$G \times G \xrightarrow{\cdot} G \qquad a \cdot b$

příklad $\qquad A \quad A$

1) $a \cdot e = a \Rightarrow f(a) \cdot f(e) = f(a \cdot e) = f(a)$

2) $a, b \in K \qquad \underset{f(K)}{\underset{\Uparrow}{f(a)}} \cdot \underset{f(K)}{\underset{\Uparrow}{f(b)}} = \underset{f(K)}{\underset{\Uparrow}{f(a \cdot b)}}$

$a \cdot b = f(x \cdot y)$

$f(a) \cdot f(a^{-1}) = f(\underset{e_G}{\underbrace{a \cdot a^{-1}}}) = e_H \qquad a = f(x)$

$\Rightarrow (f(a))^{-1} = f(a^{-1}) \qquad b = f(y)$

3) $a, b, \; f(a) \in K, \; f(b) \in K \Rightarrow f(a \cdot b) \in K$

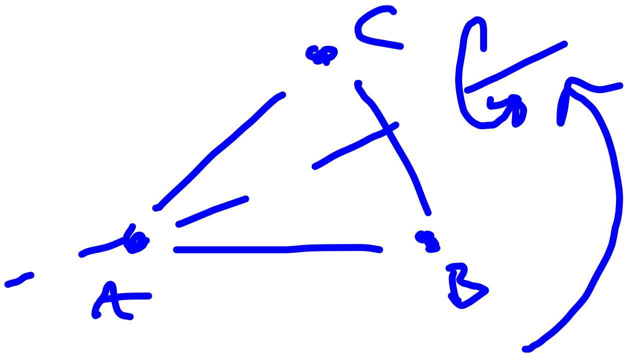
$x = f^{-1}(a), \; y = f^{-1}(b) \qquad x \cdot y \overset{?}{=} \underset{f(x)}{\underset{\|}{f^{-1}(a)}} \cdot \underset{f(y)}{\underset{\|}{f^{-1}(b)}} =$

$f(a) = f(b) \Rightarrow f(a)\cdot f(b^{-1}) = e_H = f(a\cdot b^{-1})$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \| $

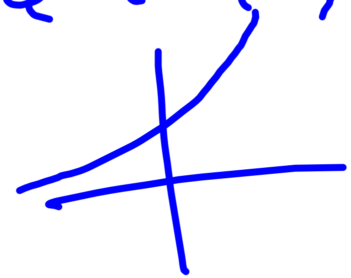
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad e_G$

$a\cdot b^{-1} = e \Leftrightarrow a = b$



Σ_3 -- permutace $\{A, B, C\}$

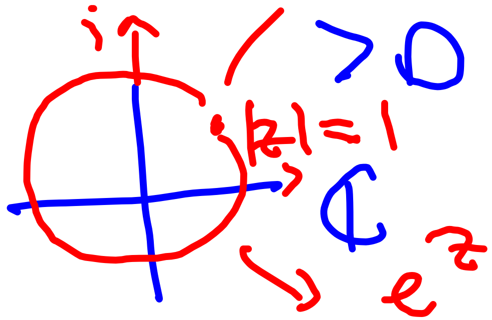
$e^{s+it} = e^s e^{it}$



$\exp x = e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \dots$

$e^{x+y} = e^x \cdot e^y$

$e^{it} = \cos t + i \sin t$



$$
\begin{tikzcd}
 & G \times H \arrow[dl, "p_G"'] \arrow[dr, "p_H"] & \\
G & & H \\
 & K \arrow[ul, "f"] \arrow[ur, "g"'] \arrow[uu, dashed, "h"] &
\end{tikzcd}
$$

$$h(k) = (f(k), g(k))$$

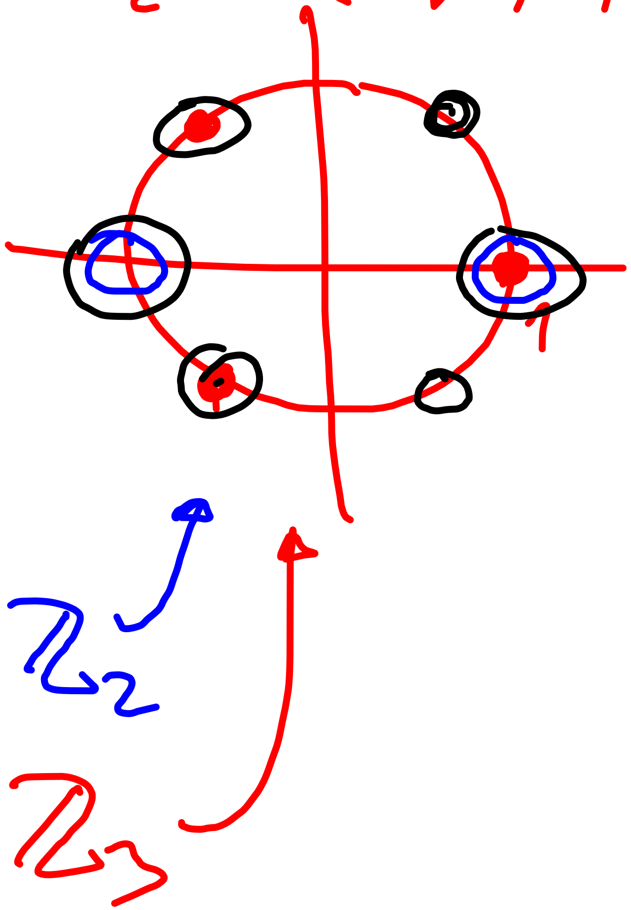
$$h(k \cdot k') = (f(k) \cdot f(k'), g(k) \cdot g(k'))$$

$$= (f(k), g(k)) \cdot (f(k'), g(k'))$$

$\mathbb{Z}_k = \{0, 1, 2, \dots, k-1\}$ sčítání mod k

$e^{it} \longleftarrow \mathbb{R} \to \mathbb{C}$

$\mathbb{Z}_k = \{a \,;\, a^k = 1\}$



$\mathbb{Z}_{p^e}$ $\qquad$ $\mathbb{Z}_6$

$$(a \cdot (a)(\cdot \ldots a \cdot \ldots \cdot a) \cdot a)$$

$$a^{\ell} = a \quad / \cdot a^{-1}$$

$$a^{\ell-1} = e \quad 1 \qquad 2$$

$$\mathbb{Z}_4 = \{0, 1, 2, 3\}$$

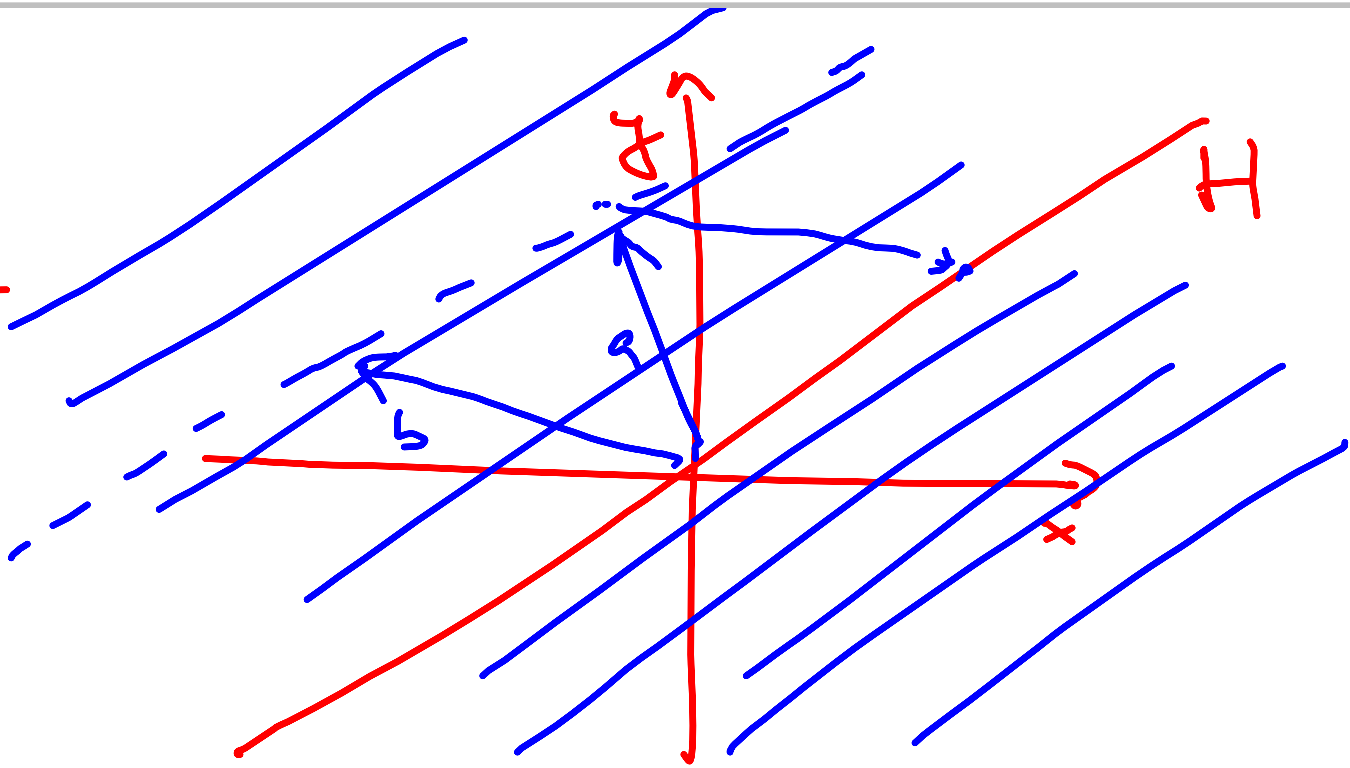
$$4 \qquad 4$$

$$9 \qquad 7$$

$\mathbb{R}^2 = G$

vektory

v rovině



v grupě $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$

$\underbrace{(b^{-1} \cdot a^{-1}) \cdot (a \cdot b)} = \underbrace{b^{-1} \cdot b} = e$

$$b^{-1} \cdot a \in H \quad \wedge \quad a \cdot b^{-1} \in H$$

$$b^{-1}\cdot a = h \qquad a\cdot b^{-1} = h' \qquad (?) \Leftarrow$$

1. $b^{-1} \cdot a = h$ $\qquad \underbrace{b \cdot b^{-1}}_{e} \cdot \overbrace{a \cdot b^{-1}}^{h \in H} \in H$

ve stejné levé $\Rightarrow$ ve stejné pravé

2. $\boxed{a \cdot H = \{a \cdot h;\ h \in H\}}$

$$a \cdot h_1 = a \cdot h_2 \Rightarrow h_1 = h_2$$

$K \subset H$, $f: G \to H$ homo

$f^{-1}(K)$ podgrupa

$$f(a \cdot f^{-1}(k) \cdot a^{-1}) = f(a) \cdot k \cdot f(a^{-1})$$

$$f(a^{-1}) = (f(a))^{-1}$$

K normální $\Rightarrow$ $f^{-1}(K)$ normální

$\{e\}$ je normální