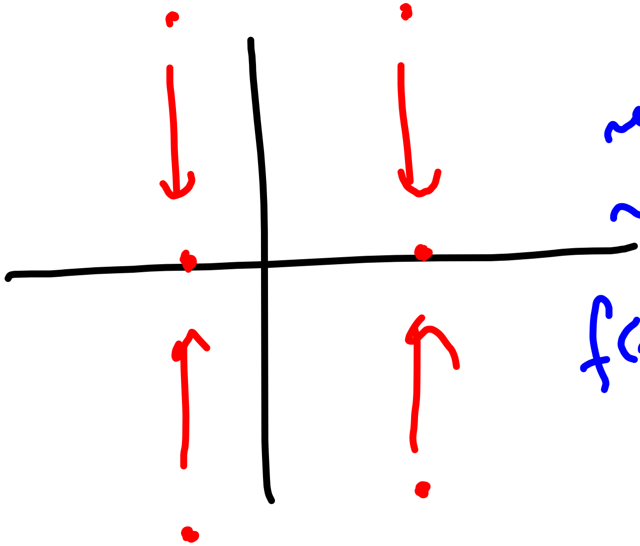


$\mathbb{R}^2$

$u = \begin{pmatrix}1\\0\end{pmatrix}$

$v = \begin{pmatrix}0\\1\end{pmatrix}$

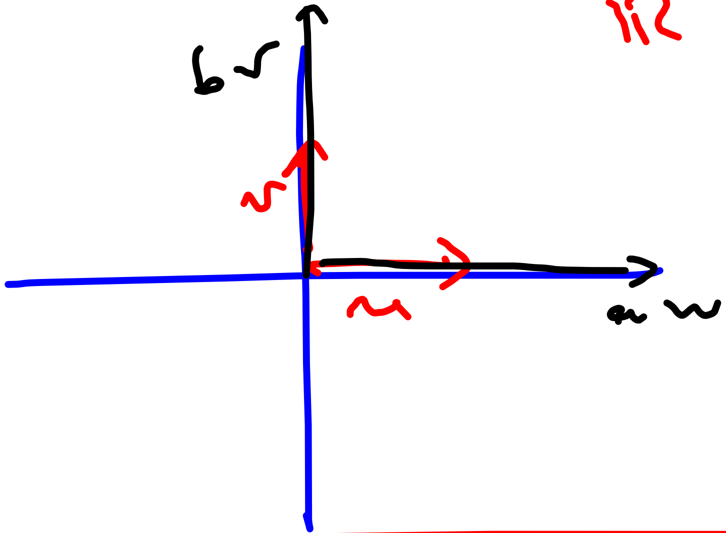
$f(v) = u$

$$\underbrace{\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}}_{= A}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\\0\end{pmatrix}$$

$$\mathbb{R}_1(x) = \{a+bx\} = \langle \overset{u}{\overset{\parallel}{1}}, \overset{v}{\overset{\parallel}{x}} \rangle$$

$$f(x) = a+bx \longmapsto f'(x) = b \qquad \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}$$

$$f(u) = 0, \quad f(v) = u$$

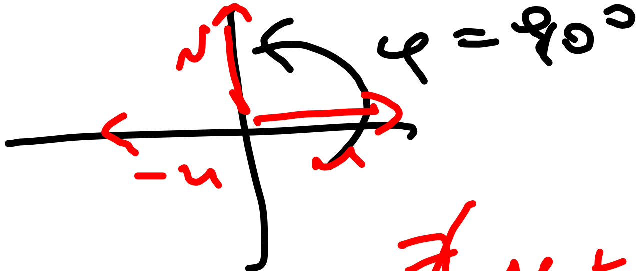


$\mathbb{R}^2 = \langle u, v \rangle$

$C = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

$f(u) = au$

$f(v) = bv$



$D = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$\nexists\, u \in \mathbb{R}^2$ nenulový

$f(u) = \lambda u, \quad \lambda \in \mathbb{R}$

no $f : \mathbb{C}^2 \to \mathbb{C}^2 \qquad \begin{pmatrix} z \\ w \end{pmatrix} \mapsto \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} z \\ w \end{pmatrix}$

$a(i, 1) = b(1, i)$

$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = A \qquad A\cdot\begin{pmatrix}1\\0\end{pmatrix} = a\begin{pmatrix}1\\0\end{pmatrix} \ \ldots$

$A\cdot x = a\cdot x \iff \overbrace{(A - a\cdot E)\cdot x = 0}$

pevné [illegible]

$x \neq 0$ singulární $\exists$ ex. $\iff$

$\det(A - aE) = 0$

hledáme $\lambda = a$

$$|A - \lambda E| = \begin{vmatrix} a-\lambda & 0 \\ 0 & b-\lambda \end{vmatrix} = (a-\lambda)(b-\lambda)$$

$= 0 \iff \lambda = a$ nebo $\lambda = b$

$$\det(A - \lambda E) = (a_{11}-\lambda)\cdots(a_{nn}-\lambda) + \ldots$$
$$= (-1)^n \lambda^n + \ldots$$

$$D = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$|D - \lambda E| = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0$$

$$\Rightarrow \quad \lambda_{1,2} = \pm i$$

$$\boxed{f(u) = a \cdot u} \rightsquigarrow \quad A \cdot x = a \cdot x$$

$\mathbb{R}^2$

$$f(u) = a \cdot u, \quad f(v) = b \cdot v, \quad a \neq b$$

$$\boxed{\alpha u + \beta v = 0} \Rightarrow \quad f(\alpha u + \beta v) = \\ = \alpha f(u) + \beta f(v) = \\ = \alpha \cdot a \cdot u + \beta \cdot b \cdot v =$$

$u = \alpha v \qquad f(u) = \alpha f(v) = \alpha \cdot b \cdot v$

$$\begin{array}{ccc} f(u) & = \alpha & f(v) \\ \| & & \| \\ a u & & \frac{\alpha}{\alpha} \cdot b \cdot u \end{array}$$

$\Rightarrow \underset{\neq 0}{(a-b)} \underset{\neq 0}{u} = 0 \Rightarrow$ spor

---

$f \longmapsto |A - \lambda E| \longmapsto a_1, \dots, a_n, \quad a_i \neq a_j \quad i \neq j$

$\Rightarrow$ ex. báze z vlastních vektorů $u_1, \dots, u_n$

$u_1 \xmapsto{f} a_1 u_1$

$\vdots$

$u_n \xmapsto{f} a_n u_n$

tj. s touto bází je

$$A = \begin{pmatrix} a_1 & 0 & \dots & 0 \\ 0 & a_2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & a_n \end{pmatrix}$$

$\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix} = B \qquad \begin{vmatrix}-\lambda & 1\\ 0 & -\lambda\end{vmatrix} = \lambda^2 = 0$

$\lambda = 0$ alg. násobnost 2

$(A - \lambda_0 E)\cdot x = 0$

$A\cdot x = 0$

$$\left.\begin{aligned} 0\cdot x_1 + x_2 &= 0\\ 0\cdot x_1 + 0\cdot x_2 &= 0\end{aligned}\right\} \Rightarrow \boxed{x_2 = 0}$$

řešení je $\left\langle \begin{pmatrix}1\\0\end{pmatrix} \right\rangle$

geom. násobnost 1

$\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix} = B' \qquad \begin{vmatrix}-\lambda & 0\\ 0 & -\lambda\end{vmatrix} = \lambda^2 = 0$

obojí násobnost 2

$$\left(\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)\left(\begin{array}{c} \square \\ \oplus \\ \square \end{array}\right) \qquad V = V_1 \oplus V_2$$

$$\underbrace{f \cdots f}_{k\times}(v) = a^k \cdot v \implies \text{nulová a platí } f \text{ nilp.}$$

$$V = \underset{\lambda_1}{\underset{\uparrow}{V_1}} \oplus \dots \oplus \underset{\lambda_\ell}{\underset{\uparrow}{V_\ell}}$$

$$(f - \lambda_i \cdot id)|_{V_i} = \text{cyklické}$$

$\mathbb{R}^2$

$$\underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix} = f\begin{pmatrix} x \\ y \end{pmatrix}$$

$$a \cdot b + c\,d = 0$$

$$\underbrace{a^2 + c^2} = b^2 + d^2 = 1$$

$$A \cdot A^T = E \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$\underbrace{(x \;\; y) \cdot E \cdot \begin{pmatrix} x \\ y \end{pmatrix}} = x^2 + y^2 = \|u\|^2$$

$$v = \begin{pmatrix} x \\ y \end{pmatrix}, \quad v' = \begin{pmatrix} x' \\ y' \end{pmatrix} \longleftrightarrow \underbrace{xx' + yy'}$$

$$\boxed{v^T \cdot E \cdot v'} \longleftarrow \text{skalární součin}$$

$$\in \mathbb{K}$$

$\langle\ ,\ \rangle$ bilineární forma

$$\Big\langle \underbrace{\sum a_i u_i}_{u}, \underbrace{\sum b_j u_j}_{v} \Big\rangle = \sum_{i,j} a_i b_j \underbrace{\overbrace{\langle u_i, u_j \rangle}^{= s_{ij}}}_{\text{ortonormální}\ (=)}$$

$$\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & \end{cases}$$

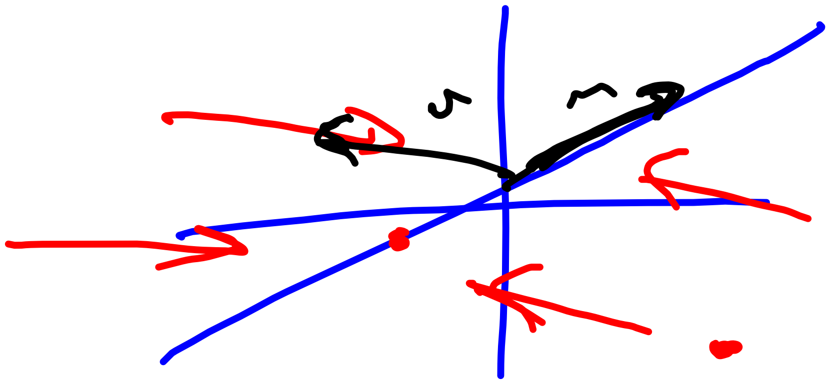
$$= \sum_i a_i b_i$$

$$= u^T \cdot v$$

$$S = (s_{ij})$$

$$= \sum_{i,j} a_i b_j s_{ij} = u^T \cdot S \cdot v$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

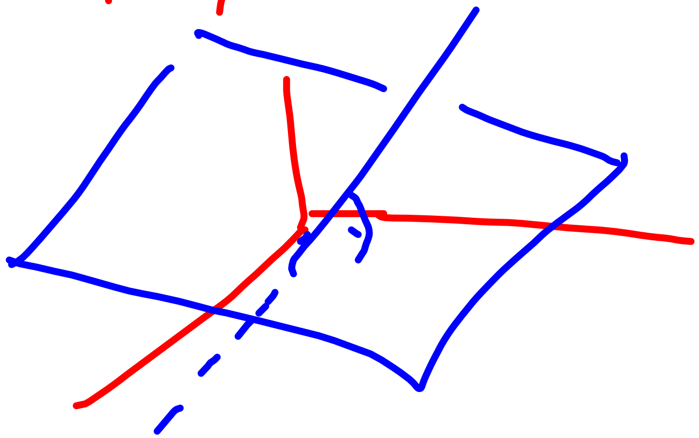


$$f \circ f = f$$

$$f(v - f(v)) = f(v) - \underbrace{f(f(v))}_{= f(v)} = 0$$

$v \in W$ právě, když $u, w \in W^{\perp}$

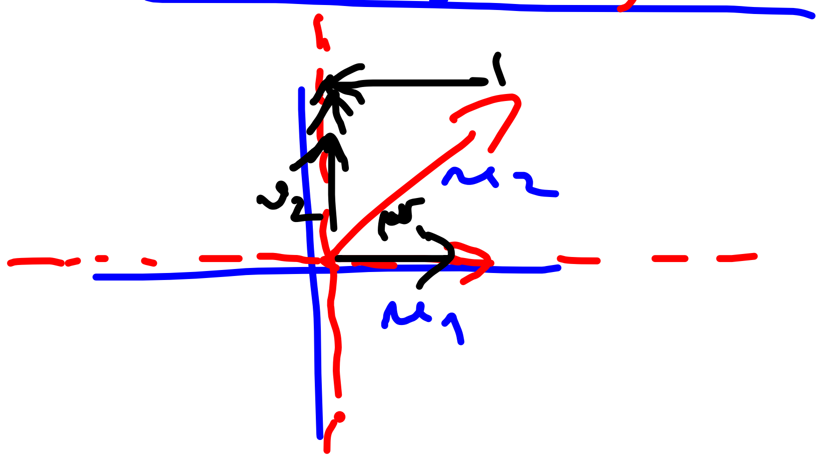
$$\langle u + w, v \rangle = \langle u, v \rangle + \langle w, v \rangle = 0 + 0 = 0$$



dim 1

$u \mapsto \frac{u}{\|u\|} = v$

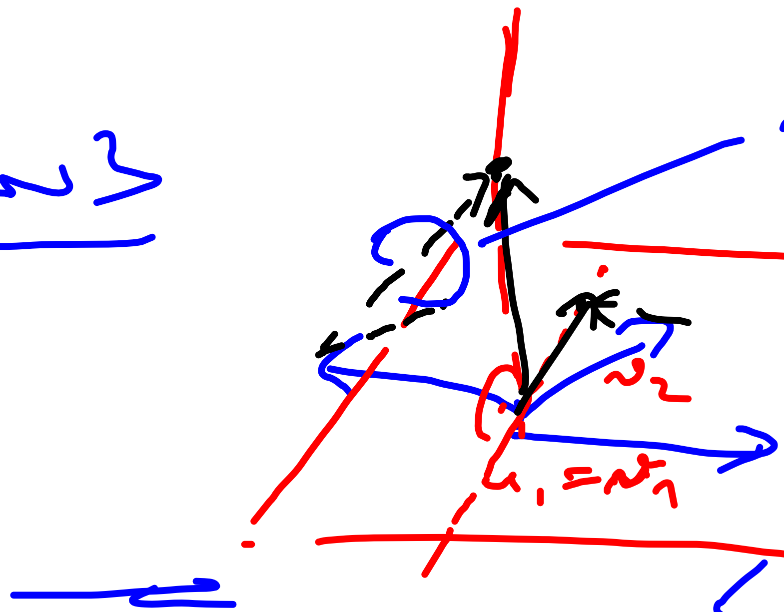
dim 2



1) projekce do $\langle u_1 \rangle^{\perp}$

2) normování

dim 3



lineární kombinace v_1, v_2

+ normovat

$$a_i = -\frac{\langle u_{k+1}, v_i \rangle}{\|v_i\|^2}$$

$W < V \qquad W = \langle \underline{e_1, \dots, e_k} \rangle$

$u \in V \Rightarrow \boxed{u = \underset{\in W}{w} + \underset{\in W^\perp}{v}}$

$V = \langle e_1, \dots, e_k, e_{k+1}, \dots, e_n \rangle$

$u = a_1 e_1 + \dots + a_k e_k + a_{k+1} e_{k+1} + \dots + a_n e_n$

$a_i = \underbrace{\langle u, e_i \rangle} = \underset{0}{\langle a_1 e_1, e_i \rangle} + \dots + \underset{a_i}{\langle a_n e_n, e_i \rangle}$

Sledujme! projekce $u \mapsto w = a_1 e_1 + \dots + a_k e_k$

$= \langle u, e_1 \rangle e_1 + \dots + \langle u, e_k \rangle e_k$

$v = u - w$

$$(A\cdot x)^T (A\cdot y) = x^T \cdot y$$

$$x^T \underbrace{(A^T A)}_{= E} y = x^T \cdot y$$

j. $\boxed{A^{-1} = A^T}$