$$P[X > 2Y] = \frac{1}{2}\int_0^\infty \int_{2y}^\infty e^{-x-\frac{y}{2}}\, dx\, dy =$$

$$= \frac{1}{2}\int_0^\infty \left[-e^{-x-\frac{y}{2}}\right]_{2y}^\infty = \frac{1}{2}\int_0^\infty e^{-\frac{5}{2}y}\, dy = \frac{1}{2}\left[-\frac{2}{5}e^{-\frac{5}{2}y}\right]_0^\infty =$$

$$= \frac{1}{5}$$

$$Z = X + Y$$

$$F(z) = P[Z < z] = P[X+Y < z] = \frac{1}{2}\int_0^z \int_0^{z-y} e^{-x-\frac{y}{2}}\, dx\, dy =$$

$$= \frac{1}{2}\int_0^z \left[-e^{-x-\frac{y}{2}}\right]_0^{z-y} = \frac{1}{2}\int_0^z \left(-e^{\frac{y}{2}-z} + e^{-\frac{y}{2}}\right) dy =$$

$$= \frac{1}{2}\left[-2e^{\frac{y}{2}-z} - 2e^{-\frac{y}{2}}\right]_0^z = \frac{1}{2}\left[-2e^{-\frac{z}{2}} - 2e^{-\frac{z}{2}}\right.$$

$$+2e^{-z}+2\Big] = 1-2e^{-\frac{z}{2}}+e^{-z} \qquad 0 \le z \le \infty$$

$$F(z) = 0 \quad \text{pro} \quad z \le 0$$

$$Z = X - Y$$

$$P[Z < z] = F(z) = P[X - Y < z] =$$

a) $z \le 0$

$$= \frac{1}{2}\int_{-z}^{\infty}\int_{0}^{z+y} e^{-x-\frac{y}{2}}\,dx\,dy$$

b) $z \ge 0$

$$= \frac{1}{2}\int_{0}^{\infty}\int_{0}^{z+y} e^{-x-\frac{y}{2}}\,dx\,dy$$

$X_1, X_2, \dots, X_n$ náhodný výběr z rozdělení

$N(\mu, \sigma^2)$, rodou

$$Z = \frac{\bar{X} - \mu}{\sigma} \cdot \sqrt{n} \sim N(0,1)$$

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$\alpha = 0{,}05$$

$$1-\alpha = P\left[|Z| < \underbrace{z\left(\frac{\alpha}{2}\right)}_{\Phi^{-1}\left(1-\frac{\alpha}{2}\right)} \right] =$$

$$P\left[\left| \frac{\bar{X} - \mu}{\sigma} \sqrt{n} \right| < z\left(\frac{\alpha}{2}\right) \right] =$$

$$= P\left[|\bar{X} - \mu| < \frac{z\left(\frac{\alpha}{2}\right) \cdot \sigma}{\sqrt{n}} \right] =$$

$$P\left[\bar{X} - \frac{z(\frac{\alpha}{2})\sigma}{\sqrt{n}} < \mu < \bar{X} + \frac{z(\frac{\alpha}{2})\sigma}{\sqrt{n}}\right] \Rightarrow$$

95 % interval spolehlivosti pro skutečnou rychlost letadla je

$$\left(870{,}3 - \frac{1{,}96 \cdot 2{,}1}{\sqrt{5}},\ 870{,}3 + \frac{1{,}96 \cdot 2{,}1}{\sqrt{5}}\right) =$$

$$\doteq (868{,}47 ; 872{,}13)$$

$$z(0{,}025) = \Phi(0{,}975) \doteq 1{,}96$$

Neznámý rozptyl měření odhadneme veličinou

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$\frac{\bar{X} - \mu}{\sigma} \cdot \sqrt{n} \sim N(0,1)$$

$$\frac{\bar{X} - \mu}{S} \sqrt{n} \sim t_{n-1}$$

$0{,}03\,‰$; $0{,}05\,‰$; $0{,}07\,‰$

v desetinách promile jsou měření:

$3, 5, 7.$

$$\bar{X} = 5$$

$$s^2 = \frac{1}{2}\left((7-5)^2 + (5-5)^2 + (3-5)^2\right) = 4$$

$$s = 2$$

t_{n-1} – kvantilová funkce Studentova rozdělení

$$Z = \frac{\bar{X} - \mu}{s} \cdot \sqrt{n} \sim t_{n-1}$$

$$0{,}95 = P\left(|Z| < t_{n-1}\left(\tfrac{\alpha}{2}\right)\right) =$$

$$= P\left(\left|\frac{\bar{X} - \mu}{s} \cdot \sqrt{n}\right| < t_{n-1}\left(\tfrac{\alpha}{2}\right)\right) =$$

$$= P\left(\bar{X} - \frac{t_{n-1}\left(\frac{\alpha}{2}\right)\cdot S}{\sqrt{n}} < \mu < \bar{X} + \frac{t_{n-1}\left(\frac{\alpha}{2}\right)\cdot S}{\sqrt{n}}\right)$$

zbývá dosadit číselné hodnoty. 95% interval spolehlivosti pro skutečný objem alkoholu v daném vzorku je tedy:

$$t_2(0{,}025) = 4{,}3$$

$$\left(5 - \frac{4{,}3\cdot 2}{\sqrt{3}}, 5 + \frac{4{,}3\cdot 2}{\sqrt{3}}\right) = (0{,}15; 9{,}85)$$

v promilích tedy:

$$(0{,}015, 0{,}985)$$