vektor = "n-tice čísel?"

$$a_{11}x_1 + a_{21}x_2 + \dots + a_{1n}x_n = 0$$

$\vdots$

komutativní grupa:

$$u + v = v + u$$

$\forall u \ \exists! -u, \quad u + (-u) = 0$

$$u + (v + w) = (u + v) + w$$

$\exists 0! \quad u + 0 = u \quad \forall u$

$0 \cdot u = 0$?

$(1 + (-1)) \cdot u = u + (-1)u \overset{(2)}{=} u + (-u) = 0$

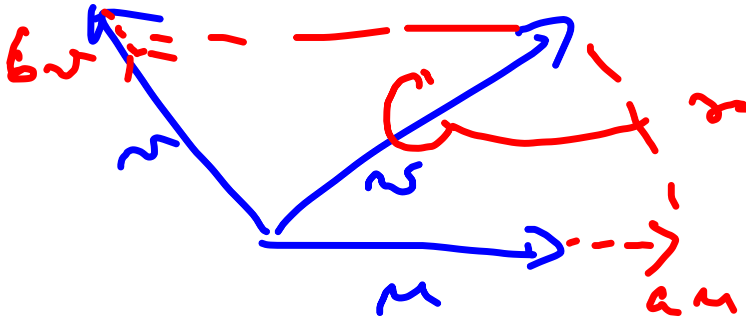
$$a_1 \cdot v_1 + \dots + a_k v_k = 0$$

$\dots$: $a_i \neq 0$

$\Rightarrow a_i \cdot v_i = -a_1 \cdot v_1 - \dots - \widehat{i} - \dots - a_k v_k$

$$\Rightarrow v_i = \underbrace{-a_i^{-1} (a_1 v_1 + \dots + \widehat{i} + \dots + a_k v_k)}$$

$\mathbb{R}^2$:



závisí na $a, b \in \mathbb{R}$

$w = au + bv$

$$a(1,0) + b(0,1) = (a,b) = (0,0)$$

$$\sqrt{2}\cdot(1,0) - 1\cdot(\sqrt{2},0) = (0,0)$$

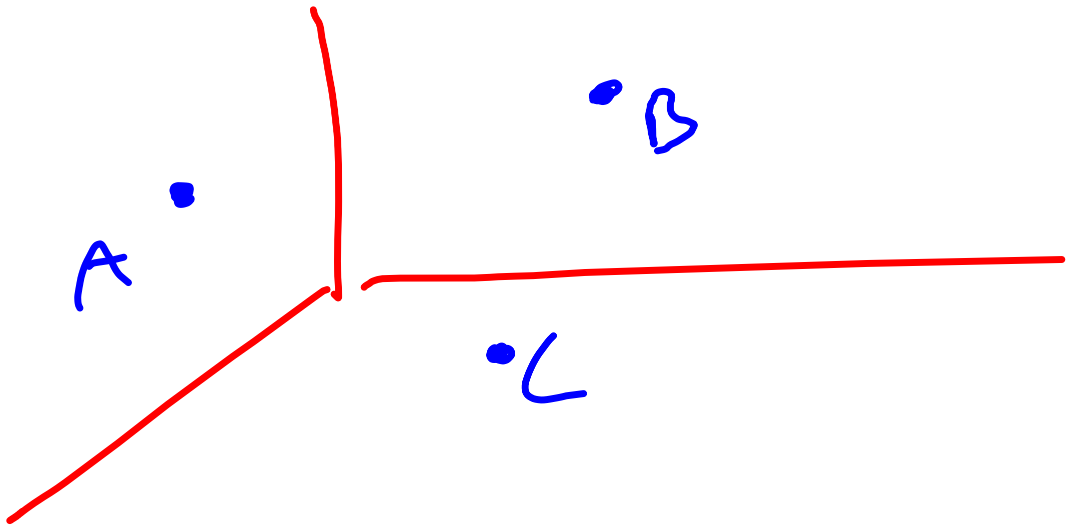
$W_1, W_2 \subset V$

$a, b \in K, \quad v, u \in W_1 \cap W_2$

$av + bu \overset{?}{\in} W_1 \cap W_2$

$U \subset \mathbb{R}^3$

$U \supset \{A, B, C\}$



$\{a_1 u_1 + \dots + a_k u_k ; \dots\} \subset \langle M \rangle$

$a_1 u_1 + \dots + a_k u_k + b_1 v_1 + \dots + b_l v_l$

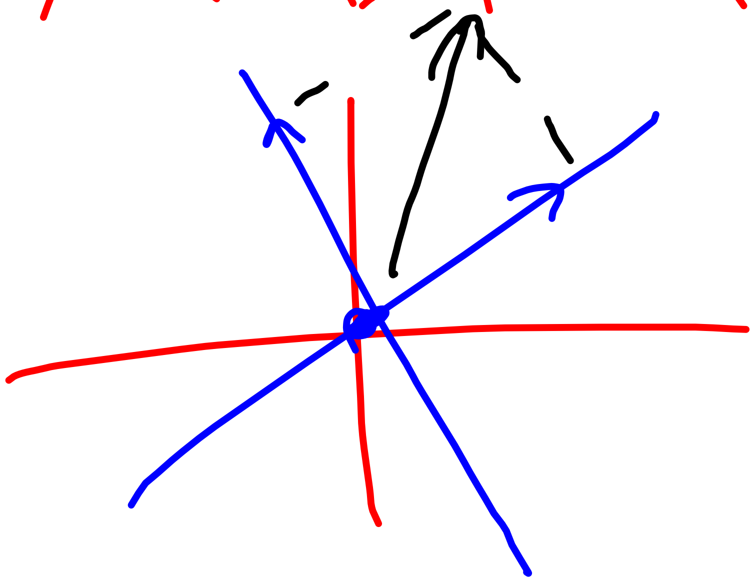
opět konečná lineární komb. prvků z M

$\Rightarrow \in \{ \ldots \}$

$b(a_1 u_1 + \dots + a_k u_k) = (b a_1) u_1 + \dots + (b a_k) u_k$

---

být lineární



$$u \in \langle V_1 \cup \dots \cup V_k \rangle \Rightarrow$$

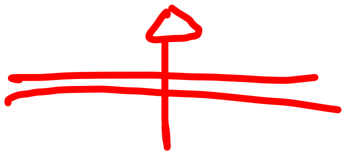
$$u = \underbrace{a_1 v_1 + \dots + a_k v_k}_{\in V_1} + \underbrace{b_1 w_1 + \dots + b_\ell w_\ell}_{\in V_2} + \dots$$

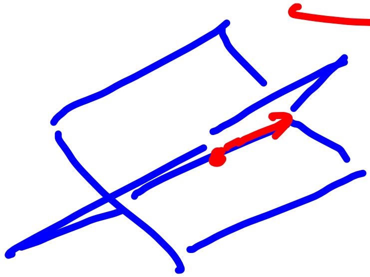
(pořadí není podstatné v součtu)

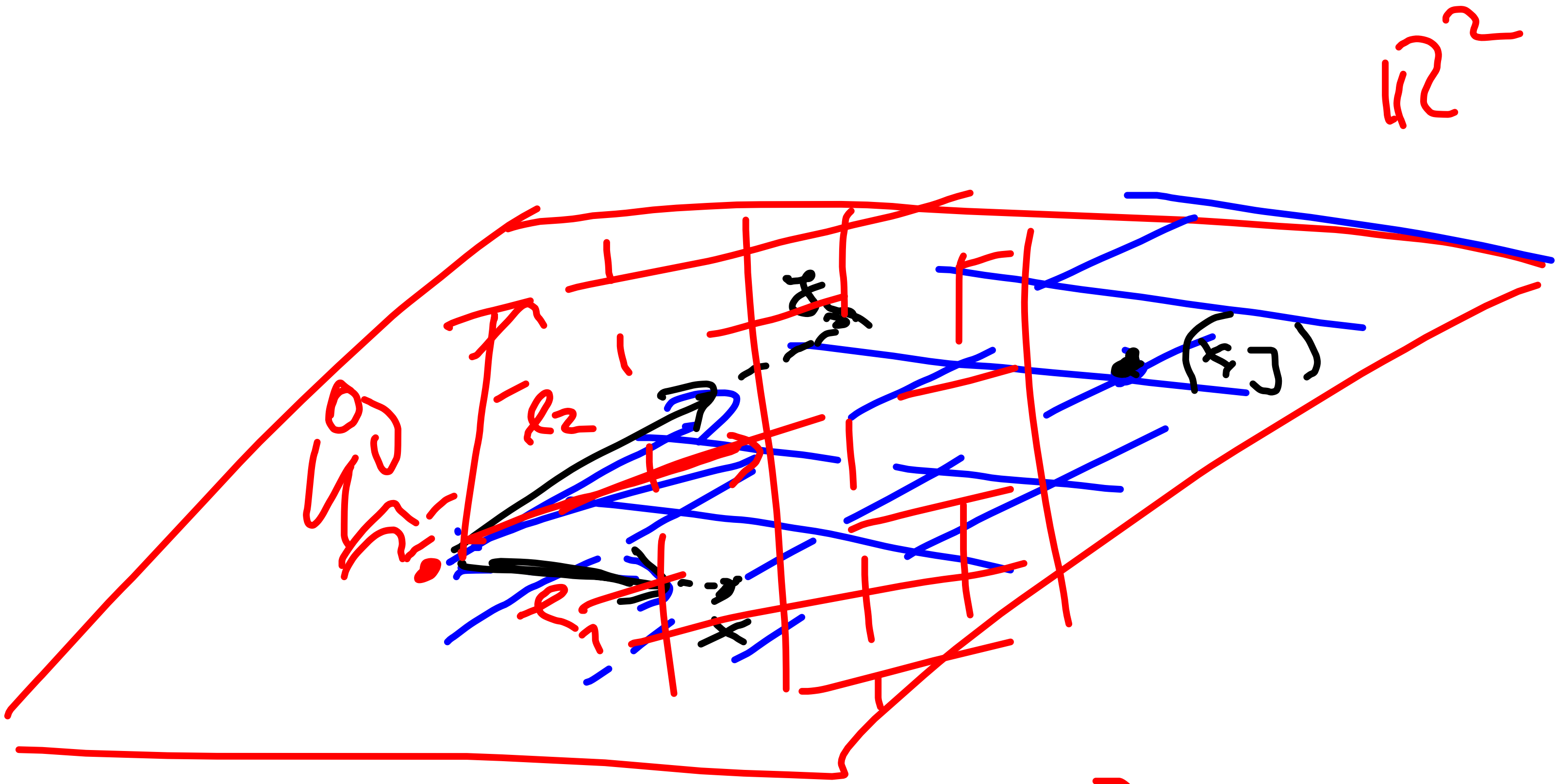
$$= \underline{v_1' + v_2' + \dots + v_k'}$$

pro přímé sumy je to jednoznačné

$$u = w_1' + w_2' + \dots + w_k'$$



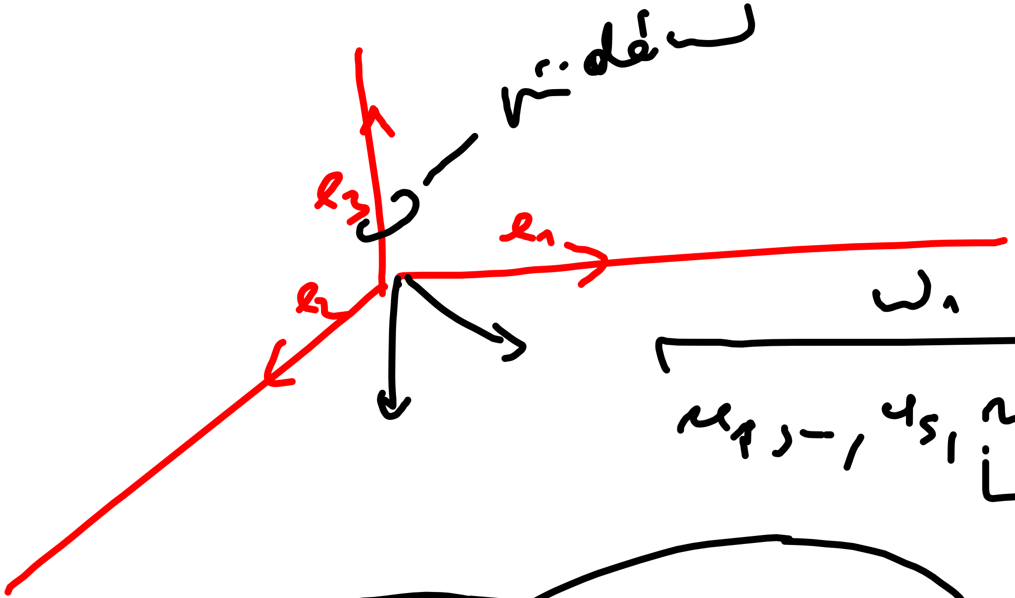




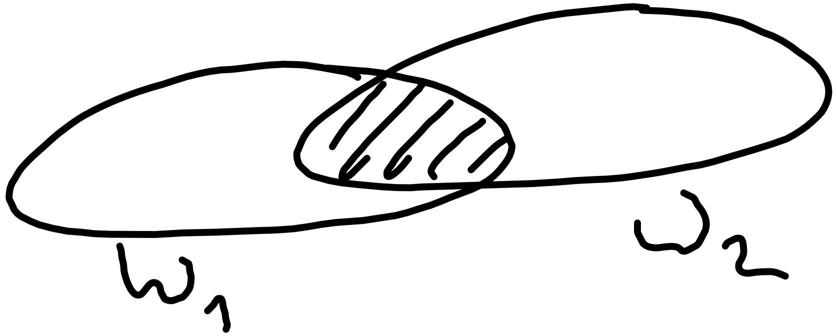
$\langle e_1 \rangle \oplus \langle e_2 \rangle = \mathbb{R}^2$

$V = \langle M \rangle \qquad M = \{u_1, \dots, u_k\}$

$V_1 = \langle u_1 \rangle \quad \dots \quad V_i = \langle V_{i-1} \cup \{u_i\} \rangle$



$\mathrlap{\overbrace{}^{W_1}}u_1,\dots,u_s,\ \underbrace{w_1,\dots,w_t,\ v_1,\dots,v_r}_{W_2}$



Souřadnice : $\underbrace{M \to \mathbb{K}}$

$$\underline{v} = (v_1, \dots, v_n)$$

$$\underline{v} : \underbrace{V \longrightarrow \mathbb{K}^n}_{n = \dim V} \qquad u \longmapsto \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$$

$$u = a_1 v_1 + \dots + a_n v_n$$

$$u + u' = a_1 v_1 + \dots + a_n v_n + a_1' v_1 + \dots + a_n' v_n$$
$$= (a_1 + a_1') v_1 + \dots + (a_n + a_n') v_n$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \longmapsto A \cdot x \in \mathbb{K}^m$$

$$A \cdot (x + y) = Ax + Ay$$
$$A \cdot (bx) = b\,Ax$$

$$a \cdot f(u) + b \cdot f(v) = f(a \cdot u + b \cdot v)$$

$$f(u) = 0 = f(v) \in W$$

$$f(au + bv) = a f(u) + b f(v) = a \cdot 0 + b \cdot 0 = 0$$

f prosté $\iff$ $f(u) = 0$ pouze pro $u = 0$

$$\left(f(u) = f(v) \implies f(u - v) = 0 \right)$$

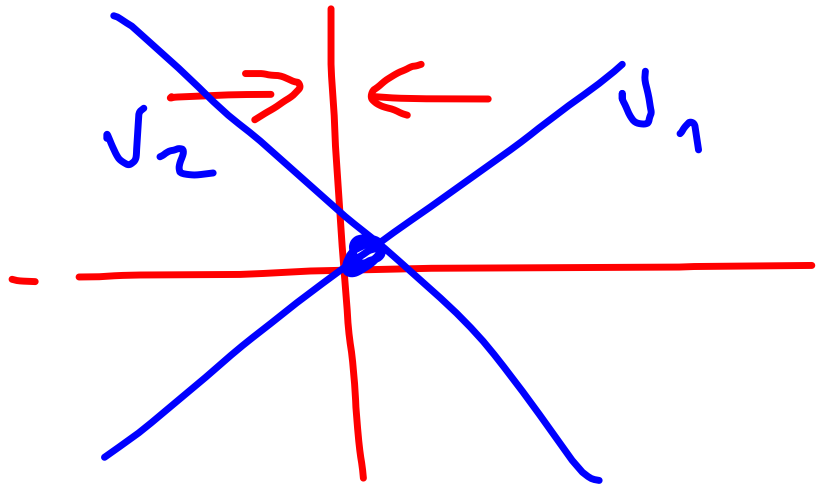
4) $u, v \in V_1 \implies au + bv \in V_1 \implies f(au + bv) \in f(V_1)$

$$= a \cdot f(u) + b \cdot f(v)$$

$$(g\circ f)(au+bv) = g(a f(u) + b f(v))$$
$$= a\,(g\circ f)(u) + b\,(g\circ f)(v)$$

$$f(u+v) = f(u) + f(v)$$

$u \in V_1,\quad v \in V_2,\qquad f(u) \in f(V_1),\quad f(v) \in f(V_2)$

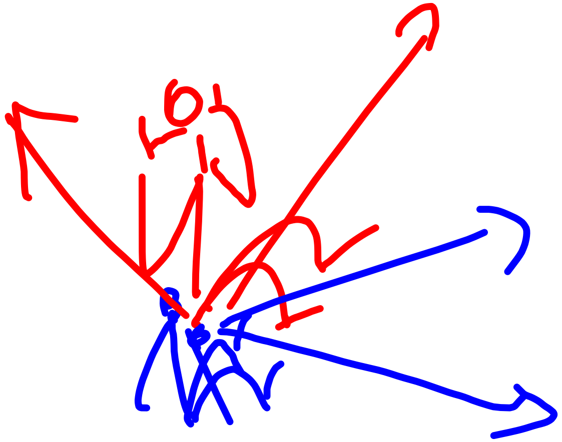


$f(x,y) = (0,y)$

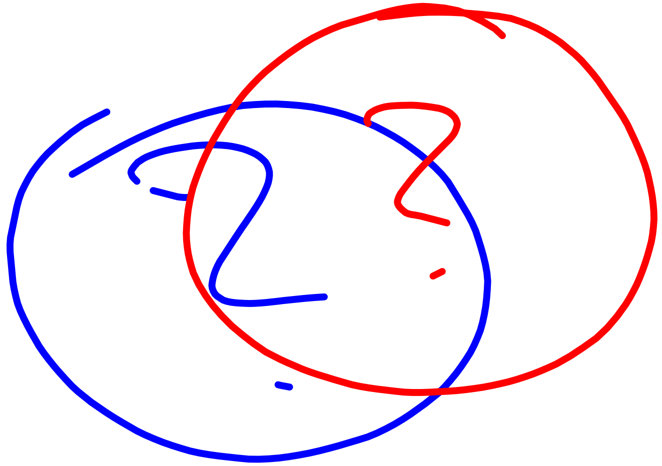
$V_1 \cap V_2 = 0$

$$V \xrightarrow{\underline{u}} K^n \xleftarrow{\underline{v}} W$$

$\underline{v}^{-1}\circ\underline{u}$ je izomorfis



- $(x,y) = (x',y')$



matice přechodu

$$X = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \xmapsto{u^{-1}} a_1 u_1 + \dots + a_n u_n \in V$$

$$\Big\downarrow f$$

obraz bázového vektoru

$$a_1 f(u_1) + \dots + a_n f(u_n) \in W$$

$$f(u_1) = a_{11} v_1 + a_{21} v_2 + \dots + a_{m1} v_m$$

$$\vdots$$

$$f(u_n) = a_{1n} v_1 + a_{2n} v_2 + \dots + a_{mn} v_m$$

$$= a_1 (a_{11} v_1 + \dots + a_{m1} v_m) + \dots + a_n (a_{1n} v_1 + \dots)$$

$$= (a_{11} a_1 + a_{12} a_2 + \dots + a_{1n} a_n) v_1 + \dots$$

$$= y = A \cdot x, \quad A = (a_{ij})$$

$$\begin{tikzcd}
V \arrow[r, "f"] \arrow[d, "\underline{u}"'] & W \arrow[r, "g"] \arrow[d, "\underline{v}"] & U \arrow[d, "\underline{w}"] \\
K^n \arrow[r, dashed, "A"] & K^m \arrow[r, dashed, "B"] & K^s
\end{tikzcd}$$

$$x \longmapsto A\cdot x \longmapsto B\cdot(A\cdot x)$$

$$(B\cdot A)\cdot x$$