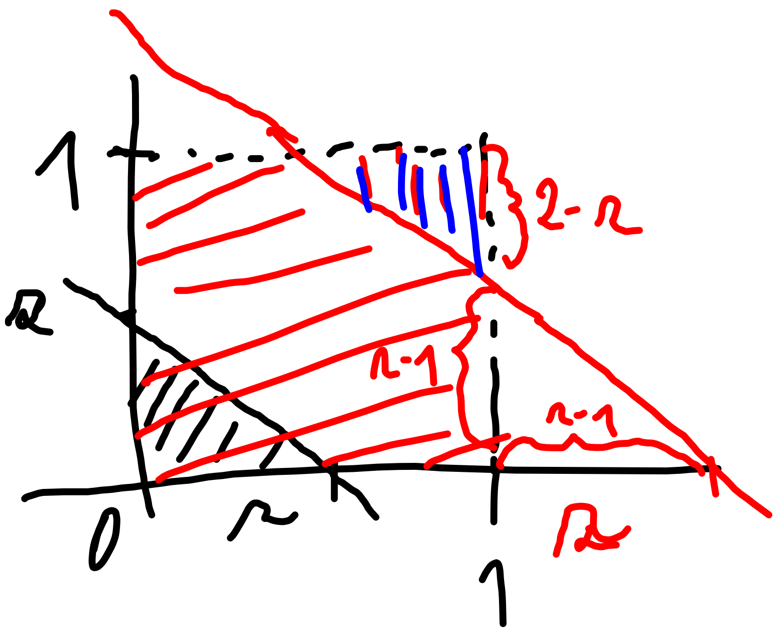
1)



$0 \le r \le 1$

$P[X+Y < r] =$

$P[X < r - Y] =$

$= \frac{r^2}{2}$

$1 \le r \le 2$:

$= 1 - \frac{(2-r)^2}{2} =$

$= 2r - \frac{r^2}{2} - 1$

$$P[X+\cancel{Y}<z] = \int_0^z \int_0^{z-y} f_{X,Y}(x,y)\,dx\,dy \doteq \%$$

$$f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y) = \begin{cases} 1 & \text{pro } 0\le x,y\le 1 \\ 0 & \text{jinak} \end{cases}$$

$$f_X(x) = \begin{cases} 1 & 0\le x\le 1 \\ 0 & \text{jinde} \end{cases}$$

$$\% = \int_0^z \int_0^{z-y} f_X(x)\,f_Y(y)\,dx\,dy = \int_0^z f_Y(y) \int_0^{z-y} f_X(x)\,dx\,dy$$

$$=$$

$$\int_0^r f_Y(y)\left(\int_0^{r-y} \underline{1}\, dx\right) dy =$$

$r \leq 1$

$$= \int_0^r \underline{1}(r-y)\, dy = r^2 - \frac{r^2}{2} = \frac{r^2}{2}$$

$r \geq 1$

$$P([X+Y<r]) = \int_0^r \int_0^{r-y} f_X(x) \cdot f_Y(y)\, dx\, dy =$$

$$= \int_0^{r-1} f_Y(y)\left(\int_0^{r-y} f_X(x)\, dx\right) dy + \int_{r-1}^{r} f_Y(y)\left(\int_0^{r-y} f_X(x)\, dx\right) dy$$

$$\int_0^{a-1} f_Y(y)\underbrace{\int_0^{a-y} f_X(x)\,dx}_{1}\,dy \;+\; \int_{a-1}^{a} f_Y(y)\underbrace{\int_0^{a-y} \underbrace{f_X(x)}_{1}\,dx}_{a-y}\,dy =$$

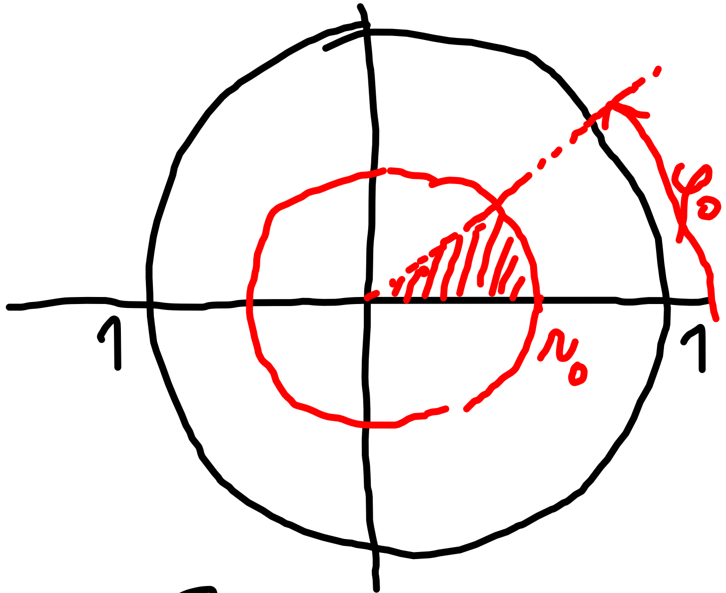
$[\,y \le a-1 \Rightarrow \underline{a-y \ge 1}\,]$, $a-1 \le y \le a \Rightarrow a-y \le 1$

$$= a-1+\left[ay-\frac{y^2}{2}\right]_{a-1}^{1} =$$

$$= a-1+\left[a-\frac{1}{2}+\frac{(a-1)^2}{2}-a(a-1)\right]=$$

$$= 2a-\frac{a^2}{2}-1$$

X, Y nejsou nezávislé



R, Φ .... polární souřadnice

$X = r\cos\varphi$

$y = r\sin\varphi$

$$F_{R,\Phi}(r_0, \varphi_0) = P[R < r_0, \Phi < \varphi_0] = \frac{\pi r_0^2 \varphi_0}{2\pi} \cdot \frac{1}{\pi} =$$

$$= \frac{1}{2\pi}(r_0^2 \varphi_0), \quad 0 \le r_0 \le 1, \quad 0 \le \varphi_0 \le 2\pi$$

$$f(r, \varphi) = \frac{\partial F(r_0, \varphi)}{\partial r \partial \varphi} = \frac{r}{\pi} \quad \begin{array}{l} 0 \le r \le 1 \\ 0 \le \varphi_0 < 2\pi \end{array}$$

$= 0$ jinak

Marginální rozdělení pravděpodobnosti:

$$F_X(x_0) = \lim_{y \to \infty} F_{X,Y}(x_0, y)$$

$$\int_{-\infty}^{x_0} f_X(s)\,ds = \lim_{y \to \infty} \left(\int_{-\infty}^{x_0} \int_{-\infty}^{y} f_{X,Y}(s, w)\,ds\,dw \right) =$$

$$= \int_{-\infty}^{x_0} \underbrace{\int_{-\infty}^{\infty} f_{X,Y}(s, w)\,dw}_{f_X(s)}\,ds$$

$$f_X(t) = \int_{-\infty}^{\infty} f_{X,Y}(t,u)\,du$$

$$g(r) = \int_{-\infty}^{\infty} \frac{r}{\pi}\,d\varphi = \int_{0}^{2\pi} \frac{r}{\pi}\,d\varphi = 2r \qquad 0 \le r \le 1$$

$$= 0 \qquad \text{jinak}$$

$$h(\varphi) = \int_{-\infty}^{\infty} \frac{r}{\pi}\,dr = \int_{0}^{1} \frac{r}{\pi}\,dr = \frac{1}{2\pi}$$

$$f_{R,\Phi}(r,\varphi) = g(r)\cdot h(\varphi)$$

$$ER = \int_0^1 r \cdot 2r \, dr = \left[\frac{2r^3}{3} \right]_0^1 = \frac{2}{3}$$

$$\operatorname{var} R = \int_0^1 r^2 \cdot 2r \, dr - \left(\frac{2}{3}\right)^2 = \frac{1}{18}$$