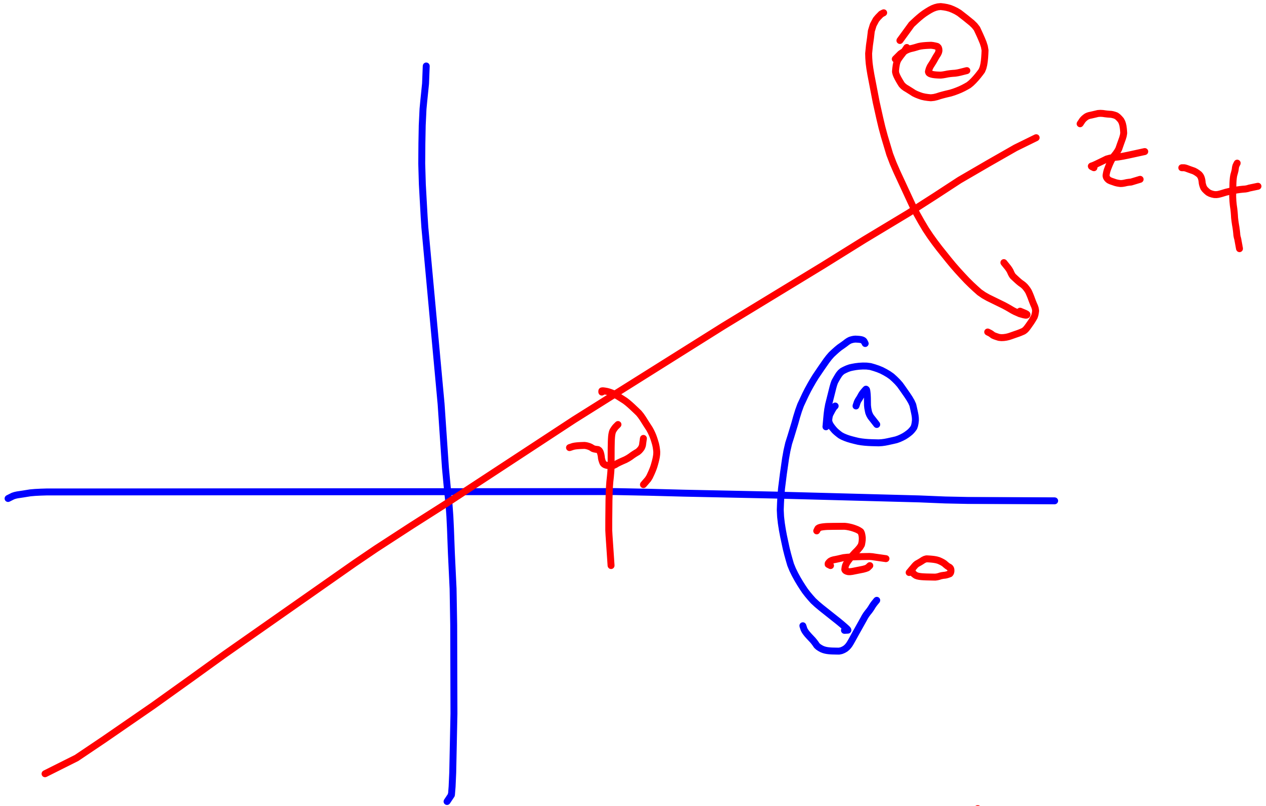
$$= 1\cdot x^2 + 1\cdot y^2$$



w
$v-w$
R_φ
0
w
$0+w$
$R_\varphi(v-w)$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$





$2 \circ 1$ je rotace o 2φ

složení

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix} \qquad \| \quad \|^2$$

$$(ax+by)^2 + (cx+dy)^2 = x^2+y^2$$

$$\underbrace{(a^2+c^2)}_{=1}x^2 + 2\underbrace{(ab+cd)}_{=0}xy + \underbrace{(b^2+d^2)}_{=1}y^2$$

$$a^2+c^2=1 \Rightarrow a=\cos\varphi,\ c=\pm\sin\varphi$$

$$\dots \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \quad \text{nebo} \quad \begin{pmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{pmatrix}$$

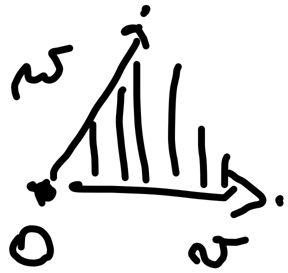
otočení   otočení + zrcadlení!

$$\det A = \begin{vmatrix} \cos\varphi & \pm \sin\varphi \\ \sin\varphi & \mp \cos\varphi \end{vmatrix}$$

$$= \mp \cos^2\varphi \mp \sin^2\varphi = \mp 1$$

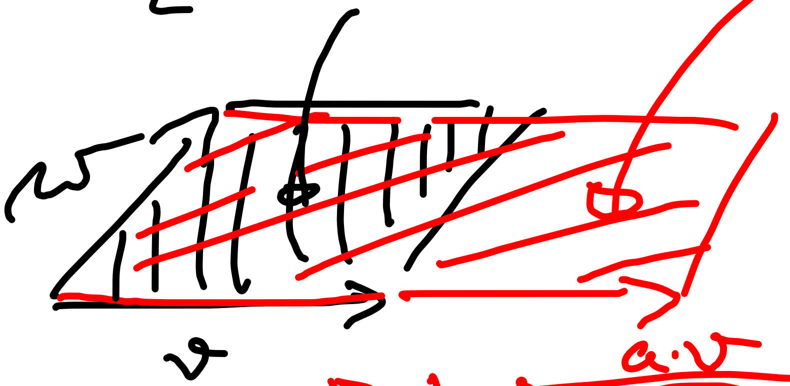
$$\det \begin{vmatrix} \frac{1}{4} & 0 \\ 0 & 4 \end{vmatrix} = 1$$

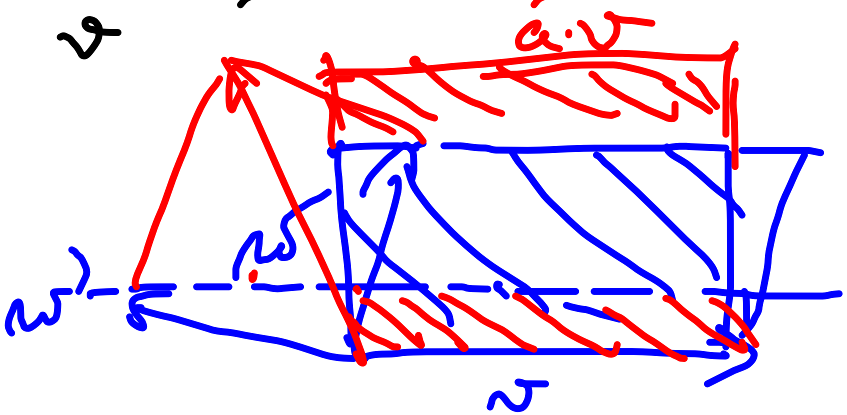
$$\mathrm{vol}(\triangle) = \mathrm{vol}(v, w) \in \mathbb{R}$$

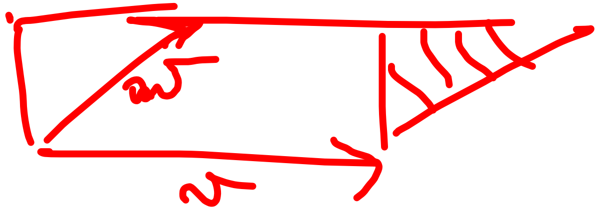


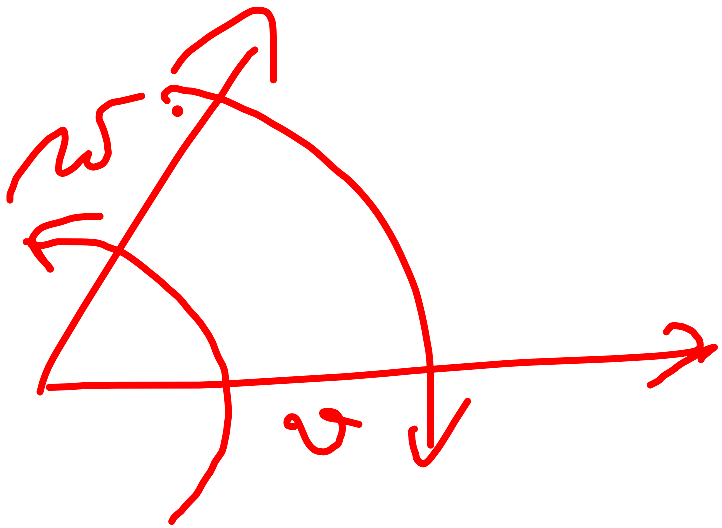
$$2\,\mathrm{vol}(v, w)$$

$$a \cdot 2\,\mathrm{vol}(v, w)$$





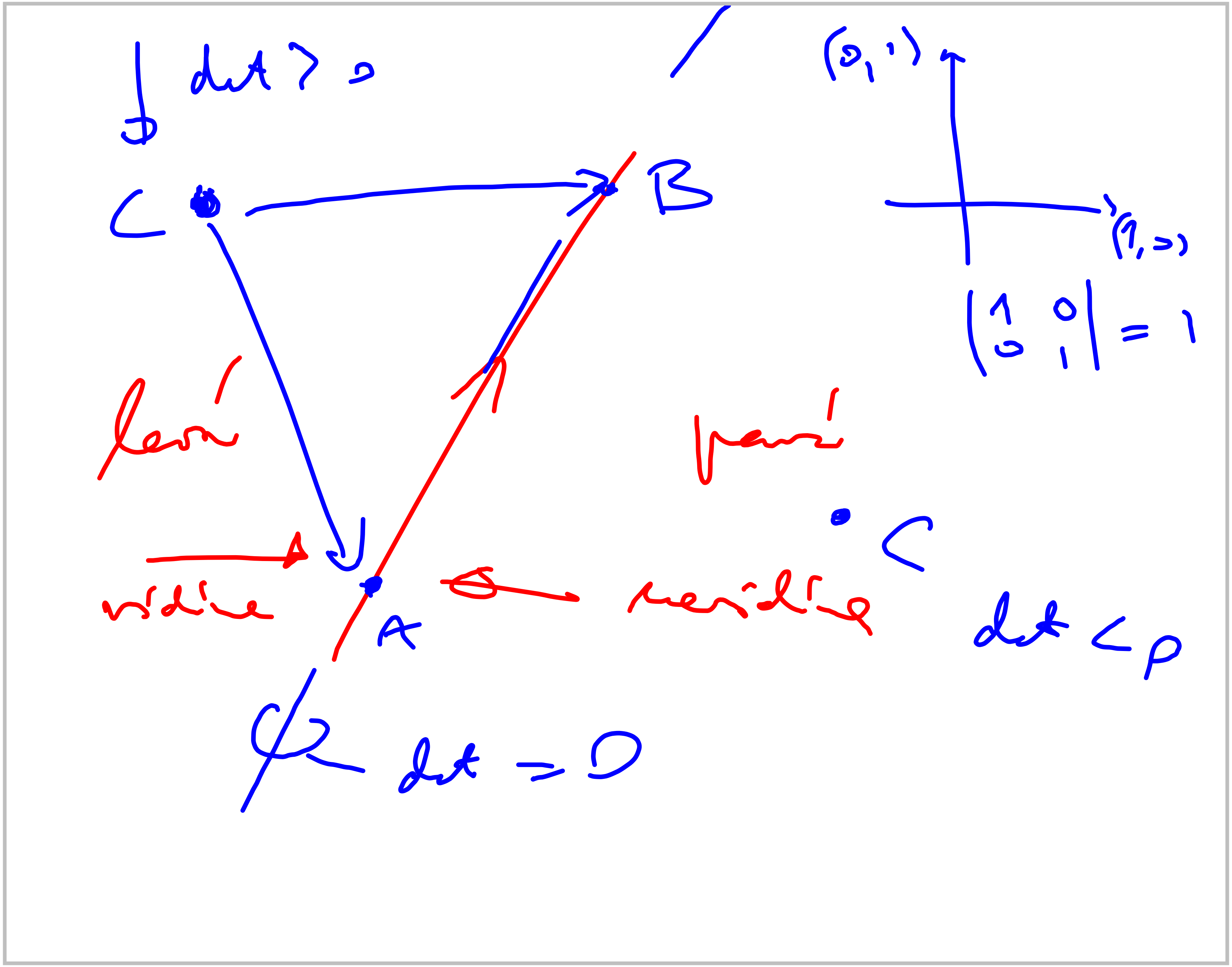




$$\mathrm{vol}(v, w) = -\mathrm{vol}(w, v)$$

$$\begin{vmatrix} x(v) & x(w) \\ y(v) & y(w) \end{vmatrix} = x(v)\,y(w) - x(w)\,y(v)$$

tedy: vol tzv. determinant


det > 0
C
B
A
levý
pravý
vidíme
nevidíme
det < 0
det = 0
(0,1)
(1,0)
$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$