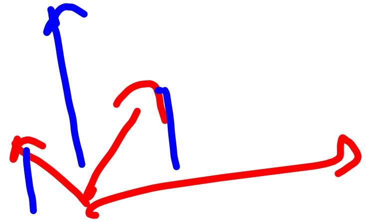
$$A = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longmapsto A \cdot x = \begin{pmatrix} x_1 \cos\varphi - x_2 \sin\varphi \\ \cdot \; - \end{pmatrix}$$

$$A \cdot x = y \quad \text{známý}$$

$\uparrow$

neznámý



$$A^{-1} \cdot A \cdot x = x = A^{-1} y$$

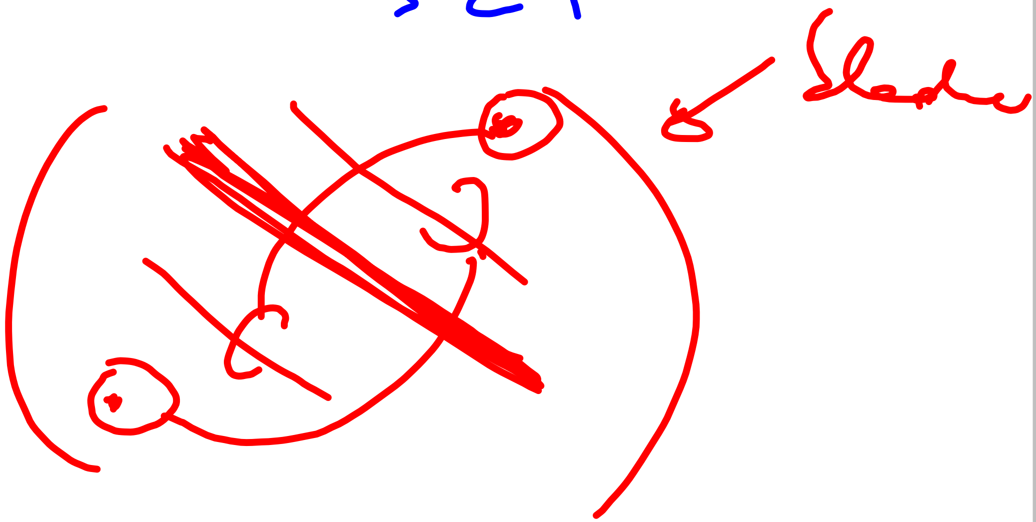
$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

1 2 3
1 3 2
2 1 3
2 3 1
3 1 2
3 2 1

$a_{11}a_{22}a_{33}$

$a_{11}a_{23}a_{32}$

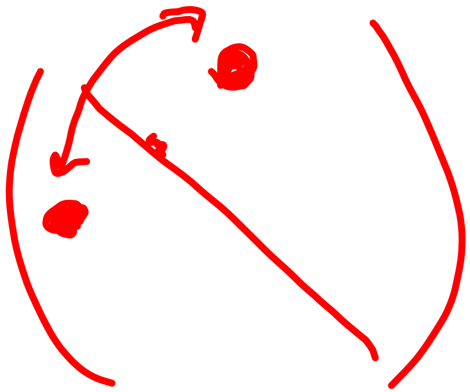
$a_{12}a_{21}a_{33}$



$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}^T = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}$$

$$(x_1 \ldots x_n)^T = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$A \overset{?}{=} A^T$$



$$\det A = \sum_{\sigma \in \Sigma_n} \operatorname{sgn} \sigma \cdot a_{1\sigma(1)} \cdot \ldots \cdot a_{n\sigma(n)}$$

$$a_{\sigma(1)1} = c_{\sigma(1)1} + d_{\sigma(1)1}$$

$$\det A^T = \sum_{\sigma \in \Sigma_n} \underbrace{\operatorname{sgn} \sigma}_{\operatorname{sgn} \sigma^{-1}} \underbrace{\boxed{a_{\sigma(1)1}} \cdot \ldots \cdot a_{\sigma(n)n}}_{a_{1\sigma^{-1}(1)} \cdot \ldots \cdot a_{n\sigma^{-1}(n)}}$$

beze zm.

$$\det B = \sum_{\sigma \in \Sigma_n} \underbrace{\operatorname{sgn} \sigma}_{\operatorname{sgn} \sigma'} \underbrace{a_{2\sigma(1)} \cdot a_{1\sigma(2)} \cdot \ldots \cdot a_{n\sigma(n)}}_{a_{1\sigma'(1)}\, a_{2\sigma'(2)} \cdot \ldots}$$

↑ opačné

$$= -\det A$$

$$\underset{A}{\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}} \xrightarrow[\text{elim.}]{\text{Gauss.}} \underset{B}{\begin{pmatrix} b_{11} & \cdots & b_{1n} \\ 0 & \ddots & \\ \vdots & & \cdots \\ 0 & \cdots\ 0 & b_{nn} \end{pmatrix}}$$

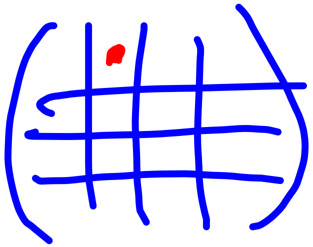
$$b_{ij} = 0 \quad \forall\, i > j$$

$$\sigma \begin{pmatrix} 1 & 2 & \cdots & n \\ \sigma(1) & \sigma(2) & & \sigma(n) \end{pmatrix}$$

$$b_{1\sigma(1)} \cdot b_{2\sigma(2)} \cdots b_{n\sigma(n)}$$

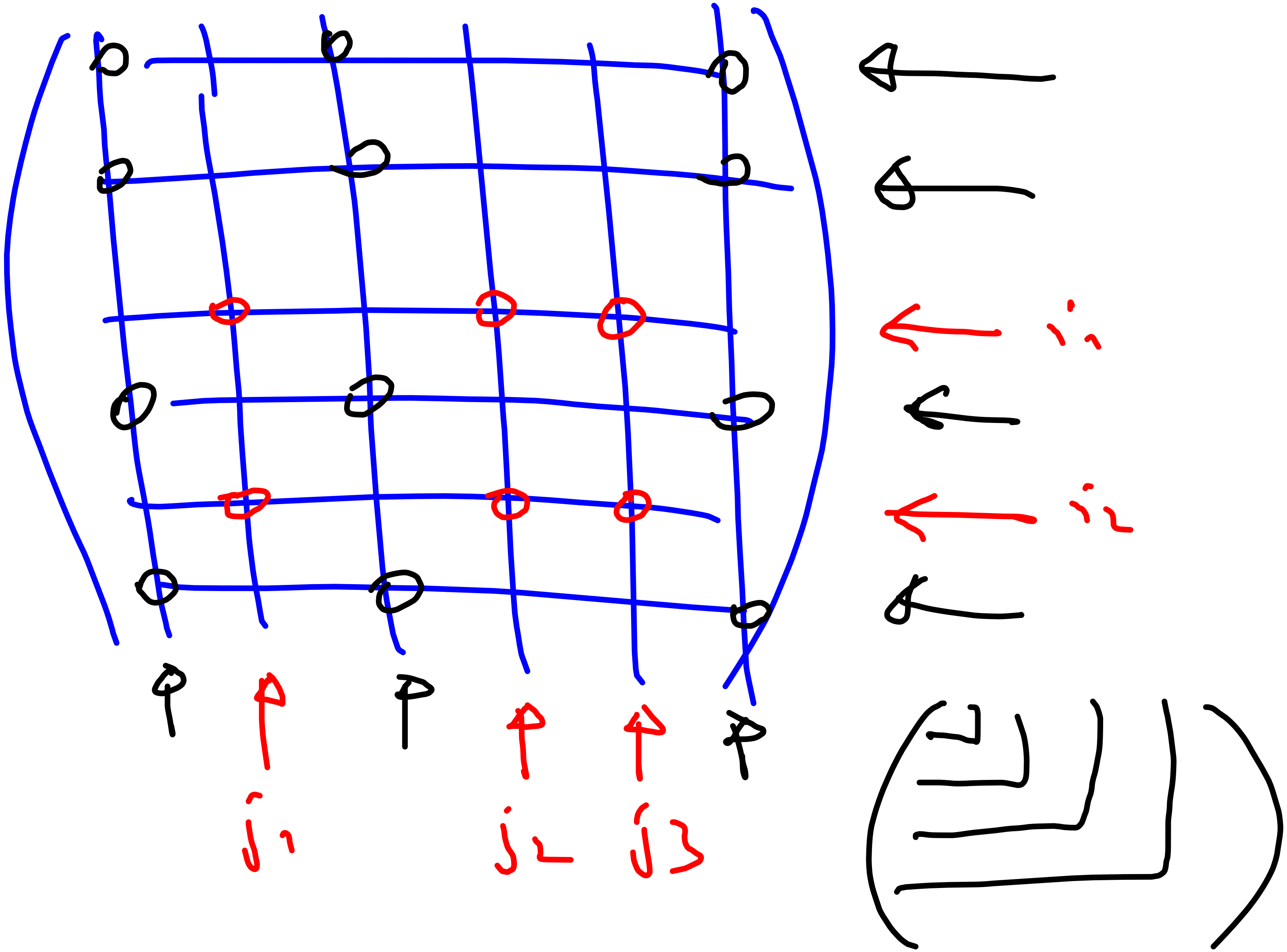
$$\boxed{\sigma(i) < i \ ?}$$

$$(\ \sigma(j) > j\ ) \longrightarrow$$



$$\boxed{\det B = b_{11} \cdots b_{nn}}$$

$$x \xmapsto{A} A\cdot x \xmapsto{B} (B\cdot A)\,x$$

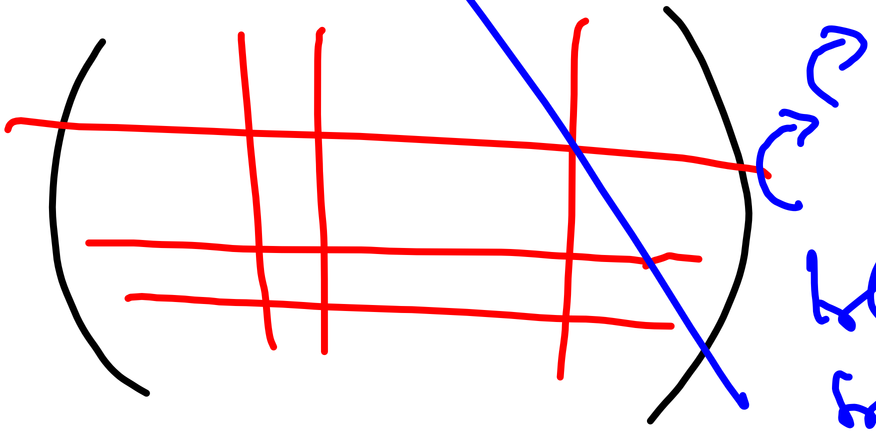


i1
i2
j1
j2
j3

člen v $|M|$: $\sigma' \in S_k$ na $\{1, \dots, k\}$
$\sigma'' \in S_{n-k}$ na $\{\dots\}$

$$\boxed{\operatorname{sgn} \sigma' \, a_{i_1 j_{\sigma'(1)}} \cdot \ldots \cdot a_{i_k j_{\sigma'(k)}}}$$

$$\boxed{(-1)^{i_1 + \dots + i_k + j_1 + \dots + j_k} \operatorname{sgn} \sigma'' \, a_{..} \cdot \ldots \cdot a_{..}}$$

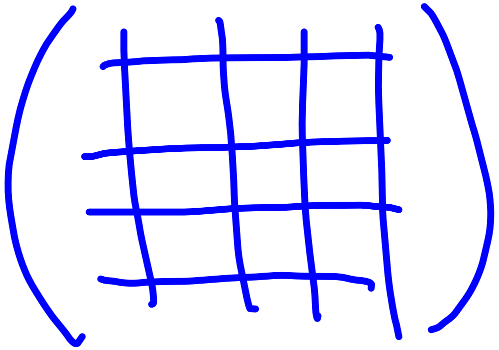


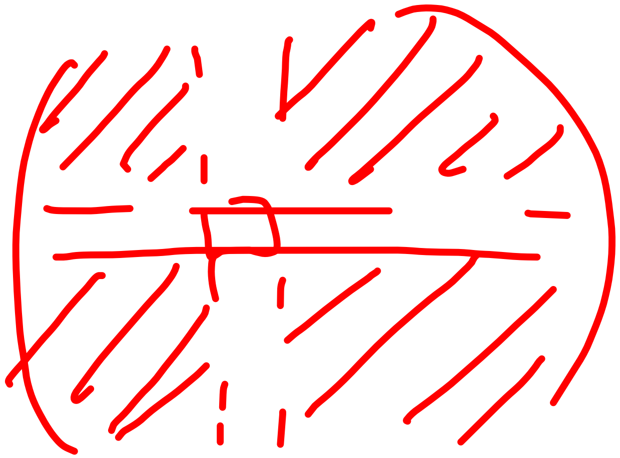
kolik transpozic řádků
posunutí ř & sloupců
převede na vrchol

$j_1 = 1, \dots, j_k = k$, $i_1 = 1, i_2 = 2, \dots, i_k = k$

$$\begin{array}{ccc} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \\ \begin{array}{ccc} 1 & 2 & \end{array} \end{array}$$

$$(-1)^{1+1} a \cdot (ei - fh) + (-1)^{1+2} b (di - fg)$$
$$+ (-1)^{1+3} c (dh - ge)$$





$A, B, \quad A \cdot B = (c_{ik})$

$a_{ij} \quad b_{jk} \qquad c_{ik} = \sum_j a_{ij} b_{jk}$

$$H = \left(\begin{array}{ccc|ccc} a_{11} & \cdots & a_{1n} & & & \\ \vdots & & & & 0 & \\ a_{n1} & \cdots & a_{nn} & & & \\ \hline -1 & \cdots & 0 & b_{11} & \cdots & b_{1n} \\ \vdots & -1 & \vdots & \vdots & & \\ 0 & \cdots & -1 & b_{n1} & \cdots & b_{nn} \end{array}\right)$$

$\det H = \boxed{\det A \cdot \det B}$

$-b_{11} + b_{11} = 0$

$\boxed{b_{11} a_{11} + a_{12} \cdot b_{21} + \cdots + a_{1n} b_{n1}} = c_{11}$

$$\begin{pmatrix} A & 0 \\ -E & B \end{pmatrix} \rightsquigarrow \begin{pmatrix} A & \boxed{A\cdot B} \\ -E & 0 \end{pmatrix} = K$$

$$\rightsquigarrow \underline{|K| = |A\cdot B|}$$

$$|A\cdot A^{-1}| = |A|\cdot|A^{-1}| = 1$$

$$\Rightarrow |A^{-1}| = (|A|)^{-1}$$

$$\exists\, (|A|)^{-1} \Rightarrow$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad A^* = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A \cdot A^* = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \det A$$

$$A \cdot A^* = (\det A) \, E$$