$$\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$$

$$\begin{vmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix} = (1-\lambda)(4-\lambda)+2 =$$

$$= 4-\lambda-4\lambda+\lambda^2+2 = \lambda^2 \underline{-5}\lambda \underline{+6}$$

$$(\lambda+a)(\lambda+b) = \lambda^2+(a+b)\lambda+a.b$$

$$= (\lambda-2)(\lambda-3)$$

$\lambda_1 = 2$ $\qquad\qquad\qquad\qquad$ $\lambda_2 = 3$

$$\left(\begin{array}{cc|c} -1 & -1 & 0 \\ 2 & 2 & 0 \end{array}\right) \Rightarrow \begin{pmatrix} t \\ -t \end{pmatrix} \qquad \left(\begin{array}{cc|c} -2 & -1 & 0 \\ 2 & 1 & 0 \end{array}\right) \Rightarrow \begin{pmatrix} t \\ -\frac{t}{2} \end{pmatrix}$$

$$\boxed{2 \ldots \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad\qquad 3 \ldots \begin{pmatrix} 1 \\ -1/2 \end{pmatrix}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{3} & -\sin\pi/3 \\ 0 & \sin\pi/3 & \cos\pi/3 \end{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

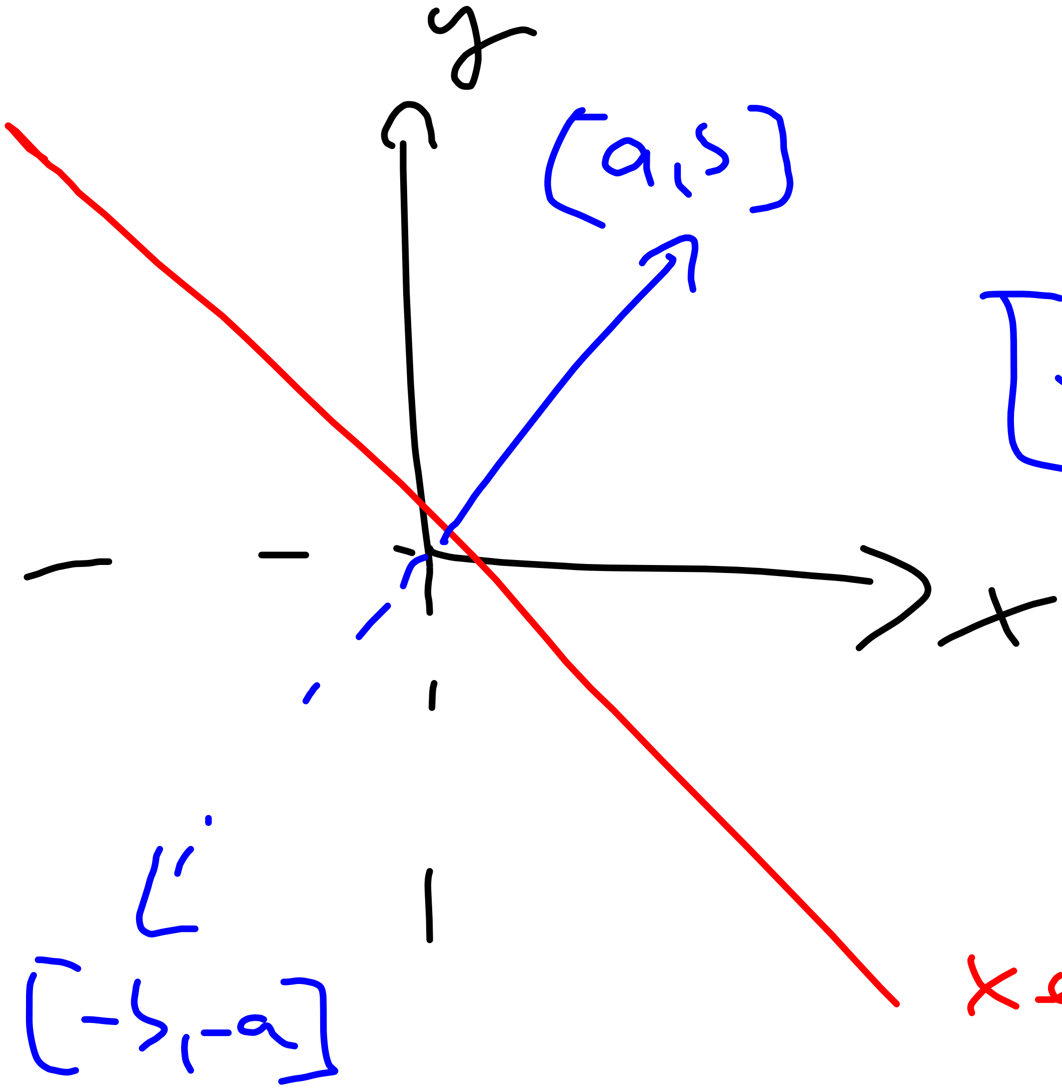
$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$x + y = 0$

$[1, 1, 2]$

$\downarrow$

$[-1, -1, 2]$



$x + y = 0$

$$\begin{pmatrix} 5 & 2 & -3 \\ 4 & 5 & -4 \\ 6 & 4 & -4 \end{pmatrix}$$

$$\begin{vmatrix} 5-\lambda & 2 & -3 \\ 4 & 5-\lambda & -4 \\ 6 & 4 & -4-\lambda \end{vmatrix} =$$

$$(5-\lambda)(5-\lambda)(-4-\lambda) - 48 - 48 + 18(5-\lambda) + 16(5-\lambda) - 8(-4-\lambda) =$$

$$= (25 - 10\lambda + \lambda^2)(-4-\lambda) =$$

$$= -100 + 40\lambda - 4\lambda^2 - 25\lambda + 10\lambda^2 - \lambda^3$$

$$-96 + 90 - 18\lambda + 80 - 16\lambda + 32 + 8\lambda =$$

$$= -\lambda^3 + 6\lambda^2 - 11\lambda + 6$$

$\pm 6, \pm 3, \pm 2, \pm 1,$ $\frac{p}{q}$ $p \mid 6$ $q \mid -1$

	-1	6	-11	6
1	-1	5	-6	0
~~1~~	~~-1~~	~~4~~	~~-2~~	
2	-1	4	-3	0
~~2~~	~~-1~~	~~2~~	~~1~~	

$(x-2)(-x^2+4x-3)$

$x^2 - 4x + 3$ $D = 16 - 12 = 4$

$x_{1,2} = \frac{4 \pm 2}{2} = \begin{cases} 3 \\ 1 \end{cases}$

1, 3, 2

$$\begin{pmatrix} 4 & 2 & -3 \\ 4 & 4 & -4 \\ 6 & 4 & -5 \end{pmatrix} \sim \quad \boxed{\begin{pmatrix} 0 \\ \frac{3}{2}t \\ t \end{pmatrix}}$$

$$\begin{pmatrix} 4 & 2 & -3 \\ -4 & 0 & 2 \\ -2 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 4 & 2 & -3 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}$$

$$PAP^{-1} = B$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 7 & -5 \\ -4 & 5 & 0 \\ 1 & 9 & -4 \end{pmatrix}$$

$$\begin{vmatrix} 4-\lambda & 7 & -5 \\ -4 & 5-\lambda & 0 \\ 1 & 9 & -4-\lambda \end{vmatrix} =$$

$$(4-\lambda)(5-\lambda)(-4-\lambda) + 180 + 5(5-\lambda)$$
$$+ 28(-4-\lambda) =$$
$$= (20+\lambda^2-9\lambda)(-4-\lambda) =$$
$$-80 - 4\lambda^2 + \boxed{36\lambda} \boxed{-20\lambda} - \lambda^3 + 9\lambda^2 -$$
$$+ 180 + 25 \boxed{-5\lambda} - 112 -$$
$$\boxed{-28}\lambda = -\lambda^3 + 5\lambda^2 - 17\lambda + 13$$

$\pm 13, \pm 1$

	-1	5	-17	13
1	-1	4	-13	0
1	-1	3	-10	

$$x^2 - 4x + 13$$

$$D = 16 - 4 \cdot 13 < 0$$

$$\partial\left(ax^2+bx+c\right)=$$

$$=2ax+b$$

$$\alpha=\left(x^2,x,1\right)$$

$$\boxed{\begin{pmatrix}0&0&0\\2&0&0\\0&1&0\end{pmatrix}}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}0\\2a\\b\end{pmatrix}$$

$$\beta=\left(1+x^2+3x,\ x+3x^2,\ 10\right)$$

$$\begin{vmatrix} 1/2-\lambda & 1/2 & \sqrt{2}/2 \\ 1/2 & 1/2-\lambda & -\sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 1/2-\lambda \end{vmatrix} =$$

$$\underbrace{(1/2-\lambda)^3} + \underbrace{1/4 + 1/4}$$

$$+\ \frac{1}{2}\left(\frac{1}{2}-\lambda\right) + \frac{1}{2}\left(\frac{1}{2}-\lambda\right)$$
$$-\ \frac{1}{4}\left(\frac{1}{2}-\lambda\right)$$

$$\underbrace{\frac{1}{8} - 3\cdot\frac{1}{4}\lambda + 3\cdot\frac{1}{2}\lambda^2 - \lambda^3} + \frac{1}{2} + \underbrace{\frac{3}{4}\left(\frac{1}{2}-\lambda\right)}_{\frac{3}{8}-\frac{3}{4}\lambda} =$$

$$1 - \frac{3}{2}\lambda + \frac{3}{2}\lambda^2 - \lambda^3$$

	1	$-\frac{3}{2}$	$\frac{3}{2}$	-1
1	1	$-\frac{1}{2}$	1	0

$$x^2 - \frac{1}{2}x + 1$$

$$2x^2 - x + 2 \qquad D = 1 - 16 = = -15$$

$$\begin{pmatrix} -1/2 & 1/2 & \sqrt{2}/2 \\ 1/2 & -1/2 & -\sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 & -1/2 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} -1 & 1 & \sqrt{2} \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix} \qquad 1 \qquad \begin{pmatrix} 0 \\ t \\ t \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$2x^2 - x + 2 \qquad D = -15$$

$$x_{1,2} = \frac{1 \pm \sqrt{-15}}{4} \begin{cases} \frac{1+\sqrt{15}\,i}{4} \leftarrow \\ \frac{1-\sqrt{15}\,i}{4} \leftarrow \end{cases}$$

$$\frac{1+\sqrt{15}\,i}{4} = |x|(\sin\varphi + i\cos\varphi)$$

$\mathbb{R}^3$, $\mathbb{E}^3$, $\langle\, , \rangle$.

$$\langle\, , \rangle : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$$

$$(f_1, f_2, f_3) = \alpha$$

$$\langle f_i, f_j \rangle = \delta_{ij}$$

$$\langle f_i, f_i \rangle = 1$$

$$\langle f_i, f_j \rangle = 0 \quad i \neq j$$

$$\left\langle \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \right\rangle = \cancel{x_1^2 + x_2^2} =$$

$$= x_1 x_2 + y_1 y_2 + z_1 z_2$$

$v_1=(2,0,-1)\quad v_2=(-1,1,1)\quad v_3=(1,1,1)$

$u_1=(2,0,-1)$

$u_2=u_1+b\,v_2$

$0=\langle u_1,u_2\rangle=\langle u_1,u_1\rangle+b\langle u_1,v_2\rangle$

$0=5-3\cdot b$

$b=\frac{5}{3}\qquad u_1=(2,0,-1)$

$u_2=u_1+\frac{5}{3}u_2=(2,0,-1)+\frac{5}{3}(-1,1,1)=$

$=\left(\frac{1}{3},\frac{5}{3},\frac{2}{3}\right)$

$u_3=u_1+b\,u_2+c\,v_3$

$0=\langle u_1,u_3\rangle=5+b\left(-\frac{1}{3}\right)+c\,1$

$0=\langle u_2,u_3\rangle=0+b\frac{10}{3}+c\,\frac{8}{3}$

$v_3=(1,1,1)$

$15-b+3c=0$

$10b+8c=0$

$$\left(\begin{array}{cc|c}-1&3&-15\\5&4&0\end{array}\right)\sim\left(\begin{array}{cc|c}-1&3&-15\\0&19&-75\end{array}\right)$$

$c=-\frac{4}{15}\qquad b=-\frac{1}{5}$

$u_3=u_1-\frac{1}{5}u_2-\frac{4}{15}v_3$

$u_3=(2,0,-1)-\frac{1}{5}\left(\frac{1}{3},\frac{5}{3},\frac{2}{3}\right)-\frac{4}{15}(1,1,1)$

$=\left(2-\frac{1}{3},-\frac{9}{15},-1-\frac{6}{15}\right)=$

$=\left(\frac{5}{3},-\frac{9}{15},-\frac{21}{15}\right)$

$u_1=(2,0,-1)\qquad w_1=\frac{1}{\sqrt{5}}(2,0,-1)$

$u_2=(1,5,2)\qquad w_2=\ldots$

$u_3=(25,-9,-21)\qquad w_3=\ldots$

$\mathbb{R}^3$ $(1,0,0)\,(0,1,0)\,(0,0,1)$

$W = \langle (1,1,1)\,(1,2,0) \rangle$

$u_1 = (1,1,1)$

$u_2 = (1,1,1) + b(1,2,0)$

$0 = \langle u_1, u_2 \rangle =$

$= \langle u_1, (1,1,1) + b(1,2,0) \rangle =$

$= \langle u_1, (1,1,1) \rangle + b\langle u_1, (1,2,0) \rangle =$

$= \langle (1,1,1),(1,1,1) \rangle + b\langle (1,1,1),(1,2,0) \rangle$

$= 3 + b\,3$

$u_2 = (1,1,1) - (1,2,0) =$ $b = -1$

$= (0,-1,1)$

$\langle (1,1,1),(0,-1,1) \rangle = W$

$\langle (1,1,1),(1,1,1) \rangle = 3$

$\boxed{w_1 = \frac{1}{\sqrt{3}}(1,1,1)}$

$\langle \frac{1}{\sqrt{3}}(1,1,1), \frac{1}{\sqrt{3}}(1,1,1) \rangle = \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} \langle \,,\, \rangle =$

$= 1$

$\boxed{w_2 = \frac{1}{\sqrt{2}}(0,-1,1)}$

$\langle w_1, w_2 \rangle = W$

$u_1 = (1,1,1)$, $u_2 = (0,-1,1)$ | $(0,1,0)$

$W' = \langle (1,1,1),(0,1,-1),(0,1,0) \rangle$

$\Rightarrow u_3 = (1,1,1) + b(0,1,-1) + c(0,1,0)$

$(u_1, u_3) = 0 = 3 + b\cdot 0 + c\cdot 1$

$(u_2, u_3) = 0 = 0 + b\cdot 2 + c(-1)$

$0 = 3 + c \quad \Rightarrow c = -3$

$0 = 2b - c \quad \Rightarrow b = -\frac{3}{2}$

$u_3 = (1,1,1) - \frac{3}{2}(0,1,-1) - 3(0,1,0)$